HEN axioms

HEN001-0.ax Henkin model axioms

less_equal(x, y) $\Rightarrow$ quotient($x, y, 0$) cnf(quotient_less_equal, axiom)

quotient($x, y, 0$) $\Rightarrow$ less_equal(x, y) cnf(less_equal_quotient, axiom)

quotient(x, y, z) $\Rightarrow$ less_equal(z, x) cnf(divisor_existence, axiom)

(quotient(x, y, v_1) and quotient(y, z, v_2) and quotient(x, z, v_3) and quotient(v_3, v_2, v_4) and quotient(v_1, z, v_5)) $\Rightarrow$ less_equal(

less_equal($0, x$) cnf(zero_is_smallest, axiom)

(less_equal(x, y) and less_equal(y, x)) $\Rightarrow$ $x = y$ cnf(less_equal_and_equal, axiom)

less_equal(x, identity) cnf(identity_is_largest, axiom)

quotient(x, y, divide(x, y)) cnf(closure, axiom)

(quotient(x, y, z) and quotient(x, y, w)) $\Rightarrow$ $z = w$ cnf(well_defined, axiom)

HEN002-0.ax Henkin model axioms

less_equal(x, y) $\Rightarrow$ divide(x, y) $= 0$ cnf(quotient_less_equal$_1$, axiom)

divide(x, y) $= 0$ $\Rightarrow$ less_equal(x, y) cnf(quotient_less_equal$_2$, axiom)

less_equal(divide(x, y), x) cnf(quotient_smaller_than_numerator, axiom)

less_equal(divide(divide(x, z), divide(y, z)), divide(divide(x, y), z)) cnf(quotient_property, axiom)

less_equal($0, x$) cnf(zero_is_smallest, axiom)

(less_equal(x, y) and less_equal(y, x)) $\Rightarrow$ $x = y$ cnf(less_equal_and_equal, axiom)

less_equal(x, identity) cnf(identity_is_largest, axiom)

HEN003-0.ax Henkin model (equality) axioms

divide(divide(x, y), x) $= 0$ cnf(quotient_smaller_than_numerator, axiom)

divide(divide(divide(x, z), divide(y, z)), divide(divide(x, y), z)) $= 0$ cnf(quotient_property, axiom)

divide($0, x$) $= 0$ cnf(zero_is_smallest, axiom)

(divide(x, y) $= 0$ and divide(y, x) $= 0$) $\Rightarrow$ $x = y$ cnf(divide_and_equal, axiom)

divide(x, identity) $= 0$ cnf(identity_is_largest, axiom)

HEN problems

HEN001-1.p X/identity = zero

include('Axioms/HEN001-0.ax')

$\neg$ quotient(x, identity, 0) cnf(prove_everything_divide_identity_is_zero, negated_conjecture)

HEN001-3.p X/identity = zero

include('Axioms/HEN002-0.ax')

divide(a, identity) $\neq 0$ cnf(prove_a_divide_id_is_zero, negated_conjecture)

HEN001-5.p X/identity = zero

include('Axioms/HEN003-0.ax')

divide(a, identity) $\neq 0$ cnf(prove_everything_divide_id_is_zero, negated_conjecture)

HEN002-1.p zero/X = zero

include('Axioms/HEN001-0.ax')

$\neg$ quotient($0, x, 0$) cnf(prove_zero_divide_anything_is_zero, negated_conjecture)

HEN002-2.p zero/X = zero

include('Axioms/HEN001-0.ax')

quotient(x, identity, 0) cnf(everything_divide_identity_is_zero, axiom)

$\neg$ quotient($0, x, 0$) cnf(prove_zero_divide_anything_is_zero, negated_conjecture)

HEN002-3.p zero/X = zero

include('Axioms/HEN002-0.ax')

divide($0, a$) $\neq 0$ cnf(prove_zero_over_a_is_zero, negated_conjecture)

HEN002-4.p zero/X = zero

include('Axioms/HEN002-0.ax')

divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)

divide($0, a$) $\neq 0$ cnf(prove_zero_over_a_is_zero, negated_conjecture)

HEN002-5.p zero/X = zero

include('Axioms/HEN003-0.ax')

divide($0, a$) $\neq 0$ cnf(prove_zero_divide_anything_is_zero, negated_conjecture)

HEN003-1.p X/X = zero
include('Axioms/HEN001-0.ax')
$\neg$ quotient($x, x, 0$)   cnf(prove_x_divide_x_is_zero, negated_conjecture)

HEN003-2.p X/X = zero
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$)   cnf(zero_divide_anything_is_zero, axiom)
$\neg$ quotient($x, x, 0$)   cnf(prove_x_divide_x_is_zero, negated_conjecture)

HEN003-3.p X/X = zero
include('Axioms/HEN002-0.ax')
divide(a, a) $\neq 0$ cnf(prove_x_divide_x_is_zero, negated_conjecture)

HEN003-4.p X/X = zero
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
divide(a, a) $\neq 0$ cnf(prove_x_divide_x_is_zero, negated_conjecture)

HEN003-5.p X/X = zero
include('Axioms/HEN003-0.ax')
divide(a, a) $\neq 0$ cnf(prove_x_divide_x_is_zero, negated_conjecture)

HEN004-1.p X/zero = X
include('Axioms/HEN001-0.ax')
$\neg$ quotient($x, 0, x$)   cnf(prove_x_divde_zero_is_x, negated_conjecture)

HEN004-2.p X/zero = X
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$)   cnf(zero_divide_anything_is_zero, axiom)
quotient($x, x, 0$)   cnf(x_divide_x_is_zero, axiom)
$\neg$ quotient($x, 0, x$)   cnf(prove_x_divde_zero_is_x, negated_conjecture)

HEN004-3.p X/zero = X
include('Axioms/HEN002-0.ax')
divide($a, 0$) $\neq a$ cnf(prove_x_divide_zero_is_x, negated_conjecture)

HEN004-4.p X/zero = X
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
divide(x, x) $= 0$ cnf(x_divide_x_is_zero, axiom)
divide($a, 0$) $\neq a$ cnf(prove_x_divide_zero_is_x, negated_conjecture)

HEN004-5.p X/zero = X
include('Axioms/HEN003-0.ax')
divide(x, x) $= 0$ cnf(x_divide_x_is_zero, axiom)
divide($a, 0$) $\neq a$ cnf(prove_x_divide_zero_is_x, negated_conjecture)

HEN004-6.p X/zero = X
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
divide($a, 0$) $\neq a$ cnf(prove_x_divide_zero_is_x, negated_conjecture)

HEN005-1.p The relation less_equal is transitive
include('Axioms/HEN001-0.ax')
less_equal(x, y)   cnf(xLEy, hypothesis)
less_equal(y, z)   cnf(yLEz, hypothesis)
$\neg$ less_equal(x, z)   cnf(prove_transitivity_of_less_equal, negated_conjecture)

HEN005-2.p The relation less_equal is transitive
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$)   cnf(zero_divide_anything_is_zero, axiom)

quotient($x, x, 0$) cnf(x_divide_x_is_zero, axiom)
quotient($x, 0, x$) cnf(x_divde_zero_is_x, axiom)
less_equal(x, y) cnf(xLEy, hypothesis)
less_equal(y, z) cnf(yLEz, hypothesis)
$\neg$ less_equal(x, z) cnf(prove_transitivity_of_less_equal, negated_conjecture)

HEN005-3.p The relation less_equal is transitive
include('Axioms/HEN002-0.ax')
less_equal(a, b) cnf(a_LE_b, hypothesis)
less_equal(b, c) cnf(b_LE_c, hypothesis)
$\neg$ less_equal(a, c) cnf(prove_a_LE_c, negated_conjecture)

HEN005-4.p The relation less_equal is transitive
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
divide(x, x) $= 0$ cnf(x_divide_x_is_zero, axiom)
divide($x, 0$) $= x$ cnf(x_divide_zero_is_x, axiom)
less_equal(a, b) cnf(a_LE_b, hypothesis)
less_equal(b, c) cnf(b_LE_c, hypothesis)
$\neg$ less_equal(a, c) cnf(prove_a_LE_c, negated_conjecture)

HEN005-5.p The relation less_equal is transitive
include('Axioms/HEN003-0.ax')
divide(a, b) $= 0$ cnf(a_divide_b_is_zero, hypothesis)
divide(b, c) $= 0$ cnf(b_divide_c_is_zero, hypothesis)
divide(a, c) $\neq 0$ cnf(prove_transitivity_of_divide_to_zero, negated_conjecture)

HEN005-6.p The relation less_equal is transitive
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
less_equal(a, b) cnf(a_LE_b, hypothesis)
less_equal(b, c) cnf(b_LE_c, hypothesis)
$\neg$ less_equal(a, c) cnf(prove_a_LE_c, negated_conjecture)

HEN006-1.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN001-0.ax')
quotient(x, y, xQy) cnf(xQy, hypothesis)
less_equal(xQy, z) cnf(xQyLEz, hypothesis)
quotient(x, z, xQz) cnf(xQz, hypothesis)
$\neg$ less_equal(xQz, y) cnf(prove_xQzLEy, negated_conjecture)

HEN006-2.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0) cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$) cnf(zero_divide_anything_is_zero, axiom)
quotient($x, x, 0$) cnf(x_divide_x_is_zero, axiom)
quotient($x, 0, x$) cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)
quotient(x, y, xQy) cnf(xQy, hypothesis)
less_equal(xQy, z) cnf(xQyLEz, hypothesis)
quotient(x, z, xQz) cnf(xQz, hypothesis)
$\neg$ less_equal(xQz, y) cnf(prove_xQzLEy, negated_conjecture)

HEN006-3.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN002-0.ax')
less_equal(divide(a, b), d) cnf(a_divide_b_LE_d, hypothesis)
$\neg$ less_equal(divide(a, d), b) cnf(prove_a_divide_d_LE_b, negated_conjecture)

HEN006-4.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)

divide$(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
divide$(a, 0) = a$ cnf(x_divide_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)
less_equal(divide$(a, b), d$) cnf(a_divide_b_LE_d, hypothesis)
$\neg$ less_equal(divide$(a, d), b$) cnf(prove_a_divide_d_LE_b, negated_conjecture)

HEN006-5.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN003-0.ax')
divide$(x, 0) = x$ cnf(x_divide_zero_is_x, axiom)
divide(divide$(a, b), d) = 0$ cnf(a_divide_b_divide_d_is_zero, hypothesis)
divide(divide$(a, d), b) \neq 0$ cnf(prove_property_of_divide$_1$, negated_conjecture)

HEN006-6.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN002-0.ax')
divide$(x$, identity$) = 0$ cnf(everything_divide_id_is_zero, axiom)
divide$(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)
divide$(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
less_equal(divide$(a, b), d$) cnf(a_divide_b_LE_d, hypothesis)
$\neg$ less_equal(divide$(a, d), b$) cnf(prove_a_divide_d_LE_b, negated_conjecture)

HEN006-7.p X/Y $\leq$ Z => X/Z $\leq$ Y
include('Axioms/HEN001-0.ax')
quotient$(x$, identity$, 0)$ cnf(everything_divide_identity_is_zero, axiom)
quotient$(0, x, 0)$ cnf(zero_divide_anything_is_zero, axiom)
quotient$(x, x, 0)$ cnf(x_divide_x_is_zero, axiom)
quotient$(x, 0, x)$ cnf(x_divde_zero_is_x, axiom)
quotient$(x, y$, xQy) cnf(xQy, hypothesis)
less_equal(xQy, z) cnf(xQyLEz, hypothesis)
quotient$(x, z$, xQz) cnf(xQz, hypothesis)
$\neg$ less_equal(xQz, y) cnf(prove_xQzLEy, negated_conjecture)

HEN007-1.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN001-0.ax')
less_equal(x, y) cnf(xLEy, hypothesis)
quotient$(z, y$, zQy) cnf(zQy, hypothesis)
quotient$(z, x$, zQx) cnf(zQx, hypothesis)
$\neg$ less_equal(zQy, zQx) cnf(prove_zQyLEzQx, negated_conjecture)

HEN007-2.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN001-0.ax')
quotient$(x$, identity$, 0)$ cnf(everything_divide_identity_is_zero, axiom)
quotient$(0, x, 0)$ cnf(zero_divide_anything_is_zero, axiom)
quotient$(x, x, 0)$ cnf(x_divide_x_is_zero, axiom)
quotient$(x, 0, x)$ cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)
(quotient(x, y, w_1) and less_equal(w_1, z) and quotient(x, z, w_2)) $\Rightarrow$ less_equal(w_2, y) cnf(xQyLEz_implies_xQzLEy, axiom
less_equal(x, y) cnf(xLEy, hypothesis)
quotient$(z, y$, zQy) cnf(zQy, hypothesis)
quotient$(z, x$, zQx) cnf(zQx, hypothesis)
$\neg$ less_equal(zQy, zQx) cnf(prove_zQyLEzQx, negated_conjecture)

HEN007-3.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN002-0.ax')
less_equal(a, b) cnf(a_LE_b, hypothesis)
$\neg$ less_equal(divide(c, b), divide(c, a)) cnf(prove_c_divide_b_LE_c_divide_a, negated_conjecture)

HEN007-4.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN002-0.ax')
divide$(x$, identity$) = 0$ cnf(everything_divide_id_is_zero, axiom)
divide$(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)
divide$(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
divide$(a, 0) = a$ cnf(x_divide_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)

less_equal(divide(x, y), z) $\Rightarrow$ less_equal(divide(x, z), y)   cnf(property_of_divide$_1$, axiom)
less_equal(a, b)   cnf(a_LE_b, hypothesis)
$\neg$ less_equal(divide(c, b), divide(c, a))   cnf(prove_c_divide_b_LE_c_divide_a, negated_conjecture)

HEN007-5.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN003-0.ax')
(divide(x, y) $= 0$ and divide(y, z) $= 0$) $\Rightarrow$ divide(x, z) $= 0$ cnf(transitivity_of_divide_to_zero, axiom)
divide(a, b) $= 0$ cnf(a_divide_b_is_zero, hypothesis)
divide(divide(c, b), divide(c, a)) $\neq 0$ cnf(prove_property_of_divide$_2$, negated_conjecture)

HEN007-6.p X $\leq$ Y => Z/Y $\leq$ Z/X
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$)   cnf(zero_divide_anything_is_zero, axiom)
quotient($x, x, 0$)   cnf(x_divide_x_is_zero, axiom)
quotient($x, 0, x$)   cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z)   cnf(transitivity_of_less_equal, axiom)
less_equal(x, y)   cnf(xLEy, hypothesis)
quotient(z, y, zQy)   cnf(zQy, hypothesis)
quotient(z, x, zQx)   cnf(zQx, hypothesis)
$\neg$ less_equal(zQy, zQx)   cnf(prove_zQyLEzQx, negated_conjecture)

HEN008-1.p X $\leq$ Y => X/Z $\leq$ Y/Z
include('Axioms/HEN001-0.ax')
less_equal(a, b)   cnf(aLEb, hypothesis)
quotient(a, c, aQc)   cnf(aQc, hypothesis)
quotient(b, c, bQc)   cnf(bQc, hypothesis)
$\neg$ less_equal(aQc, bQc)   cnf(prove_aQcLEbQc, negated_conjecture)

HEN008-2.p X $\leq$ Y => X/Z $\leq$ Y/Z
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient($0, x, 0$)   cnf(zero_divide_anything_is_zero, axiom)
quotient($x, x, 0$)   cnf(x_divide_x_is_zero, axiom)
quotient($x, 0, x$)   cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z)   cnf(transitivity_of_less_equal, axiom)
(quotient(x, y, w_1) and less_equal(w_1, z) and quotient(x, z, w_2)) $\Rightarrow$ less_equal(w_2, y)   cnf(xQyLEz_implies_xQzLEy, axiom
(less_equal(x, y) and quotient(z, y, w_1) and quotient(z, x, w_2)) $\Rightarrow$ less_equal(w_1, w_2)   cnf(xLEy_implies_zQyLEzQx, axiom
less_equal(a, b)   cnf(aLEb, hypothesis)
quotient(a, c, aQc)   cnf(aQc, hypothesis)
quotient(b, c, bQc)   cnf(bQc, hypothesis)
$\neg$ less_equal(aQc, bQc)   cnf(prove_aQcLEbQc, negated_conjecture)

HEN008-3.p X $\leq$ Y => X/Z $\leq$ Y/Z
include('Axioms/HEN002-0.ax')
less_equal(a, b)   cnf(a_LE_b, hypothesis)
$\neg$ less_equal(divide(a, c), divide(b, c))   cnf(prove_a_divide_c_LE_b_divide_c, negated_conjecture)

HEN008-4.p X $\leq$ Y => X/Z $\leq$ Y/Z
include('Axioms/HEN002-0.ax')
divide(x, identity) $= 0$ cnf(everything_divide_id_is_zero, axiom)
divide($0, x$) $= 0$ cnf(zero_divide_anything_is_zero, axiom)
divide(x, x) $= 0$ cnf(x_divide_x_is_zero, axiom)
divide($a, 0$) $= a$ cnf(x_divide_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) $\Rightarrow$ less_equal(x, z)   cnf(transitivity_of_less_equal, axiom)
less_equal(divide(x, y), z) $\Rightarrow$ less_equal(divide(x, z), y)   cnf(property_of_divide$_1$, axiom)
less_equal(x, y) $\Rightarrow$ less_equal(divide(z, y), divide(z, x))   cnf(property_of_divide$_2$, axiom)
less_equal(a, b)   cnf(a_LE_b, hypothesis)
$\neg$ less_equal(divide(a, c), divide(b, c))   cnf(prove_a_divide_c_LE_b_divide_c, negated_conjecture)

HEN008-5.p X $\leq$ Y => X/Z $\leq$ Y/Z
include('Axioms/HEN003-0.ax')
divide(a, b) $= 0$ cnf(a_divide_b_is_zero, hypothesis)

$\text{divide}(\text{divide}(a, c), \text{divide}(b, c)) \neq 0$ cnf(prove_property_of_divide$_3$, negated_conjecture)

HEN008-6.p X ≤ Y => X/Z ≤ Y/Z
include('Axioms/HEN002-0.ax')
$\text{divide}(x, \text{identity}) = 0$ cnf(everything_divide_id_is_zero, axiom)
$\text{divide}(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)
$\text{divide}(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
$\text{less\_equal}(a, b)$ cnf(a_LE_b, hypothesis)
$\neg\,\text{less\_equal}(\text{divide}(a, c), \text{divide}(b, c))$ cnf(prove_a_divide_c_LE_b_divide_c, negated_conjecture)

HEN009-1.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN001-0.ax')
$\text{quotient}(\text{identity}, a, \text{idQa})$ cnf(identity_divide_a, hypothesis)
$\text{quotient}(\text{identity}, \text{idQa}, \text{idQ\_idQa})$ cnf(identity_divide_idQa, hypothesis)
$\text{quotient}(\text{identity}, \text{idQ\_idQa}, \text{idQ\_\_idQ\_idQa})$ cnf(identity_divide_idQ_idQa, hypothesis)
$\text{idQa} \neq \text{idQ\_\_idQ\_idQa}$ cnf(prove_one_inversion_equals_three, negated_conjecture)

HEN009-2.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN001-0.ax')
$\text{quotient}(x, \text{identity}, 0)$ cnf(everything_divide_identity_is_zero, axiom)
$\text{quotient}(0, x, 0)$ cnf(zero_divide_anything_is_zero, axiom)
$\text{quotient}(x, x, 0)$ cnf(x_divide_x_is_zero, axiom)
$\text{quotient}(x, 0, x)$ cnf(x_divde_zero_is_x, axiom)
$(\text{less\_equal}(x, y) \text{ and } \text{less\_equal}(y, z)) \Rightarrow \text{less\_equal}(x, z)$ cnf(transitivity_of_less_equal, axiom)
$(\text{quotient}(x, y, w_1) \text{ and } \text{less\_equal}(w_1, z) \text{ and } \text{quotient}(x, z, w_2)) \Rightarrow \text{less\_equal}(w_2, y)$ cnf(xQyLEz_implies_xQzLEy, axiom
$(\text{less\_equal}(x, y) \text{ and } \text{quotient}(z, y, w_1) \text{ and } \text{quotient}(z, x, w_2)) \Rightarrow \text{less\_equal}(w_1, w_2)$ cnf(xLEy_implies_zQyLEzQx, axiom
$(\text{less\_equal}(x, y) \text{ and } \text{quotient}(x, z, w_1) \text{ and } \text{quotient}(y, z, w_2)) \Rightarrow \text{less\_equal}(w_1, w_2)$ cnf(xLEy_implies_xQzLEyQz, axiom
$\text{quotient}(\text{identity}, a, \text{idQa})$ cnf(identity_divide_a, hypothesis)
$\text{quotient}(\text{identity}, \text{idQa}, \text{idQ\_idQa})$ cnf(identity_divide_idQa, hypothesis)
$\text{quotient}(\text{identity}, \text{idQ\_idQa}, \text{idQ\_\_idQ\_idQa})$ cnf(identity_divide_idQ_idQa, hypothesis)
$\text{idQa} \neq \text{idQ\_\_idQ\_idQa}$ cnf(prove_one_inversion_equals_three, negated_conjecture)

HEN009-3.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN002-0.ax')
$\text{divide}(\text{identity}, a) \neq \text{divide}(\text{identity}, \text{divide}(\text{identity}, \text{divide}(\text{identity}, a)))$ cnf(part_of_theorem, hypothesis)
$\text{divide}(\text{identity}, a) = b$ cnf(id_divide_a_is_b, hypothesis)
$\text{divide}(\text{identity}, b) = c$ cnf(id_divide_b_is_c, hypothesis)
$\text{divide}(\text{identity}, c) = d$ cnf(id_divide_c_is_d, hypothesis)
$b \neq d$ cnf(prove_b_equals_d, negated_conjecture)

HEN009-4.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN002-0.ax')
$\text{divide}(x, \text{identity}) = 0$ cnf(everything_divide_id_is_zero, axiom)
$\text{divide}(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)
$\text{divide}(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
$\text{divide}(a, 0) = a$ cnf(x_divide_zero_is_x, axiom)
$(\text{less\_equal}(x, y) \text{ and } \text{less\_equal}(y, z)) \Rightarrow \text{less\_equal}(x, z)$ cnf(transitivity_of_less_equal, axiom)
$\text{less\_equal}(\text{divide}(x, y), z) \Rightarrow \text{less\_equal}(\text{divide}(x, z), y)$ cnf(property_of_divide$_1$, axiom)
$\text{less\_equal}(x, y) \Rightarrow \text{less\_equal}(\text{divide}(z, y), \text{divide}(z, x))$ cnf(property_of_divide$_2$, axiom)
$\text{less\_equal}(x, y) \Rightarrow \text{less\_equal}(\text{divide}(x, z), \text{divide}(y, z))$ cnf(property_of_divide$_3$, axiom)
$\text{divide}(\text{identity}, a) \neq \text{divide}(\text{identity}, \text{divide}(\text{identity}, \text{divide}(\text{identity}, a)))$ cnf(prove_this, negated_conjecture)

HEN009-5.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN003-0.ax')
$(\text{divide}(x, y) = 0 \text{ and } \text{divide}(y, z) = 0) \Rightarrow \text{divide}(x, z) = 0$ cnf(transitivity_of_divide_to_zero, axiom)
$\text{divide}(\text{divide}(x, y), z) = 0 \Rightarrow \text{divide}(\text{divide}(x, z), y) = 0$ cnf(property_of_divide$_1$, axiom)
$\text{divide}(x, y) = 0 \Rightarrow \text{divide}(\text{divide}(z, y), \text{divide}(z, x)) = 0$ cnf(property_of_divide$_2$, axiom)
$\text{divide}(\text{identity}, a) \neq \text{divide}(\text{identity}, \text{divide}(\text{identity}, \text{divide}(\text{identity}, a)))$ cnf(prove_this, negated_conjecture)

HEN009-6.p Define X' as identity/X. Then X' = X"'
include('Axioms/HEN002-0.ax')
$\text{divide}(x, \text{identity}) = 0$ cnf(everything_divide_id_is_zero, axiom)
$\text{divide}(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)

divide(x, x) = 0   cnf(x_divide_x_is_zero, axiom)
less_equal(divide(x, y), z) ⇒ less_equal(divide(x, z), y)   cnf(property_of_divide$_1$, axiom)
less_equal(x, y) ⇒ less_equal(divide(x, z), divide(y, z))   cnf(property_of_divide$_3$, axiom)
divide(identity, a) ≠ divide(identity, divide(identity, divide(identity, a)))   cnf(part_of_theorem, hypothesis)
divide(identity, a) = b cnf(id_divide_a_is_b, hypothesis)
divide(identity, b) = c cnf(id_divide_b_is_c, hypothesis)
divide(identity, c) = d cnf(id_divide_c_is_d, hypothesis)
$b \neq d$ cnf(prove_b_equals_d, negated_conjecture)

HEN010-1.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN001-0.ax')
quotient(identity, a, idQa)   cnf(identity_divide_a, hypothesis)
quotient(identity, idQa, idQ_idQa)   cnf(identity_divide_idQa, hypothesis)
quotient(idQa, idQ_idQa, idQa_Q__idQ_idQa)   cnf(identity_divide_idQ_idQa, hypothesis)
idQa ≠ idQa_Q__idQ_idQa   cnf(prove_idQa_equals_idQa_Q__idQ_idQa, negated_conjecture)

HEN010-2.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0)   cnf(everything_divide_identity_is_zero, axiom)
quotient(0, x, 0)   cnf(zero_divide_anything_is_zero, axiom)
quotient(x, x, 0)   cnf(x_divide_x_is_zero, axiom)
quotient(x, 0, x)   cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) ⇒ less_equal(x, z)   cnf(transitivity_of_less_equal, axiom)
(quotient(x, y, w_1) and less_equal(w_1, z) and quotient(x, z, w_2)) ⇒ less_equal(w_2, y)   cnf(xQyLEz_implies_xQzLEy, axiom
(less_equal(x, y) and quotient(z, y, w_1) and quotient(z, x, w_2)) ⇒ less_equal(w_1, w_2)   cnf(xLEy_implies_zQyLEzQx, axiom
(less_equal(x, y) and quotient(x, z, w_1) and quotient(y, z, w_2)) ⇒ less_equal(w_1, w_2)   cnf(xLEy_implies_xQzLEyQz, axiom
(quotient(identity, x, y_1) and quotient(identity, y_1, y_2) and quotient(identity, y_2, y_3)) ⇒ $y_1 = y_3$ cnf(one_inversion_equals
quotient(identity, a, idQa)   cnf(identity_divide_a, hypothesis)
quotient(identity, idQa, idQ_idQa)   cnf(identity_divide_idQa, hypothesis)
quotient(idQa, idQ_idQa, idQa_Q__idQ_idQa)   cnf(identity_divide_idQ_idQa, hypothesis)
idQa ≠ idQa_Q__idQ_idQa   cnf(prove_idQa_equals_idQa_Q__idQ_idQa, negated_conjecture)

HEN010-3.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN002-0.ax')
divide(identity, a) ≠ divide(divide(identity, a), divide(identity, divide(identity, a)))   cnf(prove_property_of_inversion, negate

HEN010-4.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN002-0.ax')
divide(x, identity) = 0   cnf(everything_divide_id_is_zero, axiom)
divide(0, x) = 0   cnf(zero_divide_anything_is_zero, axiom)
divide(x, x) = 0   cnf(x_divide_x_is_zero, axiom)
divide(a, 0) = a cnf(x_divide_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) ⇒ less_equal(x, z)   cnf(transitivity_of_less_equal, axiom)
less_equal(divide(x, y), z) ⇒ less_equal(divide(x, z), y)   cnf(property_of_divide$_1$, axiom)
less_equal(x, y) ⇒ less_equal(divide(z, y), divide(z, x))   cnf(property_of_divide$_2$, axiom)
less_equal(x, y) ⇒ less_equal(divide(x, z), divide(y, z))   cnf(property_of_divide$_3$, axiom)
divide(identity, divide(identity, divide(identity, x))) = divide(identity, x)   cnf(one_inversion_equals_three, axiom)
divide(identity, a) ≠ divide(divide(identity, a), divide(identity, divide(identity, a)))   cnf(prove_property_of_inversion, negate

HEN010-5.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN003-0.ax')
(divide(x, y) = 0 and divide(y, z) = 0) ⇒ divide(x, z) = 0   cnf(transitivity_of_divide_to_zero, axiom)
divide(divide(x, y), z) = 0 ⇒ divide(divide(x, z), y) = 0   cnf(property_of_divide$_1$, axiom)
divide(x, y) = 0 ⇒ divide(divide(x, z), divide(y, z)) = 0   cnf(property_of_divide$_3$, axiom)
divide(identity, a) ≠ divide(divide(identity, a), divide(identity, divide(identity, a)))   cnf(prove_this, negated_conjecture)

HEN010-6.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN002-0.ax')
divide(x, identity) = 0   cnf(everything_divide_id_is_zero, axiom)
divide(0, x) = 0   cnf(zero_divide_anything_is_zero, axiom)
divide(x, x) = 0   cnf(x_divide_x_is_zero, axiom)
divide(a, 0) = a cnf(x_divide_zero_is_x, axiom)
less_equal(divide(x, y), z) ⇒ less_equal(divide(x, z), y)   cnf(property_of_divide$_1$, axiom)

less_equal(x, y) ⇒ less_equal(divide(z, y), divide(z, x)) cnf(property_of_divide$_2$, axiom)
less_equal(x, y) ⇒ less_equal(divide(x, z), divide(y, z)) cnf(property_of_divide$_3$, axiom)
divide(identity, divide(identity, divide(identity, x))) = divide(identity, x) cnf(one_inversion_equals_three, axiom)
divide(identity, a) ≠ divide(divide(identity, a), divide(identity, divide(identity, a))) cnf(prove_property_of_inversion, negate

HEN010-7.p Define X' as identity/X. Then X' = X'/(identity/X')
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0) cnf(everything_divide_identity_is_zero, axiom)
quotient(0, x, 0) cnf(zero_divide_anything_is_zero, axiom)
quotient(x, x, 0) cnf(x_divide_x_is_zero, axiom)
quotient(x, 0, x) cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) ⇒ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)
(quotient(x, y, w_1) and less_equal(w_1, z) and quotient(x, z, w_2)) ⇒ less_equal(w_2, y) cnf(xQyLEz_implies_xQzLEy, axiom
(less_equal(x, y) and quotient(z, y, w_1) and quotient(z, x, w_2)) ⇒ less_equal(w_1, w_2) cnf(xLEy_implies_zQyLEzQx, axiom
(less_equal(x, y) and quotient(x, z, w_1) and quotient(y, z, w_2)) ⇒ less_equal(w_1, w_2) cnf(xLEy_implies_xQzLEyQz, axiom
quotient(identity, a, idQa) cnf(identity_divide_a, hypothesis)
quotient(identity, idQa, idQ_idQa) cnf(identity_divide_idQa, hypothesis)
quotient(idQa, idQ_idQa, idQa_Q__idQ_idQa) cnf(identity_divide_idQ_idQa, hypothesis)
idQa ≠ idQa_Q__idQ_idQa cnf(prove_idQa_equals_idQa_Q__idQ_idQa, negated_conjecture)

HEN011-1.p This operation is commutative
Define & on the set of Z', where Z' = identity/Z, by X' & Y' = X'/(identity/Y'). The operation is commutative.
include('Axioms/HEN001-0.ax')
quotient(identity, a, idQa) cnf(identity_divide_a, hypothesis)
quotient(identity, b, idQb) cnf(identity_divide_b, hypothesis)
quotient(identity, idQb, idQ_idQb) cnf(identity_divide_idQb, hypothesis)
quotient(idQa, idQ_idQb, idQa_Q__idQ_idQb) cnf(idQa_divide_idQ_idQb, hypothesis)
quotient(identity, idQa, idQ_idQa) cnf(identity_divide_idQa, hypothesis)
quotient(idQb, idQ_idQa, idQb_Q__idQ_idQa) cnf(idQb_divide_idQ_idQa, hypothesis)
idQa_Q__idQ_idQb ≠ idQb_Q__idQ_idQa cnf(prove_idQa_Q__idQ_idQb_equals_idQb_Q__idQ_idQa, negated_conjecture)

HEN011-2.p This operation is commutative
Define & on the set of Z', where Z' = identity/Z, by X' & Y' = X'/(identity/Y'). The operation is commutative.
include('Axioms/HEN001-0.ax')
quotient(x, identity, 0) cnf(everything_divide_identity_is_zero, axiom)
quotient(0, x, 0) cnf(zero_divide_anything_is_zero, axiom)
quotient(x, x, 0) cnf(x_divide_x_is_zero, axiom)
quotient(x, 0, x) cnf(x_divde_zero_is_x, axiom)
(less_equal(x, y) and less_equal(y, z)) ⇒ less_equal(x, z) cnf(transitivity_of_less_equal, axiom)
(quotient(x, y, w_1) and less_equal(w_1, z) and quotient(x, z, w_2)) ⇒ less_equal(w_2, y) cnf(xQyLEz_implies_xQzLEy, axiom
(less_equal(x, y) and quotient(z, y, w_1) and quotient(z, x, w_2)) ⇒ less_equal(w_1, w_2) cnf(xLEy_implies_zQyLEzQx, axiom
(less_equal(x, y) and quotient(x, z, w_1) and quotient(y, z, w_2)) ⇒ less_equal(w_1, w_2) cnf(xLEy_implies_xQzLEyQz, axiom
(quotient(identity, x, y_1) and quotient(identity, y_1, y_2) and quotient(identity, y_2, y_3)) ⇒ $y_1 = y_3$ cnf(one_inversion_equals
(quotient(identity, x, y_1) and quotient(identity, y_1, y_2) and quotient(y_1, y_2, y_3)) ⇒ $y_1 = y_3$ cnf(inversion_lemma, axiom)
quotient(identity, a, idQa) cnf(identity_divide_a, hypothesis)
quotient(identity, b, idQb) cnf(identity_divide_b, hypothesis)
quotient(identity, idQb, idQ_idQb) cnf(identity_divide_idQb, hypothesis)
quotient(idQa, idQ_idQb, idQa_Q__idQ_idQb) cnf(idQa_divide_idQ_idQb, hypothesis)
quotient(identity, idQa, idQ_idQa) cnf(identity_divide_idQa, hypothesis)
quotient(idQb, idQ_idQa, idQb_Q__idQ_idQa) cnf(idQb_divide_idQ_idQa, hypothesis)
idQa_Q__idQ_idQb ≠ idQb_Q__idQ_idQa cnf(prove_idQa_Q__idQ_idQb_equals_idQb_Q__idQ_idQa, negated_conjecture)

HEN011-3.p This operation is commutative
Define & on the set of Z', where Z' = identity/Z, by X' & Y'=X'/(identity/Y'). The operation is commutative.
include('Axioms/HEN002-0.ax')
divide(divide(identity, a), divide(identity, divide(identity, b))) ≠ divide(divide(identity, b), divide(identity, divide(identity, a)))
divide(identity, a) = c cnf(identity_divide_a, hypothesis)
divide(identity, b) = d cnf(identity_divide_b, hypothesis)
divide(identity, c) = e cnf(identity_divide_c, hypothesis)
divide(identity, d) = g cnf(identity_divide_d, hypothesis)
divide(c, g) ≠ divide(d, e) cnf(prove_commutativity, negated_conjecture)

HEN011-4.p This operation is commutative
Define & on the set of Z', where Z' = identity/Z, by X' & Y' = X'/(identity/Y'). The operation is commutative.
include('Axioms/HEN002-0.ax')
$\text{divide}(x, \text{identity}) = 0$ cnf(everything_divide_id_is_zero, axiom)
$\text{divide}(0, x) = 0$ cnf(zero_divide_anything_is_zero, axiom)
$\text{divide}(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
$\text{divide}(a, 0) = a$ cnf(x_divide_zero_is_x, axiom)
$(\text{less\_equal}(x, y) \text{ and } \text{less\_equal}(y, z)) \Rightarrow \text{less\_equal}(x, z)$ cnf(transitivity_of_less_equal, axiom)
$\text{less\_equal}(\text{divide}(x, y), z) \Rightarrow \text{less\_equal}(\text{divide}(x, z), y)$ cnf(property_of_divide$_1$, axiom)
$\text{less\_equal}(x, y) \Rightarrow \text{less\_equal}(\text{divide}(z, y), \text{divide}(z, x))$ cnf(property_of_divide$_2$, axiom)
$\text{less\_equal}(x, y) \Rightarrow \text{less\_equal}(\text{divide}(x, z), \text{divide}(y, z))$ cnf(property_of_divide$_3$, axiom)
$\text{divide}(\text{identity}, \text{divide}(\text{identity}, \text{divide}(\text{identity}, x))) = \text{divide}(\text{identity}, x)$ cnf(one_inversion_equals_three, axiom)
$\text{divide}(\text{divide}(\text{identity}, x), \text{divide}(\text{identity}, \text{divide}(\text{identity}, x))) = \text{divide}(\text{identity}, x)$ cnf(property_of_inversion, axiom)
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{identity}, \text{divide}(\text{identity}, b))) \neq \text{divide}(\text{divide}(\text{identity}, b), \text{divide}(\text{identity}, \text{divide}(\text{identity}, a)))$

HEN011-5.p This operation is commutative
Define & on the set of Z', where Z' = identity/Z, by X' & Y' = X'/(identity/Y'). The operation is commutative.
include('Axioms/HEN003-0.ax')
$\text{divide}(x, x) = 0$ cnf(x_divide_x_is_zero, axiom)
$\text{divide}(x, 0) = x$ cnf(x_divide_zero_is_x, axiom)
$(\text{divide}(x, y) = 0 \text{ and } \text{divide}(y, z) = 0) \Rightarrow \text{divide}(x, z) = 0$ cnf(transitivity_of_divide_to_zero, axiom)
$\text{divide}(\text{divide}(x, y), z) = 0 \Rightarrow \text{divide}(\text{divide}(x, z), y) = 0$ cnf(property_of_divide$_1$, axiom)
$\text{divide}(x, y) = 0 \Rightarrow \text{divide}(\text{divide}(z, y), \text{divide}(z, x)) = 0$ cnf(property_of_divide$_2$, axiom)
$\text{divide}(x, y) = 0 \Rightarrow \text{divide}(\text{divide}(x, z), \text{divide}(y, z)) = 0$ cnf(property_of_divide$_3$, axiom)
$\text{divide}(\text{identity}, \text{divide}(\text{identity}, \text{divide}(\text{identity}, x))) = \text{divide}(\text{identity}, x)$ cnf(one_inversion_equals_three, axiom)
$\text{divide}(\text{divide}(\text{identity}, x), \text{divide}(\text{identity}, \text{divide}(\text{identity}, x))) = \text{divide}(\text{identity}, x)$ cnf(property_of_inversion, axiom)
$\text{divide}(\text{divide}(\text{identity}, a), \text{divide}(\text{identity}, \text{divide}(\text{identity}, b))) \neq \text{divide}(\text{divide}(\text{identity}, b), \text{divide}(\text{identity}, \text{divide}(\text{identity}, a)))$

HEN012-1.p X ≤ X
include('Axioms/HEN001-0.ax')
$\neg\, \text{less\_equal}(x, x)$ cnf(prove_X_less_than_or_equal_to_X, negated_conjecture)

HEN012-3.p X ≤ X
include('Axioms/HEN002-0.ax')
$\neg\, \text{less\_equal}(a, a)$ cnf(prove_a_LE_a, negated_conjecture)

HEN013-1.p Henkin model axioms
include('Axioms/HEN001-0.ax')

HEN013-2.p Henkin model axioms
include('Axioms/HEN002-0.ax')

HEN013-3.p Henkin model (equality) axioms
include('Axioms/HEN003-0.ax')