GRP axioms

GRP001-0.ax Monoid axioms
identity $\cdot\, x{=}x$ cnf(left_identity, axiom)
$x \cdot$ identity$=x$ cnf(right_identity, axiom)
$x \cdot y{=}x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y{=}z$ and $x \cdot y{=}w) \Rightarrow z = w$ cnf(total_function$_2$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)

GRP002-0.ax Semigroup axioms
$x \cdot y{=}x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y{=}z$ and $x \cdot y{=}w) \Rightarrow z = w$ cnf(total_function$_2$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)

GRP003+0.ax Group theory axioms
$\forall x$: identity $\cdot\, x{=}x$ fof(left_identity, axiom)
$\forall x$: $x \cdot$ identity$=x$ fof(right_identity, axiom)
$\forall x$: $x' \cdot x$=identity fof(left_inverse, axiom)
$\forall x$: $x \cdot x'$=identity fof(right_inverse, axiom)
$\forall x, y$: $x \cdot y{=}x \cdot y$ fof(total_function$_1$, axiom)
$\forall w, x, y, z$: $((x \cdot y{=}z$ and $x \cdot y{=}w) \Rightarrow z = w)$ fof(total_function$_2$, axiom)
$\forall x, y, z, u, v, w$: $((x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w)$ fof(associativity$_1$, axiom)
$\forall x, y, z, u, v, w$: $((x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w)$ fof(associativity$_2$, axiom)

GRP003-0.ax Group theory axioms
identity $\cdot\, x{=}x$ cnf(left_identity, axiom)
$x \cdot$ identity$=x$ cnf(right_identity, axiom)
$x' \cdot x$=identity cnf(left_inverse, axiom)
$x \cdot x'$=identity cnf(right_inverse, axiom)
$x \cdot y{=}x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y{=}z$ and $x \cdot y{=}w) \Rightarrow z = w$ cnf(total_function$_2$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)

GRP003-1.ax Subgroup axioms for the GRP003 group theory axioms
subgroup_member$(x) \Rightarrow$ subgroup_member(x') cnf(closure_of_inverse, axiom)
(subgroup_member(a) and subgroup_member(b) and $a{\cdot}b{=}c) \Rightarrow$ subgroup_member(c) cnf(closure_of_product, axiom)

GRP003-2.ax Subgroup axioms for the GRP003 group theory axioms
(subgroup_member(a) and subgroup_member(b) and $a{\cdot}b'{=}c) \Rightarrow$ subgroup_member(c) cnf(closure_of_product_and_inverse

GRP004+0.ax Group theory (equality) axioms
$\forall x$: identity $\cdot\, x = x$ fof(left_identity, axiom)
$\forall x$: $x' \cdot x =$ identity fof(left_inverse, axiom)
$\forall x, y, z$: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ fof(associativity, axiom)

GRP004-0.ax Group theory (equality) axioms
identity $\cdot\, x = x$ cnf(left_identity, axiom)
$x' \cdot x =$ identity cnf(left_inverse, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity, axiom)

GRP004-1.ax Subgroup (equality) axioms
subgroup_member$(x) \Rightarrow$ subgroup_member(x') cnf(closure_of_inverse, axiom)
(subgroup_member(x) and subgroup_member(y) and $x{\cdot}y = z) \Rightarrow$ subgroup_member(z) cnf(closure_of_multiply, axiom)

GRP004-2.ax Lattice ordered group (equality) axioms
greatest_lower_bound(x, y) = greatest_lower_bound(y, x) cnf(symmetry_of_glb, axiom)
least_upper_bound(x, y) = least_upper_bound(y, x) cnf(symmetry_of_lub, axiom)
greatest_lower_bound$(x$, greatest_lower_bound$(y, z))$ = greatest_lower_bound(greatest_lower_bound$(x, y), z)$ cnf(associativi
least_upper_bound$(x$, least_upper_bound$(y, z))$ = least_upper_bound(least_upper_bound$(x, y), z)$ cnf(associativity_of_lub, a
least_upper_bound$(x, x) = x$ cnf(idempotence_of_lub, axiom)
greatest_lower_bound$(x, x) = x$ cnf(idempotence_of_gld, axiom)

least_upper_bound(x, greatest_lower_bound(x, y)) $= x$ cnf(lub_absorbtion, axiom)
greatest_lower_bound(x, least_upper_bound(x, y)) $= x$ cnf(glb_absorbtion, axiom)
$x \cdot$ least_upper_bound(y, z) = least_upper_bound($x \cdot y, x \cdot z$) cnf(monotony_lub$_1$, axiom)
$x \cdot$ greatest_lower_bound(y, z) = greatest_lower_bound($x \cdot y, x \cdot z$) cnf(monotony_glb$_1$, axiom)
least_upper_bound(y, z) $\cdot x$ = least_upper_bound($y \cdot x, z \cdot x$) cnf(monotony_lub$_2$, axiom)
greatest_lower_bound(y, z) $\cdot x$ = greatest_lower_bound($y \cdot x, z \cdot x$) cnf(monotony_glb$_2$, axiom)

GRP005-0.ax Group theory axioms
identity $\cdot x{=}x$ cnf(left_identity, axiom)
$x' \cdot x{=}$identity cnf(left_inverse, axiom)
$x \cdot y{=}x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y{=}z$ and $x \cdot y{=}w) \Rightarrow z{=}w$ cnf(total_function$_2$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)
$(x{=}y$ and $w \cdot z{=}x) \Rightarrow w \cdot z{=}y$ cnf(product_substitution$_3$, axiom)

GRP006-0.ax Group theory (Named groups) axioms
group_member(identity_for(xg), xg) cnf(identity_in_group, axiom)
xg $\cdot$ identity_for(xg)$=x$ cnf(left_identity, axiom)
xg $\cdot\, x{=}$identity_for(xg) cnf(right_identity, axiom)
group_member(x, xg) $\Rightarrow$ group_member(xg$'$, xg) cnf(inverse_in_group, axiom)
xg $\cdot$ xg$'{=}x$ cnf(left_inverse, axiom)
xg $\cdot\, x{=}$xg$'$ cnf(right_inverse, axiom)
(group_member(x, xg) and group_member(y, xg)) $\Rightarrow$ xg $\cdot\, x{=}y$ cnf(total_function1$_1$, axiom)
(group_member(x, xg) and group_member(y, xg)) $\Rightarrow$ group_member(m(xg, x, y), xg) cnf(total_function1$_2$, axiom)
(xg $\cdot\, x{=}y$ and xg $\cdot\, x{=}y$) $\Rightarrow w = z$ cnf(total_function$_2$, axiom)
(xg $\cdot\, x{=}y$ and xg $\cdot\, y{=}z$ and xg $\cdot$ xy$=z$) $\Rightarrow$ xg $\cdot\, x{=}$yz cnf(associativity$_1$, axiom)
(xg $\cdot\, x{=}y$ and xg $\cdot\, y{=}z$ and xg $\cdot\, x{=}$yz) $\Rightarrow$ xg $\cdot$ xy$=z$ cnf(associativity$_2$, axiom)

GRP007+0.ax Group theory (Named Semigroups) axioms
$\forall g, x, y$: ((group_member(x, g) and group_member(y, g)) $\Rightarrow$ group_member($m(g, x, y), g$)) fof(total_function, axiom)
$\forall g, x, y, z$: ((group_member(x, g) and group_member(y, g) and group_member(z, g)) $\Rightarrow m(g, m(g, x, y), z) = m(g, x, m(g, y, z$

GRP008-0.ax Semigroups axioms
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity_of_multiply, axiom)

GRP008-1.ax Cancellative semigroups axioms
$a \cdot b = c \cdot b \Rightarrow a = c$ cnf(right_cancellation, axiom)
$a \cdot b = a \cdot c \Rightarrow b = c$ cnf(left_cancellation, axiom)

GRP problems

GRP001+6.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
$\forall e$: (($\forall x, y$: $\exists z$: $x \cdot y{=}z$ and $\forall x, y, z, u, v, w$: (($x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w$) $\Rightarrow x \cdot v{=}w$) and $\forall x, y, z, u, v, w$: (($x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w$) $\Rightarrow u \cdot z{=}w$) and $\forall x$: $x \cdot e{=}x$ and $\forall x$: $e \cdot x{=}x$ and $\forall x$: $x \cdot x'{=}e$ and $\forall x$: $x' \cdot x{=}e$) $\Rightarrow$ ($\forall x$: $x \cdot x{=}e \Rightarrow \forall u, v, w$: ($u \cdot v{=}w \Rightarrow v \cdot u{=}w$))) fof(commutativity, conjecture)

GRP001-1.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
include('Axioms/GRP003-0.ax')
$x \cdot x{=}$identity cnf(square_element, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, negated_conjecture)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

GRP001-2.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
include('Axioms/GRP004-0.ax')
$x \cdot$ identity $= x$ cnf(right_identity, axiom)
$x \cdot x' =$ identity cnf(right_inverse, axiom)
$x \cdot x =$ identity cnf(squareness, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, hypothesis)
$b \cdot a \neq c$ cnf(prove_b_times_a_is_c, negated_conjecture)

GRP001-3.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
group(f_{71}, f_{72})   cnf(a_group, hypothesis)
identity(f_{71}, f_{72}, f_{73})   cnf(f73_is_the_identity, hypothesis)
$x \in f_{71} \Rightarrow$ apply_to_two_arguments(f_{72}, x, x) = f_{73} cnf(x_squared_is_identity, hypothesis)
¬ commutes(f_{71}, f_{72})   cnf(prove_the_group_is_commutative, negated_conjecture)

GRP001-4.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity, axiom)
identity $\cdot x = x$ cnf(left_identity, axiom)
$x \cdot x =$ identity   cnf(squareness, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, hypothesis)
$b \cdot a \neq c$ cnf(prove_b_times_a_is_c, negated_conjecture)

GRP001-5.p X∧2 = identity => commutativity
If the square of every element is the identity, the system is commutative.
identity $\cdot x{=}x$ cnf(left_identity, axiom)
$x \cdot$ identity$=x$ cnf(right_identity, axiom)
($x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w$) $\Rightarrow$ $x \cdot v{=}w$ cnf(associativity$_1$, axiom)
($x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w$) $\Rightarrow$ $u \cdot z{=}w$ cnf(associativity$_2$, axiom)
$x \cdot x{=}$identity   cnf(square_element, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

GRP001∧5.p TPS problem GRP-COMM2
Group is Abelian iff every element has order 2.
cP: \$i → \$i → \$i   thf(cP, type)
e: \$i   thf(e, type)
(∀xx: \$i: (cP@e@xx) = xx and ∀xy: \$i: (cP@xy@e) = xy and ∀xz: \$i: (cP@xz@xz) = e and ∀xx: \$i, xy: \$i, xz: \$i: (cP@(cP@xx
(cP@xx@(cP@xy@xz))) ⇒ ∀xa: \$i, xb: \$i: (cP@xa@xb) = (cP@xb@xa)   thf(cGRP_COMM$_2$, conjecture)

GRP002-1.p Commutator equals identity in groups of order 3
In a group, if (for all x) the cube of x is the identity (i.e. a group of order 3), then the equation [[x,y],y]= identity holds, where [x,y] is the product of x, y, the inverse of x and the inverse of y (i.e. the commutator of x and y).
include('Axioms/GRP003-0.ax')
$x \cdot x{=}y \Rightarrow x \cdot y{=}$identity   cnf(x_cubed_is_identity$_1$, hypothesis)
$x \cdot x{=}y \Rightarrow y \cdot x{=}$identity   cnf(x_cubed_is_identity$_2$, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, negated_conjecture)
$c \cdot a'{=}d$ cnf(c_times_inverse_a_is_d, negated_conjecture)
$d \cdot b'{=}h$ cnf(d_times_inverse_b_is_h, negated_conjecture)
$h \cdot b{=}j$ cnf(h_times_b_is_j, negated_conjecture)
$j \cdot h'{=}k$ cnf(j_times_inverse_h_is_k, negated_conjecture)
$\neg\, k \cdot b'{=}$identity   cnf(prove_k_times_inverse_b_is_e, negated_conjecture)

GRP002-2.p Commutator equals identity in groups of order 3
In a group, if (for all x) the cube of x is the identity (i.e. a group of order 3), then the equation [[x,y],y]= identity holds, where [x,y] is the product of x, y, the inverse of x and the inverse of y (i.e. the commutator of x and y).
include('Axioms/GRP004-0.ax')
$x \cdot$ identity $= x$ cnf(right_identity, axiom)
$x \cdot x' =$ identity   cnf(right_inverse, axiom)
$x \cdot (x \cdot x) =$ identity   cnf(x_cubed_is_identity, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$c \cdot a' = d$ cnf(c_times_inverse_a_is_d, negated_conjecture)
$d \cdot b' = h$ cnf(d_times_inverse_b_is_h, negated_conjecture)
$h \cdot b = j$ cnf(h_times_b_is_j, negated_conjecture)
$j \cdot h' = k$ cnf(j_times_inverse_h_is_k, negated_conjecture)
$k \cdot b' \neq$ identity   cnf(prove_k_times_inverse_b_is_e, negated_conjecture)

GRP002-3.p Commutator equals identity in groups of order 3

In a group, if (for all x) the cube of x is the identity (i.e. a group of order 3), then the equation [[x,y],y]= identity holds, where [x,y] is the product of x, y, the inverse of x and the inverse of y (i.e. the commutator of x and y).
include('Axioms/GRP004-0.ax')
$\text{commutator}(x, y) = x \cdot (y \cdot (x' \cdot y'))$ cnf(commutator, axiom)
$x \cdot (x \cdot x) = \text{identity}$ cnf(x_cubed_is_identity, hypothesis)
$\text{commutator}(\text{commutator}(a, b), b) \neq \text{identity}$ cnf(prove_commutator, negated_conjecture)

GRP002-4.p Commutator equals identity in groups of order 3
In a group, if (for all x) the cube of x is the identity (i.e. a group of order 3), then the equation [[x,y],y]= identity holds, where [x,y] is the product of x, y, the inverse of x and the inverse of y (i.e. the commutator of x and y).
include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$\text{commutator}(x, y) = x \cdot (y \cdot (x' \cdot y'))$ cnf(commutator, axiom)
$x \cdot (x \cdot x) = \text{identity}$ cnf(x_cubed_is_identity, hypothesis)
$\text{commutator}(\text{commutator}(a, b), b) \neq \text{identity}$ cnf(prove_commutator, negated_conjecture)

GRP003-1.p The left identity is also a right identity
$x' \cdot x{=}\text{identity}$ cnf(left_inverse, axiom)
$\text{identity} \cdot x{=}x$ cnf(left_identity, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)
$\neg\, a \cdot \text{identity}{=}a$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP003-2.p The left identity is also a right identity
include('Axioms/GRP005-0.ax')
$\neg\, a \cdot \text{identity}{=}a$ cnf(prove_right_identity, negated_conjecture)

GRP004-1.p Left inverse and identity => Right inverse exists
In a group with left inverses and left identity every element has a right inverse.
$x' \cdot x{=}\text{identity}$ cnf(left_inverse, axiom)
$\text{identity} \cdot x{=}x$ cnf(left_identity, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)
$\neg\, a \cdot x{=}\text{identity}$ cnf(prove_there_is_a_right_inverse, negated_conjecture)

GRP004-2.p Left inverse and identity => Right inverse exists
In a group with left inverses and left identity every element has a right inverse.
include('Axioms/GRP005-0.ax')
$\neg\, a \cdot a'{=}\text{identity}$ cnf(prove_right_inverse, negated_conjecture)

GRP005-1.p Identity is in this subset of a group
If S is a non-empty subset of a group such that if X, Y belong to S, the XY∧-1 belongs to S, then the identity e belongs to S.
$\text{identity} \cdot x{=}x$ cnf(left_identity, axiom)
$x \cdot \text{identity}{=}x$ cnf(right_identity, axiom)
$x \cdot x'{=}\text{identity}$ cnf(right_inverse, axiom)
$x' \cdot x{=}\text{identity}$ cnf(left_inverse, axiom)
$\text{an\_element}(a)$ cnf(element_of_set, axiom)
$(\text{an\_element}(x) \text{ and an\_element}(y) \text{ and } x \cdot y'{=}z) \Rightarrow \text{an\_element}(z)$ cnf(condition, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)
$\neg\, \text{an\_element}(\text{identity})$ cnf(prove_identity_is_an_element, negated_conjecture)

GRP006-1.p Inverse is in this group
If S is a non-empty subset of a group such that if X, Y belong to S, the XY∧-1 belongs to S, then S contains X∧-1 whenever it contains X.
$\text{identity} \cdot x{=}x$ cnf(left_identity, axiom)
$x \cdot \text{identity}{=}x$ cnf(right_identity, axiom)
$x \cdot x'{=}\text{identity}$ cnf(right_inverse, axiom)
$x' \cdot x{=}\text{identity}$ cnf(left_inverse, axiom)
$(\text{an\_element}(x) \text{ and an\_element}(y) \text{ and } x \cdot y'{=}z) \Rightarrow \text{an\_element}(z)$ cnf(condition, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)

$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \;\Rightarrow\; u \cdot z{=}w$ cnf(associativity$_2$, axiom)
an_element(the_element) cnf(element_of_set, hypothesis)
$\neg$ an_element(the_element$'$) cnf(prove_b_inverse_is_in_set, negated_conjecture)

GRP007-1.p The identity element is unique
include('Axioms/GRP003-0.ax')
$c \cdot a{=}a$ cnf(another_left_identity, hypothesis)
$a \cdot c{=}a$ cnf(another_right_identity, hypothesis)
identity $\neq c$ cnf(prove_identity_equals_c, negated_conjecture)

GRP008-1.p Unknown meaning
include('Axioms/GRP003-0.ax')
$(q(a)$ and $a \cdot b{=}c) \;\Rightarrow\; b \cdot a{=}c$ cnf(unknown_meaning$_2$, axiom)
$j(a) \cdot a{=}h(a)$ or $a \cdot j(a){=}h(a)$ or $q(a)$ cnf(unknown_meaning$_3$, axiom)
$(j(a) \cdot a{=}h(a)$ and $a \cdot j(a){=}h(a)) \;\Rightarrow\; q(a)$ cnf(unknown_meaning$_4$, axiom)
$\neg\, q$(identity) cnf(prove_identity_is_q, negated_conjecture)

GRP009-1.p The left inverse of an element is unique
include('Axioms/GRP003-0.ax')
$a \cdot b{=}$identity cnf(a_is_an_inverse_of_b, hypothesis)
$c \cdot b{=}$identity cnf(c_is_an_inverse_of_b, hypothesis)
$a \neq c$ cnf(prove_a_equals_c, negated_conjecture)

GRP010-1.p Inverse is a symmetric relationship
If a is an inverse of b then b is an inverse of a.
include('Axioms/GRP003-0.ax')
$a \cdot b{=}$identity cnf(a_multiply_b_is_identity, hypothesis)
$\neg\, b \cdot a{=}$identity cnf(prove_b_multiply_a_is_identity, negated_conjecture)

GRP010-4.p Inverse is a symmetric relationship
If a is an inverse of b then b is an inverse of a.
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity, axiom)
identity $\cdot\, x = x$ cnf(left_identity, axiom)
$x' \cdot x =$ identity cnf(left_inverse, axiom)
$c \cdot b =$ identity cnf(c_times_b_is_e, hypothesis)
$b \cdot c \neq$ identity cnf(prove_b_times_c_is_e, negated_conjecture)

GRP011-4.p Left cancellation
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity, axiom)
identity $\cdot\, x = x$ cnf(left_identity, axiom)
$x' \cdot x =$ identity cnf(left_inverse, axiom)
$b \cdot c = d \cdot c$ cnf(product_equality, hypothesis)
$b \neq d$ cnf(prove_left_cancellation, negated_conjecture)

GRP012+5.p Inverse of products = Product of inverses
The inverse of products equals the product of the inverse, in opposite order
$\forall e$: $((\forall x, y$: $\exists z$: $x \cdot y{=}z$ and $\forall x, y, z, u, v, w$: $((x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \;\Rightarrow\; x \cdot v{=}w)$ and $\forall x, y, z, u, v, w$: $((x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \;\Rightarrow\; u \cdot z{=}w)$ and $\forall x$: $x \cdot e{=}x$ and $\forall x$: $e \cdot x{=}x$ and $\forall x$: $x \cdot x'{=}e$ and $\forall x$: $x' \cdot x{=}e) \;\Rightarrow\; \forall u, v, w, x$: $((u' \cdot v'{=}w$ and $v \cdot u{=}x) \;\Rightarrow\; w' \cdot x'{=}e))$ fof(prove_distribution, conjecture)

GRP012-1.p Inverse of products = Product of inverses
The inverse of products equals the product of the inverse, in opposite order.
include('Axioms/GRP003-0.ax')
$a \cdot b{=}c$ cnf(a_multiply_b_is_c, hypothesis)
$b' \cdot a'{=}d$ cnf(inverse_b_multiply_inverse_a_is_d, hypothesis)
$\neg\, c \cdot d{=}$identity cnf(prove_c_multiply_d_is_identity, negated_conjecture)

GRP012-2.p Inverse of products = Product of inverses
The inverse of products equals the product of the inverse, in opposite order.
include('Axioms/GRP003-0.ax')
$a \cdot b{=}c$ cnf(a_multiply_b_is_c, hypothesis)
$b' \cdot a'{=}d$ cnf(inverse_b_multiply_inverse_a_is_d, hypothesis)
$c' \neq d$ cnf(prove_c_inverse_equals_d, negated_conjecture)

GRP012-3.p Inverse of products = Product of inverses

The inverse of products equals the product of the inverse, in opposite order
include('Axioms/GRP003-0.ax')
$(a \cdot b)' \neq b' \cdot a'$ cnf(prove_inverse_of_product_is_product_of_inverses, negated_conjecture)

GRP012-4.p Inverse of products = Product of inverses
The inverse of products equals the product of the inverse, in opposite order
include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$(a \cdot b)' \neq b' \cdot a'$ cnf(prove_inverse_of_product_is_product_of_inverses, negated_conjecture)

GRP013-1.p Commutator equals identity in these conditions
If X.X=identity and if X∧-1.Y∧-1 = Z then X.Z = Y, then (X.Y).(X∧-1.Y∧-1) = identity.
include('Axioms/GRP003-0.ax')
$a \cdot a = \text{identity}$ cnf(squareness, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, hypothesis)
$a' \cdot b' = d$ cnf(inverse_a_times_inverse_b_is_d, hypothesis)
$a' \cdot b' = c \Rightarrow a \cdot c = b$ cnf(inverses_have_property, hypothesis)
$\neg\, c \cdot d = \text{identity}$ cnf(prove_c_times_d_is_identity, negated_conjecture)

GRP014-1.p Product is associative in this group theory
The group theory specified by the axiom given implies the associativity of multiply.
$x \cdot (((y' \cdot (x' \cdot w))' \cdot z) \cdot (y \cdot z)')' = w$ cnf(group_axiom, axiom)
$a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$ cnf(prove_associativity, negated_conjecture)

GRP015-1.p x,<<x x,X> x,X> is a group
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
little_set(a)   cnf(a_little_set, hypothesis)
¬ group(singleton_set(a), singleton_set(ordered_pair(ordered_pair(a, a), a)))   cnf(prove_the_group, negated_conjecture)

GRP016-1.p There is a homomorphism from a group to itself
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
group(f_{69}, f_{70})   cnf(a_group, negated_conjecture)
¬ homomorphism($y, f_{69}, f_{70}, f_{69}, f_{70}$)   cnf(prove_there_is_a_homomorphism, negated_conjecture)

GRP017-1.p The inverse of each element is unique
i.e., if ab=ba=identity and ac=ca=identity then b=c
include('Axioms/GRP003-0.ax')
$a \cdot b = \text{identity}$ cnf(a_times_b_is_identity, hypothesis)
$b \cdot a = \text{identity}$ cnf(b_times_a_is_identity, hypothesis)
$a \cdot c = \text{identity}$ cnf(a_times_c_is_identity, hypothesis)
$c \cdot a = \text{identity}$ cnf(c_times_a_is_identity, hypothesis)
$b \neq c$ cnf(prove_b_equals_c, negated_conjecture)

GRP018-1.p X times identity is X
include('Axioms/GRP003-0.ax')
$a \cdot \text{identity} \neq a$ cnf(prove_X_times_id_is_X, negated_conjecture)

GRP019-1.p Identity times X is X
include('Axioms/GRP003-0.ax')
$\text{identity} \cdot a \neq a$ cnf(prove_id_times_X_is_X, negated_conjecture)

GRP020-1.p Inverse of X times X is the identity
include('Axioms/GRP003-0.ax')
$a' \cdot a \neq \text{identity}$ cnf(prove_inverse_X_times_X_is_id, negated_conjecture)

GRP021-1.p X times inverse of X is the identity
include('Axioms/GRP003-0.ax')
$a \cdot a' \neq \text{identity}$ cnf(prove_X_times_inverse_X_is_id, negated_conjecture)

GRP022-1.p Inverse is an involution
include('Axioms/GRP003-0.ax')
$(a')' \neq a$ cnf(prove_inverse_of_inverse_is_original, negated_conjecture)

GRP022-2.p Inverse is an involution

include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$(a')' \neq a$ cnf(prove_inverse_of_inverse_is_original, negated_conjecture)

GRP023-1.p The inverse of the identity is the identity
include('Axioms/GRP003-0.ax')
$\text{identity}' \neq \text{identity}$ cnf(prove_inverse_of_id_is_id, negated_conjecture)

GRP023-2.p The inverse of the identity is the identity
include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$\text{identity}' \neq \text{identity}$ cnf(prove_inverse_of_id_is_id, negated_conjecture)

GRP024-4.p Associativity of commutator
The commutator operation is associative if and only if the commutator of any two elements lies in the center of the group, i.e. [[X,Y],Z]=[X,[Y,Z]] iff [U,V]*W=W*[U,V].
include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$\text{commutator}(x, y) = x \cdot (y \cdot (x' \cdot y'))$ cnf(commutator, axiom)
$\text{commutator}(\text{commutator}(a, b), c) = \text{commutator}(a, \text{commutator}(b, c))$ or $\text{commutator}(e, f) \cdot g = g \cdot \text{commutator}(e, f)$ cnf(as
$\text{commutator}(\text{commutator}(a, b), c) = \text{commutator}(a, \text{commutator}(b, c)) \Rightarrow \text{commutator}(e, f) \cdot g \neq g \cdot \text{commutator}(e, f)$ cnf(r

GRP024-5.p Levi commutator problem.
In group theory, if the commutator [x,y] is associative, then x*[y,z] = [y,z]*x.
include('Axioms/GRP004-0.ax')
$\text{commutator}(x, y) = x' \cdot (y' \cdot (x \cdot y))$ cnf(name, axiom)
$\text{commutator}(\text{commutator}(x, y), z) = \text{commutator}(x, \text{commutator}(y, z))$ cnf(associativity_of_commutator, hypothesis)
$a \cdot \text{commutator}(b, c) \neq \text{commutator}(b, c) \cdot a$ cnf(prove_center, negated_conjecture)

GRP025-1.p All groups of order 2 are isomorphic
If G1 has exactly two elements and G2 has exactly two elements, then there exists an isomorphism [a one-to-one and onto homomorphism] between them.
include('Axioms/GRP006-0.ax')
$\text{group\_member}(a, g_1)$ cnf(a_member_of_group$_1$, hypothesis)
$\text{group\_member}(b, g_1)$ cnf(b_member_of_group$_1$, hypothesis)
$\text{group\_member}(c, g_2)$ cnf(c_member_of_group$_2$, hypothesis)
$\text{group\_member}(d, g_2)$ cnf(d_member_of_group$_2$, hypothesis)
$\text{group\_member}(x, g_1) \Rightarrow (x = a \text{ or } x = b)$ cnf(a_and_b_only_members_of_group$_1$, hypothesis)
$\text{group\_member}(x, g_2) \Rightarrow (x = c \text{ or } x = d)$ cnf(c_and_d_only_members_of_group$_2$, hypothesis)
$g_1 \cdot a{=}a$ cnf(a_times_a_is_a, hypothesis)
$g_1 \cdot a{=}b$ cnf(a_times_b_is_b, hypothesis)
$g_1 \cdot b{=}a$ cnf(b_times_a_is_b, hypothesis)
$g_1 \cdot b{=}b$ cnf(b_times_b_is_a, hypothesis)
$g_2 \cdot c{=}c$ cnf(c_times_c_is_c, hypothesis)
$g_2 \cdot c{=}d$ cnf(c_times_d_is_d, hypothesis)
$g_2 \cdot d{=}c$ cnf(d_times_c_is_d, hypothesis)
$g_2 \cdot d{=}d$ cnf(d_times_d_is_c, hypothesis)
$\text{an\_isomorphism}(a) = c$ cnf(a_maps_to_c, hypothesis)
$\text{an\_isomorphism}(b) = d$ cnf(b_maps_to_d, hypothesis)
$\text{group\_member}(d_1, g_1)$ cnf(d1_member_of_group$_1$, hypothesis)
$\text{group\_member}(d_2, g_1)$ cnf(d2_member_of_group$_1$, hypothesis)
$\text{group\_member}(d_3, g_1)$ cnf(d3_member_of_group$_1$, hypothesis)
$g_1 \cdot d_1{=}d_2$ cnf(d1_times_d2_is_d$_3$, hypothesis)
$\neg\, g_2 \cdot \text{an\_isomorphism}(d_1){=}\text{an\_isomorphism}(d_2)$ cnf(prove_product_holds_in_group$_2$, negated_conjecture)

GRP025-2.p All groups of order 2 are isomorphic
If G1 has exactly two elements and G2 has exactly two elements, then there exists an isomorphism [a one-to-one and onto homomorphism] between them.
include('Axioms/GRP006-0.ax')
$g_1 \neq g_2$ cnf(two_groups, hypothesis)

group_member(a, g_1) cnf(a_member_of_group$_1$, hypothesis)
group_member(b, g_1) cnf(b_member_of_group$_1$, hypothesis)
$a \neq b$ cnf(a_not_b, hypothesis)
group_member(c, g_2) cnf(c_member_of_group$_2$, hypothesis)
group_member(d, g_2) cnf(d_member_of_group$_2$, hypothesis)
$c \neq d$ cnf(c_not_d, hypothesis)
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq d$ cnf(a_not_d, hypothesis)
$b \neq c$ cnf(b_not_c, hypothesis)
$b \neq d$ cnf(b_not_d, hypothesis)
group_member(x, g_1) $\Rightarrow$ ($x = a$ or $x = b$) cnf(a_and_b_only_members_of_group$_1$, hypothesis)
group_member(x, g_2) $\Rightarrow$ ($x = c$ or $x = d$) cnf(c_and_d_only_members_of_group$_2$, hypothesis)
identity_for(g_1) $= a$ cnf(a_identity_of_group$_1$, hypothesis)
identity_for(g_2) $= c$ cnf(c_identity_of_group$_2$, hypothesis)
$g_1 \cdot a{=}x$ cnf(a_left_identity, hypothesis)
$g_1 \cdot x{=}a$ cnf(a_right_identity, hypothesis)
$g_2 \cdot c{=}x$ cnf(c_left_identity, hypothesis)
$g_2 \cdot x{=}c$ cnf(c_right_identity, hypothesis)
isomorphism$_1$(a) $= c$ cnf(a_maps1_to_c, hypothesis)
isomorphism$_1$(b) $= d$ cnf(b_maps1_to_d, hypothesis)
isomorphism$_2$(a) $= d$ cnf(a_maps2_to_d, hypothesis)
isomorphism$_2$(b) $= c$ cnf(b_maps2_to_c, hypothesis)
group_member(d_1, g_1) cnf(d1_member_of_group$_1$, negated_conjecture)
group_member(d_2, g_1) cnf(d2_member_of_group$_1$, negated_conjecture)
group_member(d_3, g_1) cnf(d3_member_of_group$_1$, negated_conjecture)
$g_1 \cdot d_1{=}d_2$ cnf(d1_times_d2_is_d$_3$, negated_conjecture)
g_2·isomorphism$_1$(d_1)=isomorphism$_1$(d_2) $\Rightarrow$ $\neg$ g_2·isomorphism$_2$(d_1)=isomorphism$_2$(d_2) cnf(prove_one_product_holds_in_gr

GRP025-4.p All groups of order 2 are isomorphic
If G1 has exactly two elements and G2 has exactly two elements, then there exists an isomorphism [a one-to-one and onto homomorphism] between them.
include('Axioms/GRP006-0.ax')
(xg $\cdot$ $x{=}z$ and xg $\cdot$ $x{=}w$) $\Rightarrow$ $w = z$ cnf(left_cancellation, axiom)
(xg $\cdot$ $z{=}y$ and xg $\cdot$ $w{=}y$) $\Rightarrow$ $w = z$ cnf(right_cancellation, axiom)
$g_1 \neq g_2$ cnf(two_groups, hypothesis)
group_member(a, g_1) cnf(a_member_of_group$_1$, hypothesis)
group_member(b, g_1) cnf(b_member_of_group$_1$, hypothesis)
group_member(c, g_2) cnf(c_member_of_group$_2$, hypothesis)
group_member(d, g_2) cnf(d_member_of_group$_2$, hypothesis)
$a \neq b$ cnf(a_not_b, hypothesis)
$c \neq d$ cnf(c_not_d, hypothesis)
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq d$ cnf(a_not_d, hypothesis)
$b \neq c$ cnf(b_not_c, hypothesis)
$b \neq d$ cnf(b_not_d, hypothesis)
group_member(x, g_1) $\Rightarrow$ ($x = a$ or $x = b$) cnf(a_and_b_only_members_of_group$_1$, hypothesis)
group_member(x, g_2) $\Rightarrow$ ($x = c$ or $x = d$) cnf(c_and_d_only_members_of_group$_2$, hypothesis)
identity_for(g_1) $= a$ cnf(a_identity_of_group$_1$, hypothesis)
identity_for(g_2) $= c$ cnf(c_identity_of_group$_2$, hypothesis)
$g_1 \cdot a{=}x$ cnf(a_left_identity, hypothesis)
$g_1 \cdot x{=}a$ cnf(a_right_identity, hypothesis)
$g_2 \cdot c{=}x$ cnf(c_left_identity, hypothesis)
$g_2 \cdot x{=}c$ cnf(c_right_identity, hypothesis)
isomorphism$_1$(a) $= c$ cnf(a_maps1_to_c, hypothesis)
isomorphism$_1$(b) $= d$ cnf(b_maps1_to_d, hypothesis)
isomorphism$_2$(a) $= d$ cnf(a_maps2_to_d, hypothesis)
isomorphism$_2$(b) $= c$ cnf(b_maps2_to_c, hypothesis)
group_member(d_1, g_1) cnf(d1_member_of_group$_1$, negated_conjecture)
group_member(d_2, g_1) cnf(d2_member_of_group$_1$, negated_conjecture)
group_member(d_3, g_1) cnf(d3_member_of_group$_1$, negated_conjecture)

$g_1 \cdot d_1 = d_2$ cnf(d1_times_d2_is_d$_3$, negated_conjecture)
$g_2 \cdot \text{isomorphism}_1(d_1) = \text{isomorphism}_1(d_2) \Rightarrow \neg\, g_2 \cdot \text{isomorphism}_2(d_1) = \text{isomorphism}_2(d_2)$ cnf(prove_one_product_holds_in_gr

GRP026-1.p All groups of order 3 are isomorphic
If G1 and G2 each have exactly three elements, then there exists an isomorphism [a one-to-one and onto homomorphism] between them.
include('Axioms/GRP006-0.ax')
group_member((a, g_1)) cnf(a_in_group$_1$, hypothesis)
group_member((b, g_1)) cnf(b_in_group$_1$, hypothesis)
group_member((c, g_1)) cnf(c_in_group$_1$, hypothesis)
group_member((f, g_2)) cnf(f_in_group$_2$, hypothesis)
group_member((g, g_2)) cnf(g_in_group$_2$, hypothesis)
group_member((h, g_2)) cnf(h_in_group$_2$, hypothesis)
group_member($(x, g_1) \Rightarrow (x = a$ or $x = b$ or $x = c)$ cnf(all_of_group$_1$, hypothesis)
group_member($(x, g_2) \Rightarrow (x = f$ or $x = g$ or $x = h)$ cnf(all_of_group$_2$, hypothesis)
$g_1 \cdot a = a$ cnf(a_times_a_is_a, hypothesis)
$g_1 \cdot a = b$ cnf(a_times_b_is_b, hypothesis)
$g_1 \cdot b = a$ cnf(b_times_a_is_b, hypothesis)
$g_1 \cdot a = c$ cnf(a_times_c_is_c, hypothesis)
$g_1 \cdot c = a$ cnf(c_times_a_is_c, hypothesis)
$g_1 \cdot b = b$ cnf(b_times_b_is_c, hypothesis)
$g_1 \cdot b = c$ cnf(b_times_c_is_a, hypothesis)
$g_1 \cdot c = b$ cnf(c_times_b_is_a, hypothesis)
$g_1 \cdot c = c$ cnf(c_times_c_is_b, hypothesis)
$g_2 \cdot f = f$ cnf(f_times_f_is_f, hypothesis)
$g_2 \cdot f = g$ cnf(f_times_g_is_g, hypothesis)
$g_2 \cdot g = f$ cnf(g_times_f_is_g, hypothesis)
$g_2 \cdot f = h$ cnf(f_times_h_is_h, hypothesis)
$g_2 \cdot h = f$ cnf(h_times_f_is_h, hypothesis)
$g_2 \cdot g = g$ cnf(g_times_g_is_h, hypothesis)
$g_2 \cdot g = h$ cnf(g_times_h_is_f, hypothesis)
$g_2 \cdot h = g$ cnf(h_times_g_is_f, hypothesis)
$g_2 \cdot h = h$ cnf(h_times_h_is_g, hypothesis)
an_isomorphism$(a) = f$ cnf(a_maps_to_f, hypothesis)
an_isomorphism$(b) = g$ cnf(b_maps_to_g, hypothesis)
an_isomorphism$(c) = h$ cnf(c_maps_to_h, hypothesis)
group_member(d_1, g_1) cnf(d1_in_group$_1$, hypothesis)
group_member(d_2, g_1) cnf(d2_in_group$_1$, hypothesis)
group_member(d_3, g_1) cnf(d3_in_group$_1$, hypothesis)
$g_1 \cdot d_1 = d_2$ cnf(d1_times_d2_is_d$_3$, hypothesis)
$\neg\, g_2 \cdot$ an_isomorphism(d_1)=an_isomorphism(d_2) cnf(prove_product_holds_in_group$_2$, negated_conjecture)

GRP027-1.p All groups of order 5 are cyclic
There exists an element in G that generates all other elements by taking powers of that element.
include('Axioms/GRP006-0.ax')
group_member(a, g) cnf(a_in_group, hypothesis)
group_member(b, g) cnf(b_in_group, hypothesis)
group_member(c, g) cnf(c_in_group, hypothesis)
group_member(d, g) cnf(d_in_group, hypothesis)
group_member(i, g) cnf(i_in_group, hypothesis)
identity_for$(g) = i$ cnf(i_is_identity, hypothesis)
group_member$(x, g) \Rightarrow (x = a$ or $x = b$ or $x = c$ or $x = d$ or $x = i)$ cnf(all_of_group, hypothesis)
$m(g, x, m(g, x, m(g, x, m(g, x, x)))) = i$ cnf(multiplication_to_identity, hypothesis)
not_power_of$(g, x) \neq x$ cnf(all_multiply_to_identity, hypothesis)
$\neg\, g \cdot x = x$ cnf(x2_is_not_power, negated_conjecture)
$\neg\, g \cdot x = m(g, x, x)$ cnf(x3_is_not_power, negated_conjecture)
$\neg\, g \cdot x = m(g, x, m(g, x, x))$ cnf(x4_is_not_power, negated_conjecture)
$\neg\, g \cdot x = m(g, x, m(g, x, m(g, x, x)))$ cnf(x5_is_not_power, negated_conjecture)

GRP028-1.p In semigroups, left and right solutions => right id exists
If there are left and right solutions, then there is a right identity element.

$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity, axiom)
$\text{left\_solution}(x, y) \cdot x = y$ cnf(left_soln, hypothesis)
$x \cdot \text{right\_solution}(x, y) = y$ cnf(right_soln, hypothesis)
$\neg\, \text{not\_identity}(x) \cdot x = \text{not\_identity}(x)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP028-2.p In semigroups, left and right solutions => right id exists
If there are left and right solutions, then there is a right identity element.
include('Axioms/GRP002-0.ax')
$\text{left\_solution}(x, y) \cdot x = y$ cnf(left_soln, hypothesis)
$x \cdot \text{right\_solution}(x, y) = y$ cnf(right_soln, hypothesis)
$\neg\, \text{not\_identity}(x) \cdot x = \text{not\_identity}(x)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP028-3.p In semigroups, left and right solutions => right id exists
If there are left and right solutions, then there is a right identity element.
$x \cdot y = x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity$_1$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity$_2$, axiom)
$\text{left\_solution}(x, y) \cdot x = y$ cnf(left_soln, hypothesis)
$x \cdot \text{right\_solution}(x, y) = y$ cnf(right_soln, hypothesis)
$\neg\, \text{not\_identity}(x) \cdot x = \text{not\_identity}(x)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP028-4.p In semigroups, left and right solutions => right id exists
If there are left and right solutions, then there is a right identity element.
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity$_1$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity$_2$, axiom)
$\text{left\_solution}(x, y) \cdot x = y$ cnf(left_soln, hypothesis)
$x \cdot \text{right\_solution}(x, y) = y$ cnf(right_soln, hypothesis)
$\neg\, \text{not\_identity}(x) \cdot x = \text{not\_identity}(x)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP029-1.p In semigroups, left id and inverse => right id exists
If there are a left identity and left inverse, then there is a right identity element.
include('Axioms/GRP002-0.ax')
$\text{identity} \cdot a = a$ cnf(left_identity, axiom)
$a' \cdot a = \text{identity}$ cnf(left_inverse, axiom)
$\neg\, \text{not\_right\_identity}(a) \cdot a = \text{not\_right\_identity}(a)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP029-2.p In semigroups, left id and inverse => right id exists
If there are a left identity and left inverse, then there is a right identity element.
$x = x$ cnf(reflexivity, axiom)
$x = y \Rightarrow y = x$ cnf(symmetry, axiom)
$(x = y \text{ and } y = z) \Rightarrow x = z$ cnf(transitivity, axiom)
$x = y \Rightarrow x \cdot w = y \cdot w$ cnf(multiply_substitution$_1$, axiom)
$x = y \Rightarrow w \cdot x = w \cdot y$ cnf(multiply_substitution$_2$, axiom)
$(x = y \text{ and } x \cdot w = z) \Rightarrow y \cdot w = z$ cnf(product_substitution$_1$, axiom)
$(x = y \text{ and } w \cdot x = z) \Rightarrow w \cdot y = z$ cnf(product_substitution$_2$, axiom)
$(x = y \text{ and } w \cdot z = x) \Rightarrow w \cdot z = y$ cnf(product_substitution$_3$, axiom)
$x = y \Rightarrow x' = y'$ cnf(inverse_substitution, axiom)
$x \cdot y = x \cdot y$ cnf(total_function$_1$, axiom)
$(x \cdot y = z \text{ and } x \cdot y = w) \Rightarrow z = w$ cnf(total_function$_2$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity$_1$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity$_2$, axiom)
$\text{identity} \cdot a = a$ cnf(left_identity, axiom)
$a' \cdot a = \text{identity}$ cnf(left_inverse, axiom)
$\neg\, \text{not\_right\_identity}(a) \cdot a = \text{not\_right\_identity}(a)$ cnf(prove_there_is_a_right_identity, negated_conjecture)

GRP030-1.p In semigroups, left id and inverse => left id=right id
If there are a left identity and left inverse, then the left identity is also a right identity.
include('Axioms/GRP002-0.ax')
$\text{identity} \cdot a = a$ cnf(left_identity, hypothesis)
$a' \cdot a = \text{identity}$ cnf(left_inverse, hypothesis)
$\neg\, a \cdot \text{identity} = a$ cnf(prove_identity_is_a_right_identity, negated_conjecture)

GRP031-1.p In semigroups, left inverse and id => right inverse exists

If there are left inverses and left identity, then every element has a right inverse.
include('Axioms/GRP002-0.ax')
$\text{identity} \cdot a = a$ cnf(left_identity, hypothesis)
$a' \cdot a = \text{identity}$ cnf(left_inverse, hypothesis)
$\neg\, a \cdot a = \text{identity}$ cnf(prove_a_has_an_inverse, negated_conjecture)

GRP031-2.p In semigroups, left inverse and id => right inverse exists
If there are right inverses and right identity, then every element has a left inverse.
$x \cdot y = x \cdot y$ cnf(total_function_1, axiom)
$(x \cdot y = z \text{ and } x \cdot y = w) \Rightarrow z = w$ cnf(total_function_2, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity_1, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity_2, axiom)
$a \cdot a' = \text{identity}$ cnf(right_inverse, hypothesis)
$a \cdot \text{identity} = a$ cnf(right_identity, hypothesis)
$\neg\, a \cdot a = \text{identity}$ cnf(prove_a_has_a_left_inverse, negated_conjecture)

GRP032-3.p In subgroups, there is an identity
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) cnf(a_is_in_subgroup, hypothesis)
$\neg$ subgroup_member(identity) cnf(prove_identity_is_in_subgroup, negated_conjecture)

GRP033-3.p In subgroups, the identity is the group identity
$x = x$ cnf(reflexivity, axiom)
$x = y \Rightarrow y = x$ cnf(symmetry, axiom)
$(x = y \text{ and } y = z) \Rightarrow x = z$ cnf(transitivity, axiom)
$x = y \Rightarrow x' = y'$ cnf(inverse_substitution, axiom)
$x = y \Rightarrow x \cdot w = y \cdot w$ cnf($\text{multiply\_substitution}_1$, axiom)
$x = y \Rightarrow w \cdot x = w \cdot y$ cnf($\text{multiply\_substitution}_2$, axiom)
$(x = y \text{ and } x \cdot w = z) \Rightarrow y \cdot w = z$ cnf($\text{product\_substitution}_1$, axiom)
$(x = y \text{ and } w \cdot x = z) \Rightarrow w \cdot y = z$ cnf($\text{product\_substitution}_2$, axiom)
$(x = y \text{ and } w \cdot z = x) \Rightarrow w \cdot z = y$ cnf($\text{product\_substitution}_3$, axiom)
($a = b$ and subgroup_member(a)) $\Rightarrow$ subgroup_member(b) cnf(subgroup_member_substitution, axiom)
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x' \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$x \cdot y = x \cdot y$ cnf(total_function_1, axiom)
$(x \cdot y = z \text{ and } x \cdot y = w) \Rightarrow z = w$ cnf(total_function_2, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity_1, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity_2, axiom)
(subgroup_member(a) and subgroup_member(b) and $a \cdot b' = c$) $\Rightarrow$ subgroup_member(c) cnf(closure_of_product_and_inverse
subgroup_member(a) cnf(a_is_in_subgroup, hypothesis)
subgroup_member(a) $\Rightarrow$ subgroup_member($j(a)$) cnf(subgr2_clause_1, hypothesis)
$(j(a) \cdot a = j(a) \text{ and } a \cdot j(a) = j(a)) \Rightarrow \neg$ subgroup_member(a) cnf(prove_subgr_2, negated_conjecture)

GRP033-4.p In subgroups, the identity is the group identity
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) cnf(a_is_in_subgroup, hypothesis)
subgroup_member(a) $\Rightarrow$ subgroup_member($j(a)$) cnf(subgr2_clause_1, hypothesis)
$(j(a) \cdot a = j(a) \text{ and } a \cdot j(a) = j(a)) \Rightarrow \neg$ subgroup_member(a) cnf(prove_subgr_2, negated_conjecture)

GRP034-3.p In subgroups, inverse is closed
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) cnf(a_is_in_subgroup, hypothesis)
$\neg$ subgroup_member(a') cnf(prove_a_inverse_is_in_subgroup, negated_conjecture)

GRP034-4.p In subgroups, inverse is closed
$x \cdot y = x \cdot y$ cnf(closure, axiom)
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)

$x \cdot x'$=identity   cnf(right_inverse, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity$_1$, axiom)
$(x \cdot y{=}u$ and $y \cdot z{=}v$ and $x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity$_2$, axiom)
(subgroup_member(a) and subgroup_member(b) and $b{\cdot}a'{=}c$) $\Rightarrow$ subgroup_member(c)   cnf(closure_of_subgroup, axiom)
subgroup_member(a)   cnf(a_is_in_subgroup, hypothesis)
¬ subgroup_member(a')   cnf(prove_inverse_is_in_subgroup, negated_conjecture)

GRP035-3.p In subgroups, product is closed
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a)   cnf(a_is_in_subgroup, hypothesis)
subgroup_member(b)   cnf(b_is_in_subgroup, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
¬ subgroup_member(c)   cnf(prove_c_is_in_subgroup, negated_conjecture)

GRP036-3.p In subgroups, the identity element is unique
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) $\Rightarrow$ another_identity $\cdot\, a{=}a$ cnf(another_left_identity, hypothesis)
subgroup_member(a) $\Rightarrow$ $a \,\cdot$ another_identity$=a$ cnf(another_right_identity, hypothesis)
subgroup_member(a) $\Rightarrow$ $a \,\cdot$ another_inverse(a)=another_identity   cnf(another_right_inverse, hypothesis)
subgroup_member(a) $\Rightarrow$ another_inverse(a) $\cdot\, a$=another_identity   cnf(another_left_inverse, hypothesis)
subgroup_member(a) $\Rightarrow$ subgroup_member(another_inverse(a))   cnf(another_inverse_in_subgroup, hypothesis)
subgroup_member(another_identity)   cnf(another_identity_in_subgroup, hypothesis)
identity $\neq$ another_identity   cnf(prove_identity_equals_another_identity, negated_conjecture)

GRP037-3.p In subgroups, the inverse of an element is unique
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) $\Rightarrow$ another_identity $\cdot\, a{=}a$ cnf(another_left_identity, hypothesis)
subgroup_member(a) $\Rightarrow$ $a \,\cdot$ another_identity$=a$ cnf(another_right_identity, hypothesis)
subgroup_member(a) $\Rightarrow$ $a \,\cdot$ another_inverse(a)=another_identity   cnf(another_right_inverse, hypothesis)
subgroup_member(a) $\Rightarrow$ another_inverse(a) $\cdot\, a$=another_identity   cnf(another_left_inverse, hypothesis)
subgroup_member(a) $\Rightarrow$ subgroup_member(another_inverse(a))   cnf(another_inverse_in_subgroup, hypothesis)
$(a \cdot b{=}c$ and $a \cdot d{=}c) \Rightarrow d = b$ cnf(product_right_cancellation, hypothesis)
$(a \cdot b{=}c$ and $d \cdot b{=}c) \Rightarrow d = a$ cnf(product_left_cancellation, hypothesis)
subgroup_member(a)   cnf(a_is_in_subgroup, hypothesis)
subgroup_member(another_identity)   cnf(another_identity_in_subgroup, hypothesis)
$a' \neq$ another_inverse(a)   cnf(prove_two_inverses_are_equal, negated_conjecture)

GRP038-3.p In subgroups, if a and b are members, then a.b∧-1 is a member
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(a) $\Rightarrow$ subgroup_member(a')   cnf(closure_of_inverse, axiom)
subgroup_member(identity)   cnf(identity_is_in_subgroup, axiom)
subgroup_member(a)   cnf(a_is_in_subgroup, hypothesis)
subgroup_member(b)   cnf(b_is_in_subgroup, hypothesis)
$a \cdot b'{=}c$ cnf(a_times_inverse_b_is_c, hypothesis)
¬ subgroup_member(c)   cnf(prove_c_is_in_subgroup, negated_conjecture)

GRP039-1.p Subgroups of index 2 are normal
If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-1.ax')
subgroup_member(element_in_O$_2$(a, b)) or subgroup_member(b) or subgroup_member(a)   cnf(an_element_in_O$_2$, axiom)
$a \,\cdot$ element_in_O$_2$(a, b)$=b$ or subgroup_member(b) or subgroup_member(a)   cnf(property_of_O$_2$, axiom)
subgroup_member(b)   cnf(b_is_in_subgroup, negated_conjecture)
$b \cdot a'{=}c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c{=}d$ cnf(a_times_c_is_d, negated_conjecture)
¬ subgroup_member(d)   cnf(prove_d_is_in_subgroup, negated_conjecture)

GRP039-2.p Subgroups of index 2 are normal

If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-1.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
subgroup_member(x) or subgroup_member(y) or subgroup_member(element_in_O$_2$(x, y)) cnf(an_element_in_O$_2$, axiom)
subgroup_member(x) or subgroup_member(y) or $x \cdot$ element_in_O$_2$(x, y) $= y$ cnf(property_of_O$_2$, axiom)
subgroup_member(b) cnf(b_in_O$_2$, negated_conjecture)
$b \cdot a' = c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c = d$ cnf(a_times_c_is_d, negated_conjecture)
$\neg$ subgroup_member(d) cnf(prove_d_in_O$_2$, negated_conjecture)

GRP039-3.p Subgroups of index 2 are normal
If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
($a \cdot b{=}c$ and $a \cdot d{=}c$) $\Rightarrow$ $d = b$ cnf(product_right_cancellation, axiom)
($a \cdot b{=}c$ and $d \cdot b{=}c$) $\Rightarrow$ $d = a$ cnf(product_left_cancellation, axiom)
$(a')' = a$ cnf(inverse_is_self_cancelling, axiom)
subgroup_member(identity) cnf(identity_is_in_subgroup, axiom)
subgroup_member(a) $\Rightarrow$ subgroup_member(a') cnf(subgroup_member_inverse_are_in_subgroup, axiom)
subgroup_member(element_in_O$_2$(a, b)) or subgroup_member(b) or subgroup_member(a) cnf(an_element_in_O$_2$, axiom)
$a \cdot$ element_in_O$_2$(a, b)$=b$ or subgroup_member(b) or subgroup_member(a) cnf(property_of_O$_2$, axiom)
subgroup_member(b) cnf(b_is_in_subgroup, negated_conjecture)
$b \cdot a'{=}c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c{=}d$ cnf(a_times_c_is_d, negated_conjecture)
$\neg$ subgroup_member(d) cnf(prove_d_is_in_subgroup, negated_conjecture)

GRP039-4.p Subgroups of index 2 are normal
If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-1.ax')
subgroup_member(identity) cnf(identity_is_in_subgroup, axiom)
subgroup_member(element_in_O$_2$(a, b)) or subgroup_member(b) or subgroup_member(a) cnf(an_element_in_O$_2$, axiom)
$a \cdot$ element_in_O$_2$(a, b)$=b$ or subgroup_member(b) or subgroup_member(a) cnf(property_of_O$_2$, axiom)
subgroup_member(b) cnf(b_is_in_subgroup, negated_conjecture)
$b \cdot a'{=}c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c{=}d$ cnf(a_times_c_is_d, negated_conjecture)
$\neg$ subgroup_member(d) cnf(prove_d_is_in_subgroup, negated_conjecture)

GRP039-5.p Subgroups of index 2 are normal
If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-1.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
subgroup_member(identity) cnf(identity_in_O$_2$, axiom)
subgroup_member(x) or subgroup_member(y) or subgroup_member(element_in_O$_2$(x, y)) cnf(an_element_in_O$_2$, axiom)
subgroup_member(x) or subgroup_member(y) or $x \cdot$ element_in_O$_2$(x, y) $= y$ cnf(property_of_O$_2$, axiom)
subgroup_member(b) cnf(b_in_O$_2$, negated_conjecture)
$b \cdot a' = c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c = d$ cnf(a_times_c_is_d, negated_conjecture)
$\neg$ subgroup_member(d) cnf(prove_d_in_O$_2$, negated_conjecture)

GRP039-7.p Subgroups of index 2 are normal
If O is a subgroup of G and there are exactly 2 cosets in G/O, then O is normal [that is, for all x in G and y in O, x*y*inverse(x) is back in O].

include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-1.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$(x')' = x$ cnf(inverse_inverse, axiom)
$\text{identity}' = \text{identity}$ cnf(inverse_of_identity, axiom)
subgroup_member(identity) cnf(identity_in_O_2, axiom)
subgroup_member(x) or subgroup_member(y) or subgroup_member(element_in_O_2(x, y)) cnf(an_element_in_O_2, axiom)
subgroup_member(x) or subgroup_member(y) or $x \cdot$ element_in_O_2(x, y) $= y$ cnf(property_of_O_2, axiom)
subgroup_member(b) cnf(b_in_O_2, negated_conjecture)
$b \cdot a' = c$ cnf(b_times_a_inverse_is_c, negated_conjecture)
$a \cdot c = d$ cnf(a_times_c_is_d, negated_conjecture)
$\neg$ subgroup_member(d) cnf(prove_d_in_O_2, negated_conjecture)

GRP040-3.p In subgroups of order 2, inverse is an involution
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(element_in_O_2(a, b)) or subgroup_member(b) or subgroup_member(a) cnf(an_element_in_O_2, axiom)
$a \cdot$ element_in_O_2(a, b)$=b$ or subgroup_member(b) or subgroup_member(a) cnf(property_of_O_2, axiom)
$\neg$ subgroup_member(a) cnf(a_in_subgroup, hypothesis)
subgroup_member(b) cnf(b_is_in_subgroup, hypothesis)
$\neg$ subgroup_member(d) cnf(d_in_subgroup, hypothesis)
$b \cdot a'=c$ cnf(b_times_a_inverse_is_c, hypothesis)
$a \cdot c=d$ cnf(a_times_c_is_d, hypothesis)
$(a')' = a$ cnf(prove_inverse_is_self_cancelling, negated_conjecture)

GRP040-4.p In subgroups of order 2, inverse is an involution
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')
subgroup_member(identity) cnf(identity_is_in_subgroup, axiom)
subgroup_member(a) $\Rightarrow$ subgroup_member(a') cnf(closure_of_inverse, axiom)
$(a \cdot b=c$ and $a \cdot d=c) \Rightarrow d = b$ cnf(product_right_cancellation, axiom)
$(a \cdot b=c$ and $d \cdot b=c) \Rightarrow d = a$ cnf(product_left_cancellation, axiom)
subgroup_member(element_in_O_2(a, b)) or subgroup_member(b) or subgroup_member(a) cnf(an_element_in_O_2, axiom)
$a \cdot$ element_in_O_2(a, b)$=b$ or subgroup_member(b) or subgroup_member(a) cnf(property_of_O_2, axiom)
$\neg$ subgroup_member(a) cnf(a_in_subgroup, hypothesis)
subgroup_member(b) cnf(b_is_in_subgroup, hypothesis)
$\neg$ subgroup_member(d) cnf(d_in_subgroup, hypothesis)
$b \cdot a'=c$ cnf(b_times_a_inverse_is_c, hypothesis)
$a \cdot c=d$ cnf(a_times_c_is_d, hypothesis)
$(a')' = a$ cnf(prove_inverse_is_self_cancelling, negated_conjecture)

GRP041-2.p Reflexivity is dependent
include('Axioms/GRP005-0.ax')
$\neg\, a=a$ cnf(prove_reflexivity, negated_conjecture)

GRP042-2.p Symmetry is dependent
include('Axioms/GRP005-0.ax')
$a=b$ cnf(a_equals_b, hypothesis)
$\neg\, b=a$ cnf(prove_symmetry, negated_conjecture)

GRP043-2.p Transitivity is dependent
include('Axioms/GRP005-0.ax')
$a=b$ cnf(a_equals_b, hypothesis)
$b=c$ cnf(b_equals_c, hypothesis)
$\neg\, a=c$ cnf(prove_transitivity, negated_conjecture)

GRP044-2.p Product subsitution 1 is dependent
include('Axioms/GRP005-0.ax')
$a=b$ cnf(a_equals_b, hypothesis)
$a \cdot c=\text{result}$ cnf(product_with_a, hypothesis)
$\neg\, b \cdot c=\text{result}$ cnf(prove_product_with_b, negated_conjecture)

GRP045-2.p Product subsitution 2 is dependent
include('Axioms/GRP005-0.ax')
$a{=}b$ cnf(a_equals_b, hypothesis)
$c \cdot a{=}\text{result}$ cnf(product_with_a, hypothesis)
$\neg\, c \cdot b{=}\text{result}$ cnf(prove_product_with_b, negated_conjecture)

GRP046-2.p Multiply substitution 1 is dependent
include('Axioms/GRP005-0.ax')
$a{=}b$ cnf(a_equals_b, hypothesis)
$\neg\, a \cdot c{=}b \cdot c$ cnf(prove_multiply_substitution$_1$, negated_conjecture)

GRP047-2.p Multiply substitution 2 is dependent
include('Axioms/GRP005-0.ax')
$a{=}b$ cnf(a_equals_b, hypothesis)
$\neg\, c \cdot a{=}c \cdot b$ cnf(prove_multiply_substitution$_2$, negated_conjecture)

GRP048-2.p Inverse substitution is dependent
include('Axioms/GRP005-0.ax')
$a{=}b$ cnf(a_equals_b, hypothesis)
$\neg\, a'{=}b'$ cnf(prove_inverse_substitution, negated_conjecture)

GRP049-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$z \cdot (((z \cdot y)' \cdot x)' \cdot (y \cdot (y' \cdot y))')' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP050-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$z \cdot (((z \cdot y)' \cdot x)' \cdot (y' \cdot (y' \cdot y)))' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP051-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$((z \cdot (x \cdot y)')' \cdot (z \cdot y')) \cdot (y' \cdot y)' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP052-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$z \cdot (((y' \cdot y) \cdot ((z \cdot y')' \cdot x)') \cdot y)' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP053-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$(y \cdot (((z \cdot y)' \cdot (z \cdot x'))' \cdot (y' \cdot y)')')' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP054-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$((z \cdot (x' \cdot (y \cdot (y' \cdot y)')')')' \cdot (z \cdot y))' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP055-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$((z \cdot (x' \cdot (y' \cdot (y' \cdot y)'))')' \cdot (z \cdot y))' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP056-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$((z \cdot (x' \cdot y)')' \cdot (z \cdot y'))' \cdot (y' \cdot y)' = x$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP057-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$x \cdot (((y' \cdot (x' \cdot z))' \cdot u) \cdot (y \cdot u)')' = z$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP058-1.p Single axiom for group theory, in product & inverse

This is a single axiom for group theory, in terms of product and inverse.
$x \cdot (y \cdot (((z \cdot z') \cdot (u \cdot y)') \cdot x))' = u$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP059-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$(((((x \cdot y) \cdot z)' \cdot x) \cdot y) \cdot (u \cdot u'))' = z$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP060-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$x \cdot (y \cdot (z \cdot ((z' \cdot (u \cdot y)') \cdot x)))' = u$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP061-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$(x \cdot (y \cdot ((y' \cdot z) \cdot (u \cdot (x \cdot z))')))' = u$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP062-1.p Single axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$(x \cdot (y \cdot ((z \cdot z') \cdot (u \cdot (x \cdot y))')))' = u$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP063-1.p Single axiom for group theory, in division
This is a single axiom for group theory, in terms of division
$\text{divide}(x, \text{divide}(\text{divide}(\text{divide}(\text{divide}(x, x), y), z), \text{divide}(\text{divide}(\text{divide}(x, x), x), z))) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP064-1.p Single axiom for group theory, in division
This is a single axiom for group theory, in terms of division
$\text{divide}(x, \text{divide}(\text{divide}(\text{divide}(\text{divide}(y, y), y), z), \text{divide}(\text{divide}(\text{divide}(y, y), x), z))) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP065-1.p Single axiom for group theory, in division
This is a single axiom for group theory, in terms of division
$\text{divide}(\text{divide}(\text{divide}(x, x), \text{divide}(x, \text{divide}(y, \text{divide}(\text{divide}(\text{divide}(x, x), x), z)))), z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP066-1.p Single axiom for group theory, in division and identity
This is a single axiom for group theory, in terms of division and identity
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(x, \text{divide}(y, \text{divide}(\text{divide}(\text{divide}(x, x), x), z)))), z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP067-1.p Single axiom for group theory, in division and identity
This is a single axiom for group theory, in terms of division and identity
$\text{divide}(\text{divide}(\text{divide}(x, x), \text{divide}(x, \text{divide}(y, \text{divide}(\text{divide}(\text{identity}, x), z)))), z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP068-1.p Single axiom for group theory, in division and identity
This is a single axiom for group theory, in terms of division and identity
$\text{divide}(x, \text{divide}(\text{divide}(\text{divide}(\text{identity}, y), z), \text{divide}(\text{divide}(\text{divide}(x, x), x), z))) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)

$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP069-1.p Single axiom for group theory, in division and identity
This is a single axiom for group theory, in terms of division and identity
$\text{divide}(x, \text{divide}(\text{divide}(\text{divide}(\text{divide}(x, x), y), z), \text{divide}(\text{divide}(\text{identity}, x), z))) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP070-1.p Single axiom for group theory, in division and inverse
This is a single axiom for group theory, in terms of division and inverse.
$\text{divide}(\text{divide}(x, x), \text{divide}(y, \text{divide}(\text{divide}(z, \text{divide}(u, y)), u'))) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP071-1.p Single axiom for group theory, in division and inverse
This is a single axiom for group theory, in terms of division and inverse.
$\text{divide}(\text{divide}(x, \text{divide}(y, \text{divide}(z, u)))', \text{divide}(\text{divide}(u, z), x)) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP072-1.p Single axiom for group theory, in division and inverse
This is a single axiom for group theory, in terms of division and inverse.
$\text{divide}(\text{divide}(\text{divide}(x, y)', \text{divide}(\text{divide}(z, u), x)), \text{divide}(u, z)) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP073-1.p Single axiom for group theory, in division and inverse
This is a single axiom for group theory, in terms of division and inverse.
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(x, y), z), \text{divide}(u, z))', \text{divide}(y, x)) = u$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP074-1.p Single axiom for group theory, in division and inverse
This is a single axiom for group theory, in terms of division and inverse.
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(x, x), y), \text{divide}(z, \text{divide}(y, u)))', u) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP075-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(y, \text{identity})), \text{double\_divide}(\text{double\_divide}(z, \text{double\_divide}(u, \text{doul}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP076-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y), z), \text{double\_divide}(y, \text{identity}))), \text{double\_divide}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP077-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(\text{identity}, \text{double\_divide}(\text{double\_divide}(x, \text{identity}), \text{double\_divide}($
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)

$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity} \text{ and } \text{identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP078-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(\text{identity}, x), \text{double\_divide}(\text{identity}, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y), \text{identity}), \text{do}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity} \text{ and } \text{identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP079-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(\text{identity}, x), \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(y, z), \text{double\_divide}(\text{identity}, \text{identity})), \text{d}$
y cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity} \text{ and } \text{identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP080-1.p Single axiom for group theory, in double division and identity
This is a single axiom for group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(\text{identity}, \text{double\_divide}(x, \text{double\_divide}(y, \text{identity}))), \text{double\_divide}(\text{double\_divide}(y, \text{double\_divi}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity} \text{ and } \text{identity} \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP081-1.p Single axiom for group theory, in double division and inverse
This is a single axiom for group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(y, z), \text{double\_divide}(y, \text{double\_divide}(u', z'))')), x)' = u$ cnf(sin
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP082-1.p Single axiom for group theory, in double division and inverse
This is a single axiom for group theory, in terms of double division and inverse.
$\text{double\_divide}(x', \text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(y, z))', \text{double\_divide}(u, \text{double\_divide}(y, u)))') = z$ cnf(sin
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP083-1.p Single axiom for group theory, in double division and inverse
This is a single axiom for group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(y, z)'), \text{double\_divide}(y', \text{double\_divide}(u, \text{double\_divide}(x, u))')) = z$ cnf(sin
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2) \Rightarrow (a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms, negated_conjecture)

GRP084-1.p Single axiom for Abelian group theory, in product and inverse
This is a single axiom for Abelian group theory, in terms of product and inverse.
$(((x \cdot y)' \cdot (y \cdot x))' \cdot ((z \cdot u)' \cdot (z \cdot ((v \cdot w') \cdot u')')))' \cdot w = v$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP085-1.p Single axiom for Abelian group theory, in product and inverse
This is a single axiom for Abelian group theory, in terms of product and inverse.
$((x \cdot y) \cdot z) \cdot (x \cdot z)' = y$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP086-1.p Single axiom for Abelian group theory, in product and inverse
This is a single axiom for Abelian group theory, in terms of product and inverse.
$x \cdot ((y \cdot z) \cdot (x \cdot z)') = y$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP087-1.p Single axiom for Abelian group theory, in product and inverse

This is a single axiom for Abelian group theory, in terms of product and inverse.
$x \cdot (((x \cdot y)' \cdot z) \cdot y) = z$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP088-1.p Single axiom for Abelian group theory, in division
This is a single axiom for Abelian group theory, in terms of division
$\text{divide}(x, \text{divide}(y, \text{divide}(z, \text{divide}(x, y)))) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP089-1.p Single axiom for Abelian group theory, in division
This is a single axiom for Abelian group theory, in terms of division
$\text{divide}(x, \text{divide}(\text{divide}(x, y), \text{divide}(z, y))) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP090-1.p Single axiom for Abelian group theory, in division
This is a single axiom for Abelian group theory, in terms of division
$\text{divide}(\text{divide}(x, \text{divide}(y, z)), \text{divide}(x, y)) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP091-1.p Single axiom for Abelian group theory, in division
This is a single axiom for Abelian group theory, in terms of division
$\text{divide}(\text{divide}(x, \text{divide}(\text{divide}(x, y), z)), y) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP092-1.p Single axiom for Abelian group theory, in division
This is a single axiom for Abelian group theory, in terms of division
$\text{divide}(\text{divide}(x, y), \text{divide}(\text{divide}(x, z), y)) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{divide}(z, z), y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{divide}(z, z), x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP093-1.p Single axiom for Abelian group theory, in division and identity
This is a single axiom for Abelian group theory, in terms of division and identity
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(\text{divide}(\text{divide}(x, y), z), x)), z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP094-1.p Single axiom for Abelian group theory, in division and identity
This is a single axiom for Abelian group theory, in terms of division and identity
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(x, y)), \text{divide}(\text{divide}(y, z), x)) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP095-1.p Single axiom for Abelian group theory, in division and identity
This is a single axiom for Abelian group theory, in terms of division and identity
$\text{divide}(\text{divide}(\text{identity}, x), \text{divide}(\text{divide}(\text{divide}(y, x), z), y)) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, \text{divide}(\text{identity}, y))$ cnf(multiply, axiom)
$x' = \text{divide}(\text{identity}, x)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(x, x)$ cnf(identity, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP096-1.p Single axiom for Abelian group theory, in division and inverse
This is a single axiom for Abelian group theory, in terms of division and inverse
$\text{divide}(\text{divide}(x, \text{divide}(y, \text{divide}(x, z))'), z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP097-1.p Single axiom for Abelian group theory, in division and inverse
This is a single axiom for Abelian group theory, in terms of division and inverse
$\text{divide}(x, \text{divide}(\text{divide}(y, z), \text{divide}(x, z))') = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP098-1.p Single axiom for Abelian group theory, in division and inverse
This is a single axiom for Abelian group theory, in terms of division and inverse
$\text{divide}(\text{divide}(\text{divide}(x, y'), z), \text{divide}(x, z)) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{divide}(x, y')$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP099-1.p Single axiom for Abelian group theory, in double div and id
This is a single axiom for Abelian group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(y, \text{double\_divide}(x, z)), \text{double\_divide}(\text{identity}, z))), \text{double\_divide}$
y cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conj

GRP100-1.p Single axiom for Abelian group theory, in double div and id
This is a single axiom for Abelian group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(y, \text{double\_divide}(x, z)), \text{double\_divide}(z, \text{identity}))), \text{double\_divide}$
y cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conj

GRP101-1.p Single axiom for Abelian group theory, in double div and id
This is a single axiom for Abelian group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(y, \text{double\_divide}(z, x)), \text{double\_divide}(z, \text{identity}))), \text{double\_divide}$
y cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conj

GRP102-1.p Single axiom for Abelian group theory, in double div and id
This is a single axiom for Abelian group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(y, x), z), \text{double\_divide}(y, \text{identity}))), \text{double\_divide}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conj

GRP103-1.p Single axiom for Abelian group theory, in double div and id
This is a single axiom for Abelian group theory, in terms of double division and identity.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{identity}, y), \text{double\_divide}(z, \text{double\_divide}(y, x)))), \text{double\_divide}$
z cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(\text{double\_divide}(y, x), \text{identity})$ cnf(multiply, axiom)
$x' = \text{double\_divide}(x, \text{identity})$ cnf(inverse, axiom)
$\text{identity} = \text{double\_divide}(x, x')$ cnf(identity, axiom)
$(a_1' \cdot a_1 = \text{identity and identity} \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conj

GRP104-1.p Single axiom for Abelian group theory, in double div and inv

This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(x, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y), z')', y)') = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP105-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y), \text{double\_divide}(x, z')')', y) = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP106-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, y), \text{double\_divide}(x, \text{double\_divide}(z, y)')')' = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP107-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, y), \text{double\_divide}(x, \text{double\_divide}(z', y)')') = z$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP108-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(y, \text{double\_divide}(x, z))')', z)' = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP109-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(x, \text{double\_divide}(y', \text{double\_divide}(x, z))')', z) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP110-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y)', z)', \text{double\_divide}(x, z))' = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP111-1.p Single axiom for Abelian group theory, in double div and inv
This is a single axiom for Abelian group theory, in terms of double division and inverse.
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(x, y')', z)', \text{double\_divide}(x, z)) = y$ cnf(single_axiom, axiom)
$x \cdot y = \text{double\_divide}(y, x)'$ cnf(multiply, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot b_4 \neq b_4 \cdot a_4$ cnf(prove_these_axioms, negated_conjectu

GRP112-1.p Single axiom for group theory, in product & inverse
This is a single axiom for groups in which the square of every element is the identity, in terms of product and inverse.
$((x \cdot y) \cdot z) \cdot (x \cdot z) = y$ cnf(single_axiom, axiom)
$(a_1' \cdot a_1 = b_1' \cdot b_1 \text{ and } (b_2' \cdot b_2) \cdot a_2 = a_2 \text{ and } (a_3 \cdot b_3) \cdot c_3 = a_3 \cdot (b_3 \cdot c_3)) \Rightarrow a_4 \cdot a_4 \neq b_4 \cdot b_4$ cnf(prove_these_axioms, negated_conjectu

GRP113-1.p Lemma for proving all groups of order 4 are cyclic
Prove that any group of order 4 must satisfy one of the following relations, where the elements of the group are a, b, c, and the identity. 1) the square of every element is the identity. 2) the square of a is b, the cube of a is c, and the fourth power of a is the identity. 3) the square of b is c, the cube of b is a, and the fourth power of b is the identity. 4) the square of c is a, the cube of c is b, and the fourth
include('Axioms/GRP004-0.ax')
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot x' = \text{identity}$ cnf(right_inverse, axiom)
$x = a \text{ or } x = b \text{ or } x = c \text{ or } x = \text{identity}$ cnf(all_of_group$_1$, hypothesis)
$a \neq b$ cnf(a_not_b, hypothesis)
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq \text{identity}$ cnf(a_not_identity, hypothesis)
$b \neq c$ cnf(b_not_c, hypothesis)

$b \neq$ identity cnf(b_not_identity, hypothesis)
$c \neq$ identity cnf(c_not_identity, hypothesis)
$(a \cdot a = \text{identity and } b \cdot b = \text{identity}) \Rightarrow c \cdot c \neq \text{identity}$ cnf(square_identity, negated_conjecture)
$(a \cdot a = b \text{ and } a \cdot (a \cdot a) = c) \Rightarrow a \cdot (a \cdot (a \cdot a)) \neq \text{identity}$ cnf(condition_a, negated_conjecture)
$(b \cdot b = c \text{ and } b \cdot (b \cdot b) = a) \Rightarrow b \cdot (b \cdot (b \cdot b)) \neq \text{identity}$ cnf(condition_b, negated_conjecture)
$(c \cdot c = a \text{ and } c \cdot (c \cdot c) = b) \Rightarrow c \cdot (c \cdot (c \cdot c)) \neq \text{identity}$ cnf(condition_c, negated_conjecture)

GRP114-1.p Product of positive and negative parts of X equals X
Prove that for each element X in a group, X is equal to the product of its positive part (the union with the identity) and its negative part (the intersection with the identity).
include('Axioms/GRP004-0.ax')
$\text{identity}' = \text{identity}$ cnf(inverse_of_identity, axiom)
$(x')' = x$ cnf(inverse_involution, axiom)
$(x \cdot y)' = y' \cdot x'$ cnf(inverse_product_lemma, axiom)
$\text{intersection}(x, x) = x$ cnf(intersection_idempotent, axiom)
$\text{union}(x, x) = x$ cnf(union_idempotent, axiom)
$\text{intersection}(x, y) = \text{intersection}(y, x)$ cnf(intersection_commutative, axiom)
$\text{union}(x, y) = \text{union}(y, x)$ cnf(union_commutative, axiom)
$\text{intersection}(x, \text{intersection}(y, z)) = \text{intersection}(\text{intersection}(x, y), z)$ cnf(intersection_associative, axiom)
$\text{union}(x, \text{union}(y, z)) = \text{union}(\text{union}(x, y), z)$ cnf(union_associative, axiom)
$\text{union}(\text{intersection}(x, y), y) = y$ cnf(union_intersection_absorbtion, axiom)
$\text{intersection}(\text{union}(x, y), y) = y$ cnf(intersection_union_absorbtion, axiom)
$x \cdot \text{union}(y, z) = \text{union}(x \cdot y, x \cdot z)$ cnf(multiply_union_1, axiom)
$x \cdot \text{intersection}(y, z) = \text{intersection}(x \cdot y, x \cdot z)$ cnf($\text{multiply\_intersection}_1$, axiom)
$\text{union}(y, z) \cdot x = \text{union}(y \cdot x, z \cdot x)$ cnf(multiply_union_2, axiom)
$\text{intersection}(y, z) \cdot x = \text{intersection}(y \cdot x, z \cdot x)$ cnf($\text{multiply\_intersection}_2$, axiom)
$\text{positive\_part}(x) = \text{union}(x, \text{identity})$ cnf(positive_part, axiom)
$\text{negative\_part}(x) = \text{intersection}(x, \text{identity})$ cnf(negative_part, axiom)
$\text{positive\_part}(a) \cdot \text{negative\_part}(a) \neq a$ cnf(prove_product, negated_conjecture)

GRP115-1.p Derive order 3 from a single axiom for groups order 3
$x \cdot ((x \cdot ((x \cdot y) \cdot z)) \cdot (\text{identity} \cdot (z \cdot z))) = y$ cnf(single_axiom, axiom)
$a \cdot (a \cdot a) \neq \text{identity}$ cnf(prove_order_3, negated_conjecture)

GRP116-1.p Derive left identity from a single axiom for groups order 3
$x \cdot ((x \cdot ((x \cdot y) \cdot z)) \cdot (\text{identity} \cdot (z \cdot z))) = y$ cnf(single_axiom, axiom)
$\text{identity} \cdot a \neq a$ cnf(prove_order_3, negated_conjecture)

GRP117-1.p Derive right identity from a single axiom for groups order 3
$x \cdot ((x \cdot ((x \cdot y) \cdot z)) \cdot (\text{identity} \cdot (z \cdot z))) = y$ cnf(single_axiom, axiom)
$a \cdot \text{identity} \neq a$ cnf(prove_order_3, negated_conjecture)

GRP118-1.p Derive associativity from a single axiom for groups order 3
$x \cdot ((x \cdot ((x \cdot y) \cdot z)) \cdot (\text{identity} \cdot (z \cdot z))) = y$ cnf(single_axiom, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(prove_order_3, negated_conjecture)

GRP119-1.p Derive order 4 from a single axiom for groups order 4
$y \cdot ((y \cdot ((y \cdot y) \cdot (x \cdot z))) \cdot (z \cdot (z \cdot z))) = x$ cnf(single_axiom, axiom)
$\text{identity} \cdot \text{identity} = \text{identity}$ cnf(single_axiom_2, axiom)
$a \cdot (a \cdot (a \cdot a)) \neq \text{identity}$ cnf(prove_order_4, negated_conjecture)

GRP120-1.p Derive left identity from a single axiom for groups order 4
$y \cdot ((y \cdot ((y \cdot y) \cdot (x \cdot z))) \cdot (z \cdot (z \cdot z))) = x$ cnf(single_axiom, axiom)
$\text{identity} \cdot \text{identity} = \text{identity}$ cnf(single_axiom_2, axiom)
$\text{identity} \cdot a \neq a$ cnf(prove_order_3, negated_conjecture)

GRP121-1.p Derive right identity from a single axiom for groups order 4
$y \cdot ((y \cdot ((y \cdot y) \cdot (x \cdot z))) \cdot (z \cdot (z \cdot z))) = x$ cnf(single_axiom, axiom)
$\text{identity} \cdot \text{identity} = \text{identity}$ cnf(single_axiom_2, axiom)
$a \cdot \text{identity} \neq a$ cnf(prove_order_3, negated_conjecture)

GRP122-1.p Derive associativity from a single axiom for groups order 4
$y \cdot ((y \cdot ((y \cdot y) \cdot (x \cdot z))) \cdot (z \cdot (z \cdot z))) = x$ cnf(single_axiom, axiom)
$\text{identity} \cdot \text{identity} = \text{identity}$ cnf(single_axiom_2, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(prove_order_3, negated_conjecture)

GRP123-1.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
$\neg\, e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_2=e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2=e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_3=e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3=e_2$ cnf(e_3_is_not_e_2, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$) cnf($\text{product\_total\_function}_1$, axiom)
$(x \cdot y=w$ and $x \cdot y=z) \Rightarrow w=z$ cnf($\text{product\_total\_function}_2$, axiom)
$(x \cdot w=y$ and $x \cdot z=y) \Rightarrow w=z$ cnf(product_right_cancellation, axiom)
$(w \cdot y=x$ and $z \cdot y=x) \Rightarrow w=z$ cnf(product_left_cancellation, axiom)
$x \cdot x=x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1=z_1$ and $x_2 \cdot y_2=z_1$ and $z_2 \cdot y_1=x_1$ and $z_2 \cdot y_2=x_2) \Rightarrow x_1=x_2$ cnf(qg1_1, negated_conjecture)
$(x_1 \cdot y_1=z_1$ and $x_2 \cdot y_2=z_1$ and $z_2 \cdot y_1=x_1$ and $z_2 \cdot y_2=x_2) \Rightarrow y_1=y_2$ cnf(qg1_2, negated_conjecture)

GRP123-1.005.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
group_element(e_4) cnf(element_4, axiom)
group_element(e_5) cnf(element_5, axiom)
$\neg\, e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1=e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_1=e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg\, e_2=e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2=e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2=e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_2=e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3=e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3=e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3=e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3=e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4=e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4=e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4=e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4=e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5=e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5=e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5=e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5=e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$ or $x{\cdot}y=e_4$ or $x{\cdot}y=e_5$) cnf(product_total_func
$(x \cdot y=w$ and $x \cdot y=z) \Rightarrow w=z$ cnf($\text{product\_total\_function}_2$, axiom)
$(x \cdot w=y$ and $x \cdot z=y) \Rightarrow w=z$ cnf(product_right_cancellation, axiom)
$(w \cdot y=x$ and $z \cdot y=x) \Rightarrow w=z$ cnf(product_left_cancellation, axiom)
$x \cdot x=x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1=z_1$ and $x_2 \cdot y_2=z_1$ and $z_2 \cdot y_1=x_1$ and $z_2 \cdot y_2=x_2) \Rightarrow x_1=x_2$ cnf(qg1_1, negated_conjecture)
$(x_1 \cdot y_1=z_1$ and $x_2 \cdot y_2=z_1$ and $z_2 \cdot y_1=x_1$ and $z_2 \cdot y_2=x_2) \Rightarrow y_1=y_2$ cnf(qg1_2, negated_conjecture)

GRP123-2.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)

$\text{next}(e_2, e_3)$ cnf(e_2_then_e$_3$, axiom)
$\text{greater}(e_2, e_1)$ cnf(e_2_greater_e$_1$, axiom)
$\text{greater}(e_3, e_1)$ cnf(e_3_greater_e$_1$, axiom)
$\text{greater}(e_3, e_2)$ cnf(e_3_greater_e$_2$, axiom)
$(x \cdot e_1{=}y \text{ and } \text{next}(x, x_1)) \Rightarrow \neg\,\text{greater}(y, x_1)$ cnf(no_redundancy, axiom)
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y{=}e_1 \text{ or } x{\cdot}y{=}e_2 \text{ or } x{\cdot}y{=}e_3)$ cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w \text{ and } x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y \text{ and } x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x \text{ and } z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP123-3.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
$\text{next}(e_0, e_1)$ cnf(e_0_then_e$_1$, axiom)
$\text{next}(e_1, e_2)$ cnf(e_1_then_e$_2$, axiom)
$\text{next}(e_2, e_3)$ cnf(e_2_then_e$_3$, axiom)
$\text{greater}(e_1, e_0)$ cnf(e_1_greater_e$_0$, axiom)
$\text{greater}(e_2, e_0)$ cnf(e_2_greater_e$_0$, axiom)
$\text{greater}(e_3, e_0)$ cnf(e_3_greater_e$_0$, axiom)
$\text{greater}(e_2, e_1)$ cnf(e_2_greater_e$_1$, axiom)
$\text{greater}(e_3, e_1)$ cnf(e_3_greater_e$_1$, axiom)
$\text{greater}(e_3, e_2)$ cnf(e_3_greater_e$_2$, axiom)
$(\text{cycle}(x, y) \text{ and } \text{cycle}(x, z)) \Rightarrow y{=}z$ cnf(cycle$_1$, axiom)
$\text{group\_element}(x) \Rightarrow (\text{cycle}(x, e_0) \text{ or } \text{cycle}(x, e_1) \text{ or } \text{cycle}(x, e_2))$ cnf(cycle$_2$, axiom)
$\text{cycle}(e_3, e_0)$ cnf(cycle$_3$, axiom)
$(\text{cycle}(x, y) \text{ and } \text{cycle}(w, z) \text{ and } \text{next}(x, w) \text{ and } \text{greater}(y, e_0) \text{ and } \text{next}(z, z_1)) \Rightarrow y{=}z_1$ cnf(cycle$_4$, axiom)
$(\text{cycle}(x, z_1) \text{ and } \text{cycle}(y, e_0) \text{ and } \text{cycle}(w, z_2) \text{ and } \text{next}(y, w) \text{ and } \text{greater}(y, x)) \Rightarrow \neg\,\text{greater}(z_1, z_2)$ cnf(cycle$_5$, axiom)
$(\text{cycle}(x, e_0) \text{ and } x \cdot e_1{=}y) \Rightarrow \neg\,\text{greater}(y, x)$ cnf(cycle$_6$, axiom)
$(\text{cycle}(x, y) \text{ and } x \cdot e_1{=}z \text{ and } \text{greater}(y, e_0) \text{ and } \text{next}(x, x_1)) \Rightarrow z{=}x_1$ cnf(cycle$_7$, axiom)
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y{=}e_1 \text{ or } x{\cdot}y{=}e_2 \text{ or } x{\cdot}y{=}e_3)$ cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w \text{ and } x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y \text{ and } x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x \text{ and } z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP123-4.003.p (3,2,1) conjugate orthogonality

If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.

$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (e_1 \cdot x{=}y \text{ or } e_2 \cdot x{=}y \text{ or } e_3 \cdot x{=}y)$ cnf(row_surjectivity, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x \cdot e_1{=}y \text{ or } x \cdot e_2{=}y \text{ or } x \cdot e_3{=}y)$ cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y{=}e_1 \text{ or } x{\cdot}y{=}e_2 \text{ or } x{\cdot}y{=}e_3)$ cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w \text{ and } x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y \text{ and } x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x \text{ and } z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP123-4.004.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.

$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (e_1{\cdot}x{=}y \text{ or } e_2{\cdot}x{=}y \text{ or } e_3{\cdot}x{=}y \text{ or } e_4{\cdot}x{=}y)$ cnf(row_surjectivity, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}e_1{=}y \text{ or } x{\cdot}e_2{=}y \text{ or } x{\cdot}e_3{=}y \text{ or } x{\cdot}e_4{=}y)$ cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y{=}e_1 \text{ or } x{\cdot}y{=}e_2 \text{ or } x{\cdot}y{=}e_3 \text{ or } x{\cdot}y{=}e_4)$ cnf(product_total_function$_1$, axiom
$(x \cdot y{=}w \text{ and } x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y \text{ and } x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x \text{ and } z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1 \text{ and } x_2 \cdot y_2{=}z_1 \text{ and } z_2 \cdot y_1{=}x_1 \text{ and } z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP123-6.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c Generate the multiplication table for the specified quasi- group with 3 elements.

group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)

$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1(x, y, e_1)$ or product$_1(x, y, e_2)$ or product$_1(x, y, e_3)$) cnf(product1
(product$_1(x, y, w)$ and product$_1(x, y, z)$) $\Rightarrow$ $w = z$ cnf(product1_total_function$_2$, axiom)
(product$_1(x, w, y)$ and product$_1(x, z, y)$) $\Rightarrow$ $w = z$ cnf(product1_right_cancellation, axiom)
(product$_1(w, y, x)$ and product$_1(z, y, x)$) $\Rightarrow$ $w = z$ cnf(product1_left_cancellation, axiom)
product$_1(x, x, x)$ cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2(x, y, e_1)$ or product$_2(x, y, e_2)$ or product$_2(x, y, e_3)$) cnf(product2
(product$_2(x, y, w)$ and product$_2(x, y, z)$) $\Rightarrow$ $w = z$ cnf(product2_total_function$_2$, axiom)
(product$_2(x, w, y)$ and product$_2(x, z, y)$) $\Rightarrow$ $w = z$ cnf(product2_right_cancellation, axiom)
(product$_2(w, y, x)$ and product$_2(z, y, x)$) $\Rightarrow$ $w = z$ cnf(product2_left_cancellation, axiom)
product$_2(x, x, x)$ cnf(product2_idempotence, axiom)
(product$_1(x, y, z_1)$ and product$_1(z_1, y, z_2)$) $\Rightarrow$ product$_2(z_2, x, y)$ cnf(qg1a, negated_conjecture)

GRP123-6.005.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1(x, y, e_1)$ or product$_1(x, y, e_2)$ or product$_1(x, y, e_3)$ or product$_1(x, y,$ e
(product$_1(x, y, w)$ and product$_1(x, y, z)$) $\Rightarrow$ $w = z$ cnf(product1_total_function$_2$, axiom)
(product$_1(x, w, y)$ and product$_1(x, z, y)$) $\Rightarrow$ $w = z$ cnf(product1_right_cancellation, axiom)
(product$_1(w, y, x)$ and product$_1(z, y, x)$) $\Rightarrow$ $w = z$ cnf(product1_left_cancellation, axiom)
product$_1(x, x, x)$ cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2(x, y, e_1)$ or product$_2(x, y, e_2)$ or product$_2(x, y, e_3)$ or product$_2(x, y,$ e
(product$_2(x, y, w)$ and product$_2(x, y, z)$) $\Rightarrow$ $w = z$ cnf(product2_total_function$_2$, axiom)
(product$_2(x, w, y)$ and product$_2(x, z, y)$) $\Rightarrow$ $w = z$ cnf(product2_right_cancellation, axiom)
(product$_2(w, y, x)$ and product$_2(z, y, x)$) $\Rightarrow$ $w = z$ cnf(product2_left_cancellation, axiom)
product$_2(x, x, x)$ cnf(product2_idempotence, axiom)
(product$_1(x, y, z_1)$ and product$_1(z_1, y, z_2)$) $\Rightarrow$ product$_2(z_2, x, y)$ cnf(qg1a, negated_conjecture)

GRP123-7.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)

greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x, y, e_1) or product$_1$(x, y, e_2) or product$_1$(x, y, e_3)) cnf(product1
(product$_1$(x, y, w) and product$_1$(x, y, z)) $\Rightarrow$ $w{=}z$ cnf(product1_total_function$_2$, axiom)
(product$_1$(x, w, y) and product$_1$(x, z, y)) $\Rightarrow$ $w{=}z$ cnf(product1_right_cancellation, axiom)
(product$_1$(w, y, x) and product$_1$(z, y, x)) $\Rightarrow$ $w{=}z$ cnf(product1_left_cancellation, axiom)
product$_1$(x, x, x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x, y, e_1) or product$_2$(x, y, e_2) or product$_2$(x, y, e_3)) cnf(product2
(product$_2$(x, y, w) and product$_2$(x, y, z)) $\Rightarrow$ $w{=}z$ cnf(product2_total_function$_2$, axiom)
(product$_2$(x, w, y) and product$_2$(x, z, y)) $\Rightarrow$ $w{=}z$ cnf(product2_right_cancellation, axiom)
(product$_2$(w, y, x) and product$_2$(z, y, x)) $\Rightarrow$ $w{=}z$ cnf(product2_left_cancellation, axiom)
product$_2$(x, x, x) cnf(product2_idempotence, axiom)
(product$_1$(x, y, z_1) and product$_1$(z_1, y, z_2)) $\Rightarrow$ product$_2$(z_2, x, y) cnf(qg1a, negated_conjecture)

GRP123-8.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_0, e_1) cnf(e_0_then_e$_1$, axiom)
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
greater(e_1, e_0) cnf(e_1_greater_e$_0$, axiom)
greater(e_2, e_0) cnf(e_2_greater_e$_0$, axiom)
greater(e_3, e_0) cnf(e_3_greater_e$_0$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y{=}z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2)) cnf(cycle$_2$, axiom)
cycle(e_3, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y{=}z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1{=}y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x, y, e_1) or product$_1$(x, y, e_2) or product$_1$(x, y, e_3)) cnf(product1
(product$_1$(x, y, w) and product$_1$(x, y, z)) $\Rightarrow$ $w{=}z$ cnf(product1_total_function$_2$, axiom)
(product$_1$(x, w, y) and product$_1$(x, z, y)) $\Rightarrow$ $w{=}z$ cnf(product1_right_cancellation, axiom)
(product$_1$(w, y, x) and product$_1$(z, y, x)) $\Rightarrow$ $w{=}z$ cnf(product1_left_cancellation, axiom)
product$_1$(x, x, x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x, y, e_1) or product$_2$(x, y, e_2) or product$_2$(x, y, e_3)) cnf(product2
(product$_2$(x, y, w) and product$_2$(x, y, z)) $\Rightarrow$ $w{=}z$ cnf(product2_total_function$_2$, axiom)
(product$_2$(x, w, y) and product$_2$(x, z, y)) $\Rightarrow$ $w{=}z$ cnf(product2_right_cancellation, axiom)

(product$_2$(w, y, x) and product$_2$(z, y, x)) $\Rightarrow$ w=z cnf(product2_left_cancellation, axiom)
product$_2$(x, x, x) cnf(product2_idempotence, axiom)
(product$_1$(x, y, z_1) and product$_1$(z_1, y, z_2)) $\Rightarrow$ product$_2$(z_2, x, y) cnf(qg1a, negated_conjecture)

GRP123-9.003.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 3 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x$=y or $e_2 \cdot x$=y or $e_3 \cdot x$=y) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1$=y or $x \cdot e_2$=y or $x \cdot e_3$=y) cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg$ e_1=e_2 cnf(e_1_is_not_e$_2$, axiom)
$\neg$ e_1=e_3 cnf(e_1_is_not_e$_3$, axiom)
$\neg$ e_2=e_1 cnf(e_2_is_not_e$_1$, axiom)
$\neg$ e_2=e_3 cnf(e_2_is_not_e$_3$, axiom)
$\neg$ e_3=e_1 cnf(e_3_is_not_e$_1$, axiom)
$\neg$ e_3=e_2 cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x, y, e_1) or product$_1$(x, y, e_2) or product$_1$(x, y, e_3)) cnf(product1
(product$_1$(x, y, w) and product$_1$(x, y, z)) $\Rightarrow$ w=z cnf(product1_total_function$_2$, axiom)
(product$_1$(x, w, y) and product$_1$(x, z, y)) $\Rightarrow$ w=z cnf(product1_right_cancellation, axiom)
(product$_1$(w, y, x) and product$_1$(z, y, x)) $\Rightarrow$ w=z cnf(product1_left_cancellation, axiom)
product$_1$(x, x, x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x, y, e_1) or product$_2$(x, y, e_2) or product$_2$(x, y, e_3)) cnf(product2
(product$_2$(x, y, w) and product$_2$(x, y, z)) $\Rightarrow$ w=z cnf(product2_total_function$_2$, axiom)
(product$_2$(x, w, y) and product$_2$(x, z, y)) $\Rightarrow$ w=z cnf(product2_right_cancellation, axiom)
(product$_2$(w, y, x) and product$_2$(z, y, x)) $\Rightarrow$ w=z cnf(product2_left_cancellation, axiom)
product$_2$(x, x, x) cnf(product2_idempotence, axiom)
(product$_1$(x, y, z_1) and product$_1$(z_1, y, z_2)) $\Rightarrow$ product$_2$(z_2, x, y) cnf(qg1a, negated_conjecture)

GRP123-9.004.p (3,2,1) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*b=a iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x$=y or $e_2 \cdot x$=y or $e_3 \cdot x$=y or $e_4 \cdot x$=y) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1$=y or $x \cdot e_2$=y or $x \cdot e_3$=y or $x \cdot e_4$=y) cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg$ e_1=e_2 cnf(e_1_is_not_e$_2$, axiom)
$\neg$ e_1=e_3 cnf(e_1_is_not_e$_3$, axiom)
$\neg$ e_1=e_4 cnf(e_1_is_not_e$_4$, axiom)
$\neg$ e_2=e_1 cnf(e_2_is_not_e$_1$, axiom)
$\neg$ e_2=e_3 cnf(e_2_is_not_e$_3$, axiom)
$\neg$ e_2=e_4 cnf(e_2_is_not_e$_4$, axiom)
$\neg$ e_3=e_1 cnf(e_3_is_not_e$_1$, axiom)
$\neg$ e_3=e_2 cnf(e_3_is_not_e$_2$, axiom)
$\neg$ e_3=e_4 cnf(e_3_is_not_e$_4$, axiom)
$\neg$ e_4=e_1 cnf(e_4_is_not_e$_1$, axiom)
$\neg$ e_4=e_2 cnf(e_4_is_not_e$_2$, axiom)
$\neg$ e_4=e_3 cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x, y, e_1) or product$_1$(x, y, e_2) or product$_1$(x, y, e_3) or product$_1$($x, y,$
(product$_1$(x, y, w) and product$_1$(x, y, z)) $\Rightarrow$ w=z cnf(product1_total_function$_2$, axiom)
(product$_1$(x, w, y) and product$_1$(x, z, y)) $\Rightarrow$ w=z cnf(product1_right_cancellation, axiom)
(product$_1$(w, y, x) and product$_1$(z, y, x)) $\Rightarrow$ w=z cnf(product1_left_cancellation, axiom)
product$_1$(x, x, x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x, y, e_1) or product$_2$(x, y, e_2) or product$_2$(x, y, e_3) or product$_2$($x, y,$
(product$_2$(x, y, w) and product$_2$(x, y, z)) $\Rightarrow$ w=z cnf(product2_total_function$_2$, axiom)
(product$_2$(x, w, y) and product$_2$(x, z, y)) $\Rightarrow$ w=z cnf(product2_right_cancellation, axiom)
(product$_2$(w, y, x) and product$_2$(z, y, x)) $\Rightarrow$ w=z cnf(product2_left_cancellation, axiom)

$\text{product}_2(x,x,x)$ cnf(product2_idempotence, axiom)
$(\text{product}_1(x,y,z_1)$ and $\text{product}_1(z_1,y,z_2)) \Rightarrow \text{product}_2(z_2,x,y)$ cnf(qg1a, negated_conjecture)

GRP124-1.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow x_1{=}x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow y_1{=}y_2$ cnf(qg2$_2$, negated_conjecture)

GRP124-1.005.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)

$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow x_1{=}x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow y_1{=}y_2$ cnf(qg2$_2$, negated_conjecture)

GRP124-2.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
$(x \cdot e_1{=}y$ and next$(x, x_1)) \Rightarrow \neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element$(y)) \Rightarrow (x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4)$ cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow x_1{=}x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2) \Rightarrow y_1{=}y_2$ cnf(qg2$_2$, negated_conjecture)

GRP124-4.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element$(y)) \Rightarrow (e_1{\cdot}x{=}y$ or $e_2{\cdot}x{=}y$ or $e_3{\cdot}x{=}y$ or $e_4{\cdot}x{=}y)$ cnf(row_surjectivity, axiom)
(group_element(x) and group_element$(y)) \Rightarrow (x{\cdot}e_1{=}y$ or $x{\cdot}e_2{=}y$ or $x{\cdot}e_3{=}y$ or $x{\cdot}e_4{=}y)$ cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)

$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2$) $\Rightarrow$ $x_1{=}x_2$ cnf(qg2$_1$, negated_conjecture)
($x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2$) $\Rightarrow$ $y_1{=}y_2$ cnf(qg2$_2$, negated_conjecture)

GRP124-4.005.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 5 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x{=}y$ or $e_2{\cdot}x{=}y$ or $e_3{\cdot}x{=}y$ or $e_4{\cdot}x{=}y$ or $e_5{\cdot}x{=}y$) cnf(row_surjectivity, ax
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1{=}y$ or $x{\cdot}e_2{=}y$ or $x{\cdot}e_3{=}y$ or $x{\cdot}e_4{=}y$ or $x{\cdot}e_5{=}y$) cnf(column_surjectivity
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2$) $\Rightarrow$ $x_1{=}x_2$ cnf(qg2$_1$, negated_conjecture)
($x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot x_1{=}y_1$ and $z_2 \cdot x_2{=}y_2$) $\Rightarrow$ $y_1{=}y_2$ cnf(qg2$_2$, negated_conjecture)

GRP124-6.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)

$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1(x, y, e_1)$ or product$_1(x, y, e_2)$ or product$_1(x, y, e_3)$ or product$_1(x, y,$
(product$_1(x, y, w)$ and product$_1(x, y, z)$) $\Rightarrow$ $w{=}z$ cnf(product1_total_function$_2$, axiom)
(product$_1(x, w, y)$ and product$_1(x, z, y)$) $\Rightarrow$ $w{=}z$ cnf(product1_right_cancellation, axiom)
(product$_1(w, y, x)$ and product$_1(z, y, x)$) $\Rightarrow$ $w{=}z$ cnf(product1_left_cancellation, axiom)
product$_1(x, x, x)$ cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2(x, y, e_1)$ or product$_2(x, y, e_2)$ or product$_2(x, y, e_3)$ or product$_2(x, y,$
(product$_2(x, y, w)$ and product$_2(x, y, z)$) $\Rightarrow$ $w{=}z$ cnf(product2_total_function$_2$, axiom)
(product$_2(x, w, y)$ and product$_2(x, z, y)$) $\Rightarrow$ $w{=}z$ cnf(product2_right_cancellation, axiom)
(product$_2(w, y, x)$ and product$_2(z, y, x)$) $\Rightarrow$ $w{=}z$ cnf(product2_left_cancellation, axiom)
product$_2(x, x, x)$ cnf(product2_idempotence, axiom)
(product$_1(x, y, z_1)$ and product$_1(z_1, x, z_2)$) $\Rightarrow$ product$_2(z_2, y, x)$ cnf(qg2a, negated_conjecture)

GRP124-6.005.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1(x, y, e_1)$ or product$_1(x, y, e_2)$ or product$_1(x, y, e_3)$ or product$_1(x, y,$
(product$_1(x, y, w)$ and product$_1(x, y, z)$) $\Rightarrow$ $w{=}z$ cnf(product1_total_function$_2$, axiom)
(product$_1(x, w, y)$ and product$_1(x, z, y)$) $\Rightarrow$ $w{=}z$ cnf(product1_right_cancellation, axiom)
(product$_1(w, y, x)$ and product$_1(z, y, x)$) $\Rightarrow$ $w{=}z$ cnf(product1_left_cancellation, axiom)
product$_1(x, x, x)$ cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2(x, y, e_1)$ or product$_2(x, y, e_2)$ or product$_2(x, y, e_3)$ or product$_2(x, y,$
(product$_2(x, y, w)$ and product$_2(x, y, z)$) $\Rightarrow$ $w{=}z$ cnf(product2_total_function$_2$, axiom)
(product$_2(x, w, y)$ and product$_2(x, z, y)$) $\Rightarrow$ $w{=}z$ cnf(product2_right_cancellation, axiom)
(product$_2(w, y, x)$ and product$_2(z, y, x)$) $\Rightarrow$ $w{=}z$ cnf(product2_left_cancellation, axiom)
product$_2(x, x, x)$ cnf(product2_idempotence, axiom)
(product$_1(x, y, z_1)$ and product$_1(z_1, x, z_2)$) $\Rightarrow$ product$_2(z_2, y, x)$ cnf(qg2a, negated_conjecture)

GRP124-7.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.

next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
($x \cdot e_1 = y$ and next(x, x_1)) $\Rightarrow$ ¬ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
group_element(e_4) cnf(element_4, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x, y, e_1) or product$_1$(x, y, e_2) or product$_1$(x, y, e_3) or product$_1$($x, y,$
(product$_1$(x, y, w) and product$_1$(x, y, z)) $\Rightarrow$ $w = z$ cnf(product1_total_function_2, axiom)
(product$_1$(x, w, y) and product$_1$(x, z, y)) $\Rightarrow$ $w = z$ cnf(product1_right_cancellation, axiom)
(product$_1$(w, y, x) and product$_1$(z, y, x)) $\Rightarrow$ $w = z$ cnf(product1_left_cancellation, axiom)
product$_1$(x, x, x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x, y, e_1) or product$_2$(x, y, e_2) or product$_2$(x, y, e_3) or product$_2$($x, y,$
(product$_2$(x, y, w) and product$_2$(x, y, z)) $\Rightarrow$ $w = z$ cnf(product2_total_function_2, axiom)
(product$_2$(x, w, y) and product$_2$(x, z, y)) $\Rightarrow$ $w = z$ cnf(product2_right_cancellation, axiom)
(product$_2$(w, y, x) and product$_2$(z, y, x)) $\Rightarrow$ $w = z$ cnf(product2_left_cancellation, axiom)
product$_2$(x, x, x) cnf(product2_idempotence, axiom)
(product$_1$(x, y, z_1) and product$_1$(z_1, x, z_2)) $\Rightarrow$ product$_2$(z_2, y, x) cnf(qg2a, negated_conjecture)

GRP124-9.004.p (3,1,2) conjugate orthogonality
If ab=xy and a*b = x*y then a=x and b=y, where c*a=b iff ab=c. Generate the multiplication table for the specified quasi- group with 4 elements.

(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x = y$ or $e_2 \cdot x = y$ or $e_3 \cdot x = y$ or $e_4 \cdot x = y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1 = y$ or $x \cdot e_2 = y$ or $x \cdot e_3 = y$ or $x \cdot e_4 = y$) cnf(column_surjectivity, axiom)
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
group_element(e_4) cnf(element_4, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e_1, axiom)

$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_1$(x,y,e_1) or product$_1$(x,y,e_2) or product$_1$(x,y,e_3) or product$_1$($x,y,$
(product$_1$(x,y,w) and product$_1$(x,y,z)) $\Rightarrow$ $w{=}z$ cnf(product1_total_function$_2$, axiom)
(product$_1$(x,w,y) and product$_1$(x,z,y)) $\Rightarrow$ $w{=}z$ cnf(product1_right_cancellation, axiom)
(product$_1$(w,y,x) and product$_1$(z,y,x)) $\Rightarrow$ $w{=}z$ cnf(product1_left_cancellation, axiom)
product$_1$(x,x,x) cnf(product1_idempotence, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ (product$_2$(x,y,e_1) or product$_2$(x,y,e_2) or product$_2$(x,y,e_3) or product$_2$($x,y,$
(product$_2$(x,y,w) and product$_2$(x,y,z)) $\Rightarrow$ $w{=}z$ cnf(product2_total_function$_2$, axiom)
(product$_2$(x,w,y) and product$_2$(x,z,y)) $\Rightarrow$ $w{=}z$ cnf(product2_right_cancellation, axiom)
(product$_2$(w,y,x) and product$_2$(z,y,x)) $\Rightarrow$ $w{=}z$ cnf(product2_left_cancellation, axiom)
product$_2$(x,x,x) cnf(product2_idempotence, axiom)
(product$_1$(x,y,z_1) and product$_1$(z_1,x,z_2)) $\Rightarrow$ product$_2$(z_2,y,x) cnf(qg2a, negated_conjecture)

GRP125-1.003.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}x$ cnf(qg$_3$, negated_conjecture)

GRP125-1.004.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}x$ cnf(qg$_3$, negated_conjecture)

GRP125-2.004.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)

next(e_3, e_4) cnf(e_3_then_e_4, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
($x \cdot e_1$=y and next(x, x_1)) $\Rightarrow$ ¬ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
($x \cdot y = z_1$ and $y \cdot x = z_2$) $\Rightarrow$ $z_1 \cdot z_2 = x$ cnf(qg$_3$, negated_conjecture)

GRP125-2.005.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
next(e_4, e_5) cnf(e_4_then_e_5, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_5, e_1) cnf(e_5_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_5, e_2) cnf(e_5_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
greater(e_5, e_3) cnf(e_5_greater_e_3, axiom)
greater(e_5, e_4) cnf(e_5_greater_e_4, axiom)
($x \cdot e_1$=y and next(x, x_1)) $\Rightarrow$ ¬ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg e_1 = e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e_4, axiom)

$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}x$ cnf(qg$_3$, negated_conjecture)

GRP125-3.004.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e_1, axiom)
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
greater(e_1, e_0) cnf(e_1_greater_e_0, axiom)
greater(e_2, e_0) cnf(e_2_greater_e_0, axiom)
greater(e_3, e_0) cnf(e_3_greater_e_0, axiom)
greater(e_4, e_0) cnf(e_4_greater_e_0, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y{=}z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle$_2$, axiom)
cycle(e_4, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y{=}z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1{=}y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio

$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
$(x \cdot y = z_1 \text{ and } y \cdot x = z_2) \Rightarrow z_1 \cdot z_2 = x$ cnf(qg$_3$, negated_conjecture)

GRP125-4.003.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 3 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x = y$ or $e_2 \cdot x = y$ or $e_3 \cdot x = y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1 = y$ or $x \cdot e_2 = y$ or $x \cdot e_3 = y$) cnf(column_surjectivity, axiom)
$(z_1 \cdot z_2 = x \text{ and } y \cdot x = z_2) \Rightarrow x \cdot y = z_1$ cnf(qg3$_1$, negated_conjecture)
$(z_1 \cdot z_2 = x \text{ and } x \cdot y = z_1) \Rightarrow y \cdot x = z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$) cnf(product_total_function$_1$, axiom)
$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
$(x \cdot y = z_1 \text{ and } y \cdot x = z_2) \Rightarrow z_1 \cdot z_2 = x$ cnf(qg$_3$, negated_conjecture)

GRP125-4.004.p (a.b).(b.a) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x = y$ or $e_2 \cdot x = y$ or $e_3 \cdot x = y$ or $e_4 \cdot x = y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1 = y$ or $x \cdot e_2 = y$ or $x \cdot e_3 = y$ or $x \cdot e_4 = y$) cnf(column_surjectivity, axiom)
$(z_1 \cdot z_2 = x \text{ and } y \cdot x = z_2) \Rightarrow x \cdot y = z_1$ cnf(qg3$_1$, negated_conjecture)
$(z_1 \cdot z_2 = x \text{ and } x \cdot y = z_1) \Rightarrow y \cdot x = z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
$(x \cdot y = z_1 \text{ and } y \cdot x = z_2) \Rightarrow z_1 \cdot z_2 = x$ cnf(qg$_3$, negated_conjecture)

GRP126-1.004.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)

group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP126-1.005.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP126-2.004.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)

greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
$(x \cdot e_1{=}y$ and next$(x, x_1)) \Rightarrow \neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
$(x \cdot y{=}z_1$ and $y \cdot x{=}z_2) \Rightarrow z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP126-2.005.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
next(e_4, e_5) cnf(e_4_then_e$_5$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_5, e_1) cnf(e_5_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_5, e_2) cnf(e_5_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
greater(e_5, e_3) cnf(e_5_greater_e$_3$, axiom)
greater(e_5, e_4) cnf(e_5_greater_e$_4$, axiom)
$(x \cdot e_1{=}y$ and next$(x, x_1)) \Rightarrow \neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)

$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP126-3.004.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e_1, axiom)
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
greater(e_1, e_0) cnf(e_1_greater_e_0, axiom)
greater(e_2, e_0) cnf(e_2_greater_e_0, axiom)
greater(e_3, e_0) cnf(e_3_greater_e_0, axiom)
greater(e_4, e_0) cnf(e_4_greater_e_0, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y{=}z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle$_2$, axiom)
cycle(e_4, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y{=}z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1{=}y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axior
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)

$(x \cdot w=y$ and $x \cdot z=y) \Rightarrow w=z$ cnf(product_right_cancellation, axiom)
$(w \cdot y=x$ and $z \cdot y=x) \Rightarrow w=z$ cnf(product_left_cancellation, axiom)
$x \cdot x=x$ cnf(product_idempotence, axiom)
$(x \cdot y=z_1$ and $y \cdot x=z_2) \Rightarrow z_1 \cdot z_2=y$ cnf(qg$_4$, negated_conjecture)

GRP126-4.004.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x=y$ or $e_2 \cdot x=y$ or $e_3 \cdot x=y$ or $e_4 \cdot x=y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1=y$ or $x \cdot e_2=y$ or $x \cdot e_3=y$ or $x \cdot e_4=y$) cnf(column_surjectivity, axiom)
$(z_1 \cdot z_2=y$ and $y \cdot x=z_2) \Rightarrow x \cdot y=z_1$ cnf(qg4$_1$, negated_conjecture)
$(z_1 \cdot z_2=y$ and $x \cdot y=z_1) \Rightarrow y \cdot x=z_2$ cnf(qg4$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1=e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2=e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3=e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_4=e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4=e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4=e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y=e_1$ or $x \cdot y=e_2$ or $x \cdot y=e_3$ or $x \cdot y=e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y=w$ and $x \cdot y=z) \Rightarrow w=z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w=y$ and $x \cdot z=y) \Rightarrow w=z$ cnf(product_right_cancellation, axiom)
$(w \cdot y=x$ and $z \cdot y=x) \Rightarrow w=z$ cnf(product_left_cancellation, axiom)
$x \cdot x=x$ cnf(product_idempotence, axiom)
$(x \cdot y=z_1$ and $y \cdot x=z_2) \Rightarrow z_1 \cdot z_2=y$ cnf(qg$_4$, negated_conjecture)

GRP126-4.005.p (a.b).(b.a) = b
Generate the multiplication table for the specified quasi- group with 5 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x=y$ or $e_2 \cdot x=y$ or $e_3 \cdot x=y$ or $e_4 \cdot x=y$ or $e_5 \cdot x=y$) cnf(row_surjectivity, ax
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1=y$ or $x \cdot e_2=y$ or $x \cdot e_3=y$ or $x \cdot e_4=y$ or $x \cdot e_5=y$) cnf(column_surjectivity
$(z_1 \cdot z_2=y$ and $y \cdot x=z_2) \Rightarrow x \cdot y=z_1$ cnf(qg4$_1$, negated_conjecture)
$(z_1 \cdot z_2=y$ and $x \cdot y=z_1) \Rightarrow y \cdot x=z_2$ cnf(qg4$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1=e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_1=e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2=e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_2=e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3=e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_3=e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg e_4=e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4=e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4=e_3$ cnf(e_4_is_not_e$_3$, axiom)

$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP127-1.004.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP127-1.005.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)

$\neg e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$ or $x \cdot y = e_5$) cnf(product_total_func
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
($y \cdot x = z_1$ and $z_1 \cdot y = z_2$) $\Rightarrow$ $z_2 \cdot y = x$ cnf(qg$_3$, negated_conjecture)

GRP127-2.005.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
next(e_4, e_5) cnf(e_4_then_e$_5$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_5, e_1) cnf(e_5_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_5, e_2) cnf(e_5_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
greater(e_5, e_3) cnf(e_5_greater_e$_3$, axiom)
greater(e_5, e_4) cnf(e_5_greater_e$_4$, axiom)
($x \cdot e_1 = y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$ or $x \cdot y = e_5$) cnf(product_total_func
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
$x \cdot x = x$ cnf(product_idempotence, axiom)
($y \cdot x = z_1$ and $z_1 \cdot y = z_2$) $\Rightarrow$ $z_2 \cdot y = x$ cnf(qg$_3$, negated_conjecture)

GRP127-3.004.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e_1, axiom)
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
greater(e_1, e_0) cnf(e_1_greater_e_0, axiom)
greater(e_2, e_0) cnf(e_2_greater_e_0, axiom)
greater(e_3, e_0) cnf(e_3_greater_e_0, axiom)
greater(e_4, e_0) cnf(e_4_greater_e_0, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y=z$ cnf(cycle_1, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle_2, axiom)
cycle(e_4, e_0) cnf(cycle_3, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y=z_1$ cnf(cycle_4, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle_5, axiom)
(cycle(x, e_0) and $x \cdot e_1=y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle_6, axiom)
(cycle(x, y) and $x \cdot e_1=z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z=x_1$ cnf(cycle_7, axiom)
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
group_element(e_4) cnf(element_4, axiom)
$\neg\, e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1=e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2=e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2=e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2=e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3=e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3=e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3=e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4=e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4=e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4=e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$ or $x{\cdot}y=e_4$) cnf(product_total_function$_1$, axio
($x \cdot y=w$ and $x \cdot y=z$) $\Rightarrow$ $w=z$ cnf(product_total_function$_2$, axiom)
($x \cdot w=y$ and $x \cdot z=y$) $\Rightarrow$ $w=z$ cnf(product_right_cancellation, axiom)
($w \cdot y=x$ and $z \cdot y=x$) $\Rightarrow$ $w=z$ cnf(product_left_cancellation, axiom)
$x \cdot x=x$ cnf(product_idempotence, axiom)
($y \cdot x=z_1$ and $z_1 \cdot y=z_2$) $\Rightarrow$ $z_2 \cdot y=x$ cnf(qg_3, negated_conjecture)

GRP127-4.004.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x=y$ or $e_2{\cdot}x=y$ or $e_3{\cdot}x=y$ or $e_4{\cdot}x=y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1=y$ or $x{\cdot}e_2=y$ or $x{\cdot}e_3=y$ or $x{\cdot}e_4=y$) cnf(column_surjectivity, axiom)
($z_2 \cdot y=x$ and $z_1 \cdot y=z_2$) $\Rightarrow$ $y \cdot x=z_1$ cnf(qg3_1, negated_conjecture)
($z_2 \cdot y=x$ and $y \cdot x=z_1$) $\Rightarrow$ $z_1 \cdot y=z_2$ cnf(qg3_2, negated_conjecture)
group_element(e_1) cnf(element_1, axiom)
group_element(e_2) cnf(element_2, axiom)
group_element(e_3) cnf(element_3, axiom)
group_element(e_4) cnf(element_4, axiom)
$\neg\, e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1=e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2=e_1$ cnf(e_2_is_not_e_1, axiom)

$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axior
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP127-4.005.p ((b.a).b).b) = a
Generate the multiplication table for the specified quasi- group with 5 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x{=}y$ or $e_2{\cdot}x{=}y$ or $e_3{\cdot}x{=}y$ or $e_4{\cdot}x{=}y$ or $e_5{\cdot}x{=}y$) cnf(row_surjectivity, ax
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1{=}y$ or $x{\cdot}e_2{=}y$ or $x{\cdot}e_3{=}y$ or $x{\cdot}e_4{=}y$ or $x{\cdot}e_5{=}y$) cnf(column_surjectivity
($z_2 \cdot y{=}x$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $y \cdot x{=}z_1$ cnf(qg3$_1$, negated_conjecture)
($z_2 \cdot y{=}x$ and $y \cdot x{=}z_1$) $\Rightarrow$ $z_1 \cdot y{=}z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
$x \cdot x{=}x$ cnf(product_idempotence, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP128-1.003.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)

$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $x \cdot z_1{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP128-1.004.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $x \cdot z_1{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP128-2.004.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)

$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(x \cdot y{=}z_1$ and $z_1 \cdot y{=}z_2) \Rightarrow x \cdot z_1{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP128-3.004.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e$_1$, axiom)
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_1, e_0) cnf(e_1_greater_e$_0$, axiom)
greater(e_2, e_0) cnf(e_2_greater_e$_0$, axiom)
greater(e_3, e_0) cnf(e_3_greater_e$_0$, axiom)
greater(e_4, e_0) cnf(e_4_greater_e$_0$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow y{=}z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle$_2$, axiom)
cycle(e_4, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow y{=}z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow \neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1{=}y$) $\Rightarrow \neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(x \cdot y{=}z_1$ and $z_1 \cdot y{=}z_2) \Rightarrow x \cdot z_1{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP128-4.003.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 3 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x{=}y$ or $e_2 \cdot x{=}y$ or $e_3 \cdot x{=}y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1{=}y$ or $x \cdot e_2{=}y$ or $x \cdot e_3{=}y$) cnf(column_surjectivity, axiom)
$(x \cdot z_1{=}z_2$ and $z_1 \cdot y{=}z_2) \Rightarrow x \cdot y{=}z_1$ cnf(qg3$_1$, negated_conjecture)
$(x \cdot z_1{=}z_2$ and $x \cdot y{=}z_1) \Rightarrow z_1 \cdot y{=}z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)

group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y = e_1$ or $x{\cdot}y = e_2$ or $x{\cdot}y = e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($x \cdot y = z_1$ and $z_1 \cdot y = z_2$) $\Rightarrow$ $x \cdot z_1 = z_2$ cnf(qg$_3$, negated_conjecture)

GRP128-4.004.p (a.b).b = a.(a.b)
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x = y$ or $e_2{\cdot}x = y$ or $e_3{\cdot}x = y$ or $e_4{\cdot}x = y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1 = y$ or $x{\cdot}e_2 = y$ or $x{\cdot}e_3 = y$ or $x{\cdot}e_4 = y$) cnf(column_surjectivity, axiom)
($x \cdot z_1 = z_2$ and $z_1 \cdot y = z_2$) $\Rightarrow$ $x \cdot y = z_1$ cnf(qg3$_1$, negated_conjecture)
($x \cdot z_1 = z_2$ and $x \cdot y = z_1$) $\Rightarrow$ $z_1 \cdot y = z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y = e_1$ or $x{\cdot}y = e_2$ or $x{\cdot}y = e_3$ or $x{\cdot}y = e_4$) cnf(product_total_function$_1$, axiom
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($x \cdot y = z_1$ and $z_1 \cdot y = z_2$) $\Rightarrow$ $x \cdot z_1 = z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-1.003.p a.(b.a) = (b.a).b
Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y = e_1$ or $x{\cdot}y = e_2$ or $x{\cdot}y = e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($y \cdot x = z_1$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $z_1 \cdot y = z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-1.005.p a.(b.a) = (b.a).b
Generate the multiplication table for the specified quasi- group with 5 elements.

group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$ or $x \cdot y = e_5$) cnf(product_total_func
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($y \cdot x = z_1$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $z_1 \cdot y = z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-2.004.p a.(b.a) = (b.a).a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
($x \cdot e_1 = y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$) cnf(product_total_function$_1$, axio

$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(y \cdot x{=}z_1$ and $x \cdot z_1{=}z_2) \Rightarrow z_1 \cdot y{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-2.005.p a.(b.a) = (b.a).a
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
next(e_4, e_5) cnf(e_4_then_e$_5$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_5, e_1) cnf(e_5_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_5, e_2) cnf(e_5_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
greater(e_5, e_3) cnf(e_5_greater_e$_3$, axiom)
greater(e_5, e_4) cnf(e_5_greater_e$_4$, axiom)
$(x \cdot e_1{=}y$ and next$(x, x_1)) \Rightarrow \neg$greater$(y, x_1)$ cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ $(x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5)$ cnf(product_total_func
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(y \cdot x{=}z_1$ and $x \cdot z_1{=}z_2) \Rightarrow z_1 \cdot y{=}z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-3.004.p a.(b.a) = (b.a).a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e$_1$, axiom)
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_1, e_0) cnf(e_1_greater_e$_0$, axiom)

greater(e_2, e_0) cnf(e_2_greater_e_0, axiom)
greater(e_3, e_0) cnf(e_3_greater_e_0, axiom)
greater(e_4, e_0) cnf(e_4_greater_e_0, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y=z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle$_2$, axiom)
cycle(e_4, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y=z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1=y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1=z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z=x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg e_1=e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg e_2=e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg e_3=e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg e_4=e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg e_4=e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg e_4=e_3$ cnf(e_4_is_not_e_3, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$ or $x{\cdot}y=e_4$) cnf(product_total_function$_1$, axiom)
($x \cdot y=w$ and $x \cdot y=z$) $\Rightarrow$ $w=z$ cnf(product_total_function$_2$, axiom)
($x \cdot w=y$ and $x \cdot z=y$) $\Rightarrow$ $w=z$ cnf(product_right_cancellation, axiom)
($w \cdot y=x$ and $z \cdot y=x$) $\Rightarrow$ $w=z$ cnf(product_left_cancellation, axiom)
($y \cdot x=z_1$ and $x \cdot z_1=z_2$) $\Rightarrow$ $z_1 \cdot y=z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-4.004.p a.(b.a) = (b.a).a
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x=y$ or $e_2{\cdot}x=y$ or $e_3{\cdot}x=y$ or $e_4{\cdot}x=y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1=y$ or $x{\cdot}e_2=y$ or $x{\cdot}e_3=y$ or $x{\cdot}e_4=y$) cnf(column_surjectivity, axiom)
($z_1 \cdot y=z_2$ and $x \cdot z_1=z_2$) $\Rightarrow$ $y \cdot x=z_1$ cnf(qg3$_1$, negated_conjecture)
($z_1 \cdot y=z_2$ and $y \cdot x=z_1$) $\Rightarrow$ $x \cdot z_1=z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg e_1=e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg e_2=e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg e_3=e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg e_4=e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg e_4=e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg e_4=e_3$ cnf(e_4_is_not_e_3, axiom)

(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($y \cdot x = z_1$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $z_1 \cdot y = z_2$ cnf(qg$_3$, negated_conjecture)

GRP129-4.005.p a.(b.a) = (b.a).a
Generate the multiplication table for the specified quasi- group with 5 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x = y$ or $e_2 \cdot x = y$ or $e_3 \cdot x = y$ or $e_4 \cdot x = y$ or $e_5 \cdot x = y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1 = y$ or $x \cdot e_2 = y$ or $x \cdot e_3 = y$ or $x \cdot e_4 = y$ or $x \cdot e_5 = y$) cnf(column_surjectivity, axiom)
($z_1 \cdot y = z_2$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $y \cdot x = z_1$ cnf(qg3$_1$, negated_conjecture)
($z_1 \cdot y = z_2$ and $y \cdot x = z_1$) $\Rightarrow$ $x \cdot z_1 = z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$ or $x \cdot y = e_5$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($y \cdot x = z_1$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $z_1 \cdot y = z_2$ cnf(qg$_3$, negated_conjecture)

GRP130-1.003.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y = w$ and $x \cdot y = z$) $\Rightarrow$ $w = z$ cnf(product_total_function$_2$, axiom)
($x \cdot w = y$ and $x \cdot z = y$) $\Rightarrow$ $w = z$ cnf(product_right_cancellation, axiom)
($w \cdot y = x$ and $z \cdot y = x$) $\Rightarrow$ $w = z$ cnf(product_left_cancellation, axiom)
($x \cdot y = z_1$ and $x \cdot z_1 = z_2$) $\Rightarrow$ $z_2 \cdot y = x$ cnf(qg$_3$, negated_conjecture)

GRP130-1.005.p (a.(a.b)).b = a

Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP130-2.003.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP130-2.005.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)

$\text{next}(e_4, e_5)$ cnf(e_4_then_e$_5$, axiom)
$\text{greater}(e_2, e_1)$ cnf(e_2_greater_e$_1$, axiom)
$\text{greater}(e_3, e_1)$ cnf(e_3_greater_e$_1$, axiom)
$\text{greater}(e_4, e_1)$ cnf(e_4_greater_e$_1$, axiom)
$\text{greater}(e_5, e_1)$ cnf(e_5_greater_e$_1$, axiom)
$\text{greater}(e_3, e_2)$ cnf(e_3_greater_e$_2$, axiom)
$\text{greater}(e_4, e_2)$ cnf(e_4_greater_e$_2$, axiom)
$\text{greater}(e_5, e_2)$ cnf(e_5_greater_e$_2$, axiom)
$\text{greater}(e_4, e_3)$ cnf(e_4_greater_e$_3$, axiom)
$\text{greater}(e_5, e_3)$ cnf(e_5_greater_e$_3$, axiom)
$\text{greater}(e_5, e_4)$ cnf(e_5_greater_e$_4$, axiom)
$(x \cdot e_1 = y \text{ and } \text{next}(x, x_1)) \Rightarrow \neg\, \text{greater}(y, x_1)$ cnf(no_redundancy, axiom)
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\text{group\_element}(e_4)$ cnf(element$_4$, axiom)
$\text{group\_element}(e_5)$ cnf(element$_5$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x \cdot y = e_1 \text{ or } x \cdot y = e_2 \text{ or } x \cdot y = e_3 \text{ or } x \cdot y = e_4 \text{ or } x \cdot y = e_5)$ cnf(product_total_func
$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x \cdot y = z_1 \text{ and } x \cdot z_1 = z_2) \Rightarrow z_2 \cdot y = x$ cnf(qg$_3$, negated_conjecture)

GRP130-3.003.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 3 elements.
$\text{next}(e_0, e_1)$ cnf(e_0_then_e$_1$, axiom)
$\text{next}(e_1, e_2)$ cnf(e_1_then_e$_2$, axiom)
$\text{next}(e_2, e_3)$ cnf(e_2_then_e$_3$, axiom)
$\text{greater}(e_1, e_0)$ cnf(e_1_greater_e$_0$, axiom)
$\text{greater}(e_2, e_0)$ cnf(e_2_greater_e$_0$, axiom)
$\text{greater}(e_3, e_0)$ cnf(e_3_greater_e$_0$, axiom)
$\text{greater}(e_2, e_1)$ cnf(e_2_greater_e$_1$, axiom)
$\text{greater}(e_3, e_1)$ cnf(e_3_greater_e$_1$, axiom)
$\text{greater}(e_3, e_2)$ cnf(e_3_greater_e$_2$, axiom)
$(\text{cycle}(x, y) \text{ and } \text{cycle}(x, z)) \Rightarrow y = z$ cnf(cycle$_1$, axiom)
$\text{group\_element}(x) \Rightarrow (\text{cycle}(x, e_0) \text{ or } \text{cycle}(x, e_1) \text{ or } \text{cycle}(x, e_2))$ cnf(cycle$_2$, axiom)
$\text{cycle}(e_3, e_0)$ cnf(cycle$_3$, axiom)
$(\text{cycle}(x, y) \text{ and } \text{cycle}(w, z) \text{ and } \text{next}(x, w) \text{ and } \text{greater}(y, e_0) \text{ and } \text{next}(z, z_1)) \Rightarrow y = z_1$ cnf(cycle$_4$, axiom)
$(\text{cycle}(x, z_1) \text{ and } \text{cycle}(y, e_0) \text{ and } \text{cycle}(w, z_2) \text{ and } \text{next}(y, w) \text{ and } \text{greater}(y, x)) \Rightarrow \neg\, \text{greater}(z_1, z_2)$ cnf(cycle$_5$, axiom)
$(\text{cycle}(x, e_0) \text{ and } x \cdot e_1 = y) \Rightarrow \neg\, \text{greater}(y, x)$ cnf(cycle$_6$, axiom)

(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP130-3.004.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_0, e_1) cnf(e_0_then_e$_1$, axiom)
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
next(e_3, e_4) cnf(e_3_then_e$_4$, axiom)
greater(e_1, e_0) cnf(e_1_greater_e$_0$, axiom)
greater(e_2, e_0) cnf(e_2_greater_e$_0$, axiom)
greater(e_3, e_0) cnf(e_3_greater_e$_0$, axiom)
greater(e_4, e_0) cnf(e_4_greater_e$_0$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_4, e_1) cnf(e_4_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
greater(e_4, e_2) cnf(e_4_greater_e$_2$, axiom)
greater(e_4, e_3) cnf(e_4_greater_e$_3$, axiom)
(cycle(x, y) and cycle(x, z)) $\Rightarrow$ $y{=}z$ cnf(cycle$_1$, axiom)
group_element(x) $\Rightarrow$ (cycle(x, e_0) or cycle(x, e_1) or cycle(x, e_2) or cycle(x, e_3)) cnf(cycle$_2$, axiom)
cycle(e_4, e_0) cnf(cycle$_3$, axiom)
(cycle(x, y) and cycle(w, z) and next(x, w) and greater(y, e_0) and next(z, z_1)) $\Rightarrow$ $y{=}z_1$ cnf(cycle$_4$, axiom)
(cycle(x, z_1) and cycle(y, e_0) and cycle(w, z_2) and next(y, w) and greater(y, x)) $\Rightarrow$ $\neg$ greater(z_1, z_2) cnf(cycle$_5$, axiom)
(cycle(x, e_0) and $x \cdot e_1{=}y$) $\Rightarrow$ $\neg$ greater(y, x) cnf(cycle$_6$, axiom)
(cycle(x, y) and $x \cdot e_1{=}z$ and greater(y, e_0) and next(x, x_1)) $\Rightarrow$ $z{=}x_1$ cnf(cycle$_7$, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axio
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP130-4.003.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 3 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1 \cdot x{=}y$ or $e_2 \cdot x{=}y$ or $e_3 \cdot x{=}y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot e_1{=}y$ or $x \cdot e_2{=}y$ or $x \cdot e_3{=}y$) cnf(column_surjectivity, axiom)
($z_2 \cdot y{=}x$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $x \cdot y{=}z_1$ cnf(qg3$_1$, negated_conjecture)
($z_2 \cdot y{=}x$ and $x \cdot y{=}z_1$) $\Rightarrow$ $x \cdot z_1{=}z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP130-4.004.p (a.(a.b)).b = a
Generate the multiplication table for the specified quasi- group with 4 elements.
(group_element(x) and group_element(y)) $\Rightarrow$ ($e_1{\cdot}x{=}y$ or $e_2{\cdot}x{=}y$ or $e_3{\cdot}x{=}y$ or $e_4{\cdot}x{=}y$) cnf(row_surjectivity, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}e_1{=}y$ or $x{\cdot}e_2{=}y$ or $x{\cdot}e_3{=}y$ or $x{\cdot}e_4{=}y$) cnf(column_surjectivity, axiom)
($z_2 \cdot y{=}x$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $x \cdot y{=}z_1$ cnf(qg3$_1$, negated_conjecture)
($z_2 \cdot y{=}x$ and $x \cdot y{=}z_1$) $\Rightarrow$ $x \cdot z_1{=}z_2$ cnf(qg3$_2$, negated_conjecture)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$) cnf(product_total_function$_1$, axior
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $x \cdot z_1{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP131-1.002.p (3,2,1) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y{=}e_1$ or $x \cdot y{=}e_2$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2$) $\Rightarrow$ $x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)

$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP131-1.005.p (3,2,1) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP131-2.002.p (3,2,1) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
$(x \cdot e_1{=}y$ and next$(x, x_1)) \Rightarrow \neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y{=}e_1$ or $x \cdot y{=}e_2$) cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)
$(w \cdot y{=}x$ and $z \cdot y{=}x) \Rightarrow w{=}z$ cnf(product_left_cancellation, axiom)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2) \Rightarrow x_1{=}x_2$ cnf(qg1$_1$, negated_conjecture)
$(x_1 \cdot y_1{=}z_1$ and $x_2 \cdot y_2{=}z_1$ and $z_2 \cdot y_1{=}x_1$ and $z_2 \cdot y_2{=}x_2) \Rightarrow y_1{=}y_2$ cnf(qg1$_2$, negated_conjecture)

GRP132-1.002.p (3,1,2) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y{=}e_1$ or $x \cdot y{=}e_2$) cnf(product_total_function$_1$, axiom)
$(x \cdot y{=}w$ and $x \cdot y{=}z) \Rightarrow w{=}z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w{=}y$ and $x \cdot z{=}y) \Rightarrow w{=}z$ cnf(product_right_cancellation, axiom)

$(w \cdot y = x$ and $z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow x_1 = x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow y_1 = y_2$ cnf(qg2$_2$, negated_conjecture)

GRP132-1.005.p (3,1,2) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1 = e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_1 = e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2 = e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2 = e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_2 = e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg e_3 = e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3 = e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3 = e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_3 = e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg e_4 = e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4 = e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4 = e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg e_4 = e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg e_5 = e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg e_5 = e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg e_5 = e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg e_5 = e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$ or $x \cdot y = e_3$ or $x \cdot y = e_4$ or $x \cdot y = e_5$) cnf(product_total_func
$(x \cdot y = w$ and $x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y$ and $x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x$ and $z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow x_1 = x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow y_1 = y_2$ cnf(qg2$_2$, negated_conjecture)

GRP132-2.002.p (3,1,2) conjugate orthogonality, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
($x \cdot e_1 = y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_2 = e_1$ cnf(e_2_is_not_e$_1$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y = e_1$ or $x \cdot y = e_2$) cnf(product_total_function$_1$, axiom)
$(x \cdot y = w$ and $x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y$ and $x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x$ and $z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow x_1 = x_2$ cnf(qg2$_1$, negated_conjecture)
$(x_1 \cdot y_1 = z_1$ and $x_2 \cdot y_2 = z_1$ and $z_2 \cdot x_1 = y_1$ and $z_2 \cdot x_2 = y_2) \Rightarrow y_1 = y_2$ cnf(qg2$_2$, negated_conjecture)

GRP133-1.003.p (a.b).(b.a) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 3 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg e_1 = e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1 = e_3$ cnf(e_1_is_not_e$_3$, axiom)

$\neg e_2=e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y=w$ and $x \cdot y=z$) $\Rightarrow$ $w=z$ cnf(product_total_function$_2$, axiom)
($x \cdot w=y$ and $x \cdot z=y$) $\Rightarrow$ $w=z$ cnf(product_right_cancellation, axiom)
($w \cdot y=x$ and $z \cdot y=x$) $\Rightarrow$ $w=z$ cnf(product_left_cancellation, axiom)
($x \cdot y=z_1$ and $y \cdot x=z_2$) $\Rightarrow$ $z_1 \cdot z_2=x$ cnf(qg$_3$, negated_conjecture)

GRP133-1.004.p (a.b).(b.a) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 4 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_1=e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_2=e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg e_3=e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg e_4=e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg e_4=e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg e_4=e_3$ cnf(e_4_is_not_e$_3$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$ or $x{\cdot}y=e_4$) cnf(product_total_function$_1$, axio
($x \cdot y=w$ and $x \cdot y=z$) $\Rightarrow$ $w=z$ cnf(product_total_function$_2$, axiom)
($x \cdot w=y$ and $x \cdot z=y$) $\Rightarrow$ $w=z$ cnf(product_right_cancellation, axiom)
($w \cdot y=x$ and $z \cdot y=x$) $\Rightarrow$ $w=z$ cnf(product_left_cancellation, axiom)
($x \cdot y=z_1$ and $y \cdot x=z_2$) $\Rightarrow$ $z_1 \cdot z_2=x$ cnf(qg$_3$, negated_conjecture)

GRP133-2.003.p (a.b).(b.a) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)
greater(e_2, e_1) cnf(e_2_greater_e$_1$, axiom)
greater(e_3, e_1) cnf(e_3_greater_e$_1$, axiom)
greater(e_3, e_2) cnf(e_3_greater_e$_2$, axiom)
($x \cdot e_1=y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg e_1=e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg e_1=e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg e_2=e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg e_2=e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg e_3=e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg e_3=e_2$ cnf(e_3_is_not_e$_2$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y=e_1$ or $x{\cdot}y=e_2$ or $x{\cdot}y=e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y=w$ and $x \cdot y=z$) $\Rightarrow$ $w=z$ cnf(product_total_function$_2$, axiom)
($x \cdot w=y$ and $x \cdot z=y$) $\Rightarrow$ $w=z$ cnf(product_right_cancellation, axiom)
($w \cdot y=x$ and $z \cdot y=x$) $\Rightarrow$ $w=z$ cnf(product_left_cancellation, axiom)
($x \cdot y=z_1$ and $y \cdot x=z_2$) $\Rightarrow$ $z_1 \cdot z_2=x$ cnf(qg$_3$, negated_conjecture)

GRP133-2.004.p (a.b).(b.a) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 4 elements.
next(e_1, e_2) cnf(e_1_then_e$_2$, axiom)
next(e_2, e_3) cnf(e_2_then_e$_3$, axiom)

$\text{next}(e_3, e_4)$ cnf(e_3_then_e_4, axiom)
$\text{greater}(e_2, e_1)$ cnf(e_2_greater_e_1, axiom)
$\text{greater}(e_3, e_1)$ cnf(e_3_greater_e_1, axiom)
$\text{greater}(e_4, e_1)$ cnf(e_4_greater_e_1, axiom)
$\text{greater}(e_3, e_2)$ cnf(e_3_greater_e_2, axiom)
$\text{greater}(e_4, e_2)$ cnf(e_4_greater_e_2, axiom)
$\text{greater}(e_4, e_3)$ cnf(e_4_greater_e_3, axiom)
$(x \cdot e_1 = y \text{ and } \text{next}(x, x_1)) \Rightarrow \neg\,\text{greater}(y, x_1)$ cnf(no_redundancy, axiom)
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\text{group\_element}(e_4)$ cnf(element$_4$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2 = e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3 = e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_4 = e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4 = e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4 = e_3$ cnf(e_4_is_not_e_3, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y = e_1 \text{ or } x{\cdot}y = e_2 \text{ or } x{\cdot}y = e_3 \text{ or } x{\cdot}y = e_4)$ cnf(product_total_function$_1$, axio
$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x \cdot y = z_1 \text{ and } y \cdot x = z_2) \Rightarrow z_1 \cdot z_2 = x$ cnf(qg$_3$, negated_conjecture)

GRP134-1.003.p (a.b).(b.a) = b, no idempotence
Generate the multiplication table for the specified quasi- group with 3 elements.
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_3 = e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3 = e_2$ cnf(e_3_is_not_e_2, axiom)
$(\text{group\_element}(x) \text{ and } \text{group\_element}(y)) \Rightarrow (x{\cdot}y = e_1 \text{ or } x{\cdot}y = e_2 \text{ or } x{\cdot}y = e_3)$ cnf(product_total_function$_1$, axiom)
$(x \cdot y = w \text{ and } x \cdot y = z) \Rightarrow w = z$ cnf(product_total_function$_2$, axiom)
$(x \cdot w = y \text{ and } x \cdot z = y) \Rightarrow w = z$ cnf(product_right_cancellation, axiom)
$(w \cdot y = x \text{ and } z \cdot y = x) \Rightarrow w = z$ cnf(product_left_cancellation, axiom)
$(x \cdot y = z_1 \text{ and } y \cdot x = z_2) \Rightarrow z_1 \cdot z_2 = y$ cnf(qg$_4$, negated_conjecture)

GRP134-1.005.p (a.b).(b.a) = b, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
$\text{group\_element}(e_1)$ cnf(element$_1$, axiom)
$\text{group\_element}(e_2)$ cnf(element$_2$, axiom)
$\text{group\_element}(e_3)$ cnf(element$_3$, axiom)
$\text{group\_element}(e_4)$ cnf(element$_4$, axiom)
$\text{group\_element}(e_5)$ cnf(element$_5$, axiom)
$\neg\, e_1 = e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1 = e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1 = e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_1 = e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg\, e_2 = e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2 = e_3$ cnf(e_2_is_not_e_3, axiom)

$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP134-2.003.p (a.b).(b.a) = b, no idempotence
Generate the multiplication table for the specified quasi- group with 3 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP134-2.005.p (a.b).(b.a) = b, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
next(e_4, e_5) cnf(e_4_then_e_5, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_5, e_1) cnf(e_5_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_5, e_2) cnf(e_5_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
greater(e_5, e_3) cnf(e_5_greater_e_3, axiom)
greater(e_5, e_4) cnf(e_5_greater_e_4, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)

group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e$_4$, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e$_5$, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e$_1$, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e$_2$, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e$_3$, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e$_5$, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e$_1$, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e$_2$, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e$_3$, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e$_4$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($x \cdot y{=}z_1$ and $y \cdot x{=}z_2$) $\Rightarrow$ $z_1 \cdot z_2{=}y$ cnf(qg$_4$, negated_conjecture)

GRP135-1.002.p ((b.a).b).b) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y{=}e_1$ or $x \cdot y{=}e_2$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP135-1.005.p ((b.a).b).b) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e$_2$, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e$_3$, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e$_4$, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e$_5$, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e$_1$, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e$_3$, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e$_4$, axiom)
$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e$_5$, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e$_1$, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e$_2$, axiom)

$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP135-2.002.p ((b.a).b).b) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 2 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x \cdot y{=}e_1$ or $x \cdot y{=}e_2$) cnf(product_total_function$_1$, axiom)
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP135-2.005.p ((b.a).b).b) = a, no idempotence
Generate the multiplication table for the specified quasi- group with 5 elements.
next(e_1, e_2) cnf(e_1_then_e_2, axiom)
next(e_2, e_3) cnf(e_2_then_e_3, axiom)
next(e_3, e_4) cnf(e_3_then_e_4, axiom)
next(e_4, e_5) cnf(e_4_then_e_5, axiom)
greater(e_2, e_1) cnf(e_2_greater_e_1, axiom)
greater(e_3, e_1) cnf(e_3_greater_e_1, axiom)
greater(e_4, e_1) cnf(e_4_greater_e_1, axiom)
greater(e_5, e_1) cnf(e_5_greater_e_1, axiom)
greater(e_3, e_2) cnf(e_3_greater_e_2, axiom)
greater(e_4, e_2) cnf(e_4_greater_e_2, axiom)
greater(e_5, e_2) cnf(e_5_greater_e_2, axiom)
greater(e_4, e_3) cnf(e_4_greater_e_3, axiom)
greater(e_5, e_3) cnf(e_5_greater_e_3, axiom)
greater(e_5, e_4) cnf(e_5_greater_e_4, axiom)
($x \cdot e_1{=}y$ and next(x, x_1)) $\Rightarrow$ $\neg$ greater(y, x_1) cnf(no_redundancy, axiom)
group_element(e_1) cnf(element$_1$, axiom)
group_element(e_2) cnf(element$_2$, axiom)
group_element(e_3) cnf(element$_3$, axiom)
group_element(e_4) cnf(element$_4$, axiom)
group_element(e_5) cnf(element$_5$, axiom)
$\neg\, e_1{=}e_2$ cnf(e_1_is_not_e_2, axiom)
$\neg\, e_1{=}e_3$ cnf(e_1_is_not_e_3, axiom)
$\neg\, e_1{=}e_4$ cnf(e_1_is_not_e_4, axiom)
$\neg\, e_1{=}e_5$ cnf(e_1_is_not_e_5, axiom)
$\neg\, e_2{=}e_1$ cnf(e_2_is_not_e_1, axiom)
$\neg\, e_2{=}e_3$ cnf(e_2_is_not_e_3, axiom)
$\neg\, e_2{=}e_4$ cnf(e_2_is_not_e_4, axiom)

$\neg\, e_2{=}e_5$ cnf(e_2_is_not_e_5, axiom)
$\neg\, e_3{=}e_1$ cnf(e_3_is_not_e_1, axiom)
$\neg\, e_3{=}e_2$ cnf(e_3_is_not_e_2, axiom)
$\neg\, e_3{=}e_4$ cnf(e_3_is_not_e_4, axiom)
$\neg\, e_3{=}e_5$ cnf(e_3_is_not_e_5, axiom)
$\neg\, e_4{=}e_1$ cnf(e_4_is_not_e_1, axiom)
$\neg\, e_4{=}e_2$ cnf(e_4_is_not_e_2, axiom)
$\neg\, e_4{=}e_3$ cnf(e_4_is_not_e_3, axiom)
$\neg\, e_4{=}e_5$ cnf(e_4_is_not_e_5, axiom)
$\neg\, e_5{=}e_1$ cnf(e_5_is_not_e_1, axiom)
$\neg\, e_5{=}e_2$ cnf(e_5_is_not_e_2, axiom)
$\neg\, e_5{=}e_3$ cnf(e_5_is_not_e_3, axiom)
$\neg\, e_5{=}e_4$ cnf(e_5_is_not_e_4, axiom)
(group_element(x) and group_element(y)) $\Rightarrow$ ($x{\cdot}y{=}e_1$ or $x{\cdot}y{=}e_2$ or $x{\cdot}y{=}e_3$ or $x{\cdot}y{=}e_4$ or $x{\cdot}y{=}e_5$) cnf(product_total_func
($x \cdot y{=}w$ and $x \cdot y{=}z$) $\Rightarrow$ $w{=}z$ cnf(product_total_function$_2$, axiom)
($x \cdot w{=}y$ and $x \cdot z{=}y$) $\Rightarrow$ $w{=}z$ cnf(product_right_cancellation, axiom)
($w \cdot y{=}x$ and $z \cdot y{=}x$) $\Rightarrow$ $w{=}z$ cnf(product_left_cancellation, axiom)
($y \cdot x{=}z_1$ and $z_1 \cdot y{=}z_2$) $\Rightarrow$ $z_2 \cdot y{=}x$ cnf(qg$_3$, negated_conjecture)

GRP136-1.p Prove anti-symmetry axiom using the LUB transformation
This problem proves the original anti-symmetry axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) = b cnf(ax_antisyma$_1$, hypothesis)
least_upper_bound(a, b) = a cnf(ax_antisyma$_2$, hypothesis)
$a \neq b$ cnf(prove_ax_antisyma, negated_conjecture)

GRP137-1.p Prove anti-symmetry axiom using the GLB transformation
This problem proves the original anti-symmetry axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) = a cnf(ax_antisymb$_1$, hypothesis)
greatest_lower_bound(a, b) = b cnf(ax_antisymb$_2$, hypothesis)
$a \neq b$ cnf(prove_ax_antisymb, negated_conjecture)

GRP138-1.p Prove greatest lower-bound axiom using the LUB transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, c) = a cnf(ax_glb1a$_1$, hypothesis)
least_upper_bound(b, c) = b cnf(ax_glb1a$_2$, hypothesis)
least_upper_bound(greatest_lower_bound(a, b), c) $\neq$ greatest_lower_bound(a, b) cnf(prove_ax_glb1a, negated_conjecture)

GRP139-1.p Prove greatest lower-bound axiom using the GLB transformation
This problem proves the original axiom of anti-symmetry from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, c) = c cnf(ax_glb1b_1$_1$, hypothesis)
greatest_lower_bound(b, c) = c cnf(ax_glb1b_2$_2$, hypothesis)
greatest_lower_bound(greatest_lower_bound(a, b), c) $\neq c$ cnf(prove_ax_glb1b, negated_conjecture)

GRP140-1.p Prove greatest lower-bound axiom using a transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, c) = c cnf(ax_glb1c$_1$, hypothesis)
greatest_lower_bound(b, c) = c cnf(ax_glb1c$_2$, hypothesis)
least_upper_bound(greatest_lower_bound(a, b), c) $\neq$ greatest_lower_bound(a, b) cnf(prove_ax_glb1c, negated_conjecture)

GRP141-1.p Prove greatest lower-bound axiom using a transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')

$\text{least\_upper\_bound}(a, c) = a$ cnf(ax_glb1d$_1$, hypothesis)
$\text{least\_upper\_bound}(b, c) = b$ cnf(ax_glb1d$_2$, hypothesis)
$\text{greatest\_lower\_bound}(\text{greatest\_lower\_bound}(a, b), c) \neq c$ cnf(prove_ax_glb1d, negated_conjecture)

GRP142-1.p Prove greatest lower-bound axiom using the LUB transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{greatest\_lower\_bound}(a, b), a) \neq a$ cnf(prove_ax_glb2a, negated_conjecture)

GRP143-1.p Prove greatest lower-bound axiom using the GLB transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{greatest\_lower\_bound}(a, b), a) \neq \text{greatest\_lower\_bound}(a, b)$ cnf(prove_ax_glb2b, negated_conjecture)

GRP144-1.p Prove greatest lower-bound axiom using the LUB transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{greatest\_lower\_bound}(a, b), b) \neq b$ cnf(prove_ax_glb3a, negated_conjecture)

GRP145-1.p Prove greatest lower-bound axiom using the GLB transformation
This problem proves the original greatest lower-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{greatest\_lower\_bound}(a, b), b) \neq \text{greatest\_lower\_bound}(a, b)$ cnf(prove_ax_glb3b, negated_conjecture)

GRP146-1.p Prove least upper-bound axiom using the LUB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, c) = c$ cnf(ax_lub1a$_1$, hypothesis)
$\text{least\_upper\_bound}(b, c) = c$ cnf(ax_lub1a$_2$, hypothesis)
$\text{least\_upper\_bound}(\text{least\_upper\_bound}(a, b), c) \neq c$ cnf(prove_ax_lub1a, negated_conjecture)

GRP147-1.p Prove least upper-bound axiom using the GLB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, c) = a$ cnf(ax_lub1b$_1$, hypothesis)
$\text{greatest\_lower\_bound}(b, c) = b$ cnf(ax_lub1b$_2$, hypothesis)
$\text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a, b), c) \neq \text{least\_upper\_bound}(a, b)$ cnf(prove_ax_lub1b, negated_conjecture)

GRP148-1.p Prove least upper-bound axiom using a transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, c) = c$ cnf(ax_lub1c$_1$, hypothesis)
$\text{least\_upper\_bound}(b, c) = c$ cnf(ax_lub1c$_2$, hypothesis)
$\text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a, b), c) \neq \text{least\_upper\_bound}(a, b)$ cnf(prove_ax_lub1c, negated_conjecture)

GRP149-1.p Prove least upper-bound axiom using a transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, c) = a$ cnf(ax_lub1d$_1$, hypothesis)
$\text{greatest\_lower\_bound}(b, c) = b$ cnf(ax_lub1d$_2$, hypothesis)
$\text{least\_upper\_bound}(\text{least\_upper\_bound}(a, b), c) \neq c$ cnf(prove_ax_lub1d, negated_conjecture)

GRP150-1.p Prove least upper-bound axiom using the LUB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, \text{least\_upper\_bound}(a, b)) \neq \text{least\_upper\_bound}(a, b)$ cnf(prove_ax_lub2a, negated_conjecture)

GRP151-1.p Prove least upper-bound axiom using the GLB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, least_upper_bound(a, b)) $\neq a$ cnf(prove_ax_lub2b, negated_conjecture)

GRP152-1.p Prove least upper-bound axiom using the LUB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(b, least_upper_bound(a, b)) $\neq$ least_upper_bound(a, b) cnf(prove_ax_lub3a, negated_conjecture)

GRP153-1.p Prove least upper-bound axiom using the GLB transformation
This problem proves the original least upper-bound axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(b, least_upper_bound(a, b)) $\neq b$ cnf(prove_ax_lub3b, negated_conjecture)

GRP154-1.p Prove monotonicity axiom using the LUB transformation
This problem proves the original mononicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) $= b$ cnf(ax_mono1a$_1$, hypothesis)
least_upper_bound($a \cdot c, b \cdot c$) $\neq b \cdot c$ cnf(prove_ax_mono1a, negated_conjecture)

GRP155-1.p Prove monotonicity axiom using the GLB transformation
This problem proves the original monotonicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) $= a$ cnf(ax_mono1b, hypothesis)
greatest_lower_bound($a \cdot c, b \cdot c$) $\neq a \cdot c$ cnf(prove_ax_mono1b, negated_conjecture)

GRP156-1.p Prove monotonicity axiom using a transformation
This problem proves the original monotonicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) $= b$ cnf(ax_mono1c$_1$, hypothesis)
greatest_lower_bound($a \cdot c, b \cdot c$) $\neq a \cdot c$ cnf(prove_ax_mono1c, negated_conjecture)

GRP157-1.p Prove monotonicity axiom using the LUB transformation
This problem proves the original monotonicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) $= b$ cnf(ax_mono2a$_1$, hypothesis)
least_upper_bound($c \cdot a, c \cdot b$) $\neq c \cdot b$ cnf(prove_ax_mono2a, negated_conjecture)

GRP158-1.p Prove monotonicity axiom using the GLB transformation
This problem proves the original monotonicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) $= a$ cnf(ax_mono2b$_1$, hypothesis)
greatest_lower_bound($c \cdot a, c \cdot b$) $\neq c \cdot a$ cnf(prove_ax_mono2b, negated_conjecture)

GRP159-1.p Prove monotonicity axiom using a transformation
This problem proves the original monotonicity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) $= a$ cnf(ax_mono2c$_1$, hypothesis)
least_upper_bound($c \cdot a, c \cdot b$) $\neq c \cdot b$ cnf(prove_ax_mono2c, negated_conjecture)

GRP160-1.p Prove reflexivity axiom using the LUB transformation
This problem proves the original reflexivity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, a) $\neq a$ cnf(prove_ax_refla, negated_conjecture)

GRP161-1.p Prove reflexivity axiom using the GLB transformation
This problem proves the original reflexivity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, a) \neq a$ cnf(prove_ax_reflb, negated_conjecture)

GRP162-1.p Prove transitivity axiom using the LUB transformation
This problem proves the original transitivity axiom from the equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, b) = b$ cnf(ax_transa_1, hypothesis)
$\text{least\_upper\_bound}(b, c) = c$ cnf(ax_transa_2, hypothesis)
$\text{least\_upper\_bound}(a, c) \neq c$ cnf(prove_ax_transa, negated_conjecture)

GRP163-1.p Prove transitivity axiom using the GLB transformation
This problem proves the original transitivity axiom from equational axiomatization.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, b) = a$ cnf(ax_transb_1, hypothesis)
$\text{greatest\_lower\_bound}(b, c) = b$ cnf(ax_transb_2, hypothesis)
$\text{greatest\_lower\_bound}(a, c) \neq a$ cnf(prove_ax_transb, negated_conjecture)

GRP164-1.p The lattice of each LOG is distributive
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, \text{greatest\_lower\_bound}(b, c)) \neq \text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a, b), \text{least\_upper\_bound}(a, c))$

GRP164-2.p The lattice of each LOG is distributive
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, \text{least\_upper\_bound}(b, c)) \neq \text{least\_upper\_bound}(\text{greatest\_lower\_bound}(a, b), \text{greatest\_lower\_bound}(a, c))$

GRP165-1.p An application of monotonicity
Essentially a simple application of monotonicity, more difficult when proved from the equations replacing monotonicity.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, \text{identity}) = a$ cnf(lat1a_1, hypothesis)
$\text{least\_upper\_bound}(a, a \cdot a) \neq a \cdot a$ cnf(prove_lat1a, negated_conjecture)

GRP165-2.p An application of monotonicity
Essentially a simple application of monotonicity, more difficult when proved from the equations replacing monotonicity.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, \text{identity}) = \text{identity}$ cnf(lat1b_1, hypothesis)
$\text{greatest\_lower\_bound}(a, a \cdot a) \neq a$ cnf(prove_lat1b, negated_conjecture)

GRP166-1.p Multiplication with a positive element increases a value
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, \text{identity}) = a$ cnf(lat2a_1, hypothesis)
$\text{least\_upper\_bound}(b, \text{identity}) = b$ cnf(lat2a_2, hypothesis)
$\text{least\_upper\_bound}(a, a \cdot b) \neq a \cdot b$ cnf(prove_lat2a, negated_conjecture)

GRP166-2.p Multiplication with a positive element increases a value
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, \text{identity}) = \text{identity}$ cnf(lat2b_1, hypothesis)
$\text{greatest\_lower\_bound}(b, \text{identity}) = \text{identity}$ cnf(lat2b_2, hypothesis)
$\text{greatest\_lower\_bound}(a, a \cdot b) \neq a$ cnf(prove_lat2b, negated_conjecture)

GRP166-3.p Multiplication with a positive element increases a value
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')

least_upper_bound$(a, \text{identity}) = a$ cnf(lat3a$_1$, hypothesis)
least_upper_bound$(b, \text{identity}) = b$ cnf(lat3a$_2$, hypothesis)
least_upper_bound$(a, b \cdot a) \neq b \cdot a$ cnf(prove_lat3a, negated_conjecture)

GRP166-4.p Multiplication with a positive element increases a value
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound$(a, \text{identity}) = \text{identity}$ cnf(lat3b$_1$, hypothesis)
greatest_lower_bound$(b, \text{identity}) = \text{identity}$ cnf(lat3b$_2$, hypothesis)
greatest_lower_bound$(a, b \cdot a) \neq a$ cnf(prove_lat3b, negated_conjecture)

GRP167-1.p Product of positive and negative parts
Each element in a lattice ordered group can be stated as a product of it's positive and it's negative part.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
positive_part$(x) = $ least_upper_bound$(x, \text{identity})$ cnf(lat4$_1$, axiom)
negative_part$(x) = $ greatest_lower_bound$(x, \text{identity})$ cnf(lat4$_2$, axiom)
least_upper_bound$(x,$ greatest_lower_bound$(y, z)) = $ greatest_lower_bound(least_upper_bound(x, y), least_upper_bound$(x, z))$
greatest_lower_bound$(x,$ least_upper_bound$(y, z)) = $ least_upper_bound(greatest_lower_bound(x, y), greatest_lower_bound$(x, z$
$a \neq$ positive_part$(a) \cdot$ negative_part(a) cnf(prove_lat$_4$, negated_conjecture)

GRP167-2.p Product of positive and negative parts
Each element in a lattice ordered group can be stated as a product of it's positive and it's negative part.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(lat4$_1$, axiom)
$(x')' = x$ cnf(lat4$_2$, axiom)
$(x \cdot y)' = y' \cdot x'$ cnf(lat4$_3$, axiom)
positive_part$(x) = $ least_upper_bound$(x, \text{identity})$ cnf(lat4$_4$, axiom)
negative_part$(x) = $ greatest_lower_bound$(x, \text{identity})$ cnf(lat4$_5$, axiom)
least_upper_bound$(x,$ greatest_lower_bound$(y, z)) = $ greatest_lower_bound(least_upper_bound(x, y), least_upper_bound$(x, z))$
greatest_lower_bound$(x,$ least_upper_bound$(y, z)) = $ least_upper_bound(greatest_lower_bound(x, y), greatest_lower_bound$(x, z$
$a \neq$ positive_part$(a) \cdot$ negative_part(a) cnf(prove_lat$_4$, negated_conjecture)

GRP167-3.p Product of positive and negative parts
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$a \neq$ least_upper_bound$(a, \text{identity}) \cdot$ greatest_lower_bound$(a, \text{identity})$ cnf(prove_p$_{19}$, negated_conjecture)

GRP167-4.p Product of positive and negative parts
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p19$_1$, axiom)
$(x')' = x$ cnf(p19$_2$, axiom)
$(x \cdot y)' = y' \cdot x'$ cnf(p19$_3$, axiom)
$a \neq$ least_upper_bound$(a, \text{identity}) \cdot$ greatest_lower_bound$(a, \text{identity})$ cnf(prove_p$_{19}$, negated_conjecture)

GRP167-5.p Product of positive and negative parts
Each element in a lattice ordered group can be stated as a product of it's positive and it's negative part.
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound$(a, b)' = $ greatest_lower_bound(a', b') cnf(p_{10}, axiom)
positive_part$(x) = $ least_upper_bound$(x, \text{identity})$ cnf(lat4$_1$, axiom)
negative_part$(x) = $ greatest_lower_bound$(x, \text{identity})$ cnf(lat4$_2$, axiom)
least_upper_bound$(x,$ greatest_lower_bound$(y, z)) = $ greatest_lower_bound(least_upper_bound(x, y), least_upper_bound$(x, z))$
greatest_lower_bound$(x,$ least_upper_bound$(y, z)) = $ least_upper_bound(greatest_lower_bound(x, y), greatest_lower_bound$(x, z$
$a \neq$ positive_part$(a) \cdot$ negative_part(a) cnf(prove_lat$_4$, negated_conjecture)

GRP168-1.p Inner group automorphisms are order preserving
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound$(a, b) = b$ cnf(p01a$_1$, hypothesis)
least_upper_bound$(c' \cdot (a \cdot c), c' \cdot (b \cdot c)) \neq c' \cdot (b \cdot c)$ cnf(prove_p01a, negated_conjecture)

GRP168-2.p Inner group automorphisms are order preserving
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, b) = a$ cnf(p01b_1, hypothesis)
$\text{greatest\_lower\_bound}(c' \cdot (a \cdot c), c' \cdot (b \cdot c)) \neq c' \cdot (a \cdot c)$ cnf(prove_p01b, negated_conjecture)

GRP169-1.p Inverses reverse inequalities
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a', b') = b'$ cnf(p02a_1, hypothesis)
$\text{least\_upper\_bound}(a, b) \neq a$ cnf(prove_p02a, negated_conjecture)

GRP169-2.p Inverses reverse inequalities
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a', b') = a'$ cnf(p02b_1, hypothesis)
$\text{greatest\_lower\_bound}(a, b) \neq b$ cnf(prove_p02b, negated_conjecture)

GRP170-1.p General form of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, b) = b$ cnf(p03a_1, hypothesis)
$\text{least\_upper\_bound}(c, d) = d$ cnf(p03a_2, hypothesis)
$\text{least\_upper\_bound}(a \cdot c, b \cdot d) \neq b \cdot d$ cnf(prove_p03a, negated_conjecture)

GRP170-2.p General form of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, b) = a$ cnf(p03b_1, hypothesis)
$\text{greatest\_lower\_bound}(c, d) = c$ cnf(p03b_2, hypothesis)
$\text{greatest\_lower\_bound}(a \cdot c, b \cdot d) \neq a \cdot c$ cnf(prove_p03b, negated_conjecture)

GRP170-3.p General form of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, b) = b$ cnf(p03c_1, hypothesis)
$\text{least\_upper\_bound}(c, d) = d$ cnf(p03c_2, hypothesis)
$\text{greatest\_lower\_bound}(a \cdot c, b \cdot d) \neq a \cdot c$ cnf(prove_p03c, negated_conjecture)

GRP170-4.p General form of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a, b) = b$ cnf(p03d_1, hypothesis)
$\text{greatest\_lower\_bound}(c, d) = c$ cnf(p03d_2, hypothesis)
$\text{least\_upper\_bound}(a \cdot c, b \cdot d) \neq b \cdot d$ cnf(prove_p03d, negated_conjecture)

GRP171-1.p Positive elements form a semigroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, a) = a$ cnf(p04a_1, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, b) = b$ cnf(p04a_2, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, a \cdot b) \neq a \cdot b$ cnf(prove_p04a, negated_conjecture)

GRP171-2.p Positive elements form a semigroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, a) = a$ cnf(p04c_1, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, b) = b$ cnf(p04c_2, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, a \cdot b) \neq \text{identity}$ cnf(prove_p04c, negated_conjecture)

GRP172-1.p Negative elements form a semigroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, a) = \text{identity}$ cnf(p04b_1, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, b) = \text{identity}$ cnf(p04b_2, hypothesis)

$\text{greatest\_lower\_bound}(\text{identity}, a \cdot b) \neq \text{identity}$ cnf(prove_p04b, negated_conjecture)

GRP172-2.p Negative elements form a semigroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, a) = \text{identity}$ cnf(p04d$_1$, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, b) = \text{identity}$ cnf(p04d$_2$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, a \cdot b) \neq a \cdot b$ cnf(prove_p04d, negated_conjecture)

GRP173-1.p Each subgroup of negative elements is trivial
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, a) = \text{identity}$ cnf(p05a$_1$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, a') = \text{identity}$ cnf(p05a$_2$, hypothesis)
$\text{identity} \neq a$ cnf(prove_p05a, negated_conjecture)

GRP174-1.p Each subgroup of positive elements is trivial
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, a) = a$ cnf(p05b$_1$, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, a') = a'$ cnf(p05b$_2$, hypothesis)
$\text{identity} \neq a$ cnf(prove_p05b, negated_conjecture)

GRP175-1.p Positivity is preserved under inner automorphisms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, b) = b$ cnf(p06a$_1$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, a' \cdot (b \cdot a)) \neq a' \cdot (b \cdot a)$ cnf(prove_p06a, negated_conjecture)

GRP175-2.p Positivity is preserved under inner automorphisms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, b) = \text{identity}$ cnf(p06b$_1$, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, a' \cdot (b \cdot a)) \neq \text{identity}$ cnf(prove_p06b, negated_conjecture)

GRP175-3.p Positivity is preserved under inner automorphisms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, b) = b$ cnf(p06c$_1$, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, a' \cdot (b \cdot a)) \neq \text{identity}$ cnf(prove_p06c, negated_conjecture)

GRP175-4.p Positivity is preserved under inner automorphisms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, b) = \text{identity}$ cnf(p06d$_1$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, a' \cdot (b \cdot a)) \neq a' \cdot (b \cdot a)$ cnf(prove_p06d, negated_conjecture)

GRP176-1.p General form of distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$c \cdot (\text{least\_upper\_bound}(a, b) \cdot d) \neq \text{least\_upper\_bound}(c \cdot (a \cdot d), c \cdot (b \cdot d))$ cnf(prove_p$_{07}$, negated_conjecture)

GRP176-2.p General form of distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$(x \cdot y)' = y' \cdot x'$ cnf(p07$_1$, hypothesis)
$c \cdot (\text{least\_upper\_bound}(a, b) \cdot d) \neq \text{least\_upper\_bound}(c \cdot (a \cdot d), c \cdot (b \cdot d))$ cnf(prove_p$_{07}$, negated_conjecture)

GRP177-1.p A consequence of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, a) = a$ cnf(p08a$_1$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, b) = b$ cnf(p08a$_2$, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, c) = c$ cnf(p08a$_3$, hypothesis)

least_upper_bound(greatest_lower_bound($a, b \cdot c$), greatest_lower_bound(a, b)·greatest_lower_bound(a, c)) $\neq$ greatest_lower_boun
greatest_lower_bound(a, c) cnf(prove_p08a, negated_conjecture)

GRP177-2.p A consequence of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(identity, a) = identity cnf(p08b_1, hypothesis)
greatest_lower_bound(identity, b) = identity cnf(p08b_2, hypothesis)
greatest_lower_bound(identity, c) = identity cnf(p08b_3, hypothesis)
greatest_lower_bound(greatest_lower_bound($a, b \cdot c$), greatest_lower_bound(a, b)·greatest_lower_bound(a, c)) $\neq$ greatest_lower_bo
c) cnf(prove_p08b, negated_conjecture)

GRP178-1.p A consequence of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(identity, a) = a cnf(p09a_1, hypothesis)
least_upper_bound(identity, b) = b cnf(p09a_2, hypothesis)
least_upper_bound(identity, c) = c cnf(p09a_3, hypothesis)
greatest_lower_bound(a, b) = identity cnf(p09a_4, hypothesis)
greatest_lower_bound($a, b \cdot c$) $\neq$ greatest_lower_bound(a, c) cnf(prove_p09a, negated_conjecture)

GRP178-2.p A consequence of monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(identity, a) = identity cnf(p09b_1, hypothesis)
greatest_lower_bound(identity, b) = identity cnf(p09b_2, hypothesis)
greatest_lower_bound(identity, c) = identity cnf(p09b_3, hypothesis)
greatest_lower_bound(a, b) = identity cnf(p09b_4, hypothesis)
greatest_lower_bound($a, b \cdot c$) $\neq$ greatest_lower_bound(a, c) cnf(prove_p09b, negated_conjecture)

GRP179-1.p For converting between GLB and LUB
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b)$'$ $\neq$ greatest_lower_bound(a', b') cnf(prove_p_{10}, negated_conjecture)

GRP179-2.p For converting between GLB and LUB
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a', identity) $\neq$ greatest_lower_bound(a, identity)$'$ cnf(prove_p_{18}, negated_conjecture)

GRP179-3.p For converting between GLB and LUB
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity$'$ = identity cnf(p18_1, hypothesis)
$(x')' = x$ cnf(p18_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p18_3, hypothesis)
least_upper_bound(a', identity) $\neq$ greatest_lower_bound(a, identity)$'$ cnf(prove_p_{18}, negated_conjecture)

GRP180-1.p Consequence of converting between GLB and LUB
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$a \cdot$ (greatest_lower_bound(a, b)$' \cdot b$) $\neq$ least_upper_bound(a, b) cnf(prove_p_{11}, negated_conjecture)

GRP180-2.p Consequence of converting between GLB and LUB
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity$'$ = identity cnf(p11_1, hypothesis)
$(x')' = x$ cnf(p11_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p11_3, hypothesis)
$a \cdot$ (greatest_lower_bound(a, b)$' \cdot b$) $\neq$ least_upper_bound(a, b) cnf(prove_p_{11}, negated_conjecture)

GRP181-1.p Distributivity of a lattice
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, c) = greatest_lower_bound(b, c) cnf(p12_1, hypothesis)

$\text{least\_upper\_bound}(a, c) = \text{least\_upper\_bound}(b, c)$ cnf(p12_2, hypothesis)
$a \neq b$ cnf(prove_p_{12}, negated_conjecture)

GRP181-2.p Distributivity of a lattice
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p12_1, hypothesis)
$(x')' = x$ cnf(p12_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p12_3, hypothesis)
$\text{greatest\_lower\_bound}(a, c) = \text{greatest\_lower\_bound}(b, c)$ cnf(p12_4, hypothesis)
$\text{least\_upper\_bound}(a, c) = \text{least\_upper\_bound}(b, c)$ cnf(p12_5, hypothesis)
$a \neq b$ cnf(prove_p_{12}, negated_conjecture)

GRP181-3.p Distributivity of a lattice
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(a, c) = \text{greatest\_lower\_bound}(b, c)$ cnf(p12x_1, hypothesis)
$\text{least\_upper\_bound}(a, c) = \text{least\_upper\_bound}(b, c)$ cnf(p12x_2, hypothesis)
$\text{greatest\_lower\_bound}(x, y)' = \text{least\_upper\_bound}(x', y')$ cnf(p12x_3, hypothesis)
$\text{least\_upper\_bound}(x, y)' = \text{greatest\_lower\_bound}(x', y')$ cnf(p12x_4, hypothesis)
$a \neq b$ cnf(prove_p12x, negated_conjecture)

GRP181-4.p Distributivity of a lattice
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p12x_1, hypothesis)
$(x')' = x$ cnf(p12x_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p12x_3, hypothesis)
$\text{greatest\_lower\_bound}(a, c) = \text{greatest\_lower\_bound}(b, c)$ cnf(p12x_4, hypothesis)
$\text{least\_upper\_bound}(a, c) = \text{least\_upper\_bound}(b, c)$ cnf(p12x_5, hypothesis)
$\text{greatest\_lower\_bound}(x, y)' = \text{least\_upper\_bound}(x', y')$ cnf(p12x_6, hypothesis)
$\text{least\_upper\_bound}(x, y)' = \text{greatest\_lower\_bound}(x', y')$ cnf(p12x_7, hypothesis)
$a \neq b$ cnf(prove_p12x, negated_conjecture)

GRP182-1.p Positive part of the negative part is identity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(\text{identity}, \text{greatest\_lower\_bound}(a, \text{identity})) \neq \text{identity}$ cnf(prove_p17a, negated_conjecture)

GRP182-2.p Positive part of the negative part is identity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p17a_1, hypothesis)
$(x')' = x$ cnf(p17a_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p17a_3, hypothesis)
$\text{least\_upper\_bound}(\text{identity}, \text{greatest\_lower\_bound}(a, \text{identity})) \neq \text{identity}$ cnf(prove_p17a, negated_conjecture)

GRP182-3.p Positive part of the negative part is identity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{identity}, \text{least\_upper\_bound}(a, \text{identity})) \neq \text{identity}$ cnf(prove_p17b, negated_conjecture)

GRP182-4.p Positive part of the negative part is identity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p17b_1, hypothesis)
$(x')' = x$ cnf(p17b_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p17b_3, hypothesis)
$\text{greatest\_lower\_bound}(\text{identity}, \text{least\_upper\_bound}(a, \text{identity})) \neq \text{identity}$ cnf(prove_p17b, negated_conjecture)

GRP183-1.p Orthogonal elements form a subgroup with orthogonal parts
For each X Y: X orth Y is a subgroup. Moreover, pp(a) is orthogonal to np(a).
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')

greatest_lower_bound(least_upper_bound(a, identity), greatest_lower_bound(a, identity)′) ≠ identity cnf(prove_p$_{20}$, negated

GRP183-2.p Orthogonal elements form a subgroup with orthogonal parts
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity′ = identity cnf(p20$_1$, hypothesis)
$(x')' = x$ cnf(p20$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p20$_3$, hypothesis)
greatest_lower_bound(least_upper_bound(a, identity), greatest_lower_bound(a, identity)′) ≠ identity cnf(prove_p$_{20}$, negated

GRP183-3.p Orthogonal elements form a subgroup with orthogonal parts
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(least_upper_bound(a, identity), least_upper_bound(a', identity)) ≠ identity cnf(prove_20x, negated_c

GRP183-4.p Orthogonal elements form a subgroup with orthogonal parts
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity′ = identity cnf(p20x$_1$, hypothesis)
$(x')' = x$ cnf(p20x$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p20x$_3$, hypothesis)
greatest_lower_bound(least_upper_bound(a, identity), least_upper_bound(a', identity)) ≠ identity cnf(prove_20x, negated_c

GRP184-1.p Orthogonal elements commute and form a subgroup
For each X Y: X orth Y is a subgroup. X orthogonal to Y implies that X and Y commute. Moreover, pp(a) orthogonal to np(a).
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, identity)·greatest_lower_bound(a, identity)′ ≠ greatest_lower_bound(a, identity)′·least_upper_bound(a, id

GRP184-2.p Orthogonal elements commute and form a subgroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity′ = identity cnf(p21$_1$, hypothesis)
$(x')' = x$ cnf(p21$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p21$_3$, hypothesis)
least_upper_bound(a, identity)·greatest_lower_bound(a, identity)′ ≠ greatest_lower_bound(a, identity)′·least_upper_bound(a, id

GRP184-3.p Orthogonal elements commute and form a subgroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, identity)·greatest_lower_bound(a, identity)′ ≠ greatest_lower_bound(a, identity)′·least_upper_bound(a, id

GRP184-4.p Orthogonal elements commute and form a subgroup
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity′ = identity cnf(p21x$_1$, hypothesis)
$(x')' = x$ cnf(p21x$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p21x$_3$, hypothesis)
greatest_lower_bound(x, y)′ = least_upper_bound(x', y') cnf(p21x$_4$, hypothesis)
least_upper_bound(x, y)′ = greatest_lower_bound(x', y') cnf(p21x$_5$, hypothesis)
least_upper_bound(a, identity)·greatest_lower_bound(a, identity)′ ≠ greatest_lower_bound(a, identity)′·least_upper_bound(a, id

GRP185-1.p Application of monotonicity and distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(least_upper_bound($a \cdot b$, identity), least_upper_bound(a, identity)·least_upper_bound(b, identity)) ≠ least_upper_bound(a, identity) · least_upper_bound(b, identity) cnf(prove_p22a, negated_conjecture)

GRP185-2.p Application of monotonicity and distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity′ = identity cnf(p22a$_1$, hypothesis)
$(x')' = x$ cnf(p22a$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p22a$_3$, hypothesis)

$\text{least\_upper\_bound}(\text{least\_upper\_bound}(a \cdot b, \text{identity}), \text{least\_upper\_bound}(a, \text{identity}) \cdot \text{least\_upper\_bound}(b, \text{identity})) \neq \text{least\_upper\_bound}(a, \text{identity}) \cdot \text{least\_upper\_bound}(b, \text{identity})$ cnf(prove_p22a, negated_conjecture)

GRP185-3.p Application of monotonicity and distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a \cdot b, \text{identity}), \text{least\_upper\_bound}(a, \text{identity}) \cdot \text{least\_upper\_bound}(b, \text{identity})) \neq \text{least\_upper\_bound}(a \cdot b, \text{identity})$ cnf(prove_p22b, negated_conjecture)

GRP185-4.p Application of monotonicity and distributivity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p22b_1, hypothesis)
$(x')' = x$ cnf(p22b_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p22b_3, hypothesis)
$\text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a \cdot b, \text{identity}), \text{least\_upper\_bound}(a, \text{identity}) \cdot \text{least\_upper\_bound}(b, \text{identity})) \neq \text{least\_upper\_bound}(a \cdot b, \text{identity})$ cnf(prove_p22b, negated_conjecture)

GRP186-1.p Application of distributivity and group theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a \cdot b, \text{identity}) \neq a \cdot \text{greatest\_lower\_bound}(a, b')'$ cnf(prove_p_{23}, negated_conjecture)

GRP186-2.p Application of distributivity and group theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p23_1, hypothesis)
$(x')' = x$ cnf(p23_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p23_3, hypothesis)
$\text{least\_upper\_bound}(a \cdot b, \text{identity}) \neq a \cdot \text{greatest\_lower\_bound}(a, b')'$ cnf(prove_p_{23}, negated_conjecture)

GRP186-3.p Application of distributivity and group theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(a \cdot b, \text{identity}) \neq a \cdot \text{least\_upper\_bound}(a', b)$ cnf(prove_p23x, negated_conjecture)

GRP186-4.p Application of distributivity and group theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p23x_1, hypothesis)
$(x')' = x$ cnf(p23x_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p23x_3, hypothesis)
$\text{least\_upper\_bound}(a \cdot b, \text{identity}) \neq a \cdot \text{least\_upper\_bound}(a', b)$ cnf(prove_p23x, negated_conjecture)

GRP187-1.p Orthogonal elements commute
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{greatest\_lower\_bound}(\text{least\_upper\_bound}(a, a'), \text{least\_upper\_bound}(b, b')) = \text{identity}$ cnf(p33_1, hypothesis)
$a \cdot b \neq b \cdot a$ cnf(prove_p_{33}, negated_conjecture)

GRP188-1.p Consequence of lattice theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{least\_upper\_bound}(b, \text{least\_upper\_bound}(a, b)) \neq \text{least\_upper\_bound}(a, b)$ cnf(prove_p38a, negated_conjecture)

GRP188-2.p Consequence of lattice theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
$\text{identity}' = \text{identity}$ cnf(p38a_1, hypothesis)
$(x')' = x$ cnf(p38a_2, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p38a_3, hypothesis)
$\text{least\_upper\_bound}(b, \text{least\_upper\_bound}(a, b)) \neq \text{least\_upper\_bound}(a, b)$ cnf(prove_p38a, negated_conjecture)

GRP189-1.p Consequence of lattice theory
include('Axioms/GRP004-0.ax')

include('Axioms/GRP004-2.ax')
greatest_lower_bound(b, least_upper_bound(a, b)) $\neq b$ cnf(prove_p38b, negated_conjecture)

GRP189-2.p Consequence of lattice theory
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
identity$'$ = identity cnf(p38b$_1$, hypothesis)
$(x')' = x$ cnf(p38b$_2$, hypothesis)
$(x \cdot y)' = y' \cdot x'$ cnf(p38b$_3$, hypothesis)
greatest_lower_bound(b, least_upper_bound(a, b)) $\neq b$ cnf(prove_p38b, negated_conjecture)

GRP190-1.p Something useful for estimations
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) $= a$ cnf(p39a$_1$, hypothesis)
least_upper_bound(a', b') $\neq b'$ cnf(prove_p39a, negated_conjecture)

GRP190-2.p Something useful for estimations
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(a, b) $= a$ cnf(p39c$_1$, hypothesis)
greatest_lower_bound(a', b') $\neq a'$ cnf(prove_p39c, negated_conjecture)

GRP191-1.p Something useful for estimations
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) $= b$ cnf(p39b$_1$, hypothesis)
greatest_lower_bound(a', b') $\neq a'$ cnf(prove_p39b, negated_conjecture)

GRP191-2.p Something useful for estimations
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(a, b) $= b$ cnf(p39d$_1$, hypothesis)
least_upper_bound(a', b') $\neq b'$ cnf(prove_p39d, negated_conjecture)

GRP192-1.p Even elements implies trivial group
The assumption all(X,1 =< X) even implies that the group is trivial, i.e., all(X, X = 1).
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(identity, x) $= x$ cnf(p40a$_1$, hypothesis)
$a \cdot b \neq b \cdot a$ cnf(prove_p40a, negated_conjecture)

GRP193-1.p A combination of distributivity and monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
least_upper_bound(identity, a) $= a$ cnf(p8_9a$_1$, hypothesis)
least_upper_bound(identity, b) $= b$ cnf(p8_9a$_2$, hypothesis)
least_upper_bound(identity, c) $= c$ cnf(p8_9a$_3$, hypothesis)
greatest_lower_bound(a, b) = identity cnf(p8_9a$_4$, hypothesis)
least_upper_bound(greatest_lower_bound($a, b \cdot c$), greatest_lower_bound(a, b)$\cdot$greatest_lower_bound(a, c)) = greatest_lower_boun
greatest_lower_bound(a, c) cnf(p8_9a$_5$, hypothesis)
greatest_lower_bound($a, b \cdot c$) $\neq$ greatest_lower_bound(a, c) cnf(prove_p8_9a, negated_conjecture)

GRP193-2.p A combination of distributivity and monotonicity
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')
greatest_lower_bound(identity, a) = identity cnf(p8_9b$_1$, hypothesis)
greatest_lower_bound(identity, b) = identity cnf(p8_9b$_2$, hypothesis)
greatest_lower_bound(identity, c) = identity cnf(p8_9b$_3$, hypothesis)
greatest_lower_bound(a, b) = identity cnf(p8_9b$_4$, hypothesis)
greatest_lower_bound(greatest_lower_bound($a, b \cdot c$), greatest_lower_bound(a, b)$\cdot$greatest_lower_bound(a, c)) = greatest_lower_bo
c) cnf(p8_9b$_5$, hypothesis)
greatest_lower_bound($a, b \cdot c$) $\neq$ greatest_lower_bound(a, c) cnf(prove_p8_9b, negated_conjecture)

GRP194+1.p In semigroups, a surjective homomorphism maps the zero

If (F,*) and (H,+) are two semigroups, phi is a surjective homomorphism from F to H, and id is a left zero for F, then phi(id) is a left zero for H.
include('Axioms/GRP007+0.ax')
$\forall x$: (group_member(x, f) $\Rightarrow$ group_member(phi$(x), h$)) fof($homomorphism_1$, axiom)
$\forall x, y$: ((group_member(x, f) and group_member(y, f)) $\Rightarrow$ $m(h, \text{phi}(x), \text{phi}(y)) = \text{phi}(m(f, x, y))$) fof($homomorphism_2$, ax
$\forall x$: (group_member(x, h) $\Rightarrow$ $\exists y$: (group_member(y, f) and phi$(y) = x$)) fof(surjective, axiom)
$\forall g, x$: (left_zero(g, x) $\iff$ (group_member(x, g) and $\forall y$: (group_member(y, g) $\Rightarrow$ $m(g, x, y) = x$))) fof(left_zero, axiom)
left_zero(f, f_left_zero) fof(left_zero_for_f, hypothesis)
left_zero(h, phi(f_left_zero)) fof(prove_left_zero_h, conjecture)

GRP195-1.p In semigroups, xyy=yyx → (uv)∧4 = u∧4v∧4.
In semigroups, xyy=yyx → uvuvuvuuv=uuuuvvvv.
include('Axioms/GRP008-0.ax')
$x \cdot (y \cdot y) = y \cdot (y \cdot x)$ cnf(condition, hypothesis)
$a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot b)))))) \neq a \cdot (a \cdot (a \cdot (a \cdot (b \cdot (b \cdot (b \cdot b))))))$ cnf(prove_this, negated_conjecture)

GRP196-1.p In semigroups, xyyy=yyyx → (uy)∧9 = u∧9v∧9.
include('Axioms/GRP008-0.ax')
$x \cdot (y \cdot (y \cdot y)) = y \cdot (y \cdot (y \cdot x))$ cnf(condition, hypothesis)
$a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot (b \cdot (a \cdot b)))))))))))))))) \neq a \cdot (a \cdot (a \cdot (a \cdot (a \cdot (a \cdot (a \cdot (a \cdot (a \cdot (b \cdot (b \cdot$
$(b \cdot (b \cdot (b \cdot (b \cdot (b \cdot (b \cdot b))))))))))))))))$ cnf(prove_this, negated_conjecture)

GRP197-1.p In cancellative semigroups, xxxyy=yxxyx → bbaaa = abaab.
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$x \cdot (x \cdot (x \cdot (y \cdot y))) = y \cdot (x \cdot (x \cdot (y \cdot x)))$ cnf(condition, hypothesis)
$b \cdot (b \cdot (a \cdot (a \cdot a))) \neq a \cdot (b \cdot (a \cdot (a \cdot b)))$ cnf(prove_this, negated_conjecture)

GRP198-1.p In cancellative semigroups, xyyyxy=yyyyxx → babbba=aabbbb.
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$x \cdot (y \cdot (y \cdot (y \cdot (x \cdot y)))) = y \cdot (y \cdot (y \cdot (y \cdot (x \cdot x))))$ cnf(condition, hypothesis)
$b \cdot (a \cdot (b \cdot (b \cdot (b \cdot a)))) \neq a \cdot (a \cdot (b \cdot (b \cdot (b \cdot b))))$ cnf(prove_this, negated_conjecture)

GRP199-1.p Nilpotent CS satisfy the quotient condition.
Nilpotent cancellative semigroups satisfy the quotient condition.
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$x \cdot (y \cdot (z \cdot (y \cdot x))) = y \cdot (x \cdot (z \cdot (x \cdot y)))$ cnf(nilpotency, hypothesis)
$b \cdot b_0 = a \cdot a_0$ cnf($prove\_quotient_1$, negated_conjecture)
$d \cdot b_0 = c \cdot a_0$ cnf($prove\_quotient_2$, negated_conjecture)
$b \cdot d_0 = a \cdot c_0$ cnf($prove\_quotient_3$, negated_conjecture)
$d \cdot d_0 \neq c \cdot c_0$ cnf($prove\_quotient_4$, negated_conjecture)

GRP200-1.p In Loops, Moufang-1 => Moufang-2.
identity $\cdot x = x$ cnf(left_identity, axiom)
$x \cdot$ identity $= x$ cnf(right_identity, axiom)
$x \cdot$ left_division$(x, y) = y$ cnf(multiply_left_division, axiom)
left_division$(x, x \cdot y) = y$ cnf(left_division_multiply, axiom)
right_division$(x, y) \cdot y = x$ cnf(multiply_right_division, axiom)
right_division$(x \cdot y, y) = x$ cnf(right_division_multiply, axiom)
$x \cdot$ right_inverse$(x) =$ identity cnf(right_inverse, axiom)
left_inverse$(x) \cdot x =$ identity cnf(left_inverse, axiom)
$(x \cdot (y \cdot z)) \cdot x = (x \cdot y) \cdot (z \cdot x)$ cnf($moufang_1$, axiom)
$((a \cdot b) \cdot c) \cdot b \neq a \cdot (b \cdot (c \cdot b))$ cnf($prove\_moufang_2$, negated_conjecture)

GRP201-1.p In Loops, Moufang-2 => Moufang-3.
identity $\cdot x = x$ cnf(left_identity, axiom)
$x \cdot$ identity $= x$ cnf(right_identity, axiom)
$x \cdot$ left_division$(x, y) = y$ cnf(multiply_left_division, axiom)
left_division$(x, x \cdot y) = y$ cnf(left_division_multiply, axiom)
right_division$(x, y) \cdot y = x$ cnf(multiply_right_division, axiom)
right_division$(x \cdot y, y) = x$ cnf(right_division_multiply, axiom)

$x \cdot \text{right\_inverse}(x) = \text{identity}$ cnf(right_inverse, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$((x \cdot y) \cdot z) \cdot y = x \cdot (y \cdot (z \cdot y))$ cnf(moufang$_2$, axiom)
$((a \cdot b) \cdot a) \cdot c \neq a \cdot (b \cdot (a \cdot c))$ cnf(prove_moufang$_3$, negated_conjecture)

GRP202-1.p In Loops, Moufang-3 => Moufang-1.
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot \text{left\_division}(x, y) = y$ cnf(multiply_left_division, axiom)
$\text{left\_division}(x, x \cdot y) = y$ cnf(left_division_multiply, axiom)
$\text{right\_division}(x, y) \cdot y = x$ cnf(multiply_right_division, axiom)
$\text{right\_division}(x \cdot y, y) = x$ cnf(right_division_multiply, axiom)
$x \cdot \text{right\_inverse}(x) = \text{identity}$ cnf(right_inverse, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$((x \cdot y) \cdot x) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(moufang$_3$, axiom)
$(a \cdot (b \cdot c)) \cdot a \neq (a \cdot b) \cdot (c \cdot a)$ cnf(prove_moufang$_1$, negated_conjecture)

GRP203-1.p Left identity, left inverse, Moufang-3 => Moufang-2
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$((x \cdot y) \cdot x) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(moufang$_3$, axiom)
$((a \cdot b) \cdot c) \cdot b \neq a \cdot (b \cdot (c \cdot b))$ cnf(prove_moufang$_2$, negated_conjecture)

GRP204-1.p A non-basis for Moufang loops.
Left identity, left inverse, Moufang-1 do not imply Moufang-2; that is, is not a basis for Moufang loops.
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$(x \cdot (y \cdot z)) \cdot x = (x \cdot y) \cdot (z \cdot x)$ cnf(moufang$_1$, axiom)
$((a \cdot b) \cdot c) \cdot b \neq a \cdot (b \cdot (c \cdot b))$ cnf(prove_moufang$_2$, negated_conjecture)

GRP205-1.p In Loops, Moufang-3 => Moufang-4.
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot \text{left\_division}(x, y) = y$ cnf(multiply_left_division, axiom)
$\text{left\_division}(x, x \cdot y) = y$ cnf(left_division_multiply, axiom)
$\text{right\_division}(x, y) \cdot y = x$ cnf(multiply_right_division, axiom)
$\text{right\_division}(x \cdot y, y) = x$ cnf(right_division_multiply, axiom)
$x \cdot \text{right\_inverse}(x) = \text{identity}$ cnf(right_inverse, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$((x \cdot y) \cdot x) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(moufang$_3$, axiom)
$x \cdot ((y \cdot z) \cdot x) \neq (x \cdot y) \cdot (z \cdot x)$ cnf(prove_moufang$_4$, negated_conjecture)

GRP206-1.p In Loops, Moufang-4 => Moufang-1.
$\text{identity} \cdot x = x$ cnf(left_identity, axiom)
$x \cdot \text{identity} = x$ cnf(right_identity, axiom)
$x \cdot \text{left\_division}(x, y) = y$ cnf(multiply_left_division, axiom)
$\text{left\_division}(x, x \cdot y) = y$ cnf(left_division_multiply, axiom)
$\text{right\_division}(x, y) \cdot y = x$ cnf(multiply_right_division, axiom)
$\text{right\_division}(x \cdot y, y) = x$ cnf(right_division_multiply, axiom)
$x \cdot \text{right\_inverse}(x) = \text{identity}$ cnf(right_inverse, axiom)
$\text{left\_inverse}(x) \cdot x = \text{identity}$ cnf(left_inverse, axiom)
$x \cdot ((y \cdot z) \cdot x) = (x \cdot y) \cdot (z \cdot x)$ cnf(moufang$_4$, axiom)
$(a \cdot (b \cdot c)) \cdot a \neq (a \cdot b) \cdot (c \cdot a)$ cnf(prove_moufang$_1$, negated_conjecture)

GRP207-1.p Single non-axiom for group theory, in product & inverse
This is a single axiom for group theory, in terms of product and inverse.
$u \cdot (y \cdot (((z \cdot z') \cdot (u \cdot y)') \cdot u))' = u$ cnf(single_non_axiom, axiom)
$x \cdot (y \cdot (((z \cdot z') \cdot (u \cdot y)') \cdot x))' \neq u$ cnf(try_prove_this_axiom, negated_conjecture)

GRP211-1.p An identity generated by HR, number 00349
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this$_2$, negated_conjecture)

$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_3, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_4, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_5, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_6, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_7, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_8, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_9, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{10}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_{11}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_{12}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{13}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{14}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{15}, negated_conjecture)
$\text{sk\_c}_3' = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_{16}, negated_conjecture)
$\text{sk\_c}_3' = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_{17}, negated_conjecture)
$\text{sk\_c}_3' = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{18}, negated_conjecture)
$\text{sk\_c}_3' = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{19}, negated_conjecture)
$\text{sk\_c}_3' = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{20}, negated_conjecture)
$\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_{21}, negated_conjecture)
$\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_{22}, negated_conjecture)
$\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{23}, negated_conjecture)
$\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{24}, negated_conjecture)
$\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{25}, negated_conjecture)
($x_4 \cdot x_5 = \text{sk\_c}_8$ and $x_4' = x_5$ and $x_5 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ and $x_6' = \text{sk\_c}_8$ and $x_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ and $x_1' = \text{sk\_c}_8$ and $x_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ and $\text{sk\_c}_8 \cdot x_2 = \text{sk\_c}_7$ and $x_3 \cdot \text{sk\_c}_8 = x_2$) $\Rightarrow$ $x_3' \neq \text{sk\_c}_8$ cnf(prove_this_{26}, negated_conjecture)

GRP213-1.p An identity generated by HR, number 00385
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_8' = \text{sk\_c}_7$ cnf(prove_this_1, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_2, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_3, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_4, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_5, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_6, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_8' = \text{sk\_c}_7$ cnf(prove_this_7, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_8, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_9, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{10}, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{11}, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{12}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_8' = \text{sk\_c}_7$ cnf(prove_this_{13}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_{14}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_{15}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{16}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{17}, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_3 = \text{sk\_c}_8$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{18}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_8' = \text{sk\_c}_7$ cnf(prove_this_{19}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this_{20}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ cnf(prove_this_{21}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this_{22}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_8 = \text{sk\_c}_6$ cnf(prove_this_{23}, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_3$ or $\text{sk\_c}_5' = \text{sk\_c}_8$ cnf(prove_this_{24}, negated_conjecture)
($x_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ and $x_4' = \text{sk\_c}_8$ and $x_5 \cdot x_6 = \text{sk\_c}_8$ and $x_5' = x_6$ and $\text{sk\_c}_8' = \text{sk\_c}_7$ and $x_1' = \text{sk\_c}_8$ and $x_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ and $\text{sk\_c}_8 \cdot x_2 = \text{sk\_c}_7$ and $x_3 \cdot \text{sk\_c}_8 = x_2$) $\Rightarrow$ $x_3' \neq \text{sk\_c}_8$ cnf(prove_this_{25}, negated_conjecture)

GRP214-1.p An identity generated by HR, number 00387
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this_1, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this_2, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this_3, negated_conjecture)

$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_{12}$, negated_conjecture)
$(x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot x_4 = \mathrm{sk\_c}_6$ and $x_2 \cdot \mathrm{sk\_c}_7 = x_1$ and $x_2' = \mathrm{sk\_c}_7$ and $x_5 \cdot x_1 = \mathrm{sk\_c}_6) \Rightarrow x_5' \neq x_1$ cnf(prove_this$_{13}$, negated_conjecture)

GRP215-1.p An identity generated by HR, number 00396
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
$(x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $x_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_6$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot x_2 = \mathrm{sk\_c}_6$ and $x_5 \cdot \mathrm{sk\_c}_7 = x_2) \Rightarrow x_5' \neq \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)

GRP216-1.p An identity generated by HR, number 00407
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_6$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_6$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_6$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)

$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{22}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{23}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{24}$, negated_conjecture)
($x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $x_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_6$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot x_2 = \mathrm{sk\_c}_6$ and $x_5 \cdot \mathrm{sk\_c}_7 = x_2$) $\Rightarrow$ $x_5' \neq \mathrm{sk\_c}_7$ cnf(prove_this$_{25}$, negated_conjecture)

GRP222-1.p An identity generated by HR, number 00453
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
($x_3' = \mathrm{sk\_c}_7$ and $x_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_4 \cdot x_5 = \mathrm{sk\_c}_6$ and $x_4' = x_5$ and $x_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_1' = \mathrm{sk\_c}_7$ and $x_2' = \mathrm{sk\_c}_7$) $\Rightarrow$ $x_2 \cdot \mathrm{sk\_c}_6 \neq \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)

GRP226-1.p An identity generated by HR, number 00460
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_6$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_3 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_6$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_3$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
($x_3' = \mathrm{sk\_c}_7$ and $x_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot x_4 = \mathrm{sk\_c}_6$ and $x_5 \cdot \mathrm{sk\_c}_7 = x_4$ and $x_5' = \mathrm{sk\_c}_7$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$) $\Rightarrow$ $x_2' \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{21}$, negated_conjecture)

GRP230-1.p An identity generated by HR, number 00466
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_8$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_3$, negated_conjecture)

$sk\_c_1' = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_8$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_8$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_8$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{24}$, negated_conjecture)
$(x_3' = sk\_c_8$ and $x_3 \cdot sk\_c_7 = sk\_c_8$ and $x_4 \cdot sk\_c_7 = sk\_c_8$ and $x_4' = sk\_c_7$ and $x_2 \cdot x_1 = sk\_c_8$ and $x_2' = x_1$ and $x_1 \cdot sk\_c_7 = sk\_c_8$ and $sk\_c_8 \cdot x_5 = sk\_c_7$ and $x_6 \cdot sk\_c_8 = x_5) \Rightarrow x_6' \neq sk\_c_8$ cnf(prove_this$_{25}$, negated_conjecture)

GRP234-1.p An identity generated by HR, number 00574
include('Axioms/GRP004-0.ax')
$sk\_c_7' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_7' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_7' = sk\_c_6$ or $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_7' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_7' = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{20}$, negated_conjecture)
$(sk\_c_7' = sk\_c_6$ and $x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_4 \cdot sk\_c_7 = sk\_c_6$ and $x_4' = sk\_c_7$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $sk\_c_7 \cdot x_2 = sk\_c_6$ and $x_5 \cdot sk\_c_7 = x_2) \Rightarrow x_5' \neq sk\_c_7$ cnf(prove_this$_{21}$, negated_conjecture)

GRP236-1.p An identity generated by HR, number 00602
include('Axioms/GRP004-0.ax')
$sk\_c_8' = sk\_c_7$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_9$, negated_conjecture)

$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_3 = sk\_c_7$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_3 = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_3 = sk\_c_7$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_3 = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_3 = sk\_c_7$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2' = sk\_c_3$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2' = sk\_c_3$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2' = sk\_c_3$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2' = sk\_c_3$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_3$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_8 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_5' = sk\_c_8$ cnf(prove_this$_{26}$, negated_conjecture)
($sk\_c_8' = sk\_c_7$ and $x_4' = sk\_c_8$ and $x_4 \cdot sk\_c_7 = sk\_c_8$ and $x_5 \cdot x_6 = sk\_c_7$ and $x_5' = x_6$ and $x_6 \cdot sk\_c_8 = sk\_c_7$ and $x_1' = sk\_c_8$ and $x_1 \cdot sk\_c_7 = sk\_c_8$ and $sk\_c_8 \cdot x_2 = sk\_c_7$ and $x_3 \cdot sk\_c_8 = x_2$) $\Rightarrow$ $x_3' \neq sk\_c_8$ cnf(prove_this$_{27}$, negated_conjecture)

GRP253-1.p An identity generated by HR, number 00722
include('Axioms/GRP004-0.ax')
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_3' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_3' = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_3' = sk\_c_6$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_4' = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_5' = sk\_c_6$ cnf(prove_this$_{18}$, negated_conjecture)
($x_2 \cdot sk\_c_7 = sk\_c_6$ and $x_2' = sk\_c_7$ and $x_3 \cdot sk\_c_6 = sk\_c_5$ and $x_3' = sk\_c_6$ and $x_4' = sk\_c_6$ and $x_4 \cdot sk\_c_5 = sk\_c_6$ and $x_1 \cdot sk\_c_7 = sk\_c_6$ and $x_1' = sk\_c_7$) $\Rightarrow$ $sk\_c_5' \neq sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)

GRP262-1.p An identity generated by HR, number 00951
include('Axioms/GRP004-0.ax')
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)

$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_6$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{21}$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ cnf(prove_this$_{22}$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{23}$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_6$ cnf(prove_this$_{24}$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{25}$, negated_conjecture)
$(x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $x_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ and $x_4' = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_2' = \mathrm{sk\_c}_6) \Rightarrow x_2 \cdot \mathrm{sk\_c}_5 \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP264-1.p An identity generated by HR, number 01099
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{1}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{2}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{3}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{4}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_8 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{5}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_6$ cnf(prove_this$_{6}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{7}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{8}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{9}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_8 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_8 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{21}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{22}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{23}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{24}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{25}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_8 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{26}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_6$ cnf(prove_this$_{27}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{28}$, negated_conjecture)
$(x_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ and $x_3' = \mathrm{sk\_c}_9$ and $x_4 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_8$ and $x_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ and $x_1' = \mathrm{sk\_c}_9$ and $x_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ and $x_2' = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_8 \cdot x_5 = \mathrm{sk\_c}_7$ and $x_6 \cdot \mathrm{sk\_c}_8 = x_5) \Rightarrow x_6' \neq \mathrm{sk\_c}_8$ cnf(prove_this$_{29}$, negated_conjecture

GRP265-1.p An identity generated by HR, number 01100
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{1}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{2}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{3}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{4}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_8$ cnf(prove_this$_{5}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{6}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{7}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{8}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{9}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_8$ cnf(prove_this$_{12}$, negated_conjecture)

$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_9$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_8$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ cnf(prove_this$_{22}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_9$ cnf(prove_this$_{23}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ cnf(prove_this$_{24}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_7$ cnf(prove_this$_{25}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_8$ cnf(prove_this$_{26}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{27}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_7$ cnf(prove_this$_{28}$, negated_conjecture)
$(x_3 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ and $x_3' = \mathrm{sk\_c}_9$ and $x_4 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_8$ and $x_1 \cdot \mathrm{sk\_c}_9 = \mathrm{sk\_c}_8$ and $x_1' = \mathrm{sk\_c}_9$ and $x_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_8$ and $x_2' = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot x_5 = \mathrm{sk\_c}_8$ and $x_6 \cdot \mathrm{sk\_c}_7 = x_5) \Rightarrow x_6' \neq \mathrm{sk\_c}_7$ cnf(prove_this$_{29}$, negated_conjecture

GRP267-1.p An identity generated by HR, number 01105
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{1}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{2}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{3}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{4}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{5}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{6}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{7}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{8}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{9}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_3' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)
$(x_2 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ and $x_2' = \mathrm{sk\_c}_8$ and $x_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_3' = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_7 \cdot x_4 = \mathrm{sk\_c}_6$ and $x_5 \cdot \mathrm{sk\_c}_7 = x_4$ and $x_5' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ and $x_1' = \mathrm{sk\_c}_8) \Rightarrow \mathrm{sk\_c}_6' \neq \mathrm{sk\_c}_7$ cnf(prove_this$_{22}$, negated_conjecture)

GRP268-1.p An identity generated by HR, number 01106
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{1}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{2}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{3}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{4}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{5}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_8$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{6}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{7}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{8}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{9}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5' = \mathrm{sk\_c}_8$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_8 = \mathrm{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)

sk_c$_6$ · sk_c$_4$ = sk_c$_7$ or sk_c$'_5$ = sk_c$_8$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_6$ · sk_c$_4$ = sk_c$_7$ or sk_c$'_6$ = sk_c$_7$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_3$ · sk_c$_6$ = sk_c$_4$ or sk_c$_5$ · sk_c$_8$ = sk_c$_7$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_3$ · sk_c$_6$ = sk_c$_4$ or sk_c$'_5$ = sk_c$_8$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$_3$ · sk_c$_6$ = sk_c$_4$ or sk_c$'_6$ = sk_c$_7$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$'_3$ = sk_c$_6$ or sk_c$_5$ · sk_c$_8$ = sk_c$_7$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$'_3$ = sk_c$_6$ or sk_c$'_5$ = sk_c$_8$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$'_3$ = sk_c$_6$ or sk_c$'_6$ = sk_c$_7$ cnf(prove_this$_{21}$, negated_conjecture)
(x_2 · sk_c$_8$ = sk_c$_7$ and x'_2 = sk_c$_8$ and x_3 · sk_c$_6$ = sk_c$_7$ and x'_3 = sk_c$_6$ and sk_c$_6$ · x_4 = sk_c$_7$ and x_5 · sk_c$_6$ = x_4 and x'_5 = sk_c$_6$ and x_1 · sk_c$_8$ = sk_c$_7$ and x'_1 = sk_c$_8$) $\Rightarrow$ sk_c$'_6$ $\neq$ sk_c$_7$ cnf(prove_this$_{22}$, negated_conjecture)

GRP270-1.p An identity generated by HR, number 01179
include('Axioms/GRP004-0.ax')
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_5$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_6$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_7$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_9$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ · sk_c$_7$ = sk_c$_5$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ · sk_c$_7$ = sk_c$_5$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$_2$ · sk_c$_7$ = sk_c$_5$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$_2$ · sk_c$_7$ = sk_c$_5$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$_2$ · sk_c$_7$ = sk_c$_5$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$'_2$ = sk_c$_7$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{21}$, negated_conjecture)
sk_c$'_2$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{22}$, negated_conjecture)
sk_c$'_2$ = sk_c$_7$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{23}$, negated_conjecture)
sk_c$'_2$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{24}$, negated_conjecture)
sk_c$'_2$ = sk_c$_7$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{25}$, negated_conjecture)
(x_3 · sk_c$_7$ = sk_c$_6$ and x'_3 = sk_c$_7$ and sk_c$_7$ · sk_c$_5$ = sk_c$_6$ and x_4 · sk_c$_7$ = sk_c$_5$ and x'_4 = sk_c$_7$ and sk_c$_6$ · sk_c$_7$ = sk_c$_5$ and x'_1 = sk_c$_7$ and x_1·sk_c$_6$ = sk_c$_7$ and x'_2 = sk_c$_6$) $\Rightarrow$ x_2·sk_c$_5$ $\neq$ sk_c$_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP272-1.p An identity generated by HR, number 01392
include('Axioms/GRP004-0.ax')
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_5$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_6$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_7$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_9$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_7$ · sk_c$_5$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_5$ or sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_5$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{17}$, negated_conjecture)

$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
$(x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ and $x_4 \cdot sk\_c_6 = sk\_c_5$ and $x_4' = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_2' = sk\_c_6) \Rightarrow x_2 \cdot sk\_c_5 \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP274-1.p An identity generated by HR, number 01608
include('Axioms/GRP004-0.ax')
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
$(x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ and $x_4 \cdot sk\_c_5 = sk\_c_7$ and $x_4' = sk\_c_5$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_2' = sk\_c_6) \Rightarrow x_2 \cdot sk\_c_5 \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP277-1.p An identity generated by HR, number 01755
include('Axioms/GRP004-0.ax')
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_5 = sk\_c_6$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{17}$, negated_conjecture)

$\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_2' = \text{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_2 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_4 = \text{sk\_c}_6$ cnf(prove_this$_{22}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_4$ cnf(prove_this$_{23}$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{24}$, negated_conjecture)
$(x_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ and $x_2' = \text{sk\_c}_7$ and $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ and $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ and $x_1' = \text{sk\_c}_7$ and $x_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $x_3 \cdot x_4 = \text{sk\_c}_6$ and $x_3' = x_4) \Rightarrow x_4 \cdot \text{sk\_c}_7 \neq \text{sk\_c}_6$ cnf(prove_this$_{25}$, negated_conjecture)

GRP278-1.p An identity generated by HR, number 02518
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ cnf(prove_this$_{1}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_2' = \text{sk\_c}_6$ cnf(prove_this$_{2}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{3}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_5$ cnf(prove_this$_{4}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ cnf(prove_this$_{5}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ cnf(prove_this$_{6}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_2' = \text{sk\_c}_6$ cnf(prove_this$_{7}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{8}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_5$ cnf(prove_this$_{9}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_4' = \text{sk\_c}_5$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_4' = \text{sk\_c}_5$ or $\text{sk\_c}_2' = \text{sk\_c}_6$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_4' = \text{sk\_c}_5$ or $\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_4' = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_5$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_4' = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ or $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ or $\text{sk\_c}_2' = \text{sk\_c}_6$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ or $\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_5$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ cnf(prove_this$_{20}$, negated_conjecture)
$(x_2 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ and $x_2' = \text{sk\_c}_6$ and $\text{sk\_c}_4' = \text{sk\_c}_5$ and $\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ and $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ and $x_1' = \text{sk\_c}_6$ and $x_1 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ and $x_3' = \text{sk\_c}_5) \Rightarrow x_3 \cdot \text{sk\_c}_4 \neq \text{sk\_c}_5$ cnf(prove_this$_{21}$, negated_conjecture)

GRP284-1.p An identity generated by HR, number 02999
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{1}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{2}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{3}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{4}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{5}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{6}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{7}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{8}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{9}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_{21}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{22}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{23}$, negated_conjecture)

$sk\_c_2' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
$(sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $x_4 \cdot sk\_c_7 = sk\_c_5$ and $x_4' = sk\_c_7$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_2' = sk\_c_6) \Rightarrow x_2 \cdot sk\_c_5 \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP287-1.p An identity generated by HR, number 03165
include('Axioms/GRP004-0.ax')
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_6$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_6$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_6$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3' = sk\_c_6$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
$(sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $x_4 \cdot sk\_c_6 = sk\_c_5$ and $x_4' = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_1' = sk\_c_6$ and $x_2' = sk\_c_6) \Rightarrow x_2 \cdot sk\_c_5 \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP288-1.p An identity generated by HR, number 03168
include('Axioms/GRP004-0.ax')
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{23}$, negated_conjecture)

$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
($sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $x_4 \cdot sk\_c_6 = sk\_c_5$ and $x_4' = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_2 \cdot sk\_c_6 = sk\_c_7$) $\Rightarrow$ $x_2' \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP289-1.p An identity generated by HR, number 03169
include('Axioms/GRP004-0.ax')
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)
($sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $x_3 \cdot sk\_c_7 = sk\_c_6$ and $x_3' = sk\_c_7$ and $x_4 \cdot sk\_c_6 = sk\_c_5$ and $x_4' = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_2' = sk\_c_6$) $\Rightarrow$ $x_2 \cdot sk\_c_5 \neq sk\_c_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP291-1.p An identity generated by HR, number 03387
include('Axioms/GRP004-0.ax')
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{1}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{2}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{3}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{4}$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{5}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{6}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{7}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{8}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{9}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_5 = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_2' = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{23}$, negated_conjecture)

$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_6$ cnf(prove_this$_{24}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{25}$, negated_conjecture)
($\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ and $x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $x_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_5$ and $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_2' = \mathrm{sk\_c}_6$) $\Rightarrow$ $x_2 \cdot \mathrm{sk\_c}_5 \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP292-1.p An identity generated by HR, number 03445
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)
($\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ and $x_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_3' = \mathrm{sk\_c}_7$ and $x_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ and $x_4' = \mathrm{sk\_c}_5$ and $\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_2 \cdot x_1 = \mathrm{sk\_c}_6$ and $x_2' = x_1$) $\Rightarrow$ $x_1 \cdot \mathrm{sk\_c}_5 \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{21}$, negated_conjecture)

GRP297-1.p An identity generated by HR, number 03730
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_2' = \mathrm{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf(prove_this$_{22}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_4$ cnf(prove_this$_{23}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ cnf(prove_this$_{24}$, negated_conjecture)
($\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ and $x_2 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_6$ and $x_2' = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_1' = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_3 \cdot x_4 = \mathrm{sk\_c}_6$ and $x_3' = x_4$) $\Rightarrow$ $x_4 \cdot \mathrm{sk\_c}_7 \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{25}$, negated_conjecture)

GRP299-1.p An identity generated by HR, number 03746
include('Axioms/GRP004-0.ax')

$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{24}$, negated_conjecture)
($sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $x_2 \cdot sk\_c_7 = sk\_c_6$ and $x_2' = sk\_c_7$ and $sk\_c_5 \cdot sk\_c_7 = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $x_1' = sk\_c_7$ and $x_1 \cdot sk\_c_6 = sk\_c_7$ and $x_3 \cdot x_4 = sk\_c_6$ and $x_3' = x_4$) $\Rightarrow$ $x_4 \cdot sk\_c_7 \neq sk\_c_6$ cnf(prove_this$_{25}$, negated_conjecture)

GRP300-1.p An identity generated by HR, number 03748
include('Axioms/GRP004-0.ax')
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_6' = sk\_c_4$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6' = sk\_c_4$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_6' = sk\_c_4$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_6' = sk\_c_4$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_4$ cnf(prove_this$_{24}$, negated_conjecture)
($sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ and $x_2 \cdot sk\_c_6 = sk\_c_5$ and $x_2' = sk\_c_6$ and $sk\_c_4 \cdot sk\_c_6 = sk\_c_5$ and $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ and $x_1 \cdot sk\_c_6 = sk\_c_5$ and $x_1' = sk\_c_6$ and $sk\_c_6' = sk\_c_4$ and $x_3 \cdot sk\_c_4 = sk\_c_5$) $\Rightarrow$ $x_3' \neq sk\_c_4$ cnf(prove_this$_{25}$, negated_conjecture)

GRP302-1.p An identity generated by HR, number 04549
include('Axioms/GRP004-0.ax')
$sk\_c_8 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_8' = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)

sk_c$'_1$ = sk_c$_8$ or sk_c$'_3$ = sk_c$_8$ cnf(prove_this$_3$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_3$ · sk_c$_7$ = sk_c$_8$ cnf(prove_this$_4$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$'_5$ = sk_c$_4$ cnf(prove_this$_5$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_6$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_5$ · sk_c$_8$ = sk_c$_4$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$'_8$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$'_3$ = sk_c$_8$ cnf(prove_this$_9$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$_3$ · sk_c$_7$ = sk_c$_8$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$'_5$ = sk_c$_4$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_8$ or sk_c$_5$ · sk_c$_8$ = sk_c$_4$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$'_8$ = sk_c$_7$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_8$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$_3$ · sk_c$_7$ = sk_c$_8$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$'_5$ = sk_c$_4$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$_2$ · sk_c$_8$ = sk_c$_6$ or sk_c$_5$ · sk_c$_8$ = sk_c$_4$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$'_8$ = sk_c$_7$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$'_3$ = sk_c$_8$ cnf(prove_this$_{21}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$_3$ · sk_c$_7$ = sk_c$_8$ cnf(prove_this$_{22}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$'_5$ = sk_c$_4$ cnf(prove_this$_{23}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_{24}$, negated_conjecture)
sk_c$'_2$ = sk_c$_8$ or sk_c$_5$ · sk_c$_8$ = sk_c$_4$ cnf(prove_this$_{25}$, negated_conjecture)
(sk_c$_8$ · sk_c$_7$ = sk_c$_6$ and x'_3 = sk_c$_8$ and x_3 · sk_c$_7$ = sk_c$_8$ and x_4 · sk_c$_8$ = sk_c$_6$ and x'_4 = sk_c$_8$ and sk_c$'_8$ = sk_c$_7$ and x'_1 = sk_c$_8$ and x_1·sk_c$_7$ = sk_c$_8$ and $x'_2 = x_5$ and x'_5 = sk_c$_8$) $\Rightarrow$ x_2·sk_c$_8$ $\neq x_5$ cnf(prove_this$_{26}$, negated_conje

GRP303-1.p An identity generated by HR, number 04703
include('Axioms/GRP004-0.ax')
sk_c$_7$ · sk_c$_6$ = sk_c$_5$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_7$ · sk_c$_6$ = sk_c$_5$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_7$ · sk_c$_6$ = sk_c$_5$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_7$ · sk_c$_6$ = sk_c$_5$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_6$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ · sk_c$_7$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_9$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_7$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ · sk_c$_6$ = sk_c$_7$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$'_2$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$'_2$ = sk_c$_6$ or sk_c$_3$ · sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$'_2$ = sk_c$_6$ or sk_c$'_4$ = sk_c$_6$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$'_2$ = sk_c$_6$ or sk_c$_4$ · sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{21}$, negated_conjecture)
(sk_c$_7$ · sk_c$_6$ = sk_c$_5$ and sk_c$_6$ · sk_c$_7$ = sk_c$_5$ and x_3 · sk_c$_7$ = sk_c$_6$ and x'_3 = sk_c$_7$ and x_4 · sk_c$_6$ = sk_c$_7$ and x'_4 = sk_c$_6$ and x'_1 = sk_c$_7$ and x_1·sk_c$_6$ = sk_c$_7$ and x'_2 = sk_c$_6$) $\Rightarrow$ x_2·sk_c$_5$ $\neq$ sk_c$_6$ cnf(prove_this$_{22}$, negated_conjecture)

GRP304-1.p An identity generated by HR, number 05204
include('Axioms/GRP004-0.ax')
sk_c$_7$ · sk_c$_6$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ or sk_c$_2$ · sk_c$_7$ = sk_c$_6$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ or sk_c$'_2$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ or sk_c$_7$ · sk_c$_4$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ or sk_c$_3$ · sk_c$_7$ = sk_c$_4$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_6$ · sk_c$_7$ = sk_c$_5$ or sk_c$'_3$ = sk_c$_7$ cnf(prove_this$_6$, negated_conjecture)

$sk\_c_7' = sk\_c_5$ or $sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_7' = sk\_c_5$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_7' = sk\_c_5$ or $sk\_c_7 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_7' = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_7 = sk\_c_4$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_7' = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_7$ or $sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_7$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_7$ or $sk\_c_7 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_7 = sk\_c_4$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2' = sk\_c_7$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_7 \cdot sk\_c_4 = sk\_c_6$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_7 = sk\_c_4$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{21}$, negated_conjecture)
($sk\_c_7 \cdot sk\_c_6 = sk\_c_5$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ and $sk\_c_7' = sk\_c_5$ and $x_2 \cdot sk\_c_6 = sk\_c_7$ and $x_2' = sk\_c_6$ and $x_1 \cdot sk\_c_7 = sk\_c_6$ and $x_1' = sk\_c_7$ and $sk\_c_7 \cdot x_3 = sk\_c_6$ and $x_4 \cdot sk\_c_7 = x_3$) $\Rightarrow$ $x_4' \neq sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)

GRP305-1.p An identity generated by HR, number 05617
include('Axioms/GRP004-0.ax')
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_2 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_3' = sk\_c_5$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ or $sk\_c_3' = sk\_c_5$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_6' = sk\_c_4$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_6' = sk\_c_4$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_6' = sk\_c_4$ or $sk\_c_2 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_6' = sk\_c_4$ or $sk\_c_3' = sk\_c_5$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_6' = sk\_c_4$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_4$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_4$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_4$ or $sk\_c_2 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_4$ or $sk\_c_3' = sk\_c_5$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_6 = sk\_c_4$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2' = sk\_c_6$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_2 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3' = sk\_c_5$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_1' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_4 = sk\_c_5$ cnf(prove_this$_{25}$, negated_conjecture)
($sk\_c_6 \cdot sk\_c_5 = sk\_c_4$ and $sk\_c_5 \cdot sk\_c_4 = sk\_c_6$ and $sk\_c_6' = sk\_c_4$ and $x_2 \cdot sk\_c_6 = sk\_c_4$ and $x_2' = sk\_c_6$ and $sk\_c_5 \cdot sk\_c_6 = sk\_c_4$ and $x_1' = sk\_c_6$ and $x_1 \cdot sk\_c_5 = sk\_c_6$ and $x_3' = sk\_c_5$) $\Rightarrow$ $x_3 \cdot sk\_c_4 \neq sk\_c_5$ cnf(prove_this$_{26}$, negated_conjectu

GRP306-1.p An identity generated by HR, number 17237
include('Axioms/GRP004-0.ax')
$sk\_c_8 \cdot sk\_c_7 = sk\_c_6$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_8' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_8' = sk\_c_6$ or $sk\_c_3' = sk\_c_8$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_8' = sk\_c_6$ or $sk\_c_8 \cdot sk\_c_5 = sk\_c_7$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_8' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_8 = sk\_c_5$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_8' = sk\_c_6$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_2 = sk\_c_8$ or $sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_2 = sk\_c_8$ or $sk\_c_3' = sk\_c_8$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_2 = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_5 = sk\_c_7$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_2 = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_8 = sk\_c_5$ cnf(prove_this$_{10}$, negated_conjecture)

$\text{sk\_c}_1 \cdot \text{sk\_c}_2 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_5 = \text{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_5$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_2$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_5 = \text{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_5$ cnf(prove_this$_{20}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_{21}$, negated_conjecture)
($\text{sk\_c}_8 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ and $\text{sk\_c}_8' = \text{sk\_c}_6$ and $x_3 \cdot x_4 = \text{sk\_c}_8$ and $x_3' = x_4$ and $x_4 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ and $x_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ and $x_1' = \text{sk\_c}_8$ and $\text{sk\_c}_8 \cdot x_2 = \text{sk\_c}_7$ and $x_5 \cdot \text{sk\_c}_8 = x_2$) $\Rightarrow$ $x_5' \neq \text{sk\_c}_8$ cnf(prove_this$_{22}$, negated_conjecture)

GRP309-1.p An identity generated by HR, number 17391
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_3$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\text{sk\_c}_7' = \text{sk\_c}_5$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_5$, negated_conjecture)
$\text{sk\_c}_7' = \text{sk\_c}_5$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\text{sk\_c}_7' = \text{sk\_c}_5$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_7$, negated_conjecture)
$\text{sk\_c}_7' = \text{sk\_c}_5$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_8$, negated_conjecture)
$\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_2 = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_2 = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_2 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_2 = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_2$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_2$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_2$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_2$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_{20}$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{21}$, negated_conjecture)
($\text{sk\_c}_7 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ and $\text{sk\_c}_7' = \text{sk\_c}_5$ and $\text{sk\_c}_5 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $\text{sk\_c}_7 \cdot x_3 = \text{sk\_c}_6$ and $x_4 \cdot \text{sk\_c}_7 = x_3$ and $x_4' = \text{sk\_c}_7$ and $x_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ and $x_1' = \text{sk\_c}_7$ and $x_2 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$) $\Rightarrow$ $x_2' \neq \text{sk\_c}_6$ cnf(prove_this$_{22}$, negated_conjecture)

GRP310-1.p An identity generated by HR, number 17399
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_8 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_8' = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\text{sk\_c}_8' = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_3$, negated_conjecture)
$\text{sk\_c}_8' = \text{sk\_c}_6$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_5 = \text{sk\_c}_7$ cnf(prove_this$_4$, negated_conjecture)
$\text{sk\_c}_8' = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_5$ cnf(prove_this$_5$, negated_conjecture)
$\text{sk\_c}_8' = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_6$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_7$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_8$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_5 = \text{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_5$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_8$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_8 \cdot \text{sk\_c}_2 = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_8 \cdot \text{sk\_c}_2 = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_8 \cdot \text{sk\_c}_2 = \text{sk\_c}_7$ or $\text{sk\_c}_8 \cdot \text{sk\_c}_5 = \text{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_8 \cdot \text{sk\_c}_2 = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_8 = \text{sk\_c}_5$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_8 \cdot \text{sk\_c}_2 = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_8$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_2$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_8 = \text{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_8 = \text{sk\_c}_2$ or $\text{sk\_c}_3' = \text{sk\_c}_8$ cnf(prove_this$_{18}$, negated_conjecture)

$sk\_c_1 \cdot sk\_c_8 = sk\_c_2$ or $sk\_c_8 \cdot sk\_c_5 = sk\_c_7$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_2$ or $sk\_c_4 \cdot sk\_c_8 = sk\_c_5$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_2$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_3 \cdot sk\_c_8 = sk\_c_7$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_3' = sk\_c_8$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_8 \cdot sk\_c_5 = sk\_c_7$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_8 = sk\_c_5$ cnf(prove_this$_{25}$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{26}$, negated_conjecture)
($sk\_c_8 \cdot sk\_c_7 = sk\_c_6$ and $sk\_c_8' = sk\_c_6$ and $sk\_c_6 \cdot sk\_c_7 = sk\_c_8$ and $sk\_c_8 \cdot x_3 = sk\_c_7$ and $x_4 \cdot sk\_c_8 = x_3$ and $x_4' =$ sk_c_8 and $x_1 \cdot sk\_c_8 = sk\_c_7$ and $x_1' = sk\_c_8$ and $sk\_c_8 \cdot x_2 = sk\_c_7$ and $x_5 \cdot sk\_c_8 = x_2$) $\Rightarrow$ $x_5' \neq sk\_c_8$ cnf(prove_this$_{27}$, negat

GRP314-1.p An identity generated by HR, number 18186
include('Axioms/GRP004-0.ax')
$sk\_c_7 \cdot sk\_c_8 = sk\_c_6$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_8 = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1' = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2' = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{17}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{18}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{19}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_{20}$, negated_conjecture)
$sk\_c_3 \cdot sk\_c_7 = sk\_c_8$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{21}$, negated_conjecture)
$sk\_c_3' = sk\_c_7$ or $sk\_c_4' = sk\_c_8$ cnf(prove_this$_{22}$, negated_conjecture)
$sk\_c_3' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_7 = sk\_c_8$ cnf(prove_this$_{23}$, negated_conjecture)
$sk\_c_3' = sk\_c_7$ or $sk\_c_5' = sk\_c_7$ cnf(prove_this$_{24}$, negated_conjecture)
$sk\_c_3' = sk\_c_7$ or $sk\_c_5 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{25}$, negated_conjecture)
($sk\_c_7 \cdot sk\_c_8 = sk\_c_6$ and $x_3 \cdot sk\_c_8 = sk\_c_7$ and $x_3' = sk\_c_8$ and $x_4 \cdot sk\_c_7 = sk\_c_6$ and $x_4' = sk\_c_7$ and $x_5 \cdot sk\_c_7 =$ sk_c_8 and $x_5' = sk\_c_7$ and $x_1' = sk\_c_8$ and $x_1 \cdot sk\_c_7 = sk\_c_8$ and $x_2' = sk\_c_7$) $\Rightarrow$ $x_2 \cdot sk\_c_6 \neq sk\_c_7$ cnf(prove_this$_{26}$, negated_

GRP315-1.p An identity generated by HR, number 18383
include('Axioms/GRP004-0.ax')
$sk\_c_6 \cdot sk\_c_7 = sk\_c_5$ cnf(prove_this$_1$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_2$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_3$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_4$, negated_conjecture)
$sk\_c_1 \cdot sk\_c_7 = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_5$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_6$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_7$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_8$, negated_conjecture)
$sk\_c_1' = sk\_c_7$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_9$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{10}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{11}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{12}$, negated_conjecture)
$sk\_c_2 \cdot sk\_c_6 = sk\_c_5$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{13}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3' = sk\_c_7$ cnf(prove_this$_{14}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_3 \cdot sk\_c_6 = sk\_c_7$ cnf(prove_this$_{15}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4' = sk\_c_6$ cnf(prove_this$_{16}$, negated_conjecture)
$sk\_c_2' = sk\_c_6$ or $sk\_c_4 \cdot sk\_c_5 = sk\_c_6$ cnf(prove_this$_{17}$, negated_conjecture)

$(\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ and $x_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ and $x_3' = \text{sk\_c}_7$ and $x_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ and $x_4' = \text{sk\_c}_6$ and $x_1' = \text{sk\_c}_7$ and $x_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $x_2' = \text{sk\_c}_6) \Rightarrow x_2 \cdot \text{sk\_c}_5 \neq \text{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)

GRP326-1.p An identity generated by HR, number 18694
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_2$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_3$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_7$ cnf(prove_this$_4$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_5$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_6$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_7$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_7$ cnf(prove_this$_8$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_9$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_3 = \text{sk\_c}_6$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_3 = \text{sk\_c}_6$ or $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_3 = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_7$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_7 \cdot \text{sk\_c}_3 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_3$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_3$ or $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_3$ or $\text{sk\_c}_4' = \text{sk\_c}_7$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_7 = \text{sk\_c}_3$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{17}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_{18}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ cnf(prove_this$_{19}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_7$ cnf(prove_this$_{20}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ cnf(prove_this$_{21}$, negated_conjecture)
$(\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ and $x_2 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $x_2' = \text{sk\_c}_6$ and $\text{sk\_c}_7 \cdot x_3 = \text{sk\_c}_6$ and $x_4 \cdot \text{sk\_c}_7 = x_3$ and $x_4' = \text{sk\_c}_7$ and $\text{sk\_c}_6' = \text{sk\_c}_5$ and $\text{sk\_c}_7 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ and $x_1' = \text{sk\_c}_7) \Rightarrow x_1 \cdot \text{sk\_c}_6 \neq \text{sk\_c}_7$ cnf(prove_this$_{22}$, negated_conjecture

GRP333-1.p An identity generated by HR, number 19058
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_2$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_3$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_4$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_5$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_6$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_7$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_8$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_9$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{11}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_{12}$, negated_conjecture)
$\text{sk\_c}_2' = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{13}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ cnf(prove_this$_{14}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ or $\text{sk\_c}_3' = \text{sk\_c}_7$ cnf(prove_this$_{15}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ or $\text{sk\_c}_4 \cdot \text{sk\_c}_6 = \text{sk\_c}_5$ cnf(prove_this$_{16}$, negated_conjecture)
$\text{sk\_c}_2 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ or $\text{sk\_c}_4' = \text{sk\_c}_6$ cnf(prove_this$_{17}$, negated_conjecture)
$(\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ and $x_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ and $x_3' = \text{sk\_c}_6$ and $x_4' = \text{sk\_c}_6$ and $x_4 \cdot \text{sk\_c}_5 = \text{sk\_c}_6$ and $x_1 \cdot \text{sk\_c}_7 = \text{sk\_c}_6$ and $x_1' = \text{sk\_c}_7$ and $x_2 \cdot \text{sk\_c}_6 = \text{sk\_c}_5) \Rightarrow x_2' \neq \text{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)

GRP334-1.p An identity generated by HR, number 19095
include('Axioms/GRP004-0.ax')
$\text{sk\_c}_6 \cdot \text{sk\_c}_7 = \text{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_2$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_7' = \text{sk\_c}_5$ cnf(prove_this$_3$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_6 \cdot \text{sk\_c}_4 = \text{sk\_c}_5$ cnf(prove_this$_4$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3 \cdot \text{sk\_c}_6 = \text{sk\_c}_4$ cnf(prove_this$_5$, negated_conjecture)
$\text{sk\_c}_1 \cdot \text{sk\_c}_6 = \text{sk\_c}_7$ or $\text{sk\_c}_3' = \text{sk\_c}_6$ cnf(prove_this$_6$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_6' = \text{sk\_c}_5$ cnf(prove_this$_7$, negated_conjecture)
$\text{sk\_c}_1' = \text{sk\_c}_6$ or $\text{sk\_c}_7' = \text{sk\_c}_5$ cnf(prove_this$_8$, negated_conjecture)

$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_5$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_5$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_5$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_5$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ cnf(prove_this$_{20}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{21}$, negated_conjecture)
($\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_3' = \mathrm{sk\_c}_6$ and $x_4' = \mathrm{sk\_c}_6$ and $x_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_6' = \mathrm{sk\_c}_5$ and $\mathrm{sk\_c}_7' = \mathrm{sk\_c}_5$ and $\mathrm{sk\_c}_6 \cdot x_1 = \mathrm{sk\_c}_5$ and $x_2 \cdot \mathrm{sk\_c}_6 = x_1$) $\Rightarrow$ $x_2' \neq \mathrm{sk\_c}_6$ cnf(prove_this$_{22}$, negated_conjecture)

GRP344-1.p An identity generated by HR, number 19587
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ cnf(prove_this$_{17}$, negated_conjecture)
($\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_3' = \mathrm{sk\_c}_7$ and $x_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_4' = \mathrm{sk\_c}_6$ and $x_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x_1' = \mathrm{sk\_c}_6$ and $x_2' = \mathrm{sk\_c}_5$) $\Rightarrow$ $x_2 \cdot \mathrm{sk\_c}_6 \neq \mathrm{sk\_c}_5$ cnf(prove_this$_{18}$, negated_conjecture)

GRP348-1.p An identity generated by HR, number 19633
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_1$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_2$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_3$, negated_conjecture)
$\mathrm{sk\_c}_5 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_4$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_4$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_5$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_6$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_7$, negated_conjecture)
$\mathrm{sk\_c}_1' = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_8$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_9$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{10}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{11}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{12}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_{13}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{14}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{15}$, negated_conjecture)
$\mathrm{sk\_c}_2' = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{16}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4' = \mathrm{sk\_c}_5$ cnf(prove_this$_{17}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{18}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3' = \mathrm{sk\_c}_6$ cnf(prove_this$_{19}$, negated_conjecture)
$\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf(prove_this$_{20}$, negated_conjecture)

(sk_c$_5$ $\cdot$ sk_c$_6$ = sk_c$_4$ and x_2' = sk_c$_6$ and x_2 $\cdot$ sk_c$_5$ = sk_c$_6$ and x_3' = sk_c$_5$ and x_3 $\cdot$ sk_c$_4$ = sk_c$_5$ and sk_c$_4'$ = sk_c$_5$ and sk_c$_4$ $\cdot$ sk_c$_5$ = sk_c$_6$ and x_1' = sk_c$_6$) $\Rightarrow$ x_1 $\cdot$ sk_c$_5$ $\neq$ sk_c$_6$ cnf(prove_this$_{21}$, negated_conjecture)

GRP351-1.p An identity generated by HR, number 22711
include('Axioms/GRP004-0.ax')
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_3'$ = sk_c$_6$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_4'$ = sk_c$_5$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_5$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_6$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_3'$ = sk_c$_6$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_4'$ = sk_c$_5$ cnf(prove_this$_9$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_5$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_3'$ = sk_c$_6$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_4'$ = sk_c$_5$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_5$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_3'$ = sk_c$_6$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_4'$ = sk_c$_5$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_5$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{21}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{22}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_3'$ = sk_c$_6$ cnf(prove_this$_{23}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_4'$ = sk_c$_5$ cnf(prove_this$_{24}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_5$ cnf(prove_this$_{25}$, negated_conjecture)
(sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ and x_3 $\cdot$ sk_c$_7$ = sk_c$_6$ and x_3' = sk_c$_7$ and x_4 $\cdot$ sk_c$_6$ = sk_c$_5$ and x_4' = sk_c$_6$ and sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ and $x_1$$\cdot$sk_c$_6$ = sk_c$_7$ and x_1' = sk_c$_6$ and x_2' = sk_c$_5$) $\Rightarrow$ $x_2$$\cdot$sk_c$_6$ $\neq$ sk_c$_5$ cnf(prove_this$_{26}$, negated_conjecture)

GRP352-1.p An identity generated by HR, number 22712
include('Axioms/GRP004-0.ax')
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_3'$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_4'$ = sk_c$_6$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_6$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_3'$ = sk_c$_7$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_9$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_4'$ = sk_c$_6$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_3'$ = sk_c$_7$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$_1'$ = sk_c$_7$ or sk_c$_4'$ = sk_c$_6$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_3'$ = sk_c$_7$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_5$ or sk_c$_4'$ = sk_c$_6$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{21}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_3'$ = sk_c$_7$ cnf(prove_this$_{22}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{23}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{24}$, negated_conjecture)
sk_c$_2'$ = sk_c$_6$ or sk_c$_4'$ = sk_c$_6$ cnf(prove_this$_{25}$, negated_conjecture)

(sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ and x_3 $\cdot$ sk_c$_7$ = sk_c$_6$ and x'_3 = sk_c$_7$ and x_4 $\cdot$ sk_c$_6$ = sk_c$_5$ and x'_4 = sk_c$_6$ and sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ and x'_1 = sk_c$_7$ and $x_1$$\cdot$sk_c$_6$ = sk_c$_7$ and $x_2$$\cdot$sk_c$_6$ = sk_c$_7$) $\Rightarrow$ $x'_2 \neq$ sk_c$_6$ cnf(prove_this$_{26}$, negated_conjecture)

GRP353-1.p An identity generated by HR, number 22878
include('Axioms/GRP004-0.ax')
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$'_2$ = sk_c$_7$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_4$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_4$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_7$ = sk_c$_6$ cnf(prove_this$_6$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$'_2$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_9$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_4$ = sk_c$_6$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_4$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_7$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_7$ = sk_c$_6$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_2$ = sk_c$_7$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_4$ = sk_c$_6$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_4$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$'_1$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_7$ = sk_c$_6$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$'_2$ = sk_c$_7$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$_2$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{21}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$_3$ $\cdot$ sk_c$_4$ = sk_c$_6$ cnf(prove_this$_{22}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$'_3$ = sk_c$_4$ cnf(prove_this$_{23}$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_5$ = sk_c$_6$ or sk_c$_4$ $\cdot$ sk_c$_7$ = sk_c$_6$ cnf(prove_this$_{24}$, negated_conjecture)
(sk_c$_6$$\cdot$sk_c$_5$ = sk_c$_7$ and $x_2$$\cdot$sk_c$_7$ = sk_c$_6$ and x'_2 = sk_c$_7$ and sk_c$_7$$\cdot$sk_c$_5$ = sk_c$_6$ and sk_c$_6$$\cdot$sk_c$_7$ = sk_c$_5$ and x'_1 = sk_c$_7$ and $x_1$$\cdot$sk_c$_6$ = sk_c$_7$ and $x_3$$\cdot$$x_4$ = sk_c$_6$ and $x'_3 = x_4$) $\Rightarrow$ $x_4$$\cdot$sk_c$_7 \neq$ sk_c$_6$ cnf(prove_this$_{25}$, negated_conjecture)

GRP355-1.p An identity generated by HR, number 22926
include('Axioms/GRP004-0.ax')
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$_2$ $\cdot$ sk_c$_8$ = sk_c$_7$ cnf(prove_this$_1$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$'_2$ = sk_c$_8$ cnf(prove_this$_2$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_3$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$'_3$ = sk_c$_6$ cnf(prove_this$_4$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$_8$ $\cdot$ sk_c$_5$ = sk_c$_6$ cnf(prove_this$_5$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$_4$ $\cdot$ sk_c$_8$ = sk_c$_5$ cnf(prove_this$_6$, negated_conjecture)
sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_7$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$_2$ $\cdot$ sk_c$_8$ = sk_c$_7$ cnf(prove_this$_8$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$'_2$ = sk_c$_8$ cnf(prove_this$_9$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{10}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$'_3$ = sk_c$_6$ cnf(prove_this$_{11}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$_8$ $\cdot$ sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{12}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$_4$ $\cdot$ sk_c$_8$ = sk_c$_5$ cnf(prove_this$_{13}$, negated_conjecture)
sk_c$_1$ $\cdot$ sk_c$_8$ = sk_c$_7$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_{14}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_2$ $\cdot$ sk_c$_8$ = sk_c$_7$ cnf(prove_this$_{15}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$'_2$ = sk_c$_8$ cnf(prove_this$_{16}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_3$ $\cdot$ sk_c$_6$ = sk_c$_7$ cnf(prove_this$_{17}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$'_3$ = sk_c$_6$ cnf(prove_this$_{18}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_8$ $\cdot$ sk_c$_5$ = sk_c$_6$ cnf(prove_this$_{19}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$_4$ $\cdot$ sk_c$_8$ = sk_c$_5$ cnf(prove_this$_{20}$, negated_conjecture)
sk_c$'_1$ = sk_c$_8$ or sk_c$'_4$ = sk_c$_8$ cnf(prove_this$_{21}$, negated_conjecture)
(sk_c$_7$ $\cdot$ sk_c$_6$ = sk_c$_8$ and x_2 $\cdot$ sk_c$_8$ = sk_c$_7$ and x'_2 = sk_c$_8$ and x_1 $\cdot$ sk_c$_8$ = sk_c$_7$ and x'_1 = sk_c$_8$ and x_3 $\cdot$ sk_c$_6$ = sk_c$_7$ and x'_3 = sk_c$_6$ and sk_c$_8$$\cdot$$x_4$ = sk_c$_6$ and $x_5$$\cdot$sk_c$_8$ = x_4) $\Rightarrow$ $x'_5 \neq$ sk_c$_8$ cnf(prove_this$_{22}$, negated_conjecture)

GRP360-1.p An identity generated by HR, number 23564
include('Axioms/GRP004-0.ax')
sk_c$_6$ $\cdot$ sk_c$_5$ = sk_c$_7$ or sk_c$_6$ $\cdot$ sk_c$_7$ = sk_c$_5$ cnf(prove_this$_1$, negated_conjecture)

$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_2$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_3$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_4$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_5$, negated_conjecture)
$\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_6$, negated_conjecture)
$\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_7$, negated_conjecture)
$\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_8$, negated_conjecture)
$\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_9$, negated_conjecture)
$\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{10}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_{11}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{12}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{13}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}'_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{14}$, negated_conjecture)
$\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_2 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{15}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_2$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_{16}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_2$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{17}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_2$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{18}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_2$ or $\mathrm{sk\_c}'_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{19}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_2$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{20}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_{21}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{22}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{23}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{24}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_4 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{25}$, negated_conjecture)
($\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}'_7 = \mathrm{sk\_c}_5$ and $\mathrm{sk\_c}_7 \cdot x_3 = \mathrm{sk\_c}_6$ and $x_4 \cdot \mathrm{sk\_c}_7 = x_3$ and $x'_4 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x'_1 = \mathrm{sk\_c}_7$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x'_2 = \mathrm{sk\_c}_6$) $\Rightarrow$ $x_2 \cdot \mathrm{sk\_c}_5 \neq \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{26}$, negated_conjecture)

GRP363-1.p An identity generated by HR, number 24611
include('Axioms/GRP004-0.ax')
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_1$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_2$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_2 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_3$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_4$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ cnf($\mathrm{prove\_this}_5$, negated_conjecture)
$\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_6$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_7$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_8$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_2 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_9$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{10}$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ cnf($\mathrm{prove\_this}_{11}$, negated_conjecture)
$\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{12}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_{13}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{14}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_2 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{15}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{16}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ cnf($\mathrm{prove\_this}_{17}$, negated_conjecture)
$\mathrm{sk\_c}_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{18}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ cnf($\mathrm{prove\_this}_{19}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{20}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}'_2 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{21}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_7 \cdot \mathrm{sk\_c}_4 = \mathrm{sk\_c}_6$ cnf($\mathrm{prove\_this}_{22}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}_3 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_4$ cnf($\mathrm{prove\_this}_{23}$, negated_conjecture)
$\mathrm{sk\_c}'_1 = \mathrm{sk\_c}_6$ or $\mathrm{sk\_c}'_3 = \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{24}$, negated_conjecture)
($\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_5 = \mathrm{sk\_c}_7$ and $\mathrm{sk\_c}'_6 = \mathrm{sk\_c}_5$ and $x_2 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_5$ and $x'_2 = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_6 \cdot \mathrm{sk\_c}_7 = \mathrm{sk\_c}_5$ and $x_1 \cdot \mathrm{sk\_c}_6 = \mathrm{sk\_c}_7$ and $x'_1 = \mathrm{sk\_c}_6$ and $\mathrm{sk\_c}_7 \cdot x_3 = \mathrm{sk\_c}_6$ and $x_4 \cdot \mathrm{sk\_c}_7 = x_3$) $\Rightarrow$ $x'_4 \neq \mathrm{sk\_c}_7$ cnf($\mathrm{prove\_this}_{25}$, negated_conjecture)

GRP392-1.p Monoid axioms
include('Axioms/GRP001-0.ax')

GRP393-1.p Semigroup axioms
include('Axioms/GRP002-0.ax')

GRP393-2.p Semigroups axioms
include('Axioms/GRP008-0.ax')

GRP394+3.p Group theory (equality) axioms
include('Axioms/GRP004+0.ax')

GRP394-1.p Group theory axioms
include('Axioms/GRP003-0.ax')

GRP394-2.p Group theory axioms
include('Axioms/GRP005-0.ax')

GRP394-3.p Group theory (equality) axioms
include('Axioms/GRP004-0.ax')

GRP395-1.p Group theory (Named groups) axioms
include('Axioms/GRP006-0.ax')

GRP396+1.p Group theory (Named Semigroups) axioms
include('Axioms/GRP007+0.ax')

GRP397-1.p Cancellative semigroups axioms
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')

GRP398-1.p Subgroup axioms for the GRP003 group theory axioms
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-1.ax')

GRP398-2.p Subgroup axioms for the GRP003 group theory axioms
include('Axioms/GRP003-0.ax')
include('Axioms/GRP003-2.ax')

GRP398-3.p Subgroup (equality) axioms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-1.ax')

GRP399-1.p Lattice ordered group (equality) axioms
include('Axioms/GRP004-0.ax')
include('Axioms/GRP004-2.ax')

GRP400-1.p Prove associativity implies distribution in cancellative semigroup
Assume a cancellative semigroup admits a commutator operation. Then the following three properties are equivalent: (1) commutator is associative; (2) commutator distributes over product; (3) the semigroup is nilpotent of class 2. This is a generalization of the corresponding theorem for group theory. The problem here is to prove (1) implies (2).
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$a \cdot b = b \cdot (a \cdot \text{commutator}(a, b))$ cnf(commutator, axiom)
$\text{commutator}(\text{commutator}(a, b), c) = \text{commutator}(a, \text{commutator}(b, c))$ cnf(associativity_of_commutator, axiom)
$\text{commutator}(a \cdot b, c) \neq \text{commutator}(a, c) \cdot \text{commutator}(b, c)$ cnf(prove_commutator_distributes_over_product, negated_conjec

GRP401-1.p Prove distributivity implies nilpotent in cancellative semigroup
Assume a cancellative semigroup admits a commutator operation. Then the following three properties are equivalent: (1) commutator is associative; (2) commutator distributes over product; (3) the semigroup is nilpotent of class 2. This is a generalization of the corresponding theorem for group theory. The problem here is to prove (1) implies (2).
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$a \cdot b = b \cdot (a \cdot \text{commutator}(a, b))$ cnf(commutator, axiom)
$\text{commutator}(a \cdot b, c) = \text{commutator}(a, c) \cdot \text{commutator}(b, c)$ cnf(commutator_distributes_over_product, axiom)
$\text{commutator}(a, b) \cdot c \neq c \cdot \text{commutator}(a, b)$ cnf(prove_nilpotency, negated_conjecture)

GRP402-1.p Prove nilpotent implies associativity in cancellative semigroup
Assume a cancellative semigroup admits a commutator operation. Then the following three properties are equivalent: (1) commutator is associative; (2) commutator distributes over product; (3) the semigroup is nilpotent of class 2. This is a generalization of the corresponding theorem for group theory. The problem here is to prove (1) implies (2).
include('Axioms/GRP008-0.ax')
include('Axioms/GRP008-1.ax')
$a \cdot b = b \cdot (a \cdot \text{commutator}(a, b))$ cnf(commutator, axiom)

$\mathrm{commutator}(a,b)\cdot c = c\cdot \mathrm{commutator}(a,b)$ cnf(nilpotency, axiom)
$\mathrm{commutator}(\mathrm{commutator}(a,b),c) \neq \mathrm{commutator}(a,\mathrm{commutator}(b,c))$ cnf(prove_commutator_is_associative, negated_conje

GRP403-1.p Axiom for group theory, in product & inverse, part 1
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b\cdot(b'\cdot b))')' = c$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP404-1.p Axiom for group theory, in product & inverse, part 2
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b\cdot(b'\cdot b))')' = c$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP405-1.p Axiom for group theory, in product & inverse, part 3
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b\cdot(b'\cdot b))')' = c$ cnf(single_axiom, axiom)
$(a_3\cdot b_3)\cdot c_3 \neq a_3\cdot(b_3\cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP406-1.p Axiom for group theory, in product & inverse, part 1
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b'\cdot(b'\cdot b)))' = c$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP407-1.p Axiom for group theory, in product & inverse, part 2
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b'\cdot(b'\cdot b)))' = c$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP408-1.p Axiom for group theory, in product & inverse, part 3
$a\cdot(((a\cdot b)'\cdot c)'\cdot(b'\cdot(b'\cdot b)))' = c$ cnf(single_axiom, axiom)
$(a_3\cdot b_3)\cdot c_3 \neq a_3\cdot(b_3\cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP409-1.p Axiom for group theory, in product & inverse, part 1
$((a\cdot(b\cdot c)')'\cdot(a\cdot c'))\cdot(c'\cdot c)' = b$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP410-1.p Axiom for group theory, in product & inverse, part 2
$((a\cdot(b\cdot c)')'\cdot(a\cdot c'))\cdot(c'\cdot c)' = b$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP411-1.p Axiom for group theory, in product & inverse, part 3
$((a\cdot(b\cdot c)')'\cdot(a\cdot c'))\cdot(c'\cdot c)' = b$ cnf(single_axiom, axiom)
$(a_3\cdot b_3)\cdot c_3 \neq a_3\cdot(b_3\cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP412-1.p Axiom for group theory, in product & inverse, part 1
$a\cdot(((b'\cdot b)\cdot((a\cdot b')'\cdot c)')\cdot b)' = c$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP413-1.p Axiom for group theory, in product & inverse, part 2
$a\cdot(((b'\cdot b)\cdot((a\cdot b')'\cdot c)')\cdot b)' = c$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP414-1.p Axiom for group theory, in product & inverse, part 3
$a\cdot(((b'\cdot b)\cdot((a\cdot b')'\cdot c)')\cdot b)' = c$ cnf(single_axiom, axiom)
$(a_3\cdot b_3)\cdot c_3 \neq a_3\cdot(b_3\cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP415-1.p Axiom for group theory, in product & inverse, part 1
$(a\cdot(((b\cdot a)'\cdot(b\cdot c'))'\cdot(a'\cdot a)')')' = c$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP416-1.p Axiom for group theory, in product & inverse, part 2
$(a\cdot(((b\cdot a)'\cdot(b\cdot c'))'\cdot(a'\cdot a)')')' = c$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP417-1.p Axiom for group theory, in product & inverse, part 3
$(a\cdot(((b\cdot a)'\cdot(b\cdot c'))'\cdot(a'\cdot a)')')' = c$ cnf(single_axiom, axiom)
$(a_3\cdot b_3)\cdot c_3 \neq a_3\cdot(b_3\cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP418-1.p Axiom for group theory, in product & inverse, part 1
$((a\cdot(b'\cdot(c\cdot(c'\cdot c)')')')'\cdot(a\cdot c))' = b$ cnf(single_axiom, axiom)
$a_1'\cdot a_1 \neq b_1'\cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP419-1.p Axiom for group theory, in product & inverse, part 2
$((a\cdot(b'\cdot(c\cdot(c'\cdot c)')')')'\cdot(a\cdot c))' = b$ cnf(single_axiom, axiom)
$(b_2'\cdot b_2)\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP420-1.p Axiom for group theory, in product & inverse, part 3
$((a \cdot (b' \cdot (c \cdot (c' \cdot c)')')')' \cdot (a \cdot c))' = b$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP421-1.p Axiom for group theory, in product & inverse, part 1
$((a \cdot (b' \cdot (c' \cdot (c' \cdot c)')))' \cdot (a \cdot c))' = b$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP422-1.p Axiom for group theory, in product & inverse, part 2
$((a \cdot (b' \cdot (c' \cdot (c' \cdot c)')))' \cdot (a \cdot c))' = b$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP423-1.p Axiom for group theory, in product & inverse, part 3
$((a \cdot (b' \cdot (c' \cdot (c' \cdot c)')))' \cdot (a \cdot c))' = b$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP424-1.p Axiom for group theory, in product & inverse, part 1
$((a \cdot (b' \cdot c)')' \cdot (a \cdot c'))' \cdot (c' \cdot c)' = b$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP425-1.p Axiom for group theory, in product & inverse, part 2
$((a \cdot (b' \cdot c)')' \cdot (a \cdot c'))' \cdot (c' \cdot c)' = b$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP426-1.p Axiom for group theory, in product & inverse, part 3
$((a \cdot (b' \cdot c)')' \cdot (a \cdot c'))' \cdot (c' \cdot c)' = b$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP427-1.p Axiom for group theory, in product & inverse, part 1
$a \cdot (((b' \cdot (a' \cdot c))' \cdot d) \cdot (b \cdot d)')' = c$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP428-1.p Axiom for group theory, in product & inverse, part 2
$a \cdot (((b' \cdot (a' \cdot c))' \cdot d) \cdot (b \cdot d)')' = c$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP429-1.p Axiom for group theory, in product & inverse, part 3
$a \cdot (((b' \cdot (a' \cdot c))' \cdot d) \cdot (b \cdot d)')' = c$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP430-1.p Axiom for group theory, in product & inverse, part 1
$a \cdot (b \cdot (((c \cdot c') \cdot (d \cdot b)') \cdot a))' = d$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP431-1.p Axiom for group theory, in product & inverse, part 2
$a \cdot (b \cdot (((c \cdot c') \cdot (d \cdot b)') \cdot a))' = d$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP432-1.p Axiom for group theory, in product & inverse, part 3
$a \cdot (b \cdot (((c \cdot c') \cdot (d \cdot b)') \cdot a))' = d$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP433-1.p Axiom for group theory, in product & inverse, part 1
$(((((a \cdot b) \cdot c)' \cdot a) \cdot b) \cdot (d \cdot d'))' = c$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP434-1.p Axiom for group theory, in product & inverse, part 2
$(((((a \cdot b) \cdot c)' \cdot a) \cdot b) \cdot (d \cdot d'))' = c$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP435-1.p Axiom for group theory, in product & inverse, part 3
$(((((a \cdot b) \cdot c)' \cdot a) \cdot b) \cdot (d \cdot d'))' = c$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP436-1.p Axiom for group theory, in product & inverse, part 1
$a \cdot (b \cdot (c \cdot ((c' \cdot (d \cdot b)') \cdot a)))' = d$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP437-1.p Axiom for group theory, in product & inverse, part 2
$a \cdot (b \cdot (c \cdot ((c' \cdot (d \cdot b)') \cdot a)))' = d$ cnf(single_axiom, axiom)

$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP438-1.p Axiom for group theory, in product & inverse, part 3
$a \cdot (b \cdot (c \cdot ((c' \cdot (d \cdot b)') \cdot a)))' = d$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP439-1.p Axiom for group theory, in product & inverse, part 1
$(a \cdot (b \cdot ((b' \cdot c) \cdot (d \cdot (a \cdot c))')))' = d$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP440-1.p Axiom for group theory, in product & inverse, part 2
$(a \cdot (b \cdot ((b' \cdot c) \cdot (d \cdot (a \cdot c))')))' = d$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP441-1.p Axiom for group theory, in product & inverse, part 3
$(a \cdot (b \cdot ((b' \cdot c) \cdot (d \cdot (a \cdot c))')))' = d$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP442-1.p Axiom for group theory, in product & inverse, part 1
$(a \cdot (b \cdot ((c \cdot c') \cdot (d \cdot (a \cdot b))')))' = d$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP443-1.p Axiom for group theory, in product & inverse, part 2
$(a \cdot (b \cdot ((c \cdot c') \cdot (d \cdot (a \cdot b))')))' = d$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP444-1.p Axiom for group theory, in product & inverse, part 3
$(a \cdot (b \cdot ((c \cdot c') \cdot (d \cdot (a \cdot b))')))' = d$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP445-1.p Axiom for group theory, in division, part 1
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP446-1.p Axiom for group theory, in division, part 2
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP447-1.p Axiom for group theory, in division, part 3
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP448-1.p Axiom for group theory, in division, part 1
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(b, b), b), c), \text{divide}(\text{divide}(\text{divide}(b, b), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP449-1.p Axiom for group theory, in division, part 2
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(b, b), b), c), \text{divide}(\text{divide}(\text{divide}(b, b), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP450-1.p Axiom for group theory, in division, part 3
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(b, b), b), c), \text{divide}(\text{divide}(\text{divide}(b, b), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP451-1.p Axiom for group theory, in division, part 1
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)

$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP452-1.p Axiom for group theory, in division, part 2
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP453-1.p Axiom for group theory, in division, part 3
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP454-1.p Axiom for group theory, in division and identity, part 1
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP455-1.p Axiom for group theory, in division and identity, part 2
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP456-1.p Axiom for group theory, in division and identity, part 3
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{divide}(a, a), a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP457-1.p Axiom for group theory, in division and identity, part 1
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{identity}, a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP458-1.p Axiom for group theory, in division and identity, part 2
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{identity}, a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP459-1.p Axiom for group theory, in division and identity, part 3
$\text{divide}(\text{divide}(\text{divide}(a, a), \text{divide}(a, \text{divide}(b, \text{divide}(\text{divide}(\text{identity}, a), c)))), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP460-1.p Axiom for group theory, in division and identity, part 1
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{identity}, b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP461-1.p Axiom for group theory, in division and identity, part 2
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{identity}, b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP462-1.p Axiom for group theory, in division and identity, part 3
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{identity}, b), c), \text{divide}(\text{divide}(\text{divide}(a, a), a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP463-1.p Axiom for group theory, in division and identity, part 1
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{identity}, a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP464-1.p Axiom for group theory, in division and identity, part 2
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{identity}, a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP465-1.p Axiom for group theory, in division and identity, part 3
$\text{divide}(a, \text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), c), \text{divide}(\text{divide}(\text{identity}, a), c))) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP466-1.p Axiom for group theory, in division and inverse, part 1
$\text{divide}(\text{divide}(a, a), \text{divide}(b, \text{divide}(\text{divide}(c, \text{divide}(d, b)), d'))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP467-1.p Axiom for group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(a, a), \text{divide}(b, \text{divide}(\text{divide}(c, \text{divide}(d, b)), d'))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP468-1.p Axiom for group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(a, a), \text{divide}(b, \text{divide}(\text{divide}(c, \text{divide}(d, b)), d'))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP469-1.p Axiom for group theory, in division and inverse, part 1
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(c, d)))', \text{divide}(\text{divide}(d, c), a)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP470-1.p Axiom for group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(c, d)))', \text{divide}(\text{divide}(d, c), a)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP471-1.p Axiom for group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(c, d)))', \text{divide}(\text{divide}(d, c), a)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP472-1.p Axiom for group theory, in division and inverse, part 1

$\text{divide}(\text{divide}(\text{divide}(a, b)', \text{divide}(\text{divide}(c, d), a)), \text{divide}(d, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP473-1.p Axiom for group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(\text{divide}(a, b)', \text{divide}(\text{divide}(c, d), a)), \text{divide}(d, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP474-1.p Axiom for group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(\text{divide}(a, b)', \text{divide}(\text{divide}(c, d), a)), \text{divide}(d, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP475-1.p Axiom for group theory, in division and inverse, part 1
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, b), c), \text{divide}(d, c))', \text{divide}(b, a)) = d$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP476-1.p Axiom for group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, b), c), \text{divide}(d, c))', \text{divide}(b, a)) = d$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP477-1.p Axiom for group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, b), c), \text{divide}(d, c))', \text{divide}(b, a)) = d$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP478-1.p Axiom for group theory, in division and inverse, part 1
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), \text{divide}(c, \text{divide}(b, d)))', d) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP479-1.p Axiom for group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), \text{divide}(c, \text{divide}(b, d)))', d) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP480-1.p Axiom for group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(\text{divide}(\text{divide}(a, a), b), \text{divide}(c, \text{divide}(b, d)))', d) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP481-1.p Axiom for group theory, in double division and identity, part 1
double_divide(double_divide(double_divide(a, double_divide(b, identity)), double_divide(double_divide(c, double_divide(d, doub
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP482-1.p Axiom for group theory, in double division and identity, part 2
double_divide(double_divide(double_divide(a, double_divide(b, identity)), double_divide(double_divide(c, double_divide(d, doub
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP483-1.p Axiom for group theory, in double division and identity, part 3
double_divide(double_divide(double_divide(a, double_divide(b, identity)), double_divide(double_divide(c, double_divide(d, doub
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)

$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP484-1.p Axiom for group theory, in double division and identity, part 1
double_divide(double_divide(a, double_divide(double_divide(double_divide(a, b), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP485-1.p Axiom for group theory, in double division and identity, part 2
double_divide(double_divide(a, double_divide(double_divide(double_divide(a, b), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP486-1.p Axiom for group theory, in double division and identity, part 3
double_divide(double_divide(a, double_divide(double_divide(double_divide(a, b), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP487-1.p Axiom for group theory, in double division and identity, part 1
double_divide(a, double_divide(double_divide(double_divide(identity, double_divide(double_divide(a, identity), double_divide(b
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP488-1.p Axiom for group theory, in double division and identity, part 2
double_divide(a, double_divide(double_divide(double_divide(identity, double_divide(double_divide(a, identity), double_divide(b
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP489-1.p Axiom for group theory, in double division and identity, part 3
double_divide(a, double_divide(double_divide(double_divide(identity, double_divide(double_divide(a, identity), double_divide(b
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP490-1.p Axiom for group theory, in double division and identity, part 1
double_divide(double_divide(identity, a), double_divide(identity, double_divide(double_divide(double_divide(a, b), identity), dou
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP491-1.p Axiom for group theory, in double division and identity, part 2
double_divide(double_divide(identity, a), double_divide(identity, double_divide(double_divide(double_divide(a, b), identity), dou
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)

$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP492-1.p Axiom for group theory, in double division and identity, part 3
double_divide(double_divide(identity, a), double_divide(identity, double_divide(double_divide(double_divide(a, b), identity), dou
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP493-1.p Axiom for group theory, in double division and identity, part 1
double_divide(double_divide(identity, a), double_divide(double_divide(double_divide(b, c), double_divide(identity, identity)), do
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP494-1.p Axiom for group theory, in double division and identity, part 2
double_divide(double_divide(identity, a), double_divide(double_divide(double_divide(b, c), double_divide(identity, identity)), do
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP495-1.p Axiom for group theory, in double division and identity, part 3
double_divide(double_divide(identity, a), double_divide(double_divide(double_divide(b, c), double_divide(identity, identity)), do
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP496-1.p Axiom for group theory, in double division and identity, part 1
double_divide(double_divide(identity, double_divide(a, double_divide(b, identity))), double_divide(double_divide(b, double_divid
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP497-1.p Axiom for group theory, in double division and identity, part 2
double_divide(double_divide(identity, double_divide(a, double_divide(b, identity))), double_divide(double_divide(b, double_divid
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$\text{identity} \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP498-1.p Axiom for group theory, in double division and identity, part 3
double_divide(double_divide(identity, double_divide(a, double_divide(b, identity))), double_divide(double_divide(b, double_divid
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP499-1.p Axiom for group theory, in double division and inverse, part 1
double_divide(a', double_divide(double_divide(a, double_divide(b, c))$'$, double_divide(d, double_divide(b, d)))$'$) $= c$ cnf(singl
$a \cdot b =$ double_divide(b, a)$'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP500-1.p Axiom for group theory, in double division and inverse, part 2
double_divide(a', double_divide(double_divide(a, double_divide(b, c))$'$, double_divide(d, double_divide(b, d)))$'$) $= c$ cnf(singl

$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP501-1.p Axiom for group theory, in double division and inverse, part 3
$\text{double\_divide}(a', \text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, c))', \text{double\_divide}(d, \text{double\_divide}(b, d)))') = c$ cnf(singl
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP502-1.p Axiom for group theory, in double division and inverse, part 1
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, c)'), \text{double\_divide}(b', \text{double\_divide}(d, \text{double\_divide}(a, d))')) = c$ cnf(singl
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP503-1.p Axiom for group theory, in double division and inverse, part 2
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, c)'), \text{double\_divide}(b', \text{double\_divide}(d, \text{double\_divide}(a, d))')) = c$ cnf(singl
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP504-1.p Axiom for group theory, in double division and inverse, part 3
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, c)'), \text{double\_divide}(b', \text{double\_divide}(d, \text{double\_divide}(a, d))')) = c$ cnf(singl
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP505-1.p Axiom for Abelian group theory, in product and inverse, part 1
$(((a \cdot b)' \cdot (b \cdot a))' \cdot ((c \cdot d)' \cdot (c \cdot ((e \cdot f') \cdot d')')))' \cdot f = e$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP506-1.p Axiom for Abelian group theory, in product and inverse, part 2
$(((a \cdot b)' \cdot (b \cdot a))' \cdot ((c \cdot d)' \cdot (c \cdot ((e \cdot f') \cdot d')')))' \cdot f = e$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP507-1.p Axiom for Abelian group theory, in product and inverse, part 3
$(((a \cdot b)' \cdot (b \cdot a))' \cdot ((c \cdot d)' \cdot (c \cdot ((e \cdot f') \cdot d')')))' \cdot f = e$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP508-1.p Axiom for Abelian group theory, in product and inverse, part 4
$(((a \cdot b)' \cdot (b \cdot a))' \cdot ((c \cdot d)' \cdot (c \cdot ((e \cdot f') \cdot d')')))' \cdot f = e$ cnf(single_axiom, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP509-1.p Axiom for Abelian group theory, in product and inverse, part 1
$((a \cdot b) \cdot c) \cdot (a \cdot c)' = b$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP510-1.p Axiom for Abelian group theory, in product and inverse, part 2
$((a \cdot b) \cdot c) \cdot (a \cdot c)' = b$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP511-1.p Axiom for Abelian group theory, in product and inverse, part 3
$((a \cdot b) \cdot c) \cdot (a \cdot c)' = b$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP512-1.p Axiom for Abelian group theory, in product and inverse, part 4
$((a \cdot b) \cdot c) \cdot (a \cdot c)' = b$ cnf(single_axiom, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP513-1.p Axiom for Abelian group theory, in product and inverse, part 1
$a \cdot ((b \cdot c) \cdot (a \cdot c)') = b$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP514-1.p Axiom for Abelian group theory, in product and inverse, part 2
$a \cdot ((b \cdot c) \cdot (a \cdot c)') = b$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP515-1.p Axiom for Abelian group theory, in product and inverse, part 3
$a \cdot ((b \cdot c) \cdot (a \cdot c)') = b$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP516-1.p Axiom for Abelian group theory, in product and inverse, part 4
$a \cdot ((b \cdot c) \cdot (a \cdot c)') = b$ cnf(single_axiom, axiom)

$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP517-1.p Axiom for Abelian group theory, in product and inverse, part 1
$a \cdot (((a \cdot b)' \cdot c) \cdot b) = c$ cnf(single_axiom, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP518-1.p Axiom for Abelian group theory, in product and inverse, part 2
$a \cdot (((a \cdot b)' \cdot c) \cdot b) = c$ cnf(single_axiom, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP519-1.p Axiom for Abelian group theory, in product and inverse, part 3
$a \cdot (((a \cdot b)' \cdot c) \cdot b) = c$ cnf(single_axiom, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP520-1.p Axiom for Abelian group theory, in product and inverse, part 4
$a \cdot (((a \cdot b)' \cdot c) \cdot b) = c$ cnf(single_axiom, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP521-1.p Axiom for Abelian group theory, in division, part 1
$\text{divide}(a, \text{divide}(b, \text{divide}(c, \text{divide}(a, b)))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP522-1.p Axiom for Abelian group theory, in division, part 2
$\text{divide}(a, \text{divide}(b, \text{divide}(c, \text{divide}(a, b)))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP523-1.p Axiom for Abelian group theory, in division, part 3
$\text{divide}(a, \text{divide}(b, \text{divide}(c, \text{divide}(a, b)))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP524-1.p Axiom for Abelian group theory, in division, part 4
$\text{divide}(a, \text{divide}(b, \text{divide}(c, \text{divide}(a, b)))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP525-1.p Axiom for Abelian group theory, in division, part 1
$\text{divide}(a, \text{divide}(\text{divide}(a, b), \text{divide}(c, b))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP526-1.p Axiom for Abelian group theory, in division, part 2
$\text{divide}(a, \text{divide}(\text{divide}(a, b), \text{divide}(c, b))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP527-1.p Axiom for Abelian group theory, in division, part 3
$\text{divide}(a, \text{divide}(\text{divide}(a, b), \text{divide}(c, b))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP528-1.p Axiom for Abelian group theory, in division, part 4
$\text{divide}(a, \text{divide}(\text{divide}(a, b), \text{divide}(c, b))) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP529-1.p Axiom for Abelian group theory, in division, part 1

$\text{divide}(\text{divide}(a, \text{divide}(b, c)), \text{divide}(a, b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP530-1.p Axiom for Abelian group theory, in division, part 2
$\text{divide}(\text{divide}(a, \text{divide}(b, c)), \text{divide}(a, b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP531-1.p Axiom for Abelian group theory, in division, part 3
$\text{divide}(\text{divide}(a, \text{divide}(b, c)), \text{divide}(a, b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP532-1.p Axiom for Abelian group theory, in division, part 4
$\text{divide}(\text{divide}(a, \text{divide}(b, c)), \text{divide}(a, b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP533-1.p Axiom for Abelian group theory, in division, part 1
$\text{divide}(\text{divide}(a, \text{divide}(\text{divide}(a, b), c)), b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP534-1.p Axiom for Abelian group theory, in division, part 2
$\text{divide}(\text{divide}(a, \text{divide}(\text{divide}(a, b), c)), b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP535-1.p Axiom for Abelian group theory, in division, part 3
$\text{divide}(\text{divide}(a, \text{divide}(\text{divide}(a, b), c)), b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP536-1.p Axiom for Abelian group theory, in division, part 4
$\text{divide}(\text{divide}(a, \text{divide}(\text{divide}(a, b), c)), b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP537-1.p Axiom for Abelian group theory, in division, part 1
$\text{divide}(\text{divide}(a, b), \text{divide}(\text{divide}(a, c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP538-1.p Axiom for Abelian group theory, in division, part 2
$\text{divide}(\text{divide}(a, b), \text{divide}(\text{divide}(a, c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP539-1.p Axiom for Abelian group theory, in division, part 3
$\text{divide}(\text{divide}(a, b), \text{divide}(\text{divide}(a, c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP540-1.p Axiom for Abelian group theory, in division, part 4
$\text{divide}(\text{divide}(a, b), \text{divide}(\text{divide}(a, c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{divide}(c, c), b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{divide}(b, b), a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP541-1.p Axiom for Abelian group theory, in division and identity, part 1
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(\text{divide}(\text{divide}(a, b), c), a)), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP542-1.p Axiom for Abelian group theory, in division and identity, part 2
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(\text{divide}(\text{divide}(a, b), c), a)), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP543-1.p Axiom for Abelian group theory, in division and identity, part 3
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(\text{divide}(\text{divide}(a, b), c), a)), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP544-1.p Axiom for Abelian group theory, in division and identity, part 4
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(\text{divide}(\text{divide}(a, b), c), a)), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP545-1.p Axiom for Abelian group theory, in division and identity, part 1
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, b)), \text{divide}(\text{divide}(b, c), a)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP546-1.p Axiom for Abelian group theory, in division and identity, part 2
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, b)), \text{divide}(\text{divide}(b, c), a)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP547-1.p Axiom for Abelian group theory, in division and identity, part 3
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, b)), \text{divide}(\text{divide}(b, c), a)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP548-1.p Axiom for Abelian group theory, in division and identity, part 4
$\text{divide}(\text{divide}(\text{identity}, \text{divide}(a, b)), \text{divide}(\text{divide}(b, c), a)) = c$ cnf(single_axiom, axiom)

$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP549-1.p Axiom for Abelian group theory, in division and identity, part 1
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{divide}(\text{divide}(b, a), c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP550-1.p Axiom for Abelian group theory, in division and identity, part 2
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{divide}(\text{divide}(b, a), c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP551-1.p Axiom for Abelian group theory, in division and identity, part 3
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{divide}(\text{divide}(b, a), c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP552-1.p Axiom for Abelian group theory, in division and identity, part 4
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{divide}(\text{divide}(b, a), c), b)) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, \text{divide}(\text{identity}, b))$ cnf(multiply, axiom)
$a' = \text{divide}(\text{identity}, a)$ cnf(inverse, axiom)
$\text{identity} = \text{divide}(a, a)$ cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP553-1.p Axiom for Abelian group theory, in division and inverse, part 1
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(a, c))'), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP554-1.p Axiom for Abelian group theory, in division and inverse, part 2
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(a, c))'), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP555-1.p Axiom for Abelian group theory, in division and inverse, part 3
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(a, c))'), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP556-1.p Axiom for Abelian group theory, in division and inverse, part 4
$\text{divide}(\text{divide}(a, \text{divide}(b, \text{divide}(a, c))'), c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP557-1.p Axiom for Abelian group theory, in division and inverse, part 1
$\text{divide}(a, \text{divide}(\text{divide}(b, c), \text{divide}(a, c))') = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP558-1.p Axiom for Abelian group theory, in division and inverse, part 2
$\text{divide}(a, \text{divide}(\text{divide}(b, c), \text{divide}(a, c))') = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP559-1.p Axiom for Abelian group theory, in division and inverse, part 3
$\text{divide}(a, \text{divide}(\text{divide}(b, c), \text{divide}(a, c))') = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{divide}(a, b')$ cnf(multiply, axiom)

$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP560-1.p Axiom for Abelian group theory, in division and inverse, part 4
divide(a, divide(divide(b, c), divide(a, c))$'$) $= b$ cnf(single_axiom, axiom)
$a \cdot b =$ divide(a, b') cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP561-1.p Axiom for Abelian group theory, in division and inverse, part 1
divide(divide(divide(a, b'), c), divide(a, c)) $= b$ cnf(single_axiom, axiom)
$a \cdot b =$ divide(a, b') cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP562-1.p Axiom for Abelian group theory, in division and inverse, part 2
divide(divide(divide(a, b'), c), divide(a, c)) $= b$ cnf(single_axiom, axiom)
$a \cdot b =$ divide(a, b') cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP563-1.p Axiom for Abelian group theory, in division and inverse, part 3
divide(divide(divide(a, b'), c), divide(a, c)) $= b$ cnf(single_axiom, axiom)
$a \cdot b =$ divide(a, b') cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP564-1.p Axiom for Abelian group theory, in division and inverse, part 4
divide(divide(divide(a, b'), c), divide(a, c)) $= b$ cnf(single_axiom, axiom)
$a \cdot b =$ divide(a, b') cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP565-1.p Axiom for Abelian group theory, in double div and id, part 1
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(identity, c))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq$ identity cnf(prove_these_axioms$_1$, negated_conjecture)

GRP566-1.p Axiom for Abelian group theory, in double div and id, part 2
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(identity, c))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP567-1.p Axiom for Abelian group theory, in double div and id, part 3
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(identity, c))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP568-1.p Axiom for Abelian group theory, in double div and id, part 4
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(identity, c))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP569-1.p Axiom for Abelian group theory, in double div and id, part 1
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)

$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP570-1.p Axiom for Abelian group theory, in double div and id, part 2
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP571-1.p Axiom for Abelian group theory, in double div and id, part 3
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP572-1.p Axiom for Abelian group theory, in double div and id, part 4
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(a, c)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP573-1.p Axiom for Abelian group theory, in double div and id, part 1
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(c, a)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP574-1.p Axiom for Abelian group theory, in double div and id, part 2
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(c, a)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
identity $\cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP575-1.p Axiom for Abelian group theory, in double div and id, part 3
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(c, a)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP576-1.p Axiom for Abelian group theory, in double div and id, part 4
double_divide(double_divide(a, double_divide(double_divide(b, double_divide(c, a)), double_divide(c, identity))), double_divide(
b cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP577-1.p Axiom for Abelian group theory, in double div and id, part 1
double_divide(double_divide(a, double_divide(double_divide(double_divide(b, a), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity $=$ double_divide(a, a') cnf(identity, axiom)

$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP578-1.p Axiom for Abelian group theory, in double div and id, part 2
double_divide(double_divide(a, double_divide(double_divide(double_divide(b, a), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
identity $\cdot\, a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP579-1.p Axiom for Abelian group theory, in double div and id, part 3
double_divide(double_divide(a, double_divide(double_divide(double_divide(b, a), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP580-1.p Axiom for Abelian group theory, in double div and id, part 4
double_divide(double_divide(a, double_divide(double_divide(double_divide(b, a), c), double_divide(b, identity))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP581-1.p Axiom for Abelian group theory, in double div and id, part 1
double_divide(double_divide(a, double_divide(double_divide(identity, b), double_divide(c, double_divide(b, a)))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a_1' \cdot a_1 \neq \text{identity}$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP582-1.p Axiom for Abelian group theory, in double div and id, part 2
double_divide(double_divide(a, double_divide(double_divide(identity, b), double_divide(c, double_divide(b, a)))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
identity $\cdot\, a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP583-1.p Axiom for Abelian group theory, in double div and id, part 3
double_divide(double_divide(a, double_divide(double_divide(identity, b), double_divide(c, double_divide(b, a)))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP584-1.p Axiom for Abelian group theory, in double div and id, part 4
double_divide(double_divide(a, double_divide(double_divide(identity, b), double_divide(c, double_divide(b, a)))), double_divide(
c cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(double_divide(b, a), identity) cnf(multiply, axiom)
$a' =$ double_divide(a, identity) cnf(inverse, axiom)
identity = double_divide(a, a') cnf(identity, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP585-1.p Axiom for Abelian group theory, in double div and inv, part 1
double_divide(a, double_divide(double_divide(double_divide(a, b), c')$'$, b)$'$) $= c$ cnf(single_axiom, axiom)
$a \cdot b =$ double_divide(b, a)$'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP586-1.p Axiom for Abelian group theory, in double div and inv, part 2
double_divide(a, double_divide(double_divide(double_divide(a, b), c')$'$, b)$'$) $= c$ cnf(single_axiom, axiom)

$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP587-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(a, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), c')', b)') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP588-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(a, \text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), c')', b)') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP589-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, c')')', b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP590-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, c')')', b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP591-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, c')')', b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP592-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, c')')', b) = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP593-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c, b)')')' = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP594-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c, b)')')' = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP595-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c, b)')')' = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP596-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c, b)')')' = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP597-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c', b)')') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP598-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c', b)')') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP599-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c', b)')') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP600-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(a, b), \text{double\_divide}(a, \text{double\_divide}(c', b)')') = c$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP601-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, \text{double\_divide}(a, c))')', c)' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP602-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, \text{double\_divide}(a, c))')', c)' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP603-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, \text{double\_divide}(a, c))')', c)' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP604-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b, \text{double\_divide}(a, c))')', c)' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP605-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b', \text{double\_divide}(a, c))')', c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP606-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b', \text{double\_divide}(a, c))')', c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP607-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b', \text{double\_divide}(a, c))')', c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP608-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(a, \text{double\_divide}(b', \text{double\_divide}(a, c))')', c) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP609-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b)', c)', \text{double\_divide}(a, c))' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP610-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b)', c)', \text{double\_divide}(a, c))' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP611-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b)', c)', \text{double\_divide}(a, c))' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP612-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b)', c)', \text{double\_divide}(a, c))' = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP613-1.p Axiom for Abelian group theory, in double div and inv, part 1
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b')', c)', \text{double\_divide}(a, c)) = b$ cnf(single_axiom, axiom)

$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a_1' \cdot a_1 \neq b_1' \cdot b_1$ cnf(prove_these_axioms$_1$, negated_conjecture)

GRP614-1.p Axiom for Abelian group theory, in double div and inv, part 2
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b')', c)', \text{double\_divide}(a, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(b_2' \cdot b_2) \cdot a_2 \neq a_2$ cnf(prove_these_axioms$_2$, negated_conjecture)

GRP615-1.p Axiom for Abelian group theory, in double div and inv, part 3
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b')', c)', \text{double\_divide}(a, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$(a_3 \cdot b_3) \cdot c_3 \neq a_3 \cdot (b_3 \cdot c_3)$ cnf(prove_these_axioms$_3$, negated_conjecture)

GRP616-1.p Axiom for Abelian group theory, in double div and inv, part 4
$\text{double\_divide}(\text{double\_divide}(\text{double\_divide}(a, b')', c)', \text{double\_divide}(a, c)) = b$ cnf(single_axiom, axiom)
$a \cdot b = \text{double\_divide}(b, a)'$ cnf(multiply, axiom)
$a \cdot b \neq b \cdot a$ cnf(prove_these_axioms$_4$, negated_conjecture)

GRP617-1.p PQEx lemma
Proves commutativity of mutliplication across two trivially intersecting subgroups.
include('Axioms/GRP003-0.ax')
$\text{subgroup1\_member}(x) \Rightarrow \text{subgroup1\_member}(x')$ cnf(closure_of_inverse$_1$, axiom)
$(\text{subgroup1\_member}(a) \text{ and } \text{subgroup1\_member}(b) \text{ and } a{\cdot}b{=}c) \Rightarrow \text{subgroup1\_member}(c)$ cnf(closure_of_product$_1$, axiom)
$\text{subgroup2\_member}(x) \Rightarrow \text{subgroup2\_member}(x')$ cnf(closure_of_inverse$_2$, axiom)
$(\text{subgroup2\_member}(a) \text{ and } \text{subgroup2\_member}(b) \text{ and } a{\cdot}b{=}c) \Rightarrow \text{subgroup2\_member}(c)$ cnf(closure_of_product$_2$, axiom)
$\text{subgroup1\_member}(x) \Rightarrow \text{subgroup1\_member}(a \cdot (x \cdot a'))$ cnf(normality$_1$, hypothesis)
$\text{subgroup2\_member}(x) \Rightarrow \text{subgroup2\_member}(a \cdot (x \cdot a'))$ cnf(normality$_2$, hypothesis)
$(\text{subgroup1\_member}(x) \text{ and } \text{subgroup2\_member}(x)) \Rightarrow x = \text{identity}$ cnf(trivial_intersection, hypothesis)
$\text{subgroup1\_member}(v)$ cnf(v_in_G$_1$, hypothesis)
$\text{subgroup2\_member}(u)$ cnf(u_in_G$_2$, hypothesis)
$v \cdot u \neq u \cdot v$ cnf(prove_vu_equals_uv, negated_conjecture)

GRP654+1.p A quasigroup satisfying Moufang 1 is a loop
$\forall b, a: a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a: \text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a: \text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a: \text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a: a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ fof(f_{05}, axiom)
$\exists x_0: \forall x_1: (x_1 \cdot x_0 = x_1 \text{ and } x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP654+2.p A quasigroup satisfying Moufang 1 is a loop
$\forall b, a: a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a: \text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a: \text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a: \text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a: a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ fof(f_{05}, axiom)
$\forall x_0, x_1: (x_0 \cdot \text{rd}(x_1, x_1) = x_0 \text{ and } \text{rd}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP654+3.p A quasigroup satisfying Moufang 1 is a loop
$\forall b, a: a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a: \text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a: \text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a: \text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a: a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ fof(f_{05}, axiom)
$\forall x_0, x_1: (x_0 \cdot \text{ld}(x_1, x_1) = x_0 \text{ and } \text{ld}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP655+1.p A quasigroup satisfying Moufang 2 is a loop
$\forall b, a: a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a: \text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a: \text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a: \text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a: a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b$ fof(f_{05}, axiom)
$\exists x_0: \forall x_1: (x_1 \cdot x_0 = x_1 \text{ and } x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP655+2.p A quasigroup satisfying Moufang 2 is a loop

$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b$ fof(f_{05}, axiom)
$\forall x_0, x_1 \colon (x_0 \cdot \mathrm{rd}(x_1, x_1) = x_0$ and $\mathrm{rd}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP655+3.p A quasigroup satisfying Moufang 2 is a loop
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b$ fof(f_{05}, axiom)
$\forall x_0, x_1 \colon (x_0 \cdot \mathrm{ld}(x_1, x_1) = x_0$ and $\mathrm{ld}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP656+1.p A quasigroup satisfying Moufang 3 is a loop
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon (a \cdot b) \cdot (c \cdot a) = (a \cdot (b \cdot c)) \cdot a$ fof(f_{05}, axiom)
$\exists x_0 \colon \forall x_1 \colon (x_1 \cdot x_0 = x_1$ and $x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP657+1.p A quasigroup satisfying Moufang 4 is a loop
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon (a \cdot b) \cdot (c \cdot a) = a \cdot ((b \cdot c) \cdot a)$ fof(f_{05}, axiom)
$\exists x_0 \colon \forall x_1 \colon (x_1 \cdot x_0 = x_1$ and $x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP658+1.p Bol-Moufang identity 1 implies the existence of a unit element
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon (a \cdot (b \cdot b)) \cdot c = (a \cdot b) \cdot (b \cdot c)$ fof(f_{05}, axiom)
$\exists x_0 \colon \forall x_1 \colon (x_1 \cdot x_0 = x_1$ and $x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP659+1.p Bol-Moufang identity 2 implies the existence of a unit element
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon (a \cdot (b \cdot c)) \cdot b = (a \cdot b) \cdot (c \cdot b)$ fof(f_{05}, axiom)
$\exists x_0 \colon \forall x_1 \colon (x_1 \cdot x_0 = x_1$ and $x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP660+1.p Bol-Moufang identity 3 implies a unit element
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon ((a \cdot b) \cdot c) \cdot a = a \cdot (b \cdot (c \cdot a))$ fof(f_{05}, axiom)
$\exists x_0 \colon \forall x_1 \colon (x_1 \cdot x_0 = x_1$ and $x_0 \cdot x_1 = x_1)$ fof(goals, conjecture)

GRP660+2.p Bol-Moufang identity 3 implies a unit element
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a \colon ((a \cdot b) \cdot c) \cdot a = a \cdot (b \cdot (c \cdot a))$ fof(f_{05}, axiom)
$\forall x_0, x_1 \colon (x_0 \cdot \mathrm{rd}(x_1, x_1) = x_0$ and $\mathrm{rd}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP660+3.p Bol-Moufang identity 3 implies a unit element

$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot a = a \cdot (b \cdot (c \cdot a))$ fof(f_{05}, axiom)
$\forall x_0, x_1$: $(x_0 \cdot \mathrm{ld}(x_1, x_1) = x_0$ and $\mathrm{ld}(x_1, x_1) \cdot x_0 = x_0)$ fof(goals, conjecture)

GRP661-1.p Conjugacy closed with ab = 1 implies ba is in the nucleus - a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{op\_c} \cdot \mathrm{op\_d} = 1$ cnf(c_{09}, axiom)
$(\mathrm{op\_d} \cdot \mathrm{op\_c}) \cdot (a \cdot b) \neq ((\mathrm{op\_d} \cdot \mathrm{op\_c}) \cdot a) \cdot b$ cnf(goals, negated_conjecture)

GRP662-1.p Conjugacy closed with ab = 1 implies ba is in the nucleus - b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{op\_c} \cdot \mathrm{op\_d} = 1$ cnf(c_{09}, axiom)
$(a \cdot (\mathrm{op\_d} \cdot \mathrm{op\_c})) \cdot b \neq a \cdot ((\mathrm{op\_d} \cdot \mathrm{op\_c}) \cdot b)$ cnf(goals, negated_conjecture)

GRP663-1.p Conjugacy closed with ab = 1 implies ba is in the nucleus - c
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{op\_c} \cdot \mathrm{op\_d} = 1$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot (\mathrm{op\_d} \cdot \mathrm{op\_c}) \neq a \cdot (b \cdot (\mathrm{op\_d} \cdot \mathrm{op\_c}))$ cnf(goals, negated_conjecture)

GRP664+1.p Conjugacy closed with ab = 1 implies ba is in the nucleus - a
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ fof(f_{07}, axiom)
$\forall c, b, a$: $(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ fof(f_{08}, axiom)
$\forall x_0, x_1, x_2, x_3$: $(x_0 \cdot x_1 = 1 \Rightarrow ((x_1 \cdot x_0) \cdot (x_2 \cdot x_3) = ((x_1 \cdot x_0) \cdot x_2) \cdot x_3$ and $(x_2 \cdot (x_1 \cdot x_0)) \cdot x_3 = x_2 \cdot ((x_1 \cdot x_0) \cdot x_3)$ and $(x_2 \cdot x_3) \cdot (x_1 \cdot x_0) = x_2 \cdot (x_3 \cdot (x_1 \cdot x_0))))$ fof(goals, conjecture)

GRP665+1.p Conjugacy closed implies commutant in the nucleus
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)

$\forall c, b, a$: $a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ fof(f_{07}, axiom)
$\forall c, b, a$: $(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ fof(f_{08}, axiom)
$\forall a$: $\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c}$ fof(f_{09}, axiom)
$\forall x_0, x_1$: $(\mathrm{op\_c} \cdot (x_0 \cdot x_1) = (\mathrm{op\_c} \cdot x_0) \cdot x_1$ and $(x_0 \cdot \mathrm{op\_c}) \cdot x_1 = x_0 \cdot (\mathrm{op\_c} \cdot x_1)$ and $(x_0 \cdot x_1) \cdot \mathrm{op\_c} = x_0 \cdot (x_1 \cdot \mathrm{op\_c}))$ fof(goals, conjecture)

GRP666+1.p Inverse property A-loops are Moufang
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall b, a$: $i(a) \cdot (a \cdot b) = b$ fof(f_{07}, axiom)
$\forall b, a$: $(a \cdot b) \cdot i(b) = a$ fof(f_{08}, axiom)
$\forall d, c, b, a$: $\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ fof(f_{09}, axiom)
$\forall d, c, b, a$: $\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ fof(f_{10}, axiom)
$\forall c, b, a$: $\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ fof(f_{11}, axiom)
$\forall x_0, x_1, x_2$: $(x_0 \cdot (x_1 \cdot (x_2 \cdot x_1)) = ((x_0 \cdot x_1) \cdot x_2) \cdot x_1 \Rightarrow x_1 \cdot (x_0 \cdot (x_1 \cdot x_2)) = ((x_1 \cdot x_0) \cdot x_1) \cdot x_2)$ fof(f_{12}, axiom)
$\forall x_3, x_4, x_5$: $((x_3 \cdot x_4) \cdot (x_5 \cdot x_3) = (x_3 \cdot (x_4 \cdot x_5)) \cdot x_3 \Rightarrow x_3 \cdot (x_4 \cdot (x_3 \cdot x_5)) = ((x_3 \cdot x_4) \cdot x_3) \cdot x_5)$ fof(f_{13}, axiom)
$\forall x_6, x_7, x_8$: $((x_6 \cdot x_7) \cdot (x_8 \cdot x_6) = x_6 \cdot ((x_7 \cdot x_8) \cdot x_6) \Rightarrow x_6 \cdot (x_7 \cdot (x_6 \cdot x_8)) = ((x_6 \cdot x_7) \cdot x_6) \cdot x_8)$ fof(f_{14}, axiom)
$a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ fof(goals, conjecture)

GRP666+6.p Inverse property A-loops are Moufang
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall d, c, b, a$: $\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ fof(f_{07}, axiom)
$\forall d, c, b, a$: $\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ fof(f_{08}, axiom)
$\forall c, b, a$: $\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ fof(f_{09}, axiom)
$\forall b, a$: $i(a) \cdot (a \cdot b) = b$ fof(f_{10}, axiom)
$\forall b, a$: $(a \cdot b) \cdot i(b) = a$ fof(f_{11}, axiom)
$\forall x_0, x_1, x_2$: $x_2 \cdot (x_0 \cdot (x_2 \cdot x_1)) = ((x_2 \cdot x_0) \cdot x_2) \cdot x_1$ or $\forall x_3, x_4, x_5$: $x_3 \cdot (x_5 \cdot (x_4 \cdot x_5)) = ((x_3 \cdot x_5) \cdot x_4) \cdot x_5$ or $\forall x_6, x_7, x_8$: $(x_8 \cdot x_6) \cdot (x_7 \cdot x_8) = (x_8 \cdot (x_6 \cdot x_7)) \cdot x_8$ or $\forall x_9, x_{10}, x_{11}$: $(x_{11} \cdot x_9) \cdot (x_{10} \cdot x_{11}) = x_{11} \cdot ((x_9 \cdot x_{10}) \cdot x_{11})$ fof(goals, conjecture)

GRP666-2.p Inverse property A-loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{07}, axiom)
$\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ cnf(c_{08}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{09}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{10}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{11}, axiom)
$a \cdot (b \cdot (a \cdot c)) \neq ((a \cdot b) \cdot a) \cdot c$ cnf(goals, negated_conjecture)

GRP666-3.p Inverse property A-loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{07}, axiom)
$\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ cnf(c_{08}, axiom)

$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{09}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{10}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{11}, axiom)
$a \cdot (b \cdot (c \cdot b)) \neq ((a \cdot b) \cdot c) \cdot b$ cnf(goals, negated_conjecture)

GRP666-4.p Inverse property A-loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{07}, axiom)
$\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ cnf(c_{08}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{09}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{10}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{11}, axiom)
$(a \cdot b) \cdot (c \cdot a) \neq (a \cdot (b \cdot c)) \cdot a$ cnf(goals, negated_conjecture)

GRP666-5.p Inverse property A-loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{07}, axiom)
$\mathrm{rd}(((a \cdot b) \cdot c) \cdot d, c \cdot d) = \mathrm{rd}((a \cdot c) \cdot d, c \cdot d) \cdot \mathrm{rd}((b \cdot c) \cdot d, c \cdot d)$ cnf(c_{08}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{09}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{10}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{11}, axiom)
$(a \cdot b) \cdot (c \cdot a) \neq a \cdot ((b \cdot c) \cdot a)$ cnf(goals, negated_conjecture)

GRP667+1.p 2-divisible ARIF loops are Moufang
$\forall b, a\colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a\colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a\colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a\colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a\colon a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a\colon 1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a\colon (a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ fof(f_{07}, axiom)
$\forall b, a\colon (a \cdot b) \cdot a = a \cdot (b \cdot a)$ fof(f_{08}, axiom)
$\forall a\colon f(a) \cdot f(a) = a$ fof(f_{09}, axiom)
$\forall x_0, x_1, x_2\colon (x_0 \cdot (x_1 \cdot (x_2 \cdot x_1)) = ((x_0 \cdot x_1) \cdot x_2) \cdot x_1 \Rightarrow x_1 \cdot (x_0 \cdot (x_1 \cdot x_2)) = ((x_1 \cdot x_0) \cdot x_1) \cdot x_2)$ fof(f_{10}, axiom)
$\forall x_3, x_4, x_5\colon ((x_3 \cdot x_4) \cdot (x_5 \cdot x_3) = (x_3 \cdot (x_4 \cdot x_5)) \cdot x_3 \Rightarrow x_3 \cdot (x_4 \cdot (x_3 \cdot x_5)) = ((x_3 \cdot x_4) \cdot x_3) \cdot x_5)$ fof(f_{11}, axiom)
$\forall x_6, x_7, x_8\colon ((x_6 \cdot x_7) \cdot (x_8 \cdot x_6) = x_6 \cdot ((x_7 \cdot x_8) \cdot x_6) \Rightarrow x_6 \cdot (x_7 \cdot (x_6 \cdot x_8)) = ((x_6 \cdot x_7) \cdot x_6) \cdot x_8)$ fof(f_{12}, axiom)
$a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ fof(goals, conjecture)

GRP667+6.p 2-divisible ARIF loops are Moufang
$\forall b, a\colon a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a\colon \mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a\colon \mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a\colon \mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a\colon a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a\colon 1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a\colon (a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ fof(f_{07}, axiom)
$\forall b, a\colon (a \cdot b) \cdot a = a \cdot (b \cdot a)$ fof(f_{08}, axiom)
$\forall a\colon f(a) \cdot f(a) = a$ fof(f_{09}, axiom)
$\forall x_0, x_1, x_2\colon x_2 \cdot (x_0 \cdot (x_2 \cdot x_1)) = ((x_2 \cdot x_0) \cdot x_2) \cdot x_1$ or $\forall x_3, x_4, x_5\colon x_3 \cdot (x_5 \cdot (x_4 \cdot x_5)) = ((x_3 \cdot x_5) \cdot x_4) \cdot x_5$ or $\forall x_6, x_7, x_8\colon (x_8 \cdot x_6) \cdot (x_7 \cdot x_8) = (x_8 \cdot (x_6 \cdot x_7)) \cdot x_8$ or $\forall x_9, x_{10}, x_{11}\colon (x_{11} \cdot x_9) \cdot (x_{10} \cdot x_{11}) = x_{11} \cdot ((x_9 \cdot x_{10}) \cdot x_{11})$ fof(goals, conjecture)

GRP667-2.p 2-divisible ARIF loops are Moufang

$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$(a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot a = a \cdot (b \cdot a)$ cnf(c_{08}, axiom)
$f(a) \cdot f(a) = a$ cnf(c_{09}, axiom)
$a \cdot (b \cdot (a \cdot c)) \neq ((a \cdot b) \cdot a) \cdot c$ cnf(goals, negated_conjecture)

GRP667-3.p 2-divisible ARIF loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$(a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot a = a \cdot (b \cdot a)$ cnf(c_{08}, axiom)
$f(a) \cdot f(a) = a$ cnf(c_{09}, axiom)
$a \cdot (b \cdot (c \cdot b)) \neq ((a \cdot b) \cdot c) \cdot b$ cnf(goals, negated_conjecture)

GRP667-4.p 2-divisible ARIF loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$(a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot a = a \cdot (b \cdot a)$ cnf(c_{08}, axiom)
$f(a) \cdot f(a) = a$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot (c \cdot a) \neq (a \cdot (b \cdot c)) \cdot a$ cnf(goals, negated_conjecture)

GRP667-5.p 2-divisible ARIF loops are Moufang
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$(a \cdot b) \cdot ((c \cdot b) \cdot c) = (a \cdot ((b \cdot c) \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot a = a \cdot (b \cdot a)$ cnf(c_{08}, axiom)
$f(a) \cdot f(a) = a$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot (c \cdot a) \neq a \cdot ((b \cdot c) \cdot a)$ cnf(goals, negated_conjecture)

GRP668-1.p Flexible C-loops are ARIF
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot a = a \cdot (b \cdot a)$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot ((c \cdot b) \cdot c) \neq (a \cdot ((b \cdot c) \cdot b)) \cdot c$ cnf(goals, negated_conjecture)

GRP669-1.p Moufang loops are RIF
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)

$\mathrm{rd}(a \cdot b, b) = a \qquad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \qquad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \qquad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c \qquad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b \qquad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$(a \cdot b) \cdot (c \cdot a) = (a \cdot (b \cdot c)) \cdot a \qquad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$(a \cdot b) \cdot (c \cdot a) = a \cdot ((b \cdot c) \cdot a) \qquad \mathrm{cnf}(c_{10}, \mathrm{axiom})$
$(a \cdot b) \cdot (c \cdot (a \cdot b)) \neq ((a \cdot (b \cdot c)) \cdot a) \cdot b \qquad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP670-1.p RIF loops are ARIF - a
$a \cdot \mathrm{ld}(a, b) = b \qquad \mathrm{cnf}(c_{01}, \mathrm{axiom})$
$\mathrm{ld}(a, a \cdot b) = b \qquad \mathrm{cnf}(c_{02}, \mathrm{axiom})$
$\mathrm{rd}(a, b) \cdot b = a \qquad \mathrm{cnf}(c_{03}, \mathrm{axiom})$
$\mathrm{rd}(a \cdot b, b) = a \qquad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \qquad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \qquad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$(a \cdot b) \cdot i(b) = a \qquad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$(a \cdot b) \cdot (c \cdot (a \cdot b)) = ((a \cdot (b \cdot c)) \cdot a) \cdot b \qquad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$(a \cdot b) \cdot ((c \cdot b) \cdot c) \neq (a \cdot ((b \cdot c) \cdot b)) \cdot c \qquad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP671-1.p RIF loops are ARIF - b
$a \cdot \mathrm{ld}(a, b) = b \qquad \mathrm{cnf}(c_{01}, \mathrm{axiom})$
$\mathrm{ld}(a, a \cdot b) = b \qquad \mathrm{cnf}(c_{02}, \mathrm{axiom})$
$\mathrm{rd}(a, b) \cdot b = a \qquad \mathrm{cnf}(c_{03}, \mathrm{axiom})$
$\mathrm{rd}(a \cdot b, b) = a \qquad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \qquad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \qquad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$(a \cdot b) \cdot i(b) = a \qquad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$(a \cdot b) \cdot (c \cdot (a \cdot b)) = ((a \cdot (b \cdot c)) \cdot a) \cdot b \qquad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$(a \cdot b) \cdot a \neq a \cdot (b \cdot a) \qquad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP672+1.p Extra loop commutation property 1
In an extra loop, z commutes with [x;y; t] if and only if t commutes with [x;y; z] if and only if [x;y; z][x;y; t] = [x;y; zt].
$\forall b, a\colon a \cdot \mathrm{ld}(a, b) = b \qquad \mathrm{fof}(f_{01}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{ld}(a, a \cdot b) = b \qquad \mathrm{fof}(f_{02}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{rd}(a, b) \cdot b = a \qquad \mathrm{fof}(f_{03}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{rd}(a \cdot b, b) = a \qquad \mathrm{fof}(f_{04}, \mathrm{axiom})$
$\forall a\colon a \cdot 1 = a \qquad \mathrm{fof}(f_{05}, \mathrm{axiom})$
$\forall a\colon 1 \cdot a = a \qquad \mathrm{fof}(f_{06}, \mathrm{axiom})$
$\forall c, b, a\colon a \cdot (b \cdot (c \cdot a)) = ((a \cdot b) \cdot c) \cdot a \qquad \mathrm{fof}(f_{07}, \mathrm{axiom})$
$\forall c, b, a\colon \mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c) \qquad \mathrm{fof}(f_{08}, \mathrm{axiom})$
$\mathrm{op\_z} \cdot \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_t}) = \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_t}) \cdot \mathrm{op\_z} \qquad \mathrm{fof}(f_{09}, \mathrm{axiom})$
$\mathrm{op\_t} \cdot \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_z}) = \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_z}) \cdot \mathrm{op\_t}$ and $\mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_z}) \cdot \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_t}) = \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op}$
$\mathrm{op\_t}) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

GRP673+1.p Extra loop commutation property 2
In an extra loop, z commutes with [x;y; t] if and only if t commutes with [x;y; z] if and only if [x;y; z][x;y; t] = [x;y; zt].
$\forall b, a\colon a \cdot \mathrm{ld}(a, b) = b \qquad \mathrm{fof}(f_{01}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{ld}(a, a \cdot b) = b \qquad \mathrm{fof}(f_{02}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{rd}(a, b) \cdot b = a \qquad \mathrm{fof}(f_{03}, \mathrm{axiom})$
$\forall b, a\colon \mathrm{rd}(a \cdot b, b) = a \qquad \mathrm{fof}(f_{04}, \mathrm{axiom})$
$\forall a\colon a \cdot 1 = a \qquad \mathrm{fof}(f_{05}, \mathrm{axiom})$
$\forall a\colon 1 \cdot a = a \qquad \mathrm{fof}(f_{06}, \mathrm{axiom})$
$\forall c, b, a\colon a \cdot (b \cdot (c \cdot a)) = ((a \cdot b) \cdot c) \cdot a \qquad \mathrm{fof}(f_{07}, \mathrm{axiom})$
$\forall c, b, a\colon \mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c) \qquad \mathrm{fof}(f_{08}, \mathrm{axiom})$
$\mathrm{op\_t} \cdot \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_z}) = \mathrm{asoc}(\mathrm{op\_x}, \mathrm{op\_y}, \mathrm{op\_z}) \cdot \mathrm{op\_t} \qquad \mathrm{fof}(f_{09}, \mathrm{axiom})$

op_z·asoc(op_x, op_y, op_t) = asoc(op_x, op_y, op_t)·op_z and asoc(op_x, op_y, op_z)·asoc(op_x, op_y, op_t) = asoc(op_x, op_y, op_z · op_t)   fof(goals, conjecture)

GRP674+1.p Extra loop commutation property 3
In an extra loop, z commutes with [x;y; t] if and only if t commutes with [x;y; z] if and only if [x;y; z][x;y; t] = [x;y; zt].
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (c \cdot a)) = ((a \cdot b) \cdot c) \cdot a$ fof(f_{07}, axiom)
$\forall c, b, a$: $\mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c)$ fof(f_{08}, axiom)
asoc(op_x, op_y, op_z) · asoc(op_x, op_y, op_t) = asoc(op_x, op_y, op_z · op_t)   fof(f_{09}, axiom)
op_t · asoc(op_x, op_y, op_z) = asoc(op_x, op_y, op_z) · op_t and op_z · asoc(op_x, op_y, op_t) = asoc(op_x, op_y, op_t) · op_z   fof(goals, conjecture)

GRP675-1.p In CC-loops, associators are in the center of the nucleus - 1a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c)$ cnf(c_{09}, axiom)
$\mathrm{asoc}(a, b, c) \cdot (d \cdot e) \neq (\mathrm{asoc}(a, b, c) \cdot d) \cdot e$ cnf(goals, negated_conjecture)

GRP676-1.p In CC-loops, associators are in the center of the nucleus - 1b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c)$ cnf(c_{09}, axiom)
$a \cdot (\mathrm{asoc}(b, c, d) \cdot e) \neq (a \cdot \mathrm{asoc}(b, c, d)) \cdot e$ cnf(goals, negated_conjecture)

GRP677-1.p In CC-loops, associators are in the center of the nucleus - 1c
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$\mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c)$ cnf(c_{09}, axiom)
$a \cdot (b \cdot \mathrm{asoc}(c, d, e)) \neq (a \cdot b) \cdot \mathrm{asoc}(c, d, e)$ cnf(goals, negated_conjecture)

GRP678-1.p In CC-loops, associators are in the center of the nucleus - 2
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)

$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c) \quad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$\mathrm{asoc}(a, b, c) = \mathrm{ld}(a \cdot (b \cdot c), (a \cdot b) \cdot c) \quad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$\mathrm{op\_c} \cdot (a \cdot b) = (\mathrm{op\_c} \cdot a) \cdot b \quad \mathrm{cnf}(c_{10}, \mathrm{axiom})$
$a \cdot (\mathrm{op\_c} \cdot b) = (a \cdot \mathrm{op\_c}) \cdot b \quad \mathrm{cnf}(c_{11}, \mathrm{axiom})$
$a \cdot (b \cdot \mathrm{op\_c}) = (a \cdot b) \cdot \mathrm{op\_c} \quad \mathrm{cnf}(c_{12}, \mathrm{axiom})$
$\mathrm{asoc}(a, b, c) \cdot \mathrm{op\_c} \neq \mathrm{op\_c} \cdot \mathrm{asoc}(a, b, c) \quad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP679-1.p Commutants in Bol loops 1
If Q is a Bol loop, and if a,b in C(Q), then so are a∧2, b∧-1, and a∧2b.
$a \cdot \mathrm{ld}(a, b) = b \quad \mathrm{cnf}(c_{01}, \mathrm{axiom})$
$\mathrm{ld}(a, a \cdot b) = b \quad \mathrm{cnf}(c_{02}, \mathrm{axiom})$
$\mathrm{rd}(a, b) \cdot b = a \quad \mathrm{cnf}(c_{03}, \mathrm{axiom})$
$\mathrm{rd}(a \cdot b, b) = a \quad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \quad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \quad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c \quad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c} \quad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$(\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot a \neq a \cdot (\mathrm{op\_c} \cdot \mathrm{op\_c}) \quad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP680-1.p Commutants in Bol loops 2
If Q is a Bol loop, and if a,b in C(Q), then so are a∧2, b∧-1, and a∧2b.
$a \cdot \mathrm{ld}(a, b) = b \quad \mathrm{cnf}(c_{01}, \mathrm{axiom})$
$\mathrm{ld}(a, a \cdot b) = b \quad \mathrm{cnf}(c_{02}, \mathrm{axiom})$
$\mathrm{rd}(a, b) \cdot b = a \quad \mathrm{cnf}(c_{03}, \mathrm{axiom})$
$\mathrm{rd}(a \cdot b, b) = a \quad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \quad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \quad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c \quad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$i(a) \cdot (a \cdot b) = b \quad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c} \quad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$i(\mathrm{op\_c}) \cdot a \neq a \cdot i(\mathrm{op\_c}) \quad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP681-1.p Commutants in Bol loops 3
If Q is a Bol loop, and if a,b in C(Q), then so are a∧2, b∧-1, and a∧2b.
$a \cdot \mathrm{ld}(a, b) = b \quad \mathrm{cnf}(c_{01}, \mathrm{axiom})$
$\mathrm{ld}(a, a \cdot b) = b \quad \mathrm{cnf}(c_{02}, \mathrm{axiom})$
$\mathrm{rd}(a, b) \cdot b = a \quad \mathrm{cnf}(c_{03}, \mathrm{axiom})$
$\mathrm{rd}(a \cdot b, b) = a \quad \mathrm{cnf}(c_{04}, \mathrm{axiom})$
$a \cdot 1 = a \quad \mathrm{cnf}(c_{05}, \mathrm{axiom})$
$1 \cdot a = a \quad \mathrm{cnf}(c_{06}, \mathrm{axiom})$
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c \quad \mathrm{cnf}(c_{07}, \mathrm{axiom})$
$\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c} \quad \mathrm{cnf}(c_{08}, \mathrm{axiom})$
$\mathrm{op\_d} \cdot a = a \cdot \mathrm{op\_d} \quad \mathrm{cnf}(c_{09}, \mathrm{axiom})$
$((\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot \mathrm{op\_d}) \cdot a \neq a \cdot ((\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot \mathrm{op\_d}) \quad \mathrm{cnf}(\mathrm{goals}, \mathrm{negated\_conjecture})$

GRP682+1.p Axioms of rectangular loops - a
$\forall a$: $\mathrm{ld}(a, a \cdot a) = a \quad \mathrm{fof}(f_{01}, \mathrm{axiom})$
$\forall a$: $\mathrm{rd}(a \cdot a, a) = a \quad \mathrm{fof}(f_{02}, \mathrm{axiom})$
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = \mathrm{ld}(a, a \cdot b) \quad \mathrm{fof}(f_{03}, \mathrm{axiom})$
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = \mathrm{rd}(a \cdot b, b) \quad \mathrm{fof}(f_{04}, \mathrm{axiom})$
$\forall d, c, b, a$: $\mathrm{ld}(\mathrm{ld}(a, b), \mathrm{ld}(a, b) \cdot (c \cdot d)) = \mathrm{ld}(a, a \cdot c) \cdot d \quad \mathrm{fof}(f_{05}, \mathrm{axiom})$
$\forall d, c, b, a$: $\mathrm{rd}((a \cdot b) \cdot \mathrm{rd}(c, d), \mathrm{rd}(c, d)) = a \cdot \mathrm{rd}(b \cdot d, d) \quad \mathrm{fof}(f_{06}, \mathrm{axiom})$
$\forall b, a$: $\mathrm{ld}(a, a \cdot \mathrm{ld}(b, b)) = \mathrm{rd}(\mathrm{rd}(a, a) \cdot b, b) \quad \mathrm{fof}(f_{07}, \mathrm{axiom})$
$\forall x_0, x_1, x_2$: $(\mathrm{ld}(\mathrm{ld}(x_0, x_1), \mathrm{ld}(x_0, x_1) \cdot x_2) = \mathrm{ld}(x_0, x_0 \cdot x_2)$ and $\mathrm{ld}(\mathrm{rd}(x_0, x_1), \mathrm{rd}(x_0, x_1) \cdot x_2) = \mathrm{ld}(x_0, x_0 \cdot x_2)) \quad$ fof(goals, conjectu

GRP683+1.p Axioms of rectangular loops - b
$\forall a$: $\mathrm{ld}(a, a \cdot a) = a \quad \mathrm{fof}(f_{01}, \mathrm{axiom})$
$\forall a$: $\mathrm{rd}(a \cdot a, a) = a \quad \mathrm{fof}(f_{02}, \mathrm{axiom})$
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = \mathrm{ld}(a, a \cdot b) \quad \mathrm{fof}(f_{03}, \mathrm{axiom})$
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = \mathrm{rd}(a \cdot b, b) \quad \mathrm{fof}(f_{04}, \mathrm{axiom})$
$\forall d, c, b, a$: $\mathrm{ld}(\mathrm{ld}(a, b), \mathrm{ld}(a, b) \cdot (c \cdot d)) = \mathrm{ld}(a, a \cdot c) \cdot d \quad \mathrm{fof}(f_{05}, \mathrm{axiom})$
$\forall d, c, b, a$: $\mathrm{rd}((a \cdot b) \cdot \mathrm{rd}(c, d), \mathrm{rd}(c, d)) = a \cdot \mathrm{rd}(b \cdot d, d) \quad \mathrm{fof}(f_{06}, \mathrm{axiom})$

$\forall b, a$: $\mathrm{ld}(a, a \cdot \mathrm{ld}(b, b)) = \mathrm{rd}(\mathrm{rd}(a, a) \cdot b, b)$ fof(f_{07}, axiom)
$\forall x_3, x_4, x_5$: $(x_3 \cdot \mathrm{ld}(x_4, x_4 \cdot x_5) = x_3 \cdot x_5$ and $\mathrm{rd}(x_3 \cdot x_4, x_4) \cdot x_5 = x_3 \cdot x_5)$ fof(goals, conjecture)

GRP684-1.p Axioms of rectangular loops - c
$\mathrm{ld}(a, a \cdot a) = a$ cnf(c_{01}, axiom)
$\mathrm{rd}(a \cdot a, a) = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a, b) = \mathrm{ld}(a, a \cdot b)$ cnf(c_{03}, axiom)
$\mathrm{rd}(a, b) \cdot b = \mathrm{rd}(a \cdot b, b)$ cnf(c_{04}, axiom)
$\mathrm{ld}(\mathrm{ld}(a, b), \mathrm{ld}(a, b) \cdot (c \cdot d)) = \mathrm{ld}(a, a \cdot c) \cdot d$ cnf(c_{05}, axiom)
$\mathrm{rd}((a \cdot b) \cdot \mathrm{rd}(c, d), \mathrm{rd}(c, d)) = a \cdot \mathrm{rd}(b \cdot d, d)$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, a \cdot \mathrm{ld}(b, b)) = \mathrm{rd}(\mathrm{rd}(a, a) \cdot b, b)$ cnf(c_{07}, axiom)
$\mathrm{rd}(a \cdot (b \cdot c), b \cdot c) \neq \mathrm{rd}(a \cdot c, c)$ cnf(goals, negated_conjecture)

GRP685+1.p Axioms of rectangular loops - d
$\forall a$: $\mathrm{ld}(a, a \cdot a) = a$ fof(f_{01}, axiom)
$\forall a$: $\mathrm{rd}(a \cdot a, a) = a$ fof(f_{02}, axiom)
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = \mathrm{ld}(a, a \cdot b)$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = \mathrm{rd}(a \cdot b, b)$ fof(f_{04}, axiom)
$\forall d, c, b, a$: $\mathrm{ld}(\mathrm{ld}(a, b), \mathrm{ld}(a, b) \cdot (c \cdot d)) = \mathrm{ld}(a, a \cdot c) \cdot d$ fof(f_{05}, axiom)
$\forall d, c, b, a$: $\mathrm{rd}((a \cdot b) \cdot \mathrm{rd}(c, d), \mathrm{rd}(c, d)) = a \cdot \mathrm{rd}(b \cdot d, d)$ fof(f_{06}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot \mathrm{ld}(b, b)) = \mathrm{rd}(\mathrm{rd}(a, a) \cdot b, b)$ fof(f_{07}, axiom)
$\forall x_6, x_7, x_8$: $(\mathrm{rd}(x_6 \cdot \mathrm{ld}(x_7, x_8), \mathrm{ld}(x_7, x_8)) = \mathrm{rd}(x_6 \cdot x_8, x_8)$ and $\mathrm{rd}(x_6 \cdot \mathrm{rd}(x_7, x_8), \mathrm{rd}(x_7, x_8)) = \mathrm{rd}(x_6 \cdot x_8, x_8))$ fof(goals, conjec

GRP686-1.p x(y.yz) = (x.yy)z is equivalent to xx.yz = (x.xy)z part 1
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (b \cdot c)) = (a \cdot (b \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$(a \cdot a) \cdot (b \cdot c) \neq (a \cdot (a \cdot b)) \cdot c$ cnf(goals, negated_conjecture)

GRP687-1.p x(y.yz) = (x.yy)z is equivalent to xx.yz = (x.xy)z part 2
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$(a \cdot a) \cdot (b \cdot c) = (a \cdot (a \cdot b)) \cdot c$ cnf(c_{07}, axiom)
$a \cdot (b \cdot (b \cdot c)) \neq (a \cdot (b \cdot b)) \cdot c$ cnf(goals, negated_conjecture)

GRP688-1.p Bruck loop elements of order 2∧2 commute with elements of order 3
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c$ cnf(c_{07}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{08}, axiom)
$i(a \cdot b) = i(a) \cdot i(b)$ cnf(c_{09}, axiom)
$\mathrm{op\_c} \cdot (\mathrm{op\_c} \cdot (\mathrm{op\_c} \cdot \mathrm{op\_c})) = 1$ cnf(c_{10}, axiom)
$\mathrm{op\_d} \cdot (\mathrm{op\_d} \cdot \mathrm{op\_d}) = 1$ cnf(c_{11}, axiom)
$\mathrm{op\_c} \cdot \mathrm{op\_d} \neq \mathrm{op\_d} \cdot \mathrm{op\_c}$ cnf(goals, negated_conjecture)

GRP689-1.p Bruck loop elements of order 2∧2 commute with elems of order 3∧2
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)

$1 \cdot a = a \qquad \text{cnf}(c_{06}, \text{axiom})$
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c \qquad \text{cnf}(c_{07}, \text{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(c_{08}, \text{axiom})$
$i(a \cdot b) = i(a) \cdot i(b) \qquad \text{cnf}(c_{09}, \text{axiom})$
$\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot \text{op\_c})) = 1 \qquad \text{cnf}(c_{10}, \text{axiom})$
$\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot \text{op\_d}))))))) = 1 \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_c} \cdot \text{op\_d} \neq \text{op\_d} \cdot \text{op\_c} \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP690-1.p Bruck loop elements of order 2∧4 commute with elems of order 3∧2
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(c_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(c_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(c_{06}, \text{axiom})$
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c \qquad \text{cnf}(c_{07}, \text{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(c_{08}, \text{axiom})$
$i(a \cdot b) = i(a) \cdot i(b) \qquad \text{cnf}(c_{09}, \text{axiom})$
$\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot (\text{op\_c} \cdot \text{op\_c})))))) = 1 \qquad \text{cnf}(c_{10}, \text{axiom})$
$\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot (\text{op\_d} \cdot \text{op\_d}))))))) = 1 \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_c} \cdot \text{op\_d} \neq \text{op\_d} \cdot \text{op\_c} \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP691-1.p In a power associative conjugacy closed loop, c∧3 is WIP
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(c_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(c_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(c_{06}, \text{axiom})$
$a \cdot (b \cdot c) = \text{rd}(a \cdot b, a) \cdot (a \cdot c) \qquad \text{cnf}(c_{07}, \text{axiom})$
$(a \cdot b) \cdot c = (a \cdot c) \cdot \text{ld}(c, b \cdot c) \qquad \text{cnf}(c_{08}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(c_{09}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(c_{10}, \text{axiom})$
$\text{op\_c} \cdot (\text{op\_c} \cdot \text{op\_c}) = \text{op\_d} \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_d} \cdot \text{op\_d} = \text{op\_e} \qquad \text{cnf}(c_{12}, \text{axiom})$
$\text{op\_e} \cdot \text{op\_e} = \text{op\_f} \qquad \text{cnf}(c_{13}, \text{axiom})$
$\text{op\_d} \cdot i(a \cdot \text{op\_d}) \neq i(a) \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP692-1.p In a power associative conjugacy closed loop, c∧6 is extra
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(c_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(c_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(c_{06}, \text{axiom})$
$a \cdot (b \cdot c) = \text{rd}(a \cdot b, a) \cdot (a \cdot c) \qquad \text{cnf}(c_{07}, \text{axiom})$
$(a \cdot b) \cdot c = (a \cdot c) \cdot \text{ld}(c, b \cdot c) \qquad \text{cnf}(c_{08}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(c_{09}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(c_{10}, \text{axiom})$
$\text{op\_c} \cdot (\text{op\_c} \cdot \text{op\_c}) = \text{op\_d} \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_d} \cdot \text{op\_d} = \text{op\_e} \qquad \text{cnf}(c_{12}, \text{axiom})$
$\text{op\_e} \cdot \text{op\_e} = \text{op\_f} \qquad \text{cnf}(c_{13}, \text{axiom})$
$\text{op\_e} \cdot (a \cdot (b \cdot \text{op\_e})) \neq ((\text{op\_e} \cdot a) \cdot b) \cdot \text{op\_e} \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP693-1.p In power associative conjugacy closed loop c∧12 is in nucleus - a
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(c_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(c_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(c_{06}, \text{axiom})$

$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$i(a) \cdot a = 1$ cnf(c_{09}, axiom)
$a \cdot i(a) = 1$ cnf(c_{10}, axiom)
$\mathrm{op\_c} \cdot (\mathrm{op\_c} \cdot \mathrm{op\_c}) = \mathrm{op\_d}$ cnf(c_{11}, axiom)
$\mathrm{op\_d} \cdot \mathrm{op\_d} = \mathrm{op\_e}$ cnf(c_{12}, axiom)
$\mathrm{op\_e} \cdot \mathrm{op\_e} = \mathrm{op\_f}$ cnf(c_{13}, axiom)
$\mathrm{op\_f} \cdot (a \cdot b) \neq (\mathrm{op\_f} \cdot a) \cdot b$ cnf(goals, negated_conjecture)

GRP694-1.p In power associative conjugacy closed loop c∧12 is in nucleus - b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$i(a) \cdot a = 1$ cnf(c_{09}, axiom)
$a \cdot i(a) = 1$ cnf(c_{10}, axiom)
$\mathrm{op\_c} \cdot (\mathrm{op\_c} \cdot \mathrm{op\_c}) = \mathrm{op\_d}$ cnf(c_{11}, axiom)
$\mathrm{op\_d} \cdot \mathrm{op\_d} = \mathrm{op\_e}$ cnf(c_{12}, axiom)
$\mathrm{op\_e} \cdot \mathrm{op\_e} = \mathrm{op\_f}$ cnf(c_{13}, axiom)
$a \cdot (\mathrm{op\_f} \cdot b) \neq (a \cdot \mathrm{op\_f}) \cdot b$ cnf(goals, negated_conjecture)

GRP695-1.p In power associative conjugacy closed loop c∧12 is in nucleus - c
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$i(a) \cdot a = 1$ cnf(c_{09}, axiom)
$a \cdot i(a) = 1$ cnf(c_{10}, axiom)
$\mathrm{op\_c} \cdot (\mathrm{op\_c} \cdot \mathrm{op\_c}) = \mathrm{op\_d}$ cnf(c_{11}, axiom)
$\mathrm{op\_d} \cdot \mathrm{op\_d} = \mathrm{op\_e}$ cnf(c_{12}, axiom)
$\mathrm{op\_e} \cdot \mathrm{op\_e} = \mathrm{op\_f}$ cnf(c_{13}, axiom)
$a \cdot (b \cdot \mathrm{op\_f}) \neq (a \cdot b) \cdot \mathrm{op\_f}$ cnf(goals, negated_conjecture)

GRP696-1.p Variety of power associative, WIP conjugacy closed loops - 1a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot i(b \cdot a) = i(b)$ cnf(c_{07}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{09}, axiom)
$i(a) \cdot a = 1$ cnf(c_{10}, axiom)
$a \cdot i(a) = 1$ cnf(c_{11}, axiom)
$((a \cdot b) \cdot a) \cdot (a \cdot c) \neq a \cdot (((b \cdot a) \cdot a) \cdot c)$ cnf(goals, negated_conjecture)

GRP697-1.p Variety of power associative, WIP conjugacy closed loops - 1b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)

$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot i(b \cdot a) = i(b)$ cnf(c_{07}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{09}, axiom)
$i(a) \cdot a = 1$ cnf(c_{10}, axiom)
$a \cdot i(a) = 1$ cnf(c_{11}, axiom)
$(a \cdot b) \cdot (b \cdot (c \cdot b)) \neq (a \cdot (b \cdot (b \cdot c))) \cdot b$ cnf(goals, negated_conjecture)

GRP698-1.p Variety of power associative, WIP conjugacy closed loops - 2a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$((a \cdot b) \cdot a) \cdot (a \cdot c) = a \cdot (((b \cdot a) \cdot a) \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot (b \cdot (c \cdot b)) = (a \cdot (b \cdot (b \cdot c))) \cdot b$ cnf(c_{08}, axiom)
$a \cdot (b \cdot c) \neq \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(goals, negated_conjecture)

GRP699-1.p Variety of power associative, WIP conjugacy closed loops - 2b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$((a \cdot b) \cdot a) \cdot (a \cdot c) = a \cdot (((b \cdot a) \cdot a) \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot (b \cdot (c \cdot b)) = (a \cdot (b \cdot (b \cdot c))) \cdot b$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot c \neq (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(goals, negated_conjecture)

GRP700+1.p Variety of power associative, WIP conjugacy closed loops - 2c
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot a) \cdot (a \cdot c) = a \cdot (((b \cdot a) \cdot a) \cdot c)$ fof(f_{07}, axiom)
$\forall c, b, a$: $(a \cdot b) \cdot (b \cdot (c \cdot b)) = (a \cdot (b \cdot (b \cdot c))) \cdot b$ fof(f_{08}, axiom)
$\forall x_0$: $\exists x_1$: ($x_1 \cdot x_0 = 1$ and $x_0 \cdot x_1 = 1$) fof(goals, conjecture)

GRP701-1.p Variety of power associative, WIP conjugacy closed loops - 3
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$((a \cdot b) \cdot a) \cdot (a \cdot c) = a \cdot (((b \cdot a) \cdot a) \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot (b \cdot (c \cdot b)) = (a \cdot (b \cdot (b \cdot c))) \cdot b$ cnf(c_{08}, axiom)
$a \cdot i(a) = 1$ cnf(c_{09}, axiom)
$i(a) \cdot a = 1$ cnf(c_{10}, axiom)
$a \cdot i(b \cdot a) \neq i(b)$ cnf(goals, negated_conjecture)

GRP702+1.p In C-loops the nucleus is normal - a
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ fof(f_{07}, axiom)

$\forall b, a$: $\text{op\_c} \cdot (a \cdot b) = (\text{op\_c} \cdot a) \cdot b$ fof(f_{08}, axiom)
$\forall b, a$: $a \cdot (b \cdot \text{op\_c}) = (a \cdot b) \cdot \text{op\_c}$ fof(f_{09}, axiom)
$\forall b, a$: $a \cdot (\text{op\_c} \cdot b) = (a \cdot \text{op\_c}) \cdot b$ fof(f_{10}, axiom)
$\forall a$: $\text{op\_d} = \text{ld}(a, \text{op\_c} \cdot a)$ fof(f_{11}, axiom)
$\forall b, a$: $\text{op\_e} = (\text{rd}(\text{op\_c}, a \cdot b) \cdot b) \cdot a$ fof(f_{12}, axiom)
$\forall b, a$: $\text{op\_f} = a \cdot (b \cdot \text{ld}(a \cdot b, \text{op\_c}))$ fof(f_{13}, axiom)
$\forall x_0, x_1$: $(\text{op\_d} \cdot (x_0 \cdot x_1) = (\text{op\_d} \cdot x_0) \cdot x_1$ and $x_0 \cdot (x_1 \cdot \text{op\_d}) = (x_0 \cdot x_1) \cdot \text{op\_d}$ and $x_0 \cdot (\text{op\_d} \cdot x_1) = (x_0 \cdot \text{op\_d}) \cdot x_1)$ fof(goals, conjecture)

GRP703+1.p In C-loops the nucleus is normal - b
$\forall b, a$: $a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ fof(f_{07}, axiom)
$\forall b, a$: $\text{op\_c} \cdot (a \cdot b) = (\text{op\_c} \cdot a) \cdot b$ fof(f_{08}, axiom)
$\forall b, a$: $a \cdot (b \cdot \text{op\_c}) = (a \cdot b) \cdot \text{op\_c}$ fof(f_{09}, axiom)
$\forall b, a$: $a \cdot (\text{op\_c} \cdot b) = (a \cdot \text{op\_c}) \cdot b$ fof(f_{10}, axiom)
$\forall a$: $\text{op\_d} = \text{ld}(a, \text{op\_c} \cdot a)$ fof(f_{11}, axiom)
$\forall b, a$: $\text{op\_e} = (\text{rd}(\text{op\_c}, a \cdot b) \cdot b) \cdot a$ fof(f_{12}, axiom)
$\forall b, a$: $\text{op\_f} = a \cdot (b \cdot \text{ld}(a \cdot b, \text{op\_c}))$ fof(f_{13}, axiom)
$\forall x_2, x_3$: $(\text{op\_e} \cdot (x_2 \cdot x_3) = (\text{op\_e} \cdot x_2) \cdot x_3$ and $x_2 \cdot (x_3 \cdot \text{op\_e}) = (x_2 \cdot x_3) \cdot \text{op\_e}$ and $x_2 \cdot (\text{op\_e} \cdot x_3) = (x_2 \cdot \text{op\_e}) \cdot x_3)$ fof(goals, conjecture)

GRP704+1.p In C-loops the nucleus is normal - c
$\forall b, a$: $a \cdot \text{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\text{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\text{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\text{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ fof(f_{07}, axiom)
$\forall b, a$: $\text{op\_c} \cdot (a \cdot b) = (\text{op\_c} \cdot a) \cdot b$ fof(f_{08}, axiom)
$\forall b, a$: $a \cdot (b \cdot \text{op\_c}) = (a \cdot b) \cdot \text{op\_c}$ fof(f_{09}, axiom)
$\forall b, a$: $a \cdot (\text{op\_c} \cdot b) = (a \cdot \text{op\_c}) \cdot b$ fof(f_{10}, axiom)
$\forall a$: $\text{op\_d} = \text{ld}(a, \text{op\_c} \cdot a)$ fof(f_{11}, axiom)
$\forall b, a$: $\text{op\_e} = (\text{rd}(\text{op\_c}, a \cdot b) \cdot b) \cdot a$ fof(f_{12}, axiom)
$\forall b, a$: $\text{op\_f} = a \cdot (b \cdot \text{ld}(a \cdot b, \text{op\_c}))$ fof(f_{13}, axiom)
$\forall x_4, x_5$: $(\text{op\_f} \cdot (x_4 \cdot x_5) = (\text{op\_f} \cdot x_4) \cdot x_5$ and $x_4 \cdot (x_5 \cdot \text{op\_f}) = (x_4 \cdot x_5) \cdot \text{op\_f}$ and $x_4 \cdot (\text{op\_f} \cdot x_5) = (x_4 \cdot \text{op\_f}) \cdot x_5)$ fof(goals, conjecture)

GRP705-1.p Property of commutative C-loop
In a commutative C-loop, if a has order 4 and b has order 9, then a.bx = ab.x
$a \cdot \text{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\text{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\text{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ cnf(c_{07}, axiom)
$\text{op\_a} \cdot (\text{op\_a} \cdot (\text{op\_a} \cdot \text{op\_a})) = 1$ cnf(c_{08}, axiom)
$\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot (\text{op\_b} \cdot \text{op\_b}))))))) = 1$ cnf(c_{09}, axiom)
$\text{op\_a} \cdot (\text{op\_b} \cdot a) \neq (\text{op\_a} \cdot \text{op\_b}) \cdot a$ cnf(goals, negated_conjecture)

GRP706-1.p Every F-quasigroup is isotopic to a Moufang loop
$a \cdot \text{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\text{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\text{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)

$a \cdot (b \cdot c) = (a \cdot b) \cdot (\mathrm{ld}(a, a) \cdot c)$ cnf(c_{05}, axiom)
$(a \cdot b) \cdot c = (a \cdot \mathrm{rd}(c, c)) \cdot (b \cdot c)$ cnf(c_{06}, axiom)
$f(a, b) = \mathrm{rd}(a, \mathrm{op\_c}) \cdot \mathrm{ld}(\mathrm{op\_c}, b)$ cnf(c_{07}, axiom)
$f(a, f(b, f(a, c))) \neq f(f(f(a, b), a), c)$ cnf(goals, negated_conjecture)

GRP707-1.p A C-loop of exponent four with central squares is flexible
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ cnf(c_{07}, axiom)
$a \cdot (a \cdot (a \cdot a)) = 1$ cnf(c_{08}, axiom)
$(a \cdot a) \cdot b = b \cdot (a \cdot a)$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot a \neq a \cdot (b \cdot a)$ cnf(goals, negated_conjecture)

GRP708-1.p Bol loop commutant element squared in left and right nucleus - 1
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c$ cnf(c_{07}, axiom)
$\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c}$ cnf(c_{08}, axiom)
$(\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot (a \cdot b) = ((\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot a) \cdot b$ cnf(c_{09}, axiom)
$a \cdot (b \cdot \mathrm{op\_c}) \neq (a \cdot b) \cdot \mathrm{op\_c}$ cnf(goals, negated_conjecture)

GRP709-1.p Bol loop commutant element squared in left and right nucleus - 2
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (a \cdot c)) = (a \cdot (b \cdot a)) \cdot c$ cnf(c_{07}, axiom)
$\mathrm{op\_c} \cdot a = a \cdot \mathrm{op\_c}$ cnf(c_{08}, axiom)
$a \cdot (b \cdot \mathrm{op\_c}) = (a \cdot b) \cdot \mathrm{op\_c}$ cnf(c_{09}, axiom)
$(\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot (a \cdot b) \neq ((\mathrm{op\_c} \cdot \mathrm{op\_c}) \cdot a) \cdot b$ cnf(goals, negated_conjecture)

GRP710+1.p A magma with 2-sided inverses satisfying the C-law is a loop - 1a
In a Bol loop, if c is a commutant element, then c∧2 is in the left nucleus if and only if c is in the right nucleus.
$\forall a$: $a \cdot 1 = a$ fof(f_{01}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{02}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ fof(f_{03}, axiom)
$\forall a$: $a \cdot i(a) = 1$ fof(f_{04}, axiom)
$\forall a$: $i(a) \cdot a = 1$ fof(f_{05}, axiom)
$\forall x_0, x_1$: $\exists x_2$: $x_0 \cdot x_2 = x_1$ and $\forall x_3, x_4$: $\exists x_5$: $x_5 \cdot x_4 = x_3$ fof(goals, conjecture)

GRP711+1.p A magma with 2-sided inverses satisfying the C-law is a loop - 1b
$\forall a$: $a \cdot 1 = a$ fof(f_{01}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{02}, axiom)
$\forall c, b, a$: $a \cdot (b \cdot (b \cdot c)) = ((a \cdot b) \cdot b) \cdot c$ fof(f_{03}, axiom)
$\forall a$: $a \cdot i(a) = 1$ fof(f_{04}, axiom)
$\forall a$: $i(a) \cdot a = 1$ fof(f_{05}, axiom)
$\forall x_6, x_7, x_8$: $((x_6 \cdot x_7 = x_6 \cdot x_8 \Rightarrow x_7 = x_8)$ and $(x_7 \cdot x_6 = x_8 \cdot x_6 \Rightarrow x_7 = x_8))$ fof(goals, conjecture)

GRP712-1.p In Buchsteiner loops fourth powers are nuclear - a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)

$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (a \cdot b) \cdot c) = \mathrm{rd}(b \cdot (c \cdot a), a)$ cnf(c_{07}, axiom)
$(a \cdot (a \cdot (a \cdot a))) \cdot (b \cdot c) \neq ((a \cdot (a \cdot (a \cdot a))) \cdot b) \cdot c$ cnf(goals, negated_conjecture)

GRP713-1.p In Buchsteiner loops fourth powers are nuclear - b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (a \cdot b) \cdot c) = \mathrm{rd}(b \cdot (c \cdot a), a)$ cnf(c_{07}, axiom)
$a \cdot ((b \cdot (b \cdot (b \cdot b))) \cdot c) \neq (a \cdot (b \cdot (b \cdot (b \cdot b)))) \cdot c$ cnf(goals, negated_conjecture)

GRP714-1.p In Buchsteiner loops fourth powers are nuclear - c
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (a \cdot b) \cdot c) = \mathrm{rd}(b \cdot (c \cdot a), a)$ cnf(c_{07}, axiom)
$a \cdot (b \cdot (c \cdot (c \cdot (c \cdot c)))) \neq (a \cdot b) \cdot (c \cdot (c \cdot (c \cdot c)))$ cnf(goals, negated_conjecture)

GRP715+1.p Strongly right alternative rings 1
If a has a 2-sided inverse, then R(a∧-1) = R(a)∧-1 and L(a)∧-1 = R(a)L(a∧-1)R(a∧-1).
$\forall c, b, a$: $(a + b) + c = a + (b + c)$ fof(f_{01}, axiom)
$\forall b, a$: $a + b = b + a$ fof(f_{02}, axiom)
$\forall a$: $a + \mathrm{op}_0 = a$ fof(f_{03}, axiom)
$\forall a$: $a + -a = \mathrm{op}_0$ fof(f_{04}, axiom)
$\forall c, b, a$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(f_{05}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ fof(f_{06}, axiom)
$\forall b, a$: $a \cdot (b \cdot b) = (a \cdot b) \cdot b$ fof(f_{07}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{08}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{09}, axiom)
$\mathrm{op\_a} \cdot \mathrm{op\_b} = 1$ fof(f_{10}, axiom)
$\mathrm{op\_b} \cdot \mathrm{op\_a} = 1$ fof(f_{11}, axiom)
$\forall x_0$: $((x_0 \cdot \mathrm{op\_a}) \cdot \mathrm{op\_b} = x_0$ and $(x_0 \cdot \mathrm{op\_b}) \cdot \mathrm{op\_a} = x_0)$ fof(goals, conjecture)

GRP716-1.p Strongly right alternative rings 2a
If a has a 2-sided inverse, then R(a∧-1) = R(a)∧-1 and L(a)∧-1 = R(a)L(a∧-1)R(a∧-1).
$(a + b) + c = a + (b + c)$ cnf(c_{01}, axiom)
$a + b = b + a$ cnf(c_{02}, axiom)
$a + \mathrm{op}_0 = a$ cnf(c_{03}, axiom)
$a + -a = \mathrm{op}_0$ cnf(c_{04}, axiom)
$a \cdot (b + c) = a \cdot b + a \cdot c$ cnf(c_{05}, axiom)
$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ cnf(c_{06}, axiom)
$a \cdot (b \cdot b) = (a \cdot b) \cdot b$ cnf(c_{07}, axiom)
$a \cdot 1 = a$ cnf(c_{08}, axiom)
$1 \cdot a = a$ cnf(c_{09}, axiom)
$\mathrm{op\_a} \cdot \mathrm{op\_b} = 1$ cnf(c_{10}, axiom)
$\mathrm{op\_b} \cdot \mathrm{op\_a} = 1$ cnf(c_{11}, axiom)
$\mathrm{op\_a} \cdot ((\mathrm{op\_b} \cdot (a \cdot \mathrm{op\_b})) \cdot \mathrm{op\_a}) \neq a$ cnf(goals, negated_conjecture)

GRP717-1.p Strongly right alternative rings 2b
If a has a 2-sided inverse, then R(a∧-1) = R(a)∧-1 and L(a)∧-1 = R(a)L(a∧-1)R(a∧-1).
$(a + b) + c = a + (b + c)$ cnf(c_{01}, axiom)
$a + b = b + a$ cnf(c_{02}, axiom)
$a + \mathrm{op}_0 = a$ cnf(c_{03}, axiom)
$a + -a = \mathrm{op}_0$ cnf(c_{04}, axiom)
$a \cdot (b + c) = a \cdot b + a \cdot c$ cnf(c_{05}, axiom)

$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ cnf(c_{06}, axiom)
$a \cdot (b \cdot b) = (a \cdot b) \cdot b$ cnf(c_{07}, axiom)
$a \cdot 1 = a$ cnf(c_{08}, axiom)
$1 \cdot a = a$ cnf(c_{09}, axiom)
$\text{op\_a} \cdot \text{op\_b} = 1$ cnf(c_{10}, axiom)
$\text{op\_b} \cdot \text{op\_a} = 1$ cnf(c_{11}, axiom)
$(\text{op\_b} \cdot ((\text{op\_a} \cdot a) \cdot \text{op\_b})) \cdot \text{op\_a} \neq a$ cnf(goals, negated_conjecture)

GRP718-1.p In a commutative RIF loop, all squares are Moufang elements
$a \cdot \text{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\text{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\text{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot (c \cdot (a \cdot b)) = ((a \cdot (b \cdot c)) \cdot a) \cdot b$ cnf(c_{09}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{10}, axiom)
$(a \cdot a) \cdot ((b \cdot c) \cdot (a \cdot a)) \neq ((a \cdot a) \cdot b) \cdot (c \cdot (a \cdot a))$ cnf(goals, negated_conjecture)

GRP719-1.p In a commutative RIF loop, all cubes are C-elements
$a \cdot \text{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\text{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\text{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot (c \cdot (a \cdot b)) = ((a \cdot (b \cdot c)) \cdot a) \cdot b$ cnf(c_{09}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{10}, axiom)
$a \cdot ((b \cdot (b \cdot b)) \cdot ((b \cdot (b \cdot b)) \cdot c)) \neq ((a \cdot (b \cdot (b \cdot b))) \cdot (b \cdot (b \cdot b))) \cdot c$ cnf(goals, negated_conjecture)

GRP720+1.p In commutative A-loops, squares form a subloop
$\forall a\colon a \cdot 1 = a$ fof(f_{01}, axiom)
$\forall a\colon 1 \cdot a = a$ fof(f_{02}, axiom)
$\forall b, a\colon a \cdot \text{ld}(a, b) = b$ fof(f_{03}, axiom)
$\forall b, a\colon \text{ld}(a, a \cdot b) = b$ fof(f_{04}, axiom)
$\forall b, a\colon a \cdot b = b \cdot a$ fof(f_{05}, axiom)
$\forall d, c, b, a\colon \text{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \text{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \text{ld}(a \cdot b, a \cdot (b \cdot d))$ fof(f_{06}, axiom)
$\forall c, b, a\colon \text{ld}(a, (b \cdot c) \cdot a) = \text{ld}(a, b \cdot a) \cdot \text{ld}(a, c \cdot a)$ fof(f_{07}, axiom)
$\forall x_0, x_1\colon \exists x_2\colon (x_0 \cdot x_0) \cdot (x_1 \cdot x_1) = x_2 \cdot x_2$ fof(goals, conjecture)

GRP720-2.p In commutative A-loops, squares form a subloop
$a \cdot 1 = a$ cnf(c_{01}, axiom)
$1 \cdot a = a$ cnf(c_{02}, axiom)
$a \cdot \text{ld}(a, b) = b$ cnf(c_{03}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{05}, axiom)
$\text{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \text{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \text{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{06}, axiom)
$\text{ld}(a, (b \cdot c) \cdot a) = \text{ld}(a, b \cdot a) \cdot \text{ld}(a, c \cdot a)$ cnf(c_{07}, axiom)
$(\text{op\_a} \cdot \text{op\_a}) \cdot (\text{op\_b} \cdot \text{op\_b}) \neq a \cdot a$ cnf(c_{08}, axiom)

GRP721-1.p In commutative A-loops squares form a subloop - a witnessing term
$a \cdot 1 = a$ cnf(c_{01}, axiom)
$1 \cdot a = a$ cnf(c_{02}, axiom)
$a \cdot \text{ld}(a, b) = b$ cnf(c_{03}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{05}, axiom)
$\text{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \text{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \text{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{06}, axiom)
$\text{ld}(a, (b \cdot c) \cdot a) = \text{ld}(a, b \cdot a) \cdot \text{ld}(a, c \cdot a)$ cnf(c_{07}, axiom)

$f(a,b) = \mathrm{ld}(\mathrm{ld}(a \cdot b, a) \cdot \mathrm{ld}(a \cdot b, b), 1)$ cnf(c_{08}, axiom)
$(a \cdot a) \cdot (b \cdot b) \neq f(a,b) \cdot f(a,b)$ cnf(goals, negated_conjecture)

GRP722-1.p In commutative A-loops square-subloop operation is commutative
$a \cdot 1 = a$ cnf(c_{01}, axiom)
$1 \cdot a = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{05}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{07}, axiom)
$(a \cdot a) \cdot (b \cdot b) = f(a,b) \cdot f(a,b)$ cnf(c_{08}, axiom)
$f(a,b) \neq f(b,a)$ cnf(goals, negated_conjecture)

GRP723-1.p In commutative A-loops of exp 2 square-subloop is associative
$a \cdot 1 = a$ cnf(c_{01}, axiom)
$1 \cdot a = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{05}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{07}, axiom)
$a \cdot a = 1$ cnf(c_{08}, axiom)
$f(a,b) = \mathrm{ld}(b, \mathrm{ld}(a \cdot b, b))$ cnf(c_{09}, axiom)
$f(a, f(b,c)) \neq f(f(a,b), c)$ cnf(goals, negated_conjecture)

GRP724-1.p Loops with abelian inner mapping group - associativity
Uniquely 2-divisible loops with abelian inner mapping group of exponent 2 are associative.
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{05}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{06}, axiom)
$s(a) \cdot s(a) = a$ cnf(c_{07}, axiom)
$s(a \cdot a) = a$ cnf(c_{08}, axiom)
$\mathrm{op\_l}(a,b,c) = \mathrm{ld}(c \cdot b, c \cdot (b \cdot a))$ cnf(c_{09}, axiom)
$\mathrm{op\_r}(a,b,c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{10}, axiom)
$\mathrm{op\_t}(a,b) = \mathrm{ld}(b, a \cdot b)$ cnf(c_{11}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c), d, e) = \mathrm{op\_r}(\mathrm{op\_r}(a,d,e), b, c)$ cnf(c_{12}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a,b,c), d, e) = \mathrm{op\_r}(\mathrm{op\_l}(a,d,e), b, c)$ cnf(c_{13}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c), d, e) = \mathrm{op\_l}(\mathrm{op\_l}(a,d,e), b, c)$ cnf(c_{14}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a,b,c), d) = \mathrm{op\_r}(\mathrm{op\_t}(a,d), b, c)$ cnf(c_{15}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a,b,c), d) = \mathrm{op\_l}(\mathrm{op\_t}(a,d), b, c)$ cnf(c_{16}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b), c) = \mathrm{op\_t}(\mathrm{op\_t}(a,c), b)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b), b) = a$ cnf(c_{18}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c), b, c) = a$ cnf(c_{19}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c), b, c) = a$ cnf(c_{20}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP725-1.p Loops with abelian inner mapping group - commutativity
Uniquely 2-divisible loops with abelian inner mapping group of exponent 2 are commutative.
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{05}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{06}, axiom)
$s(a) \cdot s(a) = a$ cnf(c_{07}, axiom)
$s(a \cdot a) = a$ cnf(c_{08}, axiom)
$\mathrm{op\_l}(a,b,c) = \mathrm{ld}(c \cdot b, c \cdot (b \cdot a))$ cnf(c_{09}, axiom)

$\mathrm{op\_r}(a,b,c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{10}, axiom)
$\mathrm{op\_t}(a,b) = \mathrm{ld}(b, a \cdot b)$ cnf(c_{11}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_r}(a,d,e),b,c)$ cnf(c_{12}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{13}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c),d,e) = \mathrm{op\_l}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{14}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a,b,c),d) = \mathrm{op\_r}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{15}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a,b,c),d) = \mathrm{op\_l}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{16}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b),c) = \mathrm{op\_t}(\mathrm{op\_t}(a,c),b)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b),b) = a$ cnf(c_{18}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c),b,c) = a$ cnf(c_{19}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c),b,c) = a$ cnf(c_{20}, axiom)
$a \cdot b \neq b \cdot a$ cnf(goals, negated_conjecture)

GRP726-1.p Bruck loops that are centrally nilpotent - hard part
Bruck loops with abelian inner mapping group are centrally nilpotent of class two - the hard part.
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot i(a) = 1$ cnf(c_{03}, axiom)
$i(a) \cdot a = 1$ cnf(c_{04}, axiom)
$i(a \cdot b) = i(a) \cdot i(b)$ cnf(c_{05}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{06}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{07}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{08}, axiom)
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c))$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \mathrm{asoc}(a,b,c)$ cnf(c_{10}, axiom)
$\mathrm{op\_l}(a,b,c) = i(c \cdot b) \cdot (c \cdot (b \cdot a))$ cnf(c_{11}, axiom)
$\mathrm{op\_r}(a,b,c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{12}, axiom)
$\mathrm{op\_t}(a,b) = i(b) \cdot (a \cdot b)$ cnf(c_{13}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_r}(a,d,e),b,c)$ cnf(c_{14}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{15}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c),d,e) = \mathrm{op\_l}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{16}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a,b,c),d) = \mathrm{op\_r}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a,b,c),d) = \mathrm{op\_l}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{18}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b),c) = \mathrm{op\_t}(\mathrm{op\_t}(a,c),b)$ cnf(c_{19}, axiom)
$\mathrm{asoc}(\mathrm{asoc}(a,b,c),d,e) \neq 1$ cnf(goals, negated_conjecture)

GRP727-1.p Bruck loops that are centrally nilpotent - 1st easy part
Bruck loops with abelian inner mapping group are centrally nilpotent of class two - 1st easy part.
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot i(a) = 1$ cnf(c_{03}, axiom)
$i(a) \cdot a = 1$ cnf(c_{04}, axiom)
$i(a \cdot b) = i(a) \cdot i(b)$ cnf(c_{05}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{06}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{07}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{08}, axiom)
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c))$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \mathrm{asoc}(a,b,c)$ cnf(c_{10}, axiom)
$\mathrm{op\_l}(a,b,c) = i(c \cdot b) \cdot (c \cdot (b \cdot a))$ cnf(c_{11}, axiom)
$\mathrm{op\_r}(a,b,c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{12}, axiom)
$\mathrm{op\_t}(a,b) = i(b) \cdot (a \cdot b)$ cnf(c_{13}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_r}(a,d,e),b,c)$ cnf(c_{14}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a,b,c),d,e) = \mathrm{op\_r}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{15}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a,b,c),d,e) = \mathrm{op\_l}(\mathrm{op\_l}(a,d,e),b,c)$ cnf(c_{16}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a,b,c),d) = \mathrm{op\_r}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a,b,c),d) = \mathrm{op\_l}(\mathrm{op\_t}(a,d),b,c)$ cnf(c_{18}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a,b),c) = \mathrm{op\_t}(\mathrm{op\_t}(a,c),b)$ cnf(c_{19}, axiom)
$\mathrm{asoc}(\mathrm{asoc}(a,b,c),d,e) = 1$ cnf(c_{20}, axiom)
$\mathrm{asoc}(a,b,\mathrm{asoc}(c,d,e)) \neq 1$ cnf(goals, negated_conjecture)

GRP728-1.p Bruck loops that are centrally nilpotent - 2nd easy part a

Bruck loops with abelian inner mapping group are centrally nilpotent of class two - 2nd easy part.
$1 \cdot a = a \qquad \text{cnf}(c_{01}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{02}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(c_{03}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(c_{04}, \text{axiom})$
$i(a \cdot b) = i(a) \cdot i(b) \qquad \text{cnf}(c_{05}, \text{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(c_{06}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{07}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{08}, \text{axiom})$
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c)) \qquad \text{cnf}(c_{09}, \text{axiom})$
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \text{asoc}(a, b, c) \qquad \text{cnf}(c_{10}, \text{axiom})$
$a \cdot b = (b \cdot a) \cdot \text{op\_k}(a, b) \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_l}(a, b, c) = i(c \cdot b) \cdot (c \cdot (b \cdot a)) \qquad \text{cnf}(c_{12}, \text{axiom})$
$\text{op\_r}(a, b, c) = \text{rd}((a \cdot b) \cdot c, b \cdot c) \qquad \text{cnf}(c_{13}, \text{axiom})$
$\text{op\_t}(a, b) = i(b) \cdot (a \cdot b) \qquad \text{cnf}(c_{14}, \text{axiom})$
$\text{op\_r}(\text{op\_r}(a, b, c), d, e) = \text{op\_r}(\text{op\_r}(a, d, e), b, c) \qquad \text{cnf}(c_{15}, \text{axiom})$
$\text{op\_l}(\text{op\_r}(a, b, c), d, e) = \text{op\_r}(\text{op\_l}(a, d, e), b, c) \qquad \text{cnf}(c_{16}, \text{axiom})$
$\text{op\_l}(\text{op\_l}(a, b, c), d, e) = \text{op\_l}(\text{op\_l}(a, d, e), b, c) \qquad \text{cnf}(c_{17}, \text{axiom})$
$\text{op\_t}(\text{op\_r}(a, b, c), d) = \text{op\_r}(\text{op\_t}(a, d), b, c) \qquad \text{cnf}(c_{18}, \text{axiom})$
$\text{op\_t}(\text{op\_l}(a, b, c), d) = \text{op\_l}(\text{op\_t}(a, d), b, c) \qquad \text{cnf}(c_{19}, \text{axiom})$
$\text{op\_t}(\text{op\_t}(a, b), c) = \text{op\_t}(\text{op\_t}(a, c), b) \qquad \text{cnf}(c_{20}, \text{axiom})$
$\text{asoc}(\text{asoc}(a, b, c), d, e) = 1 \qquad \text{cnf}(c_{21}, \text{axiom})$
$\text{asoc}(a, b, \text{asoc}(c, d, e)) = 1 \qquad \text{cnf}(c_{22}, \text{axiom})$
$\text{op\_k}(\text{op\_k}(a, b), c) \neq 1 \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP729-1.p Bruck loops that are centrally nilpotent - 2nd easy part b
Bruck loops with abelian inner mapping group are centrally nilpotent of class two - 2nd easy part.
$1 \cdot a = a \qquad \text{cnf}(c_{01}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{02}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(c_{03}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(c_{04}, \text{axiom})$
$i(a \cdot b) = i(a) \cdot i(b) \qquad \text{cnf}(c_{05}, \text{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(c_{06}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{07}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(c_{08}, \text{axiom})$
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c)) \qquad \text{cnf}(c_{09}, \text{axiom})$
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \text{asoc}(a, b, c) \qquad \text{cnf}(c_{10}, \text{axiom})$
$a \cdot b = (b \cdot a) \cdot \text{op\_k}(a, b) \qquad \text{cnf}(c_{11}, \text{axiom})$
$\text{op\_l}(a, b, c) = i(c \cdot b) \cdot (c \cdot (b \cdot a)) \qquad \text{cnf}(c_{12}, \text{axiom})$
$\text{op\_r}(a, b, c) = \text{rd}((a \cdot b) \cdot c, b \cdot c) \qquad \text{cnf}(c_{13}, \text{axiom})$
$\text{op\_t}(a, b) = i(b) \cdot (a \cdot b) \qquad \text{cnf}(c_{14}, \text{axiom})$
$\text{op\_r}(\text{op\_r}(a, b, c), d, e) = \text{op\_r}(\text{op\_r}(a, d, e), b, c) \qquad \text{cnf}(c_{15}, \text{axiom})$
$\text{op\_l}(\text{op\_r}(a, b, c), d, e) = \text{op\_r}(\text{op\_l}(a, d, e), b, c) \qquad \text{cnf}(c_{16}, \text{axiom})$
$\text{op\_l}(\text{op\_l}(a, b, c), d, e) = \text{op\_l}(\text{op\_l}(a, d, e), b, c) \qquad \text{cnf}(c_{17}, \text{axiom})$
$\text{op\_t}(\text{op\_r}(a, b, c), d) = \text{op\_r}(\text{op\_t}(a, d), b, c) \qquad \text{cnf}(c_{18}, \text{axiom})$
$\text{op\_t}(\text{op\_l}(a, b, c), d) = \text{op\_l}(\text{op\_t}(a, d), b, c) \qquad \text{cnf}(c_{19}, \text{axiom})$
$\text{op\_t}(\text{op\_t}(a, b), c) = \text{op\_t}(\text{op\_t}(a, c), b) \qquad \text{cnf}(c_{20}, \text{axiom})$
$\text{asoc}(\text{asoc}(a, b, c), d, e) = 1 \qquad \text{cnf}(c_{21}, \text{axiom})$
$\text{asoc}(a, b, \text{asoc}(c, d, e)) = 1 \qquad \text{cnf}(c_{22}, \text{axiom})$
$\text{asoc}(a, b, \text{op\_k}(c, d)) \neq 1 \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP730-1.p Bruck loops that are centrally nilpotent - 2nd easy part c
Bruck loops with abelian inner mapping group are centrally nilpotent of class two - 2nd easy part.
$1 \cdot a = a \qquad \text{cnf}(c_{01}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(c_{02}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(c_{03}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(c_{04}, \text{axiom})$
$i(a \cdot b) = i(a) \cdot i(b) \qquad \text{cnf}(c_{05}, \text{axiom})$
$i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(c_{06}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(c_{07}, \text{axiom})$

$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{08}, axiom)
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c))$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \mathrm{asoc}(a, b, c)$ cnf(c_{10}, axiom)
$a \cdot b = (b \cdot a) \cdot \mathrm{op\_k}(a, b)$ cnf(c_{11}, axiom)
$\mathrm{op\_l}(a, b, c) = i(c \cdot b) \cdot (c \cdot (b \cdot a))$ cnf(c_{12}, axiom)
$\mathrm{op\_r}(a, b, c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{13}, axiom)
$\mathrm{op\_t}(a, b) = i(b) \cdot (a \cdot b)$ cnf(c_{14}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a, b, c), d, e) = \mathrm{op\_r}(\mathrm{op\_r}(a, d, e), b, c)$ cnf(c_{15}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a, b, c), d, e) = \mathrm{op\_r}(\mathrm{op\_l}(a, d, e), b, c)$ cnf(c_{16}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a, b, c), d, e) = \mathrm{op\_l}(\mathrm{op\_l}(a, d, e), b, c)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a, b, c), d) = \mathrm{op\_r}(\mathrm{op\_t}(a, d), b, c)$ cnf(c_{18}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a, b, c), d) = \mathrm{op\_l}(\mathrm{op\_t}(a, d), b, c)$ cnf(c_{19}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a, b), c) = \mathrm{op\_t}(\mathrm{op\_t}(a, c), b)$ cnf(c_{20}, axiom)
$\mathrm{asoc}(\mathrm{asoc}(a, b, c), d, e) = 1$ cnf(c_{21}, axiom)
$\mathrm{asoc}(a, b, \mathrm{asoc}(c, d, e)) = 1$ cnf(c_{22}, axiom)
$\mathrm{asoc}(\mathrm{op\_k}(a, b), c, d) \neq 1$ cnf(goals, negated_conjecture)

GRP731-1.p Bruck loops that are centrally nilpotent - 2nd easy part d
Bruck loops with abelian inner mapping group are centrally nilpotent of class two - 2nd easy part.
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot i(a) = 1$ cnf(c_{03}, axiom)
$i(a) \cdot a = 1$ cnf(c_{04}, axiom)
$i(a \cdot b) = i(a) \cdot i(b)$ cnf(c_{05}, axiom)
$i(a) \cdot (a \cdot b) = b$ cnf(c_{06}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{07}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{08}, axiom)
$(a \cdot (b \cdot a)) \cdot c = a \cdot (b \cdot (a \cdot c))$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c = (a \cdot (b \cdot c)) \cdot \mathrm{asoc}(a, b, c)$ cnf(c_{10}, axiom)
$a \cdot b = (b \cdot a) \cdot \mathrm{op\_k}(a, b)$ cnf(c_{11}, axiom)
$\mathrm{op\_l}(a, b, c) = i(c \cdot b) \cdot (c \cdot (b \cdot a))$ cnf(c_{12}, axiom)
$\mathrm{op\_r}(a, b, c) = \mathrm{rd}((a \cdot b) \cdot c, b \cdot c)$ cnf(c_{13}, axiom)
$\mathrm{op\_t}(a, b) = i(b) \cdot (a \cdot b)$ cnf(c_{14}, axiom)
$\mathrm{op\_r}(\mathrm{op\_r}(a, b, c), d, e) = \mathrm{op\_r}(\mathrm{op\_r}(a, d, e), b, c)$ cnf(c_{15}, axiom)
$\mathrm{op\_l}(\mathrm{op\_r}(a, b, c), d, e) = \mathrm{op\_r}(\mathrm{op\_l}(a, d, e), b, c)$ cnf(c_{16}, axiom)
$\mathrm{op\_l}(\mathrm{op\_l}(a, b, c), d, e) = \mathrm{op\_l}(\mathrm{op\_l}(a, d, e), b, c)$ cnf(c_{17}, axiom)
$\mathrm{op\_t}(\mathrm{op\_r}(a, b, c), d) = \mathrm{op\_r}(\mathrm{op\_t}(a, d), b, c)$ cnf(c_{18}, axiom)
$\mathrm{op\_t}(\mathrm{op\_l}(a, b, c), d) = \mathrm{op\_l}(\mathrm{op\_t}(a, d), b, c)$ cnf(c_{19}, axiom)
$\mathrm{op\_t}(\mathrm{op\_t}(a, b), c) = \mathrm{op\_t}(\mathrm{op\_t}(a, c), b)$ cnf(c_{20}, axiom)
$\mathrm{asoc}(\mathrm{asoc}(a, b, c), d, e) = 1$ cnf(c_{21}, axiom)
$\mathrm{asoc}(a, b, \mathrm{asoc}(c, d, e)) = 1$ cnf(c_{22}, axiom)
$\mathrm{op\_k}(\mathrm{asoc}(a, b, c), d) \neq 1$ cnf(goals, negated_conjecture)

GRP732-1.p Basarab's theorem on CC loops
$1 \cdot a = a$ cnf(c_{01}, axiom)
$a \cdot 1 = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{05}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{06}, axiom)
$\mathrm{rd}(a \cdot b, a) \cdot (a \cdot c) = a \cdot (b \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot \mathrm{ld}(b, c \cdot b) = (a \cdot c) \cdot b$ cnf(c_{08}, axiom)
$a \cdot (b \cdot \mathrm{ld}(c \cdot d, d \cdot c)) \neq (a \cdot b) \cdot \mathrm{ld}(c \cdot d, d \cdot c)$ cnf(goals, negated_conjecture)

GRP733+1.p Non-flexible non-commutative DTS loop.
$\forall b, a \colon a \cdot \mathrm{ld}(a, b) = b$ fof(c_{01}, axiom)
$\forall b, a \colon \mathrm{ld}(a, a \cdot b) = b$ fof(c_{02}, axiom)
$\forall b, a \colon \mathrm{rd}(a, b) \cdot b = a$ fof(c_{03}, axiom)
$\forall b, a \colon \mathrm{rd}(a \cdot b, b) = a$ fof(c_{04}, axiom)
$\forall a \colon a \cdot 1 = a$ fof(c_{05}, axiom)
$\forall a \colon 1 \cdot a = a$ fof(c_{06}, axiom)

$\forall a\colon a \cdot a = 1$ fof(c_{07}, axiom)
$c \cdot d \neq d \cdot c$ fof(c_{08}, axiom)
$(a \cdot b) \cdot a \neq a \cdot (b \cdot a)$ fof(c_{09}, axiom)
$\forall x_0, x_1, x_2\colon (x_0 \cdot x_1 = x_2 \Rightarrow ((x_0 \cdot x_2 = x_1 \text{ and } x_1 \cdot x_2 = x_0) \text{ or } (x_0 \cdot x_2 = x_1 \text{ and } x_2 \cdot x_1 = x_0) \text{ or } (x_2 \cdot x_0 = x_1 \text{ and } x_2 \cdot x_1 = x_0)))$ fof(c_{10}, axiom)

GRP734+1.p Non-commutative pure DTS loop.
$\forall b, a\colon a \cdot \mathrm{ld}(a, b) = b$ fof(c_{01}, axiom)
$\forall b, a\colon \mathrm{ld}(a, a \cdot b) = b$ fof(c_{02}, axiom)
$\forall b, a\colon \mathrm{rd}(a, b) \cdot b = a$ fof(c_{03}, axiom)
$\forall b, a\colon \mathrm{rd}(a \cdot b, b) = a$ fof(c_{04}, axiom)
$\forall a\colon a \cdot 1 = a$ fof(c_{05}, axiom)
$\forall a\colon 1 \cdot a = a$ fof(c_{06}, axiom)
$\forall a\colon a \cdot a = 1$ fof(c_{07}, axiom)
$\mathrm{op\_a} \cdot \mathrm{op\_b} \neq \mathrm{op\_b} \cdot \mathrm{op\_a}$ fof(c_{08}, axiom)
$\forall x_0, x_1, x_2\colon (x_0 \cdot x_1 = x_2 \Rightarrow ((x_0 \cdot x_2 = x_1 \text{ and } x_1 \cdot x_2 = x_0) \text{ or } (x_0 \cdot x_2 = x_1 \text{ and } x_2 \cdot x_1 = x_0) \text{ or } (x_2 \cdot x_0 = x_1 \text{ and } x_2 \cdot x_1 = x_0)))$ fof(c_{09}, axiom)
$\forall x_3, x_4\colon (x_3 \cdot x_4 = x_4 \cdot x_3 \Rightarrow (x_3 = 1 \text{ or } x_4 = 1 \text{ or } x_3 \cdot x_4 = 1))$ fof(c_{10}, axiom)

GRP735-1.p Nonmedial left distributive quasigroup
$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(c_{01}, axiom)
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{02}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{05}, axiom)
$(a \cdot b) \cdot (c \cdot d) \neq (a \cdot c) \cdot (b \cdot d)$ cnf(goals, negated_conjecture)

GRP736-1.p Nonmedial left distributive left 3-symmetric quasigroup
$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(c_{01}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$a \cdot (a \cdot (a \cdot b)) = b$ cnf(c_{04}, axiom)
$(a \cdot b) \cdot (c \cdot d) \neq (a \cdot c) \cdot (b \cdot d)$ cnf(goals, negated_conjecture)

GRP737-1.p Nonmedial left distributive left 2-symmetric quasigroup
$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(c_{01}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$a \cdot (a \cdot b) = b$ cnf(c_{04}, axiom)
$(a \cdot b) \cdot (c \cdot d) \neq (a \cdot c) \cdot (b \cdot d)$ cnf(goals, negated_conjecture)

GRP738-1.p Proper Buchsteiner loop
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$\mathrm{ld}(a, (a \cdot b) \cdot c) = \mathrm{rd}(b \cdot (c \cdot a), a)$ cnf(c_{07}, axiom)
$a \cdot (b \cdot c) \neq \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(goals, negated_conjecture)

GRP739-1.p Proper commutative A-loop of odd order.
$a \cdot 1 = a$ cnf(c_{01}, axiom)
$1 \cdot a = a$ cnf(c_{02}, axiom)
$a \cdot \mathrm{ld}(a, b) = b$ cnf(c_{03}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{04}, axiom)
$\mathrm{ld}(a \cdot b, a \cdot (b \cdot (c \cdot d))) = \mathrm{ld}(a \cdot b, a \cdot (b \cdot c)) \cdot \mathrm{ld}(a \cdot b, a \cdot (b \cdot d))$ cnf(c_{05}, axiom)
$\mathrm{ld}(a, (b \cdot c) \cdot a) = \mathrm{ld}(a, b \cdot a) \cdot \mathrm{ld}(a, c \cdot a)$ cnf(c_{06}, axiom)
$s(a) \cdot s(a) = a$ cnf(c_{07}, axiom)
$s(a \cdot a) = a$ cnf(c_{08}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP740-1.p Proper commutative Moufang loop
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ cnf(c_{07}, axiom)
$a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot (c \cdot a) = (a \cdot (b \cdot c)) \cdot a$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot (c \cdot a) = a \cdot ((b \cdot c) \cdot a)$ cnf(c_{10}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{11}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP741-1.p Proper Moufang loop
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot (a \cdot c)) = ((a \cdot b) \cdot a) \cdot c$ cnf(c_{07}, axiom)
$a \cdot (b \cdot (c \cdot b)) = ((a \cdot b) \cdot c) \cdot b$ cnf(c_{08}, axiom)
$(a \cdot b) \cdot (c \cdot a) = (a \cdot (b \cdot c)) \cdot a$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot (c \cdot a) = a \cdot ((b \cdot c) \cdot a)$ cnf(c_{10}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP742-1.p Proper power associative CC loop
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (b \cdot c) = \mathrm{rd}(a \cdot b, a) \cdot (a \cdot c)$ cnf(c_{07}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot \mathrm{ld}(c, b \cdot c)$ cnf(c_{08}, axiom)
$a \cdot \mathrm{rd}(1, a) = 1$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP743-1.p Biassociative non-associative Steiner loop
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot (a \cdot b) = b$ cnf(c_{07}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{08}, axiom)
$a \cdot ((a \cdot (b \cdot c)) \cdot c) = (a \cdot ((a \cdot b) \cdot c)) \cdot c$ cnf(c_{09}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(sos, axiom)

GRP744-1.p Biassociative non-associative commutative loop of exponent 2
$a \cdot \mathrm{ld}(a,b) = b$ cnf(c_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(c_{02}, axiom)
$\mathrm{rd}(a,b) \cdot b = a$ cnf(c_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(c_{04}, axiom)
$a \cdot 1 = a$ cnf(c_{05}, axiom)
$1 \cdot a = a$ cnf(c_{06}, axiom)
$a \cdot a = 1$ cnf(c_{07}, axiom)
$a \cdot b = b \cdot a$ cnf(c_{08}, axiom)
$a \cdot ((a \cdot (b \cdot c)) \cdot c) = (a \cdot ((a \cdot b) \cdot c)) \cdot c$ cnf(c_{09}, axiom)

$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(sos, axiom)

GRP745+1.p Right alternative loop rings: the extra case
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ fof(f_{07}, axiom)
$\forall x_0, x_1, x_2$: $(((x_0 \cdot x_1) \cdot x_2 = x_0 \cdot (x_1 \cdot x_2)$ and $(x_0 \cdot x_2) \cdot x_1 = x_0 \cdot (x_2 \cdot x_1))$ or $((x_0 \cdot x_1) \cdot x_2 = x_0 \cdot (x_2 \cdot x_1)$ and $(x_0 \cdot x_2) \cdot x_1 = x_0 \cdot (x_1 \cdot x_2)))$ fof(f_{08}, axiom)
$a \cdot (b \cdot (c \cdot a)) = ((a \cdot b) \cdot c) \cdot a$ fof(goals, conjecture)

GRP746+1.p Right alternative loop rings: the group case
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ fof(f_{07}, axiom)
$\forall x_0, x_1, x_2$: $(((x_0 \cdot x_1) \cdot x_2 = x_0 \cdot (x_1 \cdot x_2)$ and $(x_0 \cdot x_2) \cdot x_1 = x_0 \cdot (x_2 \cdot x_1))$ or $((x_0 \cdot x_1) \cdot x_2 = (x_0 \cdot x_2) \cdot x_1$ and $x_0 \cdot (x_1 \cdot x_2) = x_0 \cdot (x_2 \cdot x_1)))$ fof(f_{08}, axiom)
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ fof(goals, conjecture)

GRP747+1.p Right alternative loop rings: the abelian group case
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ fof(f_{07}, axiom)
$\forall x_0, x_1, x_2$: $(((x_0 \cdot x_1) \cdot x_2 = x_0 \cdot (x_2 \cdot x_1)$ and $(x_0 \cdot x_2) \cdot x_1 = x_0 \cdot (x_1 \cdot x_2))$ or $((x_0 \cdot x_1) \cdot x_2 = (x_0 \cdot x_2) \cdot x_1$ and $x_0 \cdot (x_1 \cdot x_2) = x_0 \cdot (x_2 \cdot x_1)))$ fof(f_{08}, axiom)
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ fof(goals, conjecture)

GRP748+1.p Right alternative loop rings: a lemma
$\forall b, a$: $a \cdot \mathrm{ld}(a, b) = b$ fof(f_{01}, axiom)
$\forall b, a$: $\mathrm{ld}(a, a \cdot b) = b$ fof(f_{02}, axiom)
$\forall b, a$: $\mathrm{rd}(a, b) \cdot b = a$ fof(f_{03}, axiom)
$\forall b, a$: $\mathrm{rd}(a \cdot b, b) = a$ fof(f_{04}, axiom)
$\forall a$: $a \cdot 1 = a$ fof(f_{05}, axiom)
$\forall a$: $1 \cdot a = a$ fof(f_{06}, axiom)
$\forall c, b, a$: $((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ fof(f_{07}, axiom)
$\forall b, a$: $(a \cdot b) \cdot i(b) = a$ fof(f_{08}, axiom)
$\forall a$: $a \cdot i(a) = 1$ fof(f_{09}, axiom)
$\forall a$: $i(a) \cdot a = 1$ fof(f_{10}, axiom)
$\forall b, a$: $(a \cdot b = b \cdot a$ or $i(a) \cdot (a \cdot b) = b)$ fof(f_{11}, axiom)
$\forall x_0, x_1, x_2$: $x_2 \cdot (x_0 \cdot (x_2 \cdot x_1)) = ((x_2 \cdot x_0) \cdot x_2) \cdot x_1$ or $\forall x_3, x_4, x_5$: $x_3 \cdot (x_5 \cdot (x_4 \cdot x_5)) = ((x_3 \cdot x_5) \cdot x_4) \cdot x_5$ or $\forall x_6, x_7, x_8$: $(x_8 \cdot x_6) \cdot (x_7 \cdot x_8) = (x_8 \cdot (x_6 \cdot x_7)) \cdot x_8$ or $\forall x_9, x_{10}, x_{11}$: $(x_{11} \cdot x_9) \cdot (x_{10} \cdot x_{11}) = x_{11} \cdot ((x_9 \cdot x_{10}) \cdot x_{11})$ fof(goals, conjecture)

GRP748-2.p Right alternative loop rings: a lemma
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$a \cdot 1 = a$ cnf(f_{05}, axiom)
$1 \cdot a = a$ cnf(f_{06}, axiom)
$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b)$ cnf(f_{07}, axiom)
$(a \cdot b) \cdot i(b) = a$ cnf(f_{08}, axiom)

$a \cdot i(a) = 1 \qquad \text{cnf}(f_{09}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(f_{10}, \text{axiom})$
$a \cdot b = b \cdot a$ or $i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(f_{11}, \text{axiom})$
$a \cdot (b \cdot (a \cdot c)) \neq ((a \cdot b) \cdot a) \cdot c \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP748-3.p Right alternative loop rings: a lemma
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(f_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(f_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(f_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(f_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(f_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(f_{06}, \text{axiom})$
$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b) \qquad \text{cnf}(f_{07}, \text{axiom})$
$(a \cdot b) \cdot i(b) = a \qquad \text{cnf}(f_{08}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(f_{09}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(f_{10}, \text{axiom})$
$a \cdot b = b \cdot a$ or $i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(f_{11}, \text{axiom})$
$a \cdot (b \cdot (c \cdot b)) \neq ((a \cdot b) \cdot c) \cdot b \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP748-4.p Right alternative loop rings: a lemma
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(f_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(f_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(f_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(f_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(f_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(f_{06}, \text{axiom})$
$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b) \qquad \text{cnf}(f_{07}, \text{axiom})$
$(a \cdot b) \cdot i(b) = a \qquad \text{cnf}(f_{08}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(f_{09}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(f_{10}, \text{axiom})$
$a \cdot b = b \cdot a$ or $i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(f_{11}, \text{axiom})$
$(a \cdot b) \cdot (c \cdot a) \neq (a \cdot (b \cdot c)) \cdot a \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP748-5.p Right alternative loop rings: a lemma
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(f_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(f_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(f_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(f_{04}, \text{axiom})$
$a \cdot 1 = a \qquad \text{cnf}(f_{05}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(f_{06}, \text{axiom})$
$((a \cdot b) \cdot c) \cdot b = a \cdot ((b \cdot c) \cdot b) \qquad \text{cnf}(f_{07}, \text{axiom})$
$(a \cdot b) \cdot i(b) = a \qquad \text{cnf}(f_{08}, \text{axiom})$
$a \cdot i(a) = 1 \qquad \text{cnf}(f_{09}, \text{axiom})$
$i(a) \cdot a = 1 \qquad \text{cnf}(f_{10}, \text{axiom})$
$a \cdot b = b \cdot a$ or $i(a) \cdot (a \cdot b) = b \qquad \text{cnf}(f_{11}, \text{axiom})$
$(a \cdot b) \cdot (c \cdot a) \neq a \cdot ((b \cdot c) \cdot a) \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP749-1.p Simplifying a basis for trimedial quasigroups: part 1
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(f_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(f_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(f_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(f_{04}, \text{axiom})$
$(a \cdot (a \cdot a)) \cdot (b \cdot c) = (a \cdot b) \cdot ((a \cdot a) \cdot c) \qquad \text{cnf}(f_{05}, \text{axiom})$
$(a \cdot a) \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c) \qquad \text{cnf}(f_{06}, \text{axiom})$
$(a \cdot b) \cdot (c \cdot c) \neq (a \cdot c) \cdot (b \cdot c) \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP750-1.p Simplifying a basis for trimedial quasigroups: part 2
$a \cdot \text{ld}(a, b) = b \qquad \text{cnf}(f_{01}, \text{axiom})$
$\text{ld}(a, a \cdot b) = b \qquad \text{cnf}(f_{02}, \text{axiom})$
$\text{rd}(a, b) \cdot b = a \qquad \text{cnf}(f_{03}, \text{axiom})$
$\text{rd}(a \cdot b, b) = a \qquad \text{cnf}(f_{04}, \text{axiom})$
$(a \cdot (a \cdot a)) \cdot (b \cdot c) = (a \cdot b) \cdot ((a \cdot a) \cdot c) \qquad \text{cnf}(f_{05}, \text{axiom})$

$(a \cdot b) \cdot (c \cdot c) = (a \cdot c) \cdot (b \cdot c)$ cnf(f_{06}, axiom)
$(a \cdot a) \cdot (b \cdot c) \neq (a \cdot b) \cdot (a \cdot c)$ cnf(goals, negated_conjecture)

GRP751-1.p A new basis for trimedial quasigroups: part 1a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$(a \cdot (a \cdot a)) \cdot (b \cdot c) = (a \cdot b) \cdot ((a \cdot a) \cdot c)$ cnf(f_{05}, axiom)
$(a \cdot a) \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(f_{06}, axiom)
$(a \cdot b) \cdot (c \cdot c) = (a \cdot c) \cdot (b \cdot c)$ cnf(f_{07}, axiom)
$a \cdot (b \cdot c) \neq (\mathrm{rd}(a, a) \cdot b) \cdot (a \cdot c)$ cnf(goals, negated_conjecture)

GRP752-1.p A new basis for trimedial quasigroups: part 1b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$(a \cdot (a \cdot a)) \cdot (b \cdot c) = (a \cdot b) \cdot ((a \cdot a) \cdot c)$ cnf(f_{05}, axiom)
$(a \cdot a) \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(f_{06}, axiom)
$(a \cdot b) \cdot (c \cdot c) = (a \cdot c) \cdot (b \cdot c)$ cnf(f_{07}, axiom)
$(a \cdot b) \cdot c \neq (a \cdot c) \cdot (b \cdot \mathrm{ld}(c, c))$ cnf(goals, negated_conjecture)

GRP753-1.p A new basis for trimedial quasigroups: part 2a
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$a \cdot (b \cdot c) = (\mathrm{rd}(a, a) \cdot b) \cdot (a \cdot c)$ cnf(f_{05}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot (b \cdot \mathrm{ld}(c, c))$ cnf(f_{06}, axiom)
$(a \cdot (a \cdot a)) \cdot (b \cdot c) \neq (a \cdot b) \cdot ((a \cdot a) \cdot c)$ cnf(goals, negated_conjecture)

GRP754-1.p A new basis for trimedial quasigroups: part 2b
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$a \cdot (b \cdot c) = (\mathrm{rd}(a, a) \cdot b) \cdot (a \cdot c)$ cnf(f_{05}, axiom)
$(a \cdot b) \cdot c = (a \cdot c) \cdot (b \cdot \mathrm{ld}(c, c))$ cnf(f_{06}, axiom)
$(a \cdot a) \cdot (b \cdot c) \neq (a \cdot b) \cdot (a \cdot c)$ cnf(goals, negated_conjecture)

GRP755-1.p In char>2, right alternative loop rings are left alternative
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$a \cdot 1 = a$ cnf(f_{05}, axiom)
$1 \cdot a = a$ cnf(f_{06}, axiom)
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ or $a \cdot (b \cdot c) = (a \cdot c) \cdot b$ cnf(f_{07}, axiom)
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ or $a \cdot (c \cdot b) = (a \cdot b) \cdot c$ cnf(f_{08}, axiom)
$i(a) = \mathrm{ld}(a, 1)$ cnf(f_{09}, axiom)
$i(a \cdot b) \neq i(b) \cdot i(a)$ cnf(goals, negated_conjecture)

GRP756-1.p Quasigroups satisfying certain Bol-Moufang identity are groups
$a \cdot \mathrm{ld}(a, b) = b$ cnf(f_{01}, axiom)
$\mathrm{ld}(a, a \cdot b) = b$ cnf(f_{02}, axiom)
$\mathrm{rd}(a, b) \cdot b = a$ cnf(f_{03}, axiom)
$\mathrm{rd}(a \cdot b, b) = a$ cnf(f_{04}, axiom)
$a \cdot ((b \cdot b) \cdot c) = (a \cdot b) \cdot (b \cdot c)$ cnf(f_{05}, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(goals, negated_conjecture)

GRP759+1.p A 4-element non-abelian group
$\forall a$: $a \cdot i(a) = e$ fof(f_{01}, axiom)

$\forall a$: $a \cdot e = a$ fof(f_{02}, axiom)
$\forall b, a, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{03}, axiom)
op_a · op_b ≠ op_b · op_a fof(f_{04}, axiom)
$\forall x$: ($x = c_1$ or $x = c_2$ or $x = c_3$ or $x = c_4$) fof(a, axiom)
$c_1 \neq c_2$ fof(c1_not_c$_2$, axiom)
$c_1 \neq c_3$ fof(c1_not_c$_3$, axiom)
$c_1 \neq c_4$ fof(c1_not_c$_4$, axiom)
$c_2 \neq c_3$ fof(c2_not_c$_3$, axiom)
$c_2 \neq c_4$ fof(c2_not_c$_4$, axiom)
$c_3 \neq c_4$ fof(c3_not_c$_4$, axiom)

GRP760+1.p A group that must be infinite
A group containing an element of order 2 and having square roots must be infinite.
$\forall a$: $a \cdot i(a) = e$ fof(f_{01}, axiom)
$\forall a$: $a \cdot e = a$ fof(f_{02}, axiom)
$\forall b, a, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{03}, axiom)
$a \cdot a = e$ and $a \neq e$ fof(f_{04}, axiom)
$\forall a$: $\exists b$: $b \cdot b = a$ fof(f_{05}, axiom)

GRP761+1.p Non-discrete partially ordered group
$\forall a$: $a \cdot i(a) = e$ fof(f_{01}, axiom)
$\forall a$: $a \cdot e = a$ fof(f_{02}, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{03}, axiom)
$\forall a$: $o(a, a)$ fof(f_{04}, axiom)
$\forall a, b$: (($a \neq b$ and $o(a, b)$) $\Rightarrow$ $\neg\, o(b, a)$) fof(f_{05}, axiom)
$\forall a, b, c$: (($o(a, b)$ and $o(b, c)$) $\Rightarrow$ $o(a, c)$) fof(f_{06}, axiom)
$\forall a, b, c, d$: (($o(a, b)$ and $o(c, d)$) $\Rightarrow$ $o(a \cdot c, b \cdot d)$) fof(f_{07}, axiom)
$a \neq e$ and $o(e, a)$ fof(f_{08}, axiom)

GRP762+1.p Linearly ordered group
$\forall a$: $a \cdot i(a) = e$ fof(f_{01}, axiom)
$\forall a$: $a \cdot e = a$ fof(f_{02}, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{03}, axiom)
$\forall a, b$: $a \cdot b = b \cdot a$ fof(f_{04}, axiom)
$\forall a$: $o(a, a)$ fof(f_{05}, axiom)
$\forall a, b$: (($a \neq b$ and $o(a, b)$) $\Rightarrow$ $\neg\, o(b, a)$) fof(f_{06}, axiom)
$\forall a, b, c$: (($o(a, b)$ and $o(b, c)$) $\Rightarrow$ $o(a, c)$) fof(f_{07}, axiom)
$\forall a, b, c, d$: (($o(a, b)$ and $o(c, d)$) $\Rightarrow$ $o(a \cdot c, b \cdot d)$) fof(f_{08}, axiom)
$\forall a, b$: ($o(a, b)$ or $o(b, a)$) fof(f_{09}, axiom)
$a \neq e$ fof(f_{10}, axiom)

GRP763+1.p Lattice ordered group
$\forall a$: $a \cdot i(a) = e$ fof(f_{01}, axiom)
$\forall a$: $a \cdot e = a$ fof(f_{02}, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{03}, axiom)
$\forall a$: $m(a, a) = a$ fof(f_{04}, axiom)
$\forall a, b$: $m(a, b) = m(b, a)$ fof(f_{05}, axiom)
$\forall a, b, c$: $m(a, m(b, c)) = m(m(a, b), c)$ fof(f_{06}, axiom)
$\forall a$: $j(a, a) = a$ fof(f_{07}, axiom)
$\forall a, b$: $j(a, b) = j(b, a)$ fof(f_{08}, axiom)
$\forall a, b, c$: $j(a, j(b, c)) = j(j(a, b), c)$ fof(f_{09}, axiom)
$\forall a, b$: $m(a, j(a, b)) = a$ fof(f_{10}, axiom)
$\forall a, b$: $j(a, m(a, b)) = a$ fof(f_{11}, axiom)
$\forall a, b, c$: $a \cdot j(b, c) = j(a \cdot b, a \cdot c)$ fof(f_{12}, axiom)
$\forall a, b, c$: $j(b, c) \cdot a = j(b \cdot a, c \cdot a)$ fof(f_{13}, axiom)
$a \neq e$ fof(f_{14}, axiom)

GRP764-1.p Buchsteiner loop lemma 1
$a \cdot 1 = a$ cnf(sos$_{01}$, axiom)
$1 \cdot a = a$ cnf(sos$_{02}$, axiom)
$a \cdot (a \setminus b) = b$ cnf(sos$_{03}$, axiom)
$a \setminus a \cdot b = b$ cnf(sos$_{04}$, axiom)

$\text{quotient}(a \cdot b, b) = a \qquad \text{cnf}(\text{sos}_{05}, \text{axiom})$
$\text{quotient}(a, b) \cdot b = a \qquad \text{cnf}(\text{sos}_{06}, \text{axiom})$
$a \setminus (a \cdot b) \cdot c = \text{quotient}(b \cdot (c \cdot a), a) \qquad \text{cnf}(\text{sos}_{07}, \text{axiom})$
$a \cdot b \setminus a \cdot (b \cdot c) = \text{quotient}(\text{quotient}(c \cdot (a \cdot b), b), a) \qquad \text{cnf}(\text{sos}_{08}, \text{axiom})$
$i(a) = a \setminus 1 \qquad \text{cnf}(\text{sos}_{09}, \text{axiom})$
$j(a) = \text{quotient}(1, a) \qquad \text{cnf}(\text{sos}_{10}, \text{axiom})$
$i(a) \cdot a = a \cdot j(a) \qquad \text{cnf}(\text{sos}_{11}, \text{axiom})$
$\text{eta}(a) = i(a) \cdot a \qquad \text{cnf}(\text{sos}_{12}, \text{axiom})$
$x_0 \cdot (\text{eta}(x_0) \cdot x_1) \neq j(j(x_0)) \cdot x_1 \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP765-1.p Buchsteiner loop lemma 2
$a \cdot 1 = a \qquad \text{cnf}(\text{sos}_{01}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(\text{sos}_{02}, \text{axiom})$
$a \cdot (a \setminus b) = b \qquad \text{cnf}(\text{sos}_{03}, \text{axiom})$
$a \setminus a \cdot b = b \qquad \text{cnf}(\text{sos}_{04}, \text{axiom})$
$\text{quotient}(a \cdot b, b) = a \qquad \text{cnf}(\text{sos}_{05}, \text{axiom})$
$\text{quotient}(a, b) \cdot b = a \qquad \text{cnf}(\text{sos}_{06}, \text{axiom})$
$a \setminus (a \cdot b) \cdot c = \text{quotient}(b \cdot (c \cdot a), a) \qquad \text{cnf}(\text{sos}_{07}, \text{axiom})$
$a \cdot b \setminus a \cdot (b \cdot c) = \text{quotient}(\text{quotient}(c \cdot (a \cdot b), b), a) \qquad \text{cnf}(\text{sos}_{08}, \text{axiom})$
$i(a) = a \setminus 1 \qquad \text{cnf}(\text{sos}_{09}, \text{axiom})$
$j(a) = \text{quotient}(1, a) \qquad \text{cnf}(\text{sos}_{10}, \text{axiom})$
$i(a) \cdot a = a \cdot j(a) \qquad \text{cnf}(\text{sos}_{11}, \text{axiom})$
$\text{eta}(a) = i(a) \cdot a \qquad \text{cnf}(\text{sos}_{12}, \text{axiom})$
$i(i(x_0)) \cdot x_1 \neq \text{eta}(x_0) \cdot (x_0 \cdot x_1) \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP766-1.p Buchsteiner loop lemma 3
$a \cdot 1 = a \qquad \text{cnf}(\text{sos}_{01}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(\text{sos}_{02}, \text{axiom})$
$a \cdot (a \setminus b) = b \qquad \text{cnf}(\text{sos}_{03}, \text{axiom})$
$a \setminus a \cdot b = b \qquad \text{cnf}(\text{sos}_{04}, \text{axiom})$
$\text{quotient}(a \cdot b, b) = a \qquad \text{cnf}(\text{sos}_{05}, \text{axiom})$
$\text{quotient}(a, b) \cdot b = a \qquad \text{cnf}(\text{sos}_{06}, \text{axiom})$
$a \setminus (a \cdot b) \cdot c = \text{quotient}(b \cdot (c \cdot a), a) \qquad \text{cnf}(\text{sos}_{07}, \text{axiom})$
$a \cdot b \setminus a \cdot (b \cdot c) = \text{quotient}(\text{quotient}(c \cdot (a \cdot b), b), a) \qquad \text{cnf}(\text{sos}_{08}, \text{axiom})$
$i(a) = a \setminus 1 \qquad \text{cnf}(\text{sos}_{09}, \text{axiom})$
$j(a) = \text{quotient}(1, a) \qquad \text{cnf}(\text{sos}_{10}, \text{axiom})$
$i(a) \cdot a = a \cdot j(a) \qquad \text{cnf}(\text{sos}_{11}, \text{axiom})$
$\text{eta}(a) = i(a) \cdot a \qquad \text{cnf}(\text{sos}_{12}, \text{axiom})$
$l(a, b, c) = a \cdot b \setminus a \cdot (b \cdot c) \qquad \text{cnf}(\text{sos}_{13}, \text{axiom})$
$l(a, a, b \cdot c) = l(a, a, b) \cdot l(a, a, c) \qquad \text{cnf}(\text{sos}_{14}, \text{axiom})$
$i(i(a)) \cdot b = \text{eta}(a) \cdot (a \cdot b) \qquad \text{cnf}(\text{sos}_{15}, \text{axiom})$
$a \cdot (\text{eta}(a) \cdot b) = j(j(a)) \cdot b \qquad \text{cnf}(\text{sos}_{16}, \text{axiom})$
$a \cdot (b \cdot \text{eta}(a)) = (a \cdot b) \cdot \text{eta}(a) \qquad \text{cnf}(\text{sos}_{17}, \text{axiom})$
$t(a, b) = \text{quotient}(a \cdot b, a) \qquad \text{cnf}(\text{sos}_{18}, \text{axiom})$
$t(\text{eta}(a), b \cdot c) = t(\text{eta}(a), b) \cdot t(\text{eta}(a), c) \qquad \text{cnf}(\text{sos}_{19}, \text{axiom})$
$\text{eta}(x_0) \cdot (x_1 \cdot x_2) \neq (\text{eta}(x_0) \cdot x_1) \cdot x_2 \qquad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

GRP767-1.p Buchsteiner loop lemma 4
$a \cdot 1 = a \qquad \text{cnf}(\text{sos}_{01}, \text{axiom})$
$1 \cdot a = a \qquad \text{cnf}(\text{sos}_{02}, \text{axiom})$
$a \cdot (a \setminus b) = b \qquad \text{cnf}(\text{sos}_{03}, \text{axiom})$
$a \setminus a \cdot b = b \qquad \text{cnf}(\text{sos}_{04}, \text{axiom})$
$\text{quotient}(a \cdot b, b) = a \qquad \text{cnf}(\text{sos}_{05}, \text{axiom})$
$\text{quotient}(a, b) \cdot b = a \qquad \text{cnf}(\text{sos}_{06}, \text{axiom})$
$a \setminus (a \cdot b) \cdot c = \text{quotient}(b \cdot (c \cdot a), a) \qquad \text{cnf}(\text{sos}_{07}, \text{axiom})$
$a \cdot b \setminus a \cdot (b \cdot c) = \text{quotient}(\text{quotient}(c \cdot (a \cdot b), b), a) \qquad \text{cnf}(\text{sos}_{08}, \text{axiom})$
$i(a) = a \setminus 1 \qquad \text{cnf}(\text{sos}_{09}, \text{axiom})$
$j(a) = \text{quotient}(1, a) \qquad \text{cnf}(\text{sos}_{10}, \text{axiom})$
$i(a) \cdot a = a \cdot j(a) \qquad \text{cnf}(\text{sos}_{11}, \text{axiom})$
$\text{eta}(a) = i(a) \cdot a \qquad \text{cnf}(\text{sos}_{12}, \text{axiom})$
$i(i(a)) \cdot b = \text{eta}(a) \cdot (a \cdot b) \qquad \text{cnf}(\text{sos}_{13}, \text{axiom})$

$a \cdot (\mathrm{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos$_{14}$, axiom)
$a \cdot (b \cdot \mathrm{eta}(a)) = (a \cdot b) \cdot \mathrm{eta}(a)$ cnf(sos$_{15}$, axiom)
$\mathrm{eta}(a) \cdot (b \cdot c) = (\mathrm{eta}(a) \cdot b) \cdot c$ cnf(sos$_{16}$, axiom)
$l(a, b, c) = a \cdot b \setminus a \cdot (b \cdot c)$ cnf(sos$_{17}$, axiom)
$l(a, a, b \cdot c) = l(a, a, b) \cdot l(a, a, c)$ cnf(sos$_{18}$, axiom)
$t(a, b) = \mathrm{quotient}(a \cdot b, a)$ cnf(sos$_{19}$, axiom)
$t(\mathrm{eta}(a), b \cdot c) = t(\mathrm{eta}(a), b) \cdot t(\mathrm{eta}(a), c)$ cnf(sos$_{20}$, axiom)
$j(j(x_0)) \cdot j(x_1 \cdot x_0) \neq j(x_1)$ cnf(goals, negated_conjecture)

GRP768-1.p Buchsteiner loop lemma 5
$a \cdot 1 = a$ cnf(sos$_{01}$, axiom)
$1 \cdot a = a$ cnf(sos$_{02}$, axiom)
$a \cdot (a \setminus b) = b$ cnf(sos$_{03}$, axiom)
$a \setminus a \cdot b = b$ cnf(sos$_{04}$, axiom)
$\mathrm{quotient}(a \cdot b, b) = a$ cnf(sos$_{05}$, axiom)
$\mathrm{quotient}(a, b) \cdot b = a$ cnf(sos$_{06}$, axiom)
$a \setminus (a \cdot b) \cdot c = \mathrm{quotient}(b \cdot (c \cdot a), a)$ cnf(sos$_{07}$, axiom)
$a \cdot b \setminus a \cdot (b \cdot c) = \mathrm{quotient}(\mathrm{quotient}(c \cdot (a \cdot b), b), a)$ cnf(sos$_{08}$, axiom)
$i(a) = a \setminus 1$ cnf(sos$_{09}$, axiom)
$j(a) = \mathrm{quotient}(1, a)$ cnf(sos$_{10}$, axiom)
$i(a) \cdot a = a \cdot j(a)$ cnf(sos$_{11}$, axiom)
$\mathrm{eta}(a) = i(a) \cdot a$ cnf(sos$_{12}$, axiom)
$i(i(a)) \cdot b = \mathrm{eta}(a) \cdot (a \cdot b)$ cnf(sos$_{13}$, axiom)
$a \cdot (\mathrm{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos$_{14}$, axiom)
$a \cdot (b \cdot \mathrm{eta}(a)) = (a \cdot b) \cdot \mathrm{eta}(a)$ cnf(sos$_{15}$, axiom)
$\mathrm{eta}(a) \cdot (b \cdot c) = (\mathrm{eta}(a) \cdot b) \cdot c$ cnf(sos$_{16}$, axiom)
$\mathrm{quotient}(j(a), a) = j(a) \cdot i(a)$ cnf(sos$_{17}$, axiom)
$((\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot b) \cdot c = (\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot (b \cdot c)$ cnf(sos$_{18}$, axiom)
$t(a, b) = \mathrm{quotient}(a \cdot b, a)$ cnf(sos$_{19}$, axiom)
$t(\mathrm{eta}(x_0), x_1 \cdot x_2) \neq t(\mathrm{eta}(x_0), x_1) \cdot t(\mathrm{eta}(x_0), x_2)$ cnf(goals, negated_conjecture)

GRP769-1.p Buchsteiner loop lemma 6
$a \cdot 1 = a$ cnf(sos$_{01}$, axiom)
$1 \cdot a = a$ cnf(sos$_{02}$, axiom)
$a \cdot (a \setminus b) = b$ cnf(sos$_{03}$, axiom)
$a \setminus a \cdot b = b$ cnf(sos$_{04}$, axiom)
$\mathrm{quotient}(a \cdot b, b) = a$ cnf(sos$_{05}$, axiom)
$\mathrm{quotient}(a, b) \cdot b = a$ cnf(sos$_{06}$, axiom)
$a \setminus (a \cdot b) \cdot c = \mathrm{quotient}(b \cdot (c \cdot a), a)$ cnf(sos$_{07}$, axiom)
$a \cdot b \setminus a \cdot (b \cdot c) = \mathrm{quotient}(\mathrm{quotient}(c \cdot (a \cdot b), b), a)$ cnf(sos$_{08}$, axiom)
$i(a) = a \setminus 1$ cnf(sos$_{09}$, axiom)
$j(a) = \mathrm{quotient}(1, a)$ cnf(sos$_{10}$, axiom)
$i(a) \cdot a = a \cdot j(a)$ cnf(sos$_{11}$, axiom)
$\mathrm{eta}(a) = i(a) \cdot a$ cnf(sos$_{12}$, axiom)
$i(i(a)) \cdot b = \mathrm{eta}(a) \cdot (a \cdot b)$ cnf(sos$_{13}$, axiom)
$a \cdot (\mathrm{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos$_{14}$, axiom)
$a \cdot (b \cdot \mathrm{eta}(a)) = (a \cdot b) \cdot \mathrm{eta}(a)$ cnf(sos$_{15}$, axiom)
$\mathrm{eta}(a) \cdot (b \cdot c) = (\mathrm{eta}(a) \cdot b) \cdot c$ cnf(sos$_{16}$, axiom)
$\mathrm{quotient}(j(a), a) = j(a) \cdot i(a)$ cnf(sos$_{17}$, axiom)
$((\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot b) \cdot c = (\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot (b \cdot c)$ cnf(sos$_{18}$, axiom)
$t(a, b) = \mathrm{quotient}(a \cdot b, a)$ cnf(sos$_{19}$, axiom)
$t(\mathrm{eta}(a), b \cdot c) = t(\mathrm{eta}(a), b) \cdot t(\mathrm{eta}(a), c)$ cnf(sos$_{20}$, axiom)
$i(a \cdot b) \cdot i(i(a)) = i(b)$ cnf(sos$_{21}$, axiom)
$j(j(a)) \cdot j(b \cdot a) = j(b)$ cnf(sos$_{22}$, axiom)
$a \cdot i(b \cdot a) = i(b)$ cnf(sos$_{23}$, axiom)
$j(a \cdot b) \cdot a = j(b)$ cnf(sos$_{24}$, axiom)
$(x_0 \cdot x_1) \cdot x_2 \neq (x_0 \cdot x_2) \cdot (x_2 \setminus x_1 \cdot x_2)$ cnf(goals, negated_conjecture)

GRP770-1.p Buchsteiner loop lemma 7
$a \cdot 1 = a$ cnf(sos$_{01}$, axiom)
$1 \cdot a = a$ cnf(sos$_{02}$, axiom)

$a \cdot (a \setminus b) = b$ cnf(sos_{03}, axiom)
$a \setminus a \cdot b = b$ cnf(sos_{04}, axiom)
$\mathrm{quotient}(a \cdot b, b) = a$ cnf(sos_{05}, axiom)
$\mathrm{quotient}(a, b) \cdot b = a$ cnf(sos_{06}, axiom)
$a \setminus (a \cdot b) \cdot c = \mathrm{quotient}(b \cdot (c \cdot a), a)$ cnf(sos_{07}, axiom)
$a \cdot b \setminus a \cdot (b \cdot c) = \mathrm{quotient}(\mathrm{quotient}(c \cdot (a \cdot b), b), a)$ cnf(sos_{08}, axiom)
$i(a) = a \setminus 1$ cnf(sos_{09}, axiom)
$j(a) = \mathrm{quotient}(1, a)$ cnf(sos_{10}, axiom)
$i(a) \cdot a = a \cdot j(a)$ cnf(sos_{11}, axiom)
$\mathrm{eta}(a) = i(a) \cdot a$ cnf(sos_{12}, axiom)
$i(i(a)) \cdot b = \mathrm{eta}(a) \cdot (a \cdot b)$ cnf(sos_{13}, axiom)
$a \cdot (\mathrm{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos_{14}, axiom)
$a \cdot (b \cdot \mathrm{eta}(a)) = (a \cdot b) \cdot \mathrm{eta}(a)$ cnf(sos_{15}, axiom)
$\mathrm{eta}(a) \cdot (b \cdot c) = (\mathrm{eta}(a) \cdot b) \cdot c$ cnf(sos_{16}, axiom)
$\mathrm{quotient}(j(a), a) = j(a) \cdot i(a)$ cnf(sos_{17}, axiom)
$((\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot b) \cdot c = (\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot (b \cdot c)$ cnf(sos_{18}, axiom)
$t(a, b) = \mathrm{quotient}(a \cdot b, a)$ cnf(sos_{19}, axiom)
$t(\mathrm{eta}(a), b \cdot c) = t(\mathrm{eta}(a), b) \cdot t(\mathrm{eta}(a), c)$ cnf(sos_{20}, axiom)
$i(a \cdot b) \cdot i(i(a)) = i(b)$ cnf(sos_{21}, axiom)
$j(j(a)) \cdot j(b \cdot a) = j(b)$ cnf(sos_{22}, axiom)
$a \cdot i(b \cdot a) = i(b)$ cnf(sos_{23}, axiom)
$j(a \cdot b) \cdot a = j(b)$ cnf(sos_{24}, axiom)
$x_0 \cdot (x_1 \cdot x_2) \neq \mathrm{quotient}(x_0 \cdot x_1, x_0) \cdot (x_0 \cdot x_2)$ cnf(goals, negated_conjecture)

GRP771-1.p Buchsteiner loop lemma 8
$a \cdot 1 = a$ cnf(sos_{01}, axiom)
$1 \cdot a = a$ cnf(sos_{02}, axiom)
$a \cdot (a \setminus b) = b$ cnf(sos_{03}, axiom)
$a \setminus a \cdot b = b$ cnf(sos_{04}, axiom)
$\mathrm{quotient}(a \cdot b, b) = a$ cnf(sos_{05}, axiom)
$\mathrm{quotient}(a, b) \cdot b = a$ cnf(sos_{06}, axiom)
$a \setminus (a \cdot b) \cdot c = \mathrm{quotient}(b \cdot (c \cdot a), a)$ cnf(sos_{07}, axiom)
$a \cdot b \setminus a \cdot (b \cdot c) = \mathrm{quotient}(\mathrm{quotient}(c \cdot (a \cdot b), b), a)$ cnf(sos_{08}, axiom)
$i(a) = a \setminus 1$ cnf(sos_{09}, axiom)
$j(a) = \mathrm{quotient}(1, a)$ cnf(sos_{10}, axiom)
$i(a) \cdot a = a \cdot j(a)$ cnf(sos_{11}, axiom)
$\mathrm{eta}(a) = i(a) \cdot a$ cnf(sos_{12}, axiom)
$i(i(a)) \cdot b = \mathrm{eta}(a) \cdot (a \cdot b)$ cnf(sos_{13}, axiom)
$a \cdot (\mathrm{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos_{14}, axiom)
$a \cdot (b \cdot \mathrm{eta}(a)) = (a \cdot b) \cdot \mathrm{eta}(a)$ cnf(sos_{15}, axiom)
$\mathrm{eta}(a) \cdot (b \cdot c) = (\mathrm{eta}(a) \cdot b) \cdot c$ cnf(sos_{16}, axiom)
$\mathrm{quotient}(j(a), a) = j(a) \cdot i(a)$ cnf(sos_{17}, axiom)
$((\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot b) \cdot c = (\mathrm{quotient}(j(a), a) \cdot (a \cdot a)) \cdot (b \cdot c)$ cnf(sos_{18}, axiom)
$t(a, b) = \mathrm{quotient}(a \cdot b, a)$ cnf(sos_{19}, axiom)
$t(\mathrm{eta}(a), b \cdot c) = t(\mathrm{eta}(a), b) \cdot t(\mathrm{eta}(a), c)$ cnf(sos_{20}, axiom)
$i(a \cdot b) \cdot i(i(a)) = i(b)$ cnf(sos_{21}, axiom)
$j(j(a)) \cdot j(b \cdot a) = j(b)$ cnf(sos_{22}, axiom)
$(a \cdot (b \cdot c)) \cdot a(a, b, c) = (a \cdot b) \cdot c$ cnf(sos_{23}, axiom)
$(a \cdot b) \cdot c(b, a) = b \cdot a$ cnf(sos_{24}, axiom)
$(x_0 \cdot x_1) \cdot c(x_2, x_3) \neq x_0 \cdot (x_1 \cdot c(x_2, x_3))$ cnf(goals, negated_conjecture)

GRP772-1.p Buchsteiner loop lemma 9
$a \cdot 1 = a$ cnf(sos_{01}, axiom)
$1 \cdot a = a$ cnf(sos_{02}, axiom)
$a \cdot (a \setminus b) = b$ cnf(sos_{03}, axiom)
$a \setminus a \cdot b = b$ cnf(sos_{04}, axiom)
$\mathrm{quotient}(a \cdot b, b) = a$ cnf(sos_{05}, axiom)
$\mathrm{quotient}(a, b) \cdot b = a$ cnf(sos_{06}, axiom)
$a \setminus (a \cdot b) \cdot c = \mathrm{quotient}(b \cdot (c \cdot a), a)$ cnf(sos_{07}, axiom)
$a \cdot b \setminus a \cdot (b \cdot c) = \mathrm{quotient}(\mathrm{quotient}(c \cdot (a \cdot b), b), a)$ cnf(sos_{08}, axiom)

$i(a) = a \setminus 1$ cnf(sos_{09}, axiom)
$j(a) = \text{quotient}(1, a)$ cnf(sos_{10}, axiom)
$i(a) \cdot a = a \cdot j(a)$ cnf(sos_{11}, axiom)
$\text{eta}(a) = i(a) \cdot a$ cnf(sos_{12}, axiom)
$i(i(a)) \cdot b = \text{eta}(a) \cdot (a \cdot b)$ cnf(sos_{13}, axiom)
$a \cdot (\text{eta}(a) \cdot b) = j(j(a)) \cdot b$ cnf(sos_{14}, axiom)
$a \cdot (b \cdot \text{eta}(a)) = (a \cdot b) \cdot \text{eta}(a)$ cnf(sos_{15}, axiom)
$\text{eta}(a) \cdot (b \cdot c) = (\text{eta}(a) \cdot b) \cdot c$ cnf(sos_{16}, axiom)
$\text{quotient}(j(a), a) = j(a) \cdot i(a)$ cnf(sos_{17}, axiom)
$((\text{quotient}(j(a), a) \cdot (a \cdot a)) \cdot b) \cdot c = (\text{quotient}(j(a), a) \cdot (a \cdot a)) \cdot (b \cdot c)$ cnf(sos_{18}, axiom)
$t(a, b) = \text{quotient}(a \cdot b, a)$ cnf(sos_{19}, axiom)
$t(\text{eta}(a), b \cdot c) = t(\text{eta}(a), b) \cdot t(\text{eta}(a), c)$ cnf(sos_{20}, axiom)
$i(a \cdot b) \cdot i(i(a)) = i(b)$ cnf(sos_{21}, axiom)
$j(j(a)) \cdot j(b \cdot a) = j(b)$ cnf(sos_{22}, axiom)
$(a \cdot (b \cdot c)) \cdot a(a, b, c) = (a \cdot b) \cdot c$ cnf(sos_{23}, axiom)
$(a \cdot b) \cdot c(b, a) = b \cdot a$ cnf(sos_{24}, axiom)
$c(a, b) \cdot (c \cdot d) = (c(a, b) \cdot c) \cdot d$ cnf(sos_{25}, axiom)
$(a \cdot b) \cdot c(c, d) = a \cdot (b \cdot c(c, d))$ cnf(sos_{26}, axiom)
$a(a, b, c) \cdot (d \cdot e) = (a(a, b, c) \cdot d) \cdot e$ cnf(sos_{27}, axiom)
$(a \cdot b) \cdot a(c, d, e) = a \cdot (b \cdot a(c, d, e))$ cnf(sos_{28}, axiom)
$a(a, b, c) \cdot (c \setminus a(c, a, b) \cdot c) = 1$ cnf(sos_{29}, axiom)
$a(a, i(b), c) = a(a, j(b), c)$ cnf(sos_{30}, axiom)
$a(i(a), b, c) = a(j(a), b, c)$ cnf(sos_{31}, axiom)
$a(j(a), b, c) = a(b, c, a)$ cnf(sos_{32}, axiom)
$a(x_0, x_1, x_1) \neq a(x_1, x_1, x_0)$ cnf(goals, negated_conjecture)

GRP773-1.p Buchsteiner loop problem
$a \cdot \text{ld}(a, b) = b$ cnf(sos_{01}, axiom)
$\text{ld}(a, a \cdot b) = b$ cnf(sos_{02}, axiom)
$\text{rd}(a, b) \cdot b = a$ cnf(sos_{03}, axiom)
$\text{rd}(a \cdot b, b) = a$ cnf(sos_{04}, axiom)
$a \cdot 1 = a$ cnf(sos_{05}, axiom)
$1 \cdot a = a$ cnf(sos_{06}, axiom)
$\text{ld}(a, (a \cdot b) \cdot c) = \text{rd}(b \cdot (c \cdot a), a)$ cnf(sos_{07}, axiom)
$((a \cdot a) \cdot b) \cdot c \neq (a \cdot a) \cdot (b \cdot c)$ cnf(sos_{08}, negated_conjecture)

GRP774+1.p Green's relation D is a congruence
$\forall c, b, a\colon (a \cdot b) \cdot c = a \cdot (b \cdot c)$ fof(sos_{01}, axiom)
$\forall a\colon a \cdot a = a$ fof(sos_{02}, axiom)
$\forall x_0, x_1\colon (d(x_0, x_1) \iff (x_0 \cdot (x_1 \cdot x_0) = x_0 \text{ and } x_1 \cdot (x_0 \cdot x_1) = x_1))$ fof(sos_{03}, axiom)
$\forall x_2, x_3, x_4, x_5\colon ((d(x_2, x_3) \text{ and } d(x_4, x_5)) \Rightarrow d(x_2 \cdot x_4, x_3 \cdot x_5))$ fof(goals, conjecture)

GRP775+1.p Equivalent definition for Green's relation D
$\forall c, b, a\colon (a \cdot b) \cdot c = a \cdot (b \cdot c)$ fof(sos_{01}, axiom)
$\forall a\colon a \cdot a = a$ fof(sos_{02}, axiom)
$\forall x_0, x_1\colon (l(x_0, x_1) \iff (x_0 \cdot x_1 = x_0 \text{ and } x_1 \cdot x_0 = x_1))$ fof(sos_{03}, axiom)
$\forall x_2, x_3\colon (r(x_2, x_3) \iff (x_2 \cdot x_3 = x_3 \text{ and } x_3 \cdot x_2 = x_2))$ fof(sos_{04}, axiom)
$\forall x_4, x_5\colon (d(x_4, x_5) \iff \exists x_6\colon (r(x_4, x_6) \text{ and } l(x_6, x_5)))$ fof(sos_{05}, axiom)
$\forall x_7, x_8\colon (d(x_7, x_8) \iff (x_7 \cdot (x_8 \cdot x_7) = x_7 \text{ and } x_8 \cdot (x_7 \cdot x_8) = x_8))$ fof(goals, conjecture)

GRP776+1.p A homomorphic mapping between two groups
A mapping between two groups that respects multiplication is a homomorphism.
$\forall b, a\colon ((g(a) \text{ and } g(b)) \Rightarrow g(a \cdot b))$ fof(sos_{01}, axiom)
$\forall a\colon (g(a) \Rightarrow g(a^{-1}))$ fof(sos_{02}, axiom)
$g(\text{eh})$ fof(sos_{03}, axiom)
$\forall c, b, a\colon ((g(a) \text{ and } g(b) \text{ and } g(c)) \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c))$ fof(sos_{04}, axiom)
$\forall a\colon (g(a) \Rightarrow \text{eh} \cdot a = a)$ fof(sos_{05}, axiom)
$\forall a\colon (g(a) \Rightarrow a \cdot \text{eh} = a)$ fof(sos_{06}, axiom)
$\forall a\colon (g(a) \Rightarrow a \cdot a^{-1} = \text{eh})$ fof(sos_{07}, axiom)
$\forall a\colon (g(a) \Rightarrow a^{-1} \cdot a = \text{eh})$ fof(sos_{08}, axiom)
$\forall b, a\colon ((h(a) \text{ and } h(b)) \Rightarrow h(a + b))$ fof(sos_{09}, axiom)

$\forall b, a\colon (h(a) \Rightarrow h(\mathrm{opp}(b)))$ fof(sos_{10}, axiom)
$h(\mathrm{eg})$ fof(sos_{11}, axiom)
$\forall c, b, a\colon ((h(a) \text{ and } h(b) \text{ and } h(c)) \Rightarrow (a+b)+c = a+(b+c))$ fof(sos_{12}, axiom)
$\forall a\colon (h(a) \Rightarrow \mathrm{eg}+a = a)$ fof(sos_{13}, axiom)
$\forall a\colon (h(a) \Rightarrow a+\mathrm{eg} = a)$ fof(sos_{14}, axiom)
$\forall a\colon (h(a) \Rightarrow a+\mathrm{opp}(a) = \mathrm{eg})$ fof(sos_{15}, axiom)
$\forall a\colon (h(a) \Rightarrow \mathrm{opp}(a)+a = \mathrm{eg})$ fof(sos_{16}, axiom)
$\forall a\colon (g(a) \Rightarrow h(f(a)))$ fof(sos_{17}, axiom)
$\forall b, a\colon f(a \cdot b) = f(a)+f(b)$ fof(sos_{18}, axiom)
$\forall x_0\colon (f(\mathrm{eh}) = \mathrm{eg} \text{ and } (\neg g(x_0) \text{ or } f(x_0^{-1}) = \mathrm{opp}(f(x_0))))$ fof(goals, conjecture)

GRP777+1.p Napoleon's quasigroups: the centroid relation
$\forall b, a\colon a \setminus a \cdot b = b$ fof(sos_{01}, axiom)
$\forall b, a\colon a \cdot (a \setminus b) = b$ fof(sos_{02}, axiom)
$\forall b, a\colon \mathrm{quotient}(a \cdot b, b) = a$ fof(sos_{03}, axiom)
$\forall b, a\colon \mathrm{quotient}(a, b) \cdot b = a$ fof(sos_{04}, axiom)
$\forall d, c, b, a\colon (a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d)$ fof(sos_{05}, axiom)
$\forall a\colon a \cdot a = a$ fof(sos_{06}, axiom)
$\forall b, a\colon ((a \cdot b) \cdot b) \cdot (b \cdot (b \cdot a)) = b$ fof(sos_{07}, axiom)
$\forall c, b, a\colon \mathrm{bigC}(a, b, c) = (a \cdot b) \cdot (c \cdot a)$ fof(sos_{08}, axiom)
$(a \cdot c) \cdot (c \cdot b) = a \cdot b$ fof(sos_{09}, axiom)
$\forall x_0\colon \mathrm{bigC}(a, b, x_0) = \mathrm{bigC}(c, c, x_0)$ fof(goals, conjecture)

GRP778+1.p Napoleon's quasigroups: Gruenbaum's theorem 1
$\forall b, a\colon a \setminus a \cdot b = b$ fof(sos_{01}, axiom)
$\forall b, a\colon a \cdot (a \setminus b) = b$ fof(sos_{02}, axiom)
$\forall b, a\colon \mathrm{quotient}(a \cdot b, b) = a$ fof(sos_{03}, axiom)
$\forall b, a\colon \mathrm{quotient}(a, b) \cdot b = a$ fof(sos_{04}, axiom)
$\forall d, c, b, a\colon (a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d)$ fof(sos_{05}, axiom)
$\forall a\colon a \cdot a = a$ fof(sos_{06}, axiom)
$\forall b, a\colon ((a \cdot b) \cdot b) \cdot (b \cdot (b \cdot a)) = b$ fof(sos_{07}, axiom)
$\forall x_0, x_1, x_2\colon (d(x_0, x_1, x_2) \iff x_0 \cdot x_1 = x_1 \cdot x_2)$ fof(sos_{08}, axiom)
$\forall x_3, x_4, x_5\colon (m(x_3, x_4, x_5) \iff (x_3 \cdot x_4) \cdot (x_4 \cdot x_5) = x_3 \cdot x_5)$ fof(sos_{09}, axiom)
$d(a_1, b, c)$ fof(sos_{10}, axiom)
$d(a, b_1, c)$ fof(sos_{11}, axiom)
$d(a, b, c_1)$ fof(sos_{12}, axiom)
$d(a_2, b_1, c_1)$ fof(sos_{13}, axiom)
$d(a_1, b_2, c_1)$ fof(sos_{14}, axiom)
$d(a_1, b_1, c_2)$ fof(sos_{15}, axiom)
$m(b_1, b, b_2)$ fof(goals, conjecture)

GRP779+1.p Napoleon's quasigroups: Gruenbaum's theorem 2
$\forall b, a\colon a \setminus a \cdot b = b$ fof(sos_{01}, axiom)
$\forall b, a\colon a \cdot (a \setminus b) = b$ fof(sos_{02}, axiom)
$\forall b, a\colon \mathrm{quotient}(a \cdot b, b) = a$ fof(sos_{03}, axiom)
$\forall b, a\colon \mathrm{quotient}(a, b) \cdot b = a$ fof(sos_{04}, axiom)
$\forall d, c, b, a\colon (a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d)$ fof(sos_{05}, axiom)
$\forall a\colon a \cdot a = a$ fof(sos_{06}, axiom)
$\forall b, a\colon ((a \cdot b) \cdot b) \cdot (b \cdot (b \cdot a)) = b$ fof(sos_{07}, axiom)
$\forall x_0, x_1, x_2\colon (d(x_0, x_1, x_2) \iff x_0 \cdot x_1 = x_1 \cdot x_2)$ fof(sos_{08}, axiom)
$\forall x_3, x_4, x_5\colon (m(x_3, x_4, x_5) \iff (x_3 \cdot x_4) \cdot (x_4 \cdot x_5) = x_3 \cdot x_5)$ fof(sos_{09}, axiom)
$d(b, \mathrm{bigA}, c)$ fof(sos_{10}, axiom)
$d(\mathrm{bigB}, a, c)$ fof(sos_{11}, axiom)
$d(b, a, \mathrm{bigC})$ fof(sos_{12}, axiom)
$d(a, b_1, c_1)$ fof(sos_{13}, axiom)
$d(a_1, b, c_1)$ fof(sos_{14}, axiom)
$d(a_1, b_1, c)$ fof(sos_{15}, axiom)
$m(\mathrm{bigA}, b_1, \mathrm{bigC})$ fof(goals, conjecture)

GRP780+1.p Napoleon's quasigroups: Lamoen's theorem
$\forall b, a\colon a \setminus a \cdot b = b$ fof(sos_{01}, axiom)

$\forall b, a\colon a \cdot (a \setminus b) = b \qquad \text{fof}(\text{sos}_{02}, \text{axiom})$
$\forall b, a\colon \text{quotient}(a \cdot b, b) = a \qquad \text{fof}(\text{sos}_{03}, \text{axiom})$
$\forall b, a\colon \text{quotient}(a, b) \cdot b = a \qquad \text{fof}(\text{sos}_{04}, \text{axiom})$
$\forall d, c, b, a\colon (a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d) \qquad \text{fof}(\text{sos}_{05}, \text{axiom})$
$\forall a\colon a \cdot a = a \qquad \text{fof}(\text{sos}_{06}, \text{axiom})$
$\forall b, a\colon ((a \cdot b) \cdot b) \cdot (b \cdot (b \cdot a)) = b \qquad \text{fof}(\text{sos}_{07}, \text{axiom})$
$\forall x_0, x_1, x_2\colon (d(x_0, x_1, x_2) \iff x_0 \cdot x_1 = x_1 \cdot x_2) \qquad \text{fof}(\text{sos}_{08}, \text{axiom})$
$\forall c, b, a\colon \text{bigC}(a, b, c) = (a \cdot b) \cdot (c \cdot a) \qquad \text{fof}(\text{sos}_{09}, \text{axiom})$
$d(a_1, a_2, a_3) \qquad \text{fof}(\text{sos}_{10}, \text{axiom})$
$d(b_1, b_2, b_3) \qquad \text{fof}(\text{sos}_{11}, \text{axiom})$
$d(c_1, c_2, c_3) \qquad \text{fof}(\text{sos}_{12}, \text{axiom})$
$d(\text{bigC}(a_1, b_3, c_2), \text{bigC}(a_2, b_1, c_3), \text{bigC}(a_3, b_2, c_1)) \qquad \text{fof}(\text{goals}, \text{conjecture})$

GRP781-1.p Distributivity of commutator in cancellative semigroups
$m(x, m(y, z)) = m(m(x, y), z) \qquad \text{cnf}(\text{associativity}, \text{axiom})$
$m(x, z) = m(y, z) \;\Rightarrow\; x = y \qquad \text{cnf}(\text{cancellation}, \text{axiom})$
$m(z, x) = m(z, y) \;\Rightarrow\; x = y \qquad \text{cnf}(\text{cancellation}_{001}, \text{axiom})$
$m(y, m(x, c(x, y))) = m(x, y) \qquad \text{cnf}(\text{commutator}, \text{axiom})$
$m(x, m(y, m(z, m(y, x)))) = m(y, m(x, m(z, m(x, y)))) \qquad \text{cnf}(\text{assumption}, \text{axiom})$
$c(m(x, y), z) \neq m(c(x, z), c(y, z)) \qquad \text{cnf}(\text{distributivity}, \text{negated\_conjecture})$