

KLE axioms

KLE001+0.ax Idempotent semirings

$$\begin{aligned} \forall a, b: a + b = b + a & \quad \text{fof(additive_commutativity, axiom)} \\ \forall c, b, a: a + (b + c) = (a + b) + c & \quad \text{fof(additive_associativity, axiom)} \\ \forall a: a + 0 = a & \quad \text{fof(additive_identity, axiom)} \\ \forall a: a + a = a & \quad \text{fof(additive_idempotence, axiom)} \\ \forall a, b, c: a \cdot (b \cdot c) = (a \cdot b) \cdot c & \quad \text{fof(multiplicative_associativity, axiom)} \\ \forall a: a \cdot 1 = a & \quad \text{fof(multiplicative_right_identity, axiom)} \\ \forall a: 1 \cdot a = a & \quad \text{fof(multiplicative_left_identity, axiom)} \\ \forall a, b, c: a \cdot (b + c) = a \cdot b + a \cdot c & \quad \text{fof(right_distributivity, axiom)} \\ \forall a, b, c: (a + b) \cdot c = a \cdot c + b \cdot c & \quad \text{fof(left_distributivity, axiom)} \\ \forall a: a \cdot 0 = 0 & \quad \text{fof(right_annihilation, axiom)} \\ \forall a: 0 \cdot a = 0 & \quad \text{fof(left_annihilation, axiom)} \\ \forall a, b: (a \leq b \iff a + b = b) & \quad \text{fof(order, axiom)} \end{aligned}$$

KLE001+1.ax Characterisation of tests by complement predicate

$$\begin{aligned} \forall x_0: (\text{test}(x_0) \iff \exists x_1: x'_1 = x_0) & \quad \text{fof(test}_1, \text{axiom)} \\ \forall x_0, x_1: (x'_1 = x_0 \iff (x_0 \cdot x_1 = 0 \text{ and } x_1 \cdot x_0 = 0 \text{ and } x_0 + x_1 = 1)) & \quad \text{fof(test}_2, \text{axiom)} \\ \forall x_0, x_1: (\text{test}(x_0) \Rightarrow (c(x_0) = x_1 \iff x'_0 = x_1)) & \quad \text{fof(test}_3, \text{axiom)} \\ \forall x_0: (\neg \text{test}(x_0) \Rightarrow c(x_0) = 0) & \quad \text{fof(test}_4, \text{axiom)} \end{aligned}$$

KLE001+2.ax de Morgan's laws for tests

$$\begin{aligned} \forall x_0, x_1: ((\text{test}(x_0) \text{ and } \text{test}(x_1)) \Rightarrow c(x_0 + x_1) = c(x_0) \cdot c(x_1)) & \quad \text{fof(test_deMorgan}_1, \text{axiom)} \\ \forall x_0, x_1: ((\text{test}(x_0) \text{ and } \text{test}(x_1)) \Rightarrow c(x_0 \cdot x_1) = c(x_0) + c(x_1)) & \quad \text{fof(test_deMorgan}_2, \text{axiom)} \end{aligned}$$

KLE001+3.ax Universal characterisation of meet

$$\begin{aligned} \forall x_0, x_1, x_2: (\text{ismeet}(x_2, x_0, x_1) \iff (x_2 \leq x_0 \text{ and } x_2 \leq x_1 \text{ and } \forall x_3: ((x_3 \leq x_0 \text{ and } x_3 \leq x_1) \Rightarrow x_3 \leq x_2))) & \quad \text{fof(ismeet, axiom)} \\ \forall x_0, x_1, x_2: (\text{ismeetu}(x_2, x_0, x_1) \iff \forall x_3: ((x_3 \leq x_0 \text{ and } x_3 \leq x_1) \iff x_3 \leq x_2)) & \quad \text{fof(ismeetu, axiom)} \end{aligned}$$

KLE001+4.ax Boolean domain, antidomain, codomain, coantidomain

$$\begin{aligned} \forall x_0: \text{ad}(x_0) \cdot x_0 = 0 & \quad \text{fof(domain}_1, \text{axiom)} \\ \forall x_0, x_1: \text{ad}(x_0 \cdot x_1) + \text{ad}(x_0 \cdot \text{ad}(\text{ad}(x_1))) = \text{ad}(x_0 \cdot \text{ad}(\text{ad}(x_1))) & \quad \text{fof(domain}_2, \text{axiom)} \\ \forall x_0: \text{ad}(\text{ad}(x_0)) + \text{ad}(x_0) = 1 & \quad \text{fof(domain}_3, \text{axiom)} \\ \forall x_0: \text{dom}(x_0) = \text{ad}(\text{ad}(x_0)) & \quad \text{fof(domain}_4, \text{axiom)} \\ \forall x_0: x_0 \cdot \text{coad}(x_0) = 0 & \quad \text{fof(codomain}_1, \text{axiom)} \\ \forall x_0, x_1: \text{coad}(x_0 \cdot x_1) + \text{coad}(\text{coad}(\text{coad}(x_0)) \cdot x_1) = \text{coad}(\text{coad}(\text{coad}(x_0)) \cdot x_1) & \quad \text{fof(codomain}_2, \text{axiom)} \\ \forall x_0: \text{coad}(\text{coad}(x_0)) + \text{coad}(x_0) = 1 & \quad \text{fof(codomain}_3, \text{axiom)} \\ \forall x_0: \text{cod}(x_0) = \text{coad}(\text{coad}(x_0)) & \quad \text{fof(codomain}_4, \text{axiom)} \end{aligned}$$

KLE001+5.ax Domain (not Boolean domain!)

$$\begin{aligned} \forall x_0: x_0 + \text{dom}(x_0) \cdot x_0 = \text{dom}(x_0) \cdot x_0 & \quad \text{fof(domain}_1, \text{axiom)} \\ \forall x_0, x_1: \text{dom}(x_0 \cdot x_1) = \text{dom}(x_0 \cdot \text{dom}(x_1)) & \quad \text{fof(domain}_2, \text{axiom)} \\ \forall x_0: \text{dom}(x_0) + 1 = 1 & \quad \text{fof(domain}_3, \text{axiom)} \\ \text{dom}(0) = 0 & \quad \text{fof(domain}_4, \text{axiom)} \\ \forall x_0, x_1: \text{dom}(x_0 + x_1) = \text{dom}(x_0) + \text{dom}(x_1) & \quad \text{fof(domain}_5, \text{axiom)} \end{aligned}$$

KLE001+6.ax Modal operators

$$\begin{aligned} \forall x_0: c(x_0) = \text{ad}(\text{dom}(x_0)) & \quad \text{fof(complement, axiom)} \\ \forall x_0, x_1: \text{domain_difference}(x_0, x_1) = \text{dom}(x_0) \cdot \text{ad}(x_1) & \quad \text{fof(domain_difference, axiom)} \\ \forall x_0, x_1: \text{forward_diamond}(x_0, x_1) = \text{dom}(x_0 \cdot \text{dom}(x_1)) & \quad \text{fof(forward_diamond, axiom)} \\ \forall x_0, x_1: \text{backward_diamond}(x_0, x_1) = \text{cod}(\text{cod}(x_1) \cdot x_0) & \quad \text{fof(backward_diamond, axiom)} \\ \forall x_0, x_1: \text{forward_box}(x_0, x_1) = c(\text{forward_diamond}(x_0, c(x_1))) & \quad \text{fof(forward_box, axiom)} \\ \forall x_0, x_1: \text{backward_box}(x_0, x_1) = c(\text{backward_diamond}(x_0, c(x_1))) & \quad \text{fof(backward_box, axiom)} \end{aligned}$$

KLE001+7.ax Divergence Kleene algebras

$$\begin{aligned} \forall x_0: \text{forward_diamond}(x_0, \text{divergence}(x_0)) = \text{divergence}(x_0) & \quad \text{fof(divergence}_1, \text{axiom)} \\ \forall x_0, x_1, x_2: (\text{dom}(x_0) + (\text{forward_diamond}(x_1, \text{dom}(x_0)) + \text{dom}(x_2)) = \text{forward_diamond}(x_1, \text{dom}(x_0)) + \text{dom}(x_2) \Rightarrow \text{dom}(x_0) + (\text{divergence}(x_1) + \text{forward_diamond}(x_1^*, \text{dom}(x_2))) = \text{divergence}(x_1) + \text{forward_diamond}(x_1^*, \text{dom}(x_2))) & \quad \text{fof(divergence}_2, \text{axiom)} \end{aligned}$$

KLE002+0.ax Kleene algebra

$$\begin{aligned} \forall a, b: a + b = b + a & \quad \text{fof(additive_commutativity, axiom)} \\ \forall c, b, a: a + (b + c) = (a + b) + c & \quad \text{fof(additive_associativity, axiom)} \end{aligned}$$

$\forall a: a + 0 = a$ fof(additive_identity, axiom)
 $\forall a: a + a = a$ fof(additive_idempotence, axiom)
 $\forall a, b, c: a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
 $\forall a: a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
 $\forall a: 1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
 $\forall a, b, c: a \cdot (b + c) = a \cdot b + a \cdot c$ fof(right_distributivity, axiom)
 $\forall a, b, c: (a + b) \cdot c = a \cdot c + b \cdot c$ fof(left_distributivity, axiom)
 $\forall a: a \cdot 0 = 0$ fof(right_annihilation, axiom)
 $\forall a: 0 \cdot a = 0$ fof(left_annihilation, axiom)
 $\forall a, b: (a \leq b \iff a + b = b)$ fof(order, axiom)
 $\forall a: 1 + a \cdot a^* \leq a^*$ fof(star_unfold_right, axiom)
 $\forall a: 1 + a^* \cdot a \leq a^*$ fof(star_unfold_left, axiom)
 $\forall a, b, c: (a \cdot b + c \leq b \implies a^* \cdot c \leq b)$ fof(star_induction_left, axiom)
 $\forall a, b, c: (a \cdot b + c \leq a \implies c \cdot b^* \leq a)$ fof(star_induction_right, axiom)

KLE003+0.ax Omega algebra

$\forall a, b: a + b = b + a$ fof(additive_commutativity, axiom)
 $\forall c, b, a: a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)
 $\forall a: a + 0 = a$ fof(additive_identity, axiom)
 $\forall a: a + a = a$ fof(additive_idempotence, axiom)
 $\forall a, b, c: a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
 $\forall a: a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
 $\forall a: 1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
 $\forall a, b, c: a \cdot (b + c) = a \cdot b + a \cdot c$ fof(right_distributivity, axiom)
 $\forall a, b, c: (a + b) \cdot c = a \cdot c + b \cdot c$ fof(left_distributivity, axiom)
 $\forall a: a \cdot 0 = 0$ fof(right_annihilation, axiom)
 $\forall a: 0 \cdot a = 0$ fof(left_annihilation, axiom)
 $\forall a, b: (a \leq b \iff a + b = b)$ fof(order, axiom)
 $\forall a: 1 + a \cdot a^* \leq a^*$ fof(star_unfold_right, axiom)
 $\forall a: 1 + a^* \cdot a \leq a^*$ fof(star_unfold_left, axiom)
 $\forall a, b, c: (a \cdot b + c \leq b \implies a^* \cdot c \leq b)$ fof(star_induction_left, axiom)
 $\forall a, b, c: (a \cdot b + c \leq a \implies c \cdot b^* \leq a)$ fof(star_induction_right, axiom)
 $\forall a: a \cdot a^\omega = a^\omega$ fof(omega_unfold, axiom)
 $\forall a, b, c: (a \leq b \cdot a + c \implies a \leq b^\omega + b^* \cdot c)$ fof(omega_co_induction, axiom)

KLE004+0.ax Demonic Refinement Algebra

$\forall a, b: a + b = b + a$ fof(additive_commutativity, axiom)
 $\forall c, b, a: a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)
 $\forall a: a + 0 = a$ fof(additive_identity, axiom)
 $\forall a: a + a = a$ fof(idempotence, axiom)
 $\forall a, b, c: a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
 $\forall a: a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
 $\forall a: 1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
 $\forall a, b, c: a \cdot (b + c) = a \cdot b + a \cdot c$ fof(distributivity₁, axiom)
 $\forall a, b, c: (a + b) \cdot c = a \cdot c + b \cdot c$ fof(distributivity₂, axiom)
 $\forall a: 0 \cdot a = 0$ fof(left_annihilation, axiom)
 $\forall a: 1 + a \cdot a^* = a^*$ fof(star_unfold₁, axiom)
 $\forall a: 1 + a^* \cdot a = a^*$ fof(star_unfold₂, axiom)
 $\forall a, b, c: (a \cdot c + b \leq c \implies a^* \cdot b \leq c)$ fof(star_induction₁, axiom)
 $\forall a, b, c: (c \cdot a + b \leq c \implies b \cdot a^* \leq c)$ fof(star_induction₂, axiom)
 $\forall a: \text{strong_iteration}(a) = a \cdot \text{strong_iteration}(a) + 1$ fof(infty_unfold₁, axiom)
 $\forall a, b, c: (c \leq a \cdot c + b \implies c \leq \text{strong_iteration}(a) \cdot b)$ fof(infty_coinduction, axiom)
 $\forall a: \text{strong_iteration}(a) = a^* + \text{strong_iteration}(a) \cdot 0$ fof(isolation, axiom)
 $\forall a, b: (a \leq b \iff a + b = b)$ fof(order, axiom)

KLE problems

KLE001+1.p Addition is isotone

include('Axioms/KLE001+0.ax')

$\forall x_0, x_1, x_2: (x_0 \leq x_1 \implies x_0 + x_2 \leq x_1 + x_2)$ fof(goals, conjecture)

KLE002+1.p Multiplication is isotone

include('Axioms/KLE001+0.ax')

$\forall x_0, x_1, x_2: (x_0 \leq x_1 \Rightarrow x_0 \cdot x_2 \leq x_1 \cdot x_2)$ fof(goals, conjecture)

KLE003+1.p A semiring is idempotent iff 1 is idempotent

include('Axioms/KLE001+0.ax')

$\forall x_0: x_0 + x_0 = x_0 \iff 1 + 1 = 1$ fof(goals, conjecture)

KLE004+1.p Complement - 1 is the complement of 0

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$c(0) = 1$ fof(goals, conjecture)

KLE005+1.p Complement - 0 is the complement of 1

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$c(1) = 0$ fof(goals, conjecture)

KLE006+1.p Split 1 with p

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0: (\text{test}(x_0) \Rightarrow 1 = x_0 + c(x_0))$ fof(goals, conjecture)

KLE007+1.p Split 1 with q and split the parts with p

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1))$ fof(goals, conjecture)

KLE007+2.p Split 1 with q and split the parts with p

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1) \text{ and } (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE007+3.p Split 1 with q and split the parts with p

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1))$ fof(goals, conjecture)

KLE007+4.p Split 1 with q and split the parts with p

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1) \text{ and } (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE008+1.p A simple law to eliminate the complement

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: (\text{test}(x_2) \Rightarrow (x_0 \leq x_2 \cdot x_1 \iff (x_0 \leq x_1 \text{ and } c(x_2) \cdot x_0 \leq 0)))$ fof(goals, conjecture)

KLE009+1.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE009+2.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE009+3.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE009+4.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE010+1.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))) \quad \text{fof}(\text{goals, conjecture})$

KLE010+2.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE010+3.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))) \quad \text{fof}(\text{goals, conjecture})$

KLE010+4.p Split 1 into all combinations of p,q and their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE011+1.p Split 1 into p,q and the product of their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1)) \quad \text{fof}(\text{goals, conjecture})$

KLE011+2.p Split 1 into p,q and the product of their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE011+3.p Split 1 into p,q and the product of their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1)) \quad \text{fof}(\text{goals, conjecture})$

KLE011+4.p Split 1 into p,q and the product of their complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \text{ and } ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))) \quad \text{fof}(\text{goals, conjecture})$

KLE012+1.p The multiplication of tests is commutative

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow x_0 \cdot x_1 = x_1 \cdot x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE013+1.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (x_0 + x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE014+2.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow c(x_0 + x_1) = c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE014+3.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (c(x_0+x_1) \leq c(x_0) \cdot c(x_1) \text{ and } c(x_0) \cdot c(x_1) \leq c(x_0+x_1)))$ fof(goals, conjecture)

KLE015+1.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow ((x_0 + x_1) \cdot c(x_0)) \cdot c(x_1) = 0)$ fof(goals, conjecture)

KLE016+2.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow c(x_0 \cdot x_1) = c(x_0) + c(x_1))$ fof(goals, conjecture)

KLE016+3.p The complement of a product is the sum of the complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (c(x_0 \cdot x_1) \leq c(x_0) + c(x_1) \text{ and } c(x_0) + c(x_1) \leq c(x_0 \cdot x_1)))$ fof(goals, conjecture)

KLE017+1.p Product of tests is their meet

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_0) \text{ and } \text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_2 \leq x_0 \cdot x_1 \iff (x_2 \leq x_0 \text{ and } x_2 \leq x_1)))$ fof(goals, conjecture)

KLE018+1.p Move a term from the left of an implication to the right and back

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_0) \text{ and } \text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_0 \cdot c(x_1) \leq x_2 \Rightarrow x_0 \leq x_1 + x_2))$ fof(goals, conjecture)

KLE019+1.p Move a term from the left of an implication to the right and back

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_0) \text{ and } \text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_0 \cdot c(x_1) \leq x_2 \Leftarrow x_0 \leq x_1 + x_2))$ fof(goals, conjecture)

KLE020+1.p On tests addition distributes over multiplication

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_0) \text{ and } \text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow x_0 + x_1 \cdot x_2 = (x_0 + x_1) \cdot (x_0 + x_2))$ fof(goals, conjecture)

KLE020+2.p On tests addition distributes over multiplication

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_0) \text{ and } \text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_0 + x_1 \cdot x_2 \leq (x_0 + x_1) \cdot (x_0 + x_2) \text{ and } (x_0 + x_1) \cdot (x_0 + x_2) \leq x_0 + x_1 \cdot x_2))$ fof(goals, conjecture)

KLE021+1.p Decompose splitting

Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow x_0 = x_1 \cdot x_0 + c(x_1) \cdot x_0)$ fof(goals, conjecture)

KLE021+2.p Decompose splitting

Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow (x_0 \leq x_1 \cdot x_0 + c(x_1) \cdot x_0 \text{ and } x_1 \cdot x_0 + c(x_1) \cdot x_0 \leq x_0))$ fof(goals, conjecture)

KLE021+3.p Decompose splitting

Decompose x into the part starting in p and the one starting in p 's complement; also the rule if p then x else $x = x$ where if p then y else $z = p; y + c(p); z$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow x_0 = x_1 \cdot x_0 + c(x_1) \cdot x_0) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE021+4.p Decompose splitting

Decompose x into the part starting in p and the one starting in p 's complement; also the rule if p then x else $x = x$ where if p then y else $z = p; y + c(p); z$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow (x_0 \leq x_1 \cdot x_0 + c(x_1) \cdot x_0 \text{ and } x_1 \cdot x_0 + c(x_1) \cdot x_0 \leq x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE022+1.p Decompose x into parts ending in p and p 's complement

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow x_0 = x_0 \cdot x_1 + x_0 \cdot c(x_1)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE022+2.p Decompose x into parts ending in p and p 's complement

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow (x_0 \leq x_0 \cdot x_1 + x_0 \cdot c(x_1) \text{ and } x_0 \cdot x_1 + x_0 \cdot c(x_1) \leq x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE022+3.p Decompose x into parts ending in p and p 's complement

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow x_0 = x_0 \cdot x_1 + x_0 \cdot c(x_1)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE022+4.p Decompose x into parts ending in p and p 's complement

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1: (\text{test}(x_1) \Rightarrow (x_0 \leq x_0 \cdot x_1 + x_0 \cdot c(x_1) \text{ and } x_0 \cdot x_1 + x_0 \cdot c(x_1) \leq x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE023+1.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_1 \cdot x_0 + x_0 \cdot x_2 = x_0 \cdot x_2 \Rightarrow x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE023+2.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_1 \cdot x_0 + x_0 \cdot x_2 = x_0 \cdot x_2 \Rightarrow x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE024+1.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0 \Rightarrow (x_1 \cdot x_0) \cdot c(x_2) = 0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE024+2.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0 \Rightarrow (x_1 \cdot x_0) \cdot c(x_2) = 0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE025+1.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow ((x_1 \cdot x_0) \cdot c(x_2) = 0 \Rightarrow x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE025+2.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow ((x_1 \cdot x_0) \cdot c(x_2) = 0 \Rightarrow x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2))$ fof(goals, conjecture)

KLE026+1.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2 \Rightarrow x_1 \cdot x_0 \leq x_0 \cdot x_2))$ fof(goals, conjecture)

KLE026+2.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2)) \Rightarrow (x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2 \Rightarrow x_1 \cdot x_0 \leq x_0 \cdot x_2))$ fof(goals, conjecture)

KLE027+1.p Simplify conditional

Simplify conditional: if p then (if p then x else y) else z = if p then x else z.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_4)) \Rightarrow x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 = x_3 \cdot x_0 + c(x_3) \cdot x_2)$ fof(goals, conjecture)

KLE027+2.p Simplify conditional

Simplify conditional: if p then (if p then x else y) else z = if p then x else z.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_4)) \Rightarrow (x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 \leq x_3 \cdot x_0 + c(x_3) \cdot x_2 \text{ and } x_3 \cdot x_0 + c(x_3) \cdot x_2 \leq x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2))$ fof(goals, conjecture)

KLE027+3.p Simplify conditional

Simplify conditional: if p then (if p then x else y) else z = if p then x else z.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_4)) \Rightarrow x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 = x_3 \cdot x_0 + c(x_3) \cdot x_2)$ fof(goals, conjecture)

KLE027+4.p Simplify conditional

Simplify conditional: if p then (if p then x else y) else z = if p then x else z.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_4)) \Rightarrow (x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 \leq x_3 \cdot x_0 + c(x_3) \cdot x_2 \text{ and } x_3 \cdot x_0 + c(x_3) \cdot x_2 \leq x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2))$ fof(goals, conjecture)

KLE028+1.p Switch nested conditions and rearrange branches of conditional

If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3, x_4, x_5: ((\text{test}(x_4) \text{ and } \text{test}(x_5)) \Rightarrow x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) = x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3))$ fof(goals, conjecture)

KLE028+2.p Switch nested conditions and rearrange branches of conditional

If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3, x_4, x_5: ((\text{test}(x_4) \text{ and } \text{test}(x_5)) \Rightarrow (x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) \leq x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \text{ and } x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \leq x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3)))$ fof(goals, conjecture)

KLE028+3.p Switch nested conditions and rearrange branches of conditional

If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5: ((\text{test}(x_4) \text{ and } \text{test}(x_5)) \Rightarrow x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) = x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3))$ fof(goals, conjecture)

KLE028+4.p Switch nested conditions and rearrange branches of conditional

If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5: ((\text{test}(x_4) \text{ and } \text{test}(x_5)) \Rightarrow (x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) \leq x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \text{ and } x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \leq x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3)))$ fof(goals, conjecture)

KLE029+1.p Characterisations of meet

Equivalence of characterisation as greatest lower bound and universal characterisation.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+3.ax')

$\forall x_0, x_1, x_2: (\text{ismeet}(x_0, x_1, x_2) \iff \text{ismeetu}(x_0, x_1, x_2))$ fof(goals, conjecture)

KLE030+1.p Restriction can be pulled out of meet

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+3.ax')

$\forall x_0, x_1, x_2, x_3: ((\text{test}(x_3) \text{ and } \text{ismeet}(x_0, x_1, x_2)) \Rightarrow \text{ismeet}(x_3 \cdot x_0, x_3 \cdot x_1, x_2))$ fof(goals, conjecture)

KLE031+1.p Restriction distributes through meet

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+3.ax')

$\forall x_0, x_1, x_2, x_3: ((\text{test}(x_3) \text{ and } \text{ismeet}(x_0, x_1, x_2)) \Rightarrow \text{ismeet}(x_3 \cdot x_0, x_3 \cdot x_1, x_3 \cdot x_2))$ fof(goals, conjecture)

KLE032+1.p Meet of restrictions is restriction by meet

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+3.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_2) \text{ and } \text{test}(x_1)) \Rightarrow \text{ismeet}((x_1 \cdot x_2) \cdot x_0, x_1 \cdot x_0, x_2 \cdot x_0))$ fof(goals, conjecture)

KLE033+1.p Disjoint tests induce disjoint restrictions

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+3.ax')

$\forall x_0, x_1, x_2: ((\text{test}(x_2) \text{ and } \text{test}(x_1) \text{ and } \text{ismeet}(0, x_1, x_2)) \Rightarrow \text{ismeet}(0, x_1 \cdot x_0, x_2 \cdot x_0))$ fof(goals, conjecture)

KLE034+1.p Hoare rule product

Encoding of Hoare rule $\text{pxq} \langle \text{description} \rangle \text{qyr} \rightarrow \text{px};\text{yr}$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_2) \text{ and } \text{test}(x_4) \text{ and } (x_2 \cdot x_0) \cdot c(x_3) \leq 0 \text{ and } (x_3 \cdot x_1) \cdot c(x_4) \leq 0) \Rightarrow ((x_2 \cdot x_0) \cdot x_1) \cdot c(x_4) \leq 0)$ fof(goals, conjecture)

KLE034+2.p Hoare rule product

Encoding of Hoare rule $\text{pxq} \langle \text{description} \rangle \text{qyr} \rightarrow \text{px};\text{yr}$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3, x_4: ((\text{test}(x_3) \text{ and } \text{test}(x_2) \text{ and } \text{test}(x_4) \text{ and } (x_2 \cdot x_0) \cdot c(x_3) \leq 0 \text{ and } (x_3 \cdot x_1) \cdot c(x_4) \leq 0) \Rightarrow ((x_2 \cdot x_0) \cdot x_1) \cdot c(x_4) \leq 0)$ fof(goals, conjecture)

KLE035+1.p Hoare rule sum

Encoding of Hoare rule $\text{pxq} \langle \text{description} \rangle \text{qyr} \rightarrow \text{px};\text{yr}$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

$\forall x_0, x_1, x_2, x_3: ((\text{test}(x_3) \text{ and } \text{test}(x_2) \text{ and } (x_2 \cdot x_0) \cdot c(x_3) \leq 0 \text{ and } (x_2 \cdot x_1) \cdot c(x_3) \leq 0) \Rightarrow (x_2 \cdot (x_0 + x_1)) \cdot c(x_3) \leq 0)$ fof(goals, conjecture)

KLE035+2.p Hoare rule sum

Encoding of Hoare rule $pxq \langle \text{description} \rangle qyr \rightarrow px;yr$.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3: ((\text{test}(x_3) \text{ and } \text{test}(x_2) \text{ and } (x_2 \cdot x_0) \cdot c(x_3) \leq 0 \text{ and } (x_2 \cdot x_1) \cdot c(x_3) \leq 0) \Rightarrow (x_2 \cdot (x_0 + x_1)) \cdot c(x_3) \leq 0)$ fof(goals, conjecture)

KLE036+1.p Star recursion

Together with star unfold this yields the classical recursion equation for star.

include('Axioms/KLE002+0.ax')

$\forall x_0: x_0^* \leq 1 + x_0 \cdot x_0^*$ fof(goals, conjecture)

KLE037+1.p Star reflexivity

Star is reflexive, i.e., contains the identity.

include('Axioms/KLE002+0.ax')

$\forall x_0: 1 \leq x_0^*$ fof(goals, conjecture)

KLE038+1.p Star extensivity

Star is extensive.

include('Axioms/KLE002+0.ax')

$\forall x_0: x_0 \leq x_0^*$ fof(goals, conjecture)

KLE039+1.p Star idempotence

The star operation is idempotent.

include('Axioms/KLE002+0.ax')

$\forall x_0: (x_0^*)^* = x_0^*$ fof(goals, conjecture)

KLE039+2.p Star idempotence

The star operation is idempotent.

include('Axioms/KLE002+0.ax')

$\forall x_0: ((x_0^*)^* \leq x_0^* \text{ and } x_0^* \leq (x_0^*)^*)$ fof(goals, conjecture)

KLE040+1.p The star of an element is multiplicatively idempotent

include('Axioms/KLE002+0.ax')

$\forall x_0: x_0^* \cdot x_0^* = x_0^*$ fof(goals, conjecture)

KLE040+2.p The star of an element is multiplicatively idempotent

include('Axioms/KLE002+0.ax')

$\forall x_0: (x_0^* \cdot x_0^* \leq x_0^* \text{ and } x_0^* \leq x_0^* \cdot x_0^*)$ fof(goals, conjecture)

KLE041+1.p Star isotony

The star operation is isotone.

include('Axioms/KLE002+0.ax')

$\forall x_0, x_1: (x_0 \leq x_1 \Rightarrow x_0^* \leq x_1^*)$ fof(goals, conjecture)

KLE042+1.p Star sliding

Two ways of grouping an alternation of x's and y's that starts and ends with x.

include('Axioms/KLE002+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1)^* \cdot x_0 = x_0 \cdot (x_1 \cdot x_0)^*$ fof(goals, conjecture)

KLE042+2.p Star sliding

Two ways of grouping an alternation of x's and y's that starts and ends with x.

include('Axioms/KLE002+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1)^* \cdot x_0 \leq x_0 \cdot (x_1 \cdot x_0)^* \text{ and } x_0 \cdot (x_1 \cdot x_0)^* \leq (x_0 \cdot x_1)^* \cdot x_0)$ fof(goals, conjecture)

KLE043+1.p Star recursion

Classical recursion equation for star of x followed by y.

include('Axioms/KLE002+0.ax')

$\forall x_0, x_1: x_0^* \cdot x_1 = x_1 + (x_0 \cdot x_0^*) \cdot x_1$ fof(goals, conjecture)

KLE043+2.p Star recursion

Classical recursion equation for star of x followed by y.

include('Axioms/KLE002+0.ax')

$\forall x_0, x_1: (x_0^* \cdot x_1 \leq x_1 + (x_0 \cdot x_0^*) \cdot x_1 \text{ and } x_1 + (x_0 \cdot x_0^*) \cdot x_1 \leq x_0^* \cdot x_1)$ fof(goals, conjecture)

KLE044+1.p Star simplification

Identity can be eliminated from the star of a sum.

include('Axioms/KLE002+0.ax')
 $\forall x_0: (1 + x_0)^* = x_0^*$ fof(goals, conjecture)

KLE044+2.p Star simplification

Identity can be eliminated from the star of a sum.

include('Axioms/KLE002+0.ax')
 $\forall x_0: ((1 + x_0)^* \leq x_0^* \text{ and } x_0^* \leq (1 + x_0)^*)$ fof(goals, conjecture)

KLE045+1.p Semi-commutation

If x semi-commutes over y transforming into z then the same applies to x and y*.

include('Axioms/KLE002+0.ax')
 $\forall x_0, x_1, x_2: (x_0 \cdot x_2 \leq x_2 \cdot x_1 \Rightarrow x_0^* \cdot x_2 \leq x_2 \cdot x_1^*)$ fof(goals, conjecture)

KLE046+1.p Church-Rosser theorem

Normalise arbitrary sequences of x's and y's into x's followed by y's.

include('Axioms/KLE002+0.ax')
 $\forall x_0, x_1: (x_1^* \cdot x_0^* \leq x_0^* \cdot x_1^* \Rightarrow (x_0 + x_1)^* \leq x_0^* \cdot x_1^*)$ fof(goals, conjecture)

KLE047+1.p Star simplification

Semi-commutation of x and y allows normalizing arbitrary sequences of x's and y's.

include('Axioms/KLE002+0.ax')
 $\forall x_0, x_1: (x_1 \cdot x_0 \leq x_0 \cdot x_1 \Rightarrow (x_0 + x_1)^* \leq x_0^* \cdot x_1^*)$ fof(goals, conjecture)

KLE048+1.p The star of a test is always 1

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0: (\text{test}(x_0) \Rightarrow x_0^* = 1)$ fof(goals, conjecture)

KLE049+1.p Hoare rule while

Encoding of Hoare rule $p;qxq \rightarrow q$ while p do x c(p);q, where while p do x = (p;x)*;c(p).

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2) \text{ and } ((x_2 \cdot x_1) \cdot x_0) \cdot c(x_2) \leq 0) \Rightarrow ((x_2 \cdot (x_1 \cdot x_0)^*) \cdot c(x_1)) \cdot c(x_2) \leq 0)$ fof(goals, conjecture)

KLE049+2.p Hoare rule while

Encoding of Hoare rule $p;qxq \rightarrow q$ while p do x c(p);q, where while p do x = (p;x)*;c(p).

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2: ((\text{test}(x_1) \text{ and } \text{test}(x_2) \text{ and } ((x_2 \cdot x_1) \cdot x_0) \cdot c(x_2) \leq 0) \Rightarrow ((x_2 \cdot (x_1 \cdot x_0)^*) \cdot c(x_1)) \cdot c(x_2) \leq 0)$ fof(goals, conjecture)

KLE050+1.p Loop denesting

Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3: ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \Rightarrow (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) = (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2))$ fof(goals, conjecture)

KLE050+2.p Loop denesting

Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
 $\forall x_0, x_1, x_2, x_3: ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \Rightarrow (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \text{ and } (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2))$ fof(goals, conjecture)

KLE050+3.p Loop denesting

Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
 $\forall x_0, x_1, x_2, x_3: ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \Rightarrow (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) = (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2))$ fof(goals, conjecture)

KLE050+4.p Loop denesting

Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+1.ax')

include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3: ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \Rightarrow (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \text{ and } (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2))) \quad \text{fof}(\text{goals, conjecture})$

KLE051+1.p Every domain semiring is automatically isotone

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: x_0 + x_0 = x_0 \quad \text{fof}(\text{goals, conjecture})$

KLE052+1.p Domain is a left invariant

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(x_0) \cdot x_0 = x_0 \quad \text{fof}(\text{goals, conjecture})$

KLE053+1.p Domain is a projection

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(\text{dom}(x_0)) = \text{dom}(x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE054+1.p Domain is prefix increasing

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(x_0 \cdot x_1) + \text{dom}(x_0) = \text{dom}(x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE055+1.p Domain expands subidentities

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (x_0 + 1 = 1 \Rightarrow x_0 + \text{dom}(x_0) = \text{dom}(x_0)) \quad \text{fof}(\text{goals, conjecture})$

KLE056+1.p Domain is very strict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\text{dom}(x_0) = 0 \Rightarrow x_0 = 0) \quad \text{fof}(\text{goals, conjecture})$

KLE057+1.p Domain is very strict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\text{dom}(x_0) = 0 \Leftarrow x_0 = 0) \quad \text{fof}(\text{goals, conjecture})$

KLE058+1.p Domain is constrict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\text{dom}(1) = 1 \quad \text{fof}(\text{goals, conjecture})$

KLE059+1.p Domain is isotone

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (x_0 + x_1 = x_1 \Rightarrow \text{dom}(x_0) + \text{dom}(x_1) = \text{dom}(x_1)) \quad \text{fof}(\text{goals, conjecture})$

KLE060+1.p Domain elements can be exported

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(\text{dom}(x_0) \cdot x_1) = \text{dom}(x_0) \cdot \text{dom}(x_1) \quad \text{fof}(\text{goals, conjecture})$

KLE061+1.p Domain elements are multiplicatively idempotent

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(x_0) \cdot \text{dom}(x_0) = \text{dom}(x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE062+1.p Domain elements are multiplicatively commutative

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(x_0) \cdot \text{dom}(x_1) = \text{dom}(x_1) \cdot \text{dom}(x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE063+1.p Domain elements are least left preservers

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (x_0 + \text{dom}(x_1) \cdot x_0 = \text{dom}(x_1) \cdot x_0 \Rightarrow \text{dom}(x_0) + \text{dom}(x_1) = \text{dom}(x_1))$ fof(goals, conjecture)

KLE064+1.p Domain elements are least left preservers

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (x_0 + \text{dom}(x_1) \cdot x_0 = \text{dom}(x_1) \cdot x_0 \Leftarrow \text{dom}(x_0) + \text{dom}(x_1) = \text{dom}(x_1))$ fof(goals, conjecture)

KLE065+1.p Domain is weakly local

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot \text{dom}(x_1) = 0)$ fof(goals, conjecture)

KLE066+1.p Domain is weakly local

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 = 0 \Leftarrow x_0 \cdot \text{dom}(x_1) = 0)$ fof(goals, conjecture)

KLE067+1.p Domain elements are closed under addition

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(\text{dom}(x_0) + \text{dom}(x_1)) = \text{dom}(x_0) + \text{dom}(x_1)$ fof(goals, conjecture)

KLE068+1.p Domain elements are closed under multiplication

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(\text{dom}(x_0) \cdot \text{dom}(x_1)) = \text{dom}(x_0) \cdot \text{dom}(x_1)$ fof(goals, conjecture)

KLE069+1.p Domain elements satisfy the first lattice absorption law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(x_0) \cdot (\text{dom}(x_0) + \text{dom}(x_1)) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE070+1.p Domain elements satisfy the second lattice absorption law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: \text{dom}(x_0) + \text{dom}(x_0) \cdot \text{dom}(x_1) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE071+1.p Domain elements satisfy one of the lattice distributivity laws

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1, x_2: \text{dom}(x_0) + \text{dom}(x_1) \cdot \text{dom}(x_2) = (\text{dom}(x_0) + \text{dom}(x_1)) \cdot (\text{dom}(x_0) + \text{dom}(x_2))$ fof(goals, conjecture)

KLE072+1.p Domain elements satisfy the first axiom of Kleene modules

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1, x_2: \text{dom}((x_0 + x_1) \cdot \text{dom}(x_2)) = \text{dom}(x_0 \cdot \text{dom}(x_2)) + \text{dom}(x_1 \cdot \text{dom}(x_2))$ fof(goals, conjecture)

KLE073+1.p Domain elements satisfy the second Kleene module axiom

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1, x_2: \text{dom}(x_0 \cdot (\text{dom}(x_1) + \text{dom}(x_2))) = \text{dom}(x_0 \cdot \text{dom}(x_1)) + \text{dom}(x_0 \cdot \text{dom}(x_2))$ fof(goals, conjecture)

KLE074+1.p Domain elements satisfy the third Kleene module axiom.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1, x_2: \text{dom}((x_0 \cdot x_1) \cdot \text{dom}(x_2)) = \text{dom}(x_0 \cdot \text{dom}(x_1 \cdot \text{dom}(x_2)))$ fof(goals, conjecture)

KLE075+1.p Domain elements satisfy the fourth Kleene module axiom

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(1 \cdot \text{dom}(x_0)) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE076+1.p Domain elements satisfy the fifth Kleene module axiom

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(x_0 \cdot \text{dom}(0)) = 0 \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE077+1.p Domain elements satisfy the sixth Kleene module axiom

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1, x_2: ((\text{dom}(x_0) + \text{dom}(x_1 \cdot \text{dom}(x_2))) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow \text{dom}(x_1^* \cdot \text{dom}(x_0)) + \text{dom}(x_2) = \text{dom}(x_2)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE078+1.p Antidomain elements are domain elements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\forall x_1: (\text{dom}(x_1) + \text{ad}(x_1) = 1 \text{ and } \text{dom}(x_1) \cdot \text{ad}(x_1) = 0) \Rightarrow \text{dom}(\text{ad}(x_0)) = \text{ad}(x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE079+1.p Domain and antidomain elements are complements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\forall x_1: (\text{dom}(x_1) + \text{ad}(x_1) = 1 \text{ and } \text{dom}(x_1) \cdot \text{ad}(x_1) = 0) \Rightarrow \text{ad}(x_0) \cdot \text{dom}(x_0) = 0) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE080+1.p Another complementation property of domain and antidomain elements

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\forall x_1: (\text{dom}(x_1) + \text{ad}(x_1) = 1 \text{ and } \text{dom}(x_1) \cdot \text{ad}(x_1) = 0) \Rightarrow \text{ad}(\text{ad}(x_0)) = \text{dom}(x_0)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE081+1.p Antidomain elements are left annihilators

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: (\forall x_1: (\text{dom}(x_1) + \text{ad}(x_1) = 1 \text{ and } \text{dom}(x_1) \cdot \text{ad}(x_1) = 0) \Rightarrow \text{ad}(x_0) \cdot x_0 = 0) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE082+1.p Antidomain is local with respect to multiplication

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0, x_1: (\forall x_2: (\text{dom}(x_2) + \text{ad}(x_2) = 1 \text{ and } \text{dom}(x_2) \cdot \text{ad}(x_2) = 0) \Rightarrow \text{ad}(x_0 \cdot x_1) + \text{ad}(x_0 \cdot \text{dom}(x_1)) = \text{ad}(x_0 \cdot \text{dom}(x_1))) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE083+1.p Domain is a left invariant

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0: x_0 = \text{dom}(x_0) \cdot x_0 \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE084+1.p Domain is local with respect to multiplication

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1: \text{dom}(x_0 \cdot x_1) = \text{dom}(x_0 \cdot \text{dom}(x_1)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE085+1.p Domain elements are subidentities

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0: \text{dom}(x_0) + 1 = 1 \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE086+1.p Domain is strict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\text{dom}(0) = 0 \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE087+1.p Domain is additive

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1: \text{dom}(x_0 + x_1) = \text{dom}(x_0) + \text{dom}(x_1) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE088+1.p Antidomain elements are greatest left annihilators

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1: (\text{dom}(x_0) \cdot x_1 = 0 \Rightarrow \text{dom}(x_0) + \text{ad}(x_1) = \text{ad}(x_1)) \quad \text{fof}(\text{goals}, \text{conjecture})$

KLE089+1.p Antidomain elements are greatest left annihilators

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1: (\text{dom}(x_0) \cdot x_1 = 0 \Leftrightarrow \text{dom}(x_0) + \text{ad}(x_1) = \text{ad}(x_1))$ fof(goals, conjecture)

KLE090+1.p Antidomain elements are antitone

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0, x_1: (x_0 + x_1 = x_1 \Rightarrow \text{ad}(x_1) + \text{ad}(x_0) = \text{ad}(x_0))$ fof(goals, conjecture)

KLE091+1.p Codomain closure

In a Boolean domain semiring, codomain elements are domain elements.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0: \text{dom}(\text{cod}(x_0)) = \text{cod}(x_0)$ fof(goals, conjecture)

KLE092+1.p Coantidomain closure

In a Boolean domain semiring, coantidomain elements are domain elements.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

$\forall x_0: \text{dom}(\text{coad}(x_0)) = \text{coad}(x_0)$ fof(goals, conjecture)

KLE093+1.p Domain of star

In a Kleene algebra with domain, the domain of a star is always one.

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+5.ax')

$\forall x_0: \text{dom}(x_0^*) = 1$ fof(goals, conjecture)

KLE094+1.p Segerberg

Segerberg's formula holds in every Kleene algebra with Boolean domain.

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{domain_difference}(\text{forward_diamond}(x_0^*, \text{dom}(x_1)), \text{dom}(x_1)) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{forward_diamond}(x_0, \text{dom}(x_1)), \text{dom}(x_1))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{forward_diamond}(x_0, \text{dom}(x_1)), \text{dom}(x_1)))$ fof(goals, conjecture)

KLE095+1.p Modal operators satisfy a star unfold law

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{dom}(x_0) + \text{forward_diamond}(x_1, \text{forward_diamond}(x_1^*, \text{dom}(x_0))) = \text{forward_diamond}(x_1^*, \text{dom}(x_0))$ fof(goals, conjecture)

KLE096+1.p Modal operators satisfy a star unfold law

include('Axioms/KLE002+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{dom}(x_0) + \text{forward_diamond}(x_1^*, \text{forward_diamond}(x_1, \text{dom}(x_0))) = \text{forward_diamond}(x_1^*, \text{dom}(x_0))$ fof(goals, conjecture)

KLE097+1.p Modal operators at left hand-sides can be eliminated

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Leftrightarrow x_0 \cdot \text{dom}(x_1) + \text{dom}(x_2) \cdot x_0 = \text{dom}(x_2) \cdot x_0)$ fof(goals, conjecture)

KLE098+1.p Modal operators at left hand-sides can be eliminated

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow x_0 \cdot \text{dom}(x_1) + \text{dom}(x_2) \cdot x_0 = \text{dom}(x_2) \cdot x_0)$ fof(goals, conjecture)

KLE099+1.p Modal operators at left hand-sides can be eliminated

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow \text{ad}(x_2) \cdot (x_0 \cdot \text{dom}(x_1)) = 0)$ fof(goals, conjecture)

KLE100+1.p Modal operators at left hand-sides can be eliminated

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Leftarrow \text{ad}(x_2) \cdot (x_0 \cdot \text{dom}(x_1)) = 0) \quad \text{fof}(\text{goals, conjecture})$

KLE101+1.p Forward and backward diamonds are conjugates

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) \cdot \text{dom}(x_2) = 0 \Rightarrow \text{dom}(x_1) \cdot \text{backward_diamond}(x_0, \text{dom}(x_2)) = 0) \quad \text{fof}(\text{goals, conjecture})$

KLE102+1.p Forward and backward diamonds are conjugates

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) \cdot \text{dom}(x_2) = 0 \Leftarrow \text{dom}(x_1) \cdot \text{backward_diamond}(x_0, \text{dom}(x_2)) = 0) \quad \text{fof}(\text{goals, conjecture})$

KLE103+1.p Forward and backward boxes are conjugates

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_box}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = 1 \Rightarrow \text{dom}(x_1) + \text{backward_box}(x_0, \text{dom}(x_2)) = 1) \quad \text{fof}(\text{goals, conjecture})$

KLE104+1.p Forward and backward boxes are conjugates

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_box}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = 1 \Leftarrow \text{dom}(x_1) + \text{backward_box}(x_0, \text{dom}(x_2)) = 1) \quad \text{fof}(\text{goals, conjecture})$

KLE105+1.p Galois

Forward diamonds and backward boxes are adjoints in a Galois connection.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow \text{dom}(x_1) + \text{backward_box}(x_0, \text{dom}(x_2)) = \text{backward_box}(x_0, \text{dom}(x_2))) \quad \text{fof}(\text{goals, conjecture})$

KLE106+1.p Galois

Forward diamonds and backward boxes are adjoints in a Galois connection.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Leftarrow \text{dom}(x_1) + \text{backward_box}(x_0, \text{dom}(x_2)) = \text{backward_box}(x_0, \text{dom}(x_2))) \quad \text{fof}(\text{goals, conjecture})$

KLE107+1.p Galois

Backward diamonds and forward boxes are adjoints of a Galois connection.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{backward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow \text{dom}(x_1) + \text{forward_box}(x_0, \text{dom}(x_2)) = \text{forward_box}(x_0, \text{dom}(x_2))) \quad \text{fof}(\text{goals, conjecture})$

KLE108+1.p Galois

Backward diamonds and forward boxes are adjoints of a Galois connection.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: (\text{backward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \Leftarrow \text{dom}(x_1) + \text{forward_box}(x_0, \text{dom}(x_2)) = \text{forward_box}(x_0, \text{dom}(x_2))) \quad \text{fof}(\text{goals, conjecture})$

KLE109+1.p Forward diamonds and backward boxes satisfy a cancellation law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{forward_diamond}(x_0, \text{backward_box}(x_0, \text{dom}(x_1))) + \text{dom}(x_1) = \text{dom}(x_1)$ fof(goals, conjecture)

KLE110+1.p Forward diamonds and backward boxes satisfy a cancellation law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{dom}(x_0) + \text{backward_box}(x_1, \text{forward_diamond}(x_1, \text{dom}(x_0))) = \text{backward_box}(x_1, \text{forward_diamond}(x_1, \text{dom}(x_0)))$

KLE111+1.p Backward diamonds and forward boxes satisfy a cancellation law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{backward_diamond}(x_0, \text{forward_box}(x_0, \text{dom}(x_1))) + \text{dom}(x_1) = \text{dom}(x_1)$ fof(goals, conjecture)

KLE112+1.p Backward diamonds and forward boxes satisfy a cancellation law

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{dom}(x_0) + \text{forward_box}(x_1, \text{backward_diamond}(x_1, \text{dom}(x_0))) = \text{forward_box}(x_1, \text{backward_diamond}(x_1, \text{dom}(x_0)))$

KLE113+1.p Diamonds are strict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0: \text{forward_diamond}(x_0, 0) = 0$ fof(goals, conjecture)

KLE114+1.p Boxes are costrict

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0: \text{forward_box}(x_0, 1) = 1$ fof(goals, conjecture)

KLE115+1.p Diamonds are additive

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: \text{forward_diamond}(x_0, \text{dom}(x_1) + \text{dom}(x_2)) = \text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{forward_diamond}(x_0, \text{dom}(x_2))$ fof(goals, conjecture)

KLE116+1.p Boxes are multiplicative

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: \text{forward_box}(x_0, \text{dom}(x_1) \cdot \text{dom}(x_2)) = \text{forward_box}(x_0, \text{dom}(x_1)) \cdot \text{forward_box}(x_0, \text{dom}(x_2))$ fof(goals, conjecture)

KLE117+1.p Diamonds are isotone

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: (x_0 + x_1 = x_1 \Rightarrow \forall x_2: \text{forward_diamond}(x_0, \text{dom}(x_2)) + \text{forward_diamond}(x_1, \text{dom}(x_2)) = \text{forward_diamond}(x_1, \text{dom}(x_2)))$

KLE118+1.p Boxes are antitone

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: (x_0 + x_1 = x_1 \Rightarrow \forall x_2: \text{forward_box}(x_0, \text{dom}(x_2)) + \text{forward_box}(x_1, \text{dom}(x_2)) = \text{forward_box}(x_0, \text{dom}(x_2)))$ fof(goals, conjecture)

KLE119+1.p Validity of abort rule

The abort rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1: \text{backward_diamond}(0, \text{dom}(x_0)) + \text{dom}(x_1) = \text{dom}(x_1)$ fof(goals, conjecture)

KLE120+1.p Validity of skip rule

The skip rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
 $\forall x_0: \text{backward_diamond}(1, \text{dom}(x_0)) + \text{dom}(x_0) = \text{dom}(x_0) \quad \text{fof}(\text{goals, conjecture})$

KLE121+1.p Validity of composition rule

The composition rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{backward_diamond}(x_3, \text{dom}(x_0)) + \text{dom}(x_1) = \text{dom}(x_1) \text{ and } \text{backward_diamond}(x_4, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2)) \Rightarrow \text{backward_diamond}(x_3 \cdot x_4, \text{dom}(x_0)) + \text{dom}(x_2) = \text{dom}(x_2)) \quad \text{fof}(\text{goals, conjecture})$

KLE122+1.p Validity of conditional rule

The conditional rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
 $\forall x_0, x_1, x_2: \text{if_then_else}(x_0, x_1, x_2) = \text{dom}(x_0) \cdot x_1 + \text{ad}(x_0) \cdot x_2 \quad \text{fof}(\text{if_then_else_definition, axiom})$
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{backward_diamond}(x_0, \text{dom}(x_2)) \cdot \text{dom}(x_3)) + \text{dom}(x_4) = \text{dom}(x_4) \text{ and } \text{backward_diamond}(x_1, \text{ad}(x_2) \cdot \text{dom}(x_3)) + \text{dom}(x_4) = \text{dom}(x_4)) \Rightarrow \text{backward_diamond}(\text{if_then_else}(x_2, x_0, x_1), \text{dom}(x_3)) + \text{dom}(x_4) = \text{dom}(x_4) \quad \text{fof}(\text{goals, conjecture})$

KLE123+1.p Validity of while rule

The while rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
 $\forall x_0, x_1: \text{while_do}(x_1, x_0) = (\text{dom}(x_1) \cdot x_0)^* \cdot \text{ad}(x_1) \quad \text{fof}(\text{while_do_definition, axiom})$
 $\forall x_0, x_1, x_2: (\text{backward_diamond}(x_0, \text{dom}(x_1)) \cdot \text{dom}(x_2)) + \text{dom}(x_2) = \text{dom}(x_2) \Rightarrow \text{backward_diamond}(\text{while_do}(x_1, x_0), \text{dom}(x_2)) + \text{dom}(x_2) = \text{dom}(x_2) \quad \text{fof}(\text{goals, conjecture})$

KLE124+1.p Validity of weakening rule

The weakening rule of Hoare logic is valid with respect to the Kleene algebra semantics.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
 $\forall x_0, x_1, x_2, x_3, x_4: ((\text{dom}(x_2) + \text{dom}(x_1) = \text{dom}(x_1) \text{ and } \text{backward_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_3) = \text{dom}(x_3) \text{ and } \text{dom}(x_3) + \text{dom}(x_4) = \text{dom}(x_4)) \Rightarrow \text{backward_diamond}(x_0, \text{dom}(x_2)) + \text{dom}(x_4) = \text{dom}(x_4) \quad \text{fof}(\text{goals, conjecture})$

KLE125+1.p Quasicommutation theorem

If x quasicommutates over y, then x+y terminates if x and y individually do.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
 $\forall x_0, x_1: (x_0 \cdot x_1 + x_1 \cdot (x_1 + x_0)^* = x_1 \cdot (x_1 + x_0)^* \Rightarrow (\text{divergence}(x_1 + x_0) = 0 \iff \text{divergence}(x_1) + \text{divergence}(x_0) = 0)) \quad \text{fof}(\text{goals, conjecture})$

KLE126+1.p Lazy commutation theorem

If x lazily commutes over y, then x+y terminates if x and y individually do.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
 $\forall x_0, x_1: (x_0 \cdot x_1 + x_1 \cdot (x_1 + x_0)^* = x_1 \cdot (x_1 + x_0)^* + x_0 \Rightarrow (\text{divergence}(x_1 + x_0) = 0 \iff \text{divergence}(x_1) + \text{divergence}(x_0) = 0)) \quad \text{fof}(\text{goals, conjecture})$

KLE127+1.p Loop refinement theorem

If x quasicommutates over y, then all infinite behaviours of x+y can be separated into infinite behaviours of x and a infinite behaviour of y after finitely many x steps.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 + x_1 \cdot (x_1 + x_0)^* = x_1 \cdot (x_1 + x_0)^* \Rightarrow \text{divergence}(x_1 + x_0) = \text{divergence}(x_1) + \text{forward_diamond}(x_1^*, \text{divergence}(x_0)))$

KLE128+1.p Comparison of two different notions of termination

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: (\text{divergence}(x_0) = 0 \Rightarrow \forall x_1: (\text{dom}(x_1) + \text{forward_diamond}(x_0, \text{dom}(x_1)) = \text{forward_diamond}(x_0, \text{dom}(x_1)) \Rightarrow \text{dom}(x_1) = 0))$ fof(goals, conjecture)

KLE129+1.p Comparison of two different notions of termination

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: (\text{divergence}(x_0) = 0 \Leftarrow \forall x_1: (\text{dom}(x_1) + \text{forward_diamond}(x_0, \text{dom}(x_1)) = \text{forward_diamond}(x_0, \text{dom}(x_1)) \Rightarrow \text{dom}(x_1) = 0))$ fof(goals, conjecture)

KLE130+1.p Two notions of termination

The lemma compares absence of divergence with a third notion of termination.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: (\text{divergence}(x_0) = 0 \Rightarrow \forall x_1: \text{dom}(x_1) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1))))$ fof(goals, conjecture)

KLE131+1.p Two notions of termination

The lemma compares absence of divergence with a third notion of termination.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: (\text{divergence}(x_0) = 0 \Leftarrow \forall x_1: \text{dom}(x_1) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1))))$ fof(goals, conjecture)

KLE132+1.p Every element that satisfies Loeb's formula is wellfounded

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: ((\forall x_1: \text{forward_diamond}(x_0, \text{dom}(x_1)) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) \Rightarrow \forall x_2: (\text{dom}(x_2) + \text{forward_diamond}(x_0, \text{dom}(x_2)) \Rightarrow \text{dom}(x_2) = 0))$ fof(goals, conjecture)

KLE133+1.p Loeb's formula and wellfoundedness

Every diamond transitive wellfounded element satisfies Loeb's formula.

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

include('Axioms/KLE001+7.ax')

$\forall x_0: ((\forall x_1: \text{dom}(x_1) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_1), \text{forward_diamond}(x_0, \text{dom}(x_1)))) \Rightarrow \forall x_3: \text{forward_diamond}(x_0, \text{dom}(x_3)) + \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_3), \text{forward_diamond}(x_0, \text{dom}(x_3)))) = \text{forward_diamond}(x_0^*, \text{domain_difference}(\text{dom}(x_3), \text{forward_diamond}(x_0, \text{dom}(x_3))))$ fof(goals, conjecture)

KLE134+1.p A no name lemma

include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')

include('Axioms/KLE001+6.ax')

$\forall x_0, x_1, x_2: \text{domain_difference}(\text{forward_diamond}(x_0, \text{dom}(x_1)), \text{forward_diamond}(x_0, \text{dom}(x_2))) + \text{forward_diamond}(x_0, \text{domain_difference}(\text{dom}(x_1), \text{dom}(x_2))) = \text{domain_difference}(\text{forward_diamond}(x_0, \text{dom}(x_1)), \text{forward_diamond}(x_0, \text{dom}(x_2)))$ fof(goals, conjecture)

KLE135+1.p Two notions of termination

If an element does not diverge, then it always becomes disabled after finitely many steps.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
 $\forall x_0: (\text{divergence}(x_0) = 0 \Rightarrow \text{forward_diamond}(x_0^*, \text{ad}(x_0)) = 1)$ fof(goals, conjecture)

KLE136+1.p Newman's lemma holds in divergence Kleene algebras

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
 $\forall x_0, x_1: ((\text{divergence}(x_0 + x_1) = 0 \text{ and } x_1 \cdot x_0 + x_0^* \cdot x_1^* = x_0^* \cdot x_1^*) \Rightarrow (x_0 + x_1)^* + x_0^* \cdot x_1^* = x_0^* \cdot x_1^*)$ fof(goals, conjecture)

KLE137+1.p There is a greatest element, namely $1 \wedge \text{infty}$

include('Axioms/KLE004+0.ax')
 $\forall x_0: x_0 \leq \text{strong_iteration}(1)$ fof(goals, conjecture)

KLE138+1.p Strong iteration of a abort is miracle

include('Axioms/KLE004+0.ax')
 $\text{strong_iteration}(0) = 1$ fof(goals, conjecture)

KLE139+1.p Dual unfold

include('Axioms/KLE004+0.ax')
 $\forall x_0: \text{strong_iteration}(x_0) = \text{strong_iteration}(x_0) \cdot x_0 + 1$ fof(goals, conjecture)

KLE139+2.p Dual unfold

include('Axioms/KLE004+0.ax')
 $\forall x_0: (\text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_0) \cdot x_0 + 1 \text{ and } \text{strong_iteration}(x_0) \cdot x_0 + 1 \leq \text{strong_iteration}(x_0))$ fof(goals, conjecture)

KLE140+1.p Isotonicity of strong iteration

include('Axioms/KLE004+0.ax')
 $\forall x_0, x_1: (x_0 \leq x_1 \Rightarrow \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_1))$ fof(goals, conjecture)

KLE141+1.p The greatest is left annihilator

The greatest is left annihilator; a miracle followed by anything is again a miracle.

include('Axioms/KLE004+0.ax')
 $\forall x_0: \text{strong_iteration}(1) \cdot x_0 = \text{strong_iteration}(1)$ fof(goals, conjecture)

KLE141+2.p The greatest is left annihilator

The greatest is left annihilator; a miracle followed by anything is again a miracle.

include('Axioms/KLE004+0.ax')
 $\forall x_0: (\text{strong_iteration}(1) \cdot x_0 \leq \text{strong_iteration}(1) \text{ and } \text{strong_iteration}(1) \leq \text{strong_iteration}(1) \cdot x_0)$ fof(goals, conjecture)

KLE142+1.p If strong iteration is applied twice, a miracle occurs.

include('Axioms/KLE004+0.ax')
 $\forall x_0: \text{strong_iteration}(\text{strong_iteration}(x_0)) = \text{strong_iteration}(1)$ fof(goals, conjecture)

KLE142+2.p If strong iteration is applied twice, a miracle occurs.

include('Axioms/KLE004+0.ax')
 $\forall x_0: (\text{strong_iteration}(\text{strong_iteration}(x_0)) \leq \text{strong_iteration}(1) \text{ and } \text{strong_iteration}(1) \leq \text{strong_iteration}(\text{strong_iteration}(x_0)))$

KLE143+1.p Strong iteration is idempotent w.r.t. multiplication

include('Axioms/KLE004+0.ax')
 $\forall x_0: \text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_0) = \text{strong_iteration}(x_0)$ fof(goals, conjecture)

KLE143+2.p Strong iteration is idempotent w.r.t. multiplication

include('Axioms/KLE004+0.ax')
 $\forall x_0: (\text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_0) \text{ and } \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_0))$ fof(goals, conjecture)

KLE144+1.p Strong iteration applied after finite iteration is magic.

include('Axioms/KLE004+0.ax')
 $\forall x_0: \text{strong_iteration}(x_0^*) = \text{strong_iteration}(1)$ fof(goals, conjecture)

KLE144+2.p Strong iteration applied after finite iteration is magic.

include('Axioms/KLE004+0.ax')
 $\forall x_0: (\text{strong_iteration}(x_0^*) \leq \text{strong_iteration}(1) \text{ and } \text{strong_iteration}(1) \leq \text{strong_iteration}(x_0^*))$ fof(goals, conjecture)

KLE145+1.p Finite iteration after strong iteration is strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0: \text{strong_iteration}(x_0)^* = \text{strong_iteration}(x_0)$ fof(goals, conjecture)

KLE145+2.p Finite iteration after strong iteration is strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0: (\text{strong_iteration}(x_0)^* \leq \text{strong_iteration}(x_0) \text{ and } \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_0)^*)$ fof(goals, conjecture)

KLE146+1.p Skip is part of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0: 1 \leq \text{strong_iteration}(x_0)$ fof(goals, conjecture)

KLE147+1.p Sliding of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: \text{strong_iteration}(x_0^* \cdot x_1) = x_1^* \cdot \text{strong_iteration}(x_0^* \cdot x_1)$ fof(goals, conjecture)

KLE147+2.p Sliding of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (\text{strong_iteration}(x_0^* \cdot x_1) \leq x_1^* \cdot \text{strong_iteration}(x_0^* \cdot x_1) \text{ and } x_1^* \cdot \text{strong_iteration}(x_0^* \cdot x_1) \leq \text{strong_iteration}(x_0^* \cdot x_1))$ fof(goals, conjecture)

KLE148+1.p Blocking law

If y is blocked by x then x before strong iteration of y reduces to x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot \text{strong_iteration}(x_1) = x_0)$ fof(goals, conjecture)

KLE148+2.p Blocking law

If y is blocked by x then x before strong iteration of y reduces to x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot \text{strong_iteration}(x_1) \leq x_0) \text{ and } x_0 \leq x_0 \cdot \text{strong_iteration}(x_1))$ fof(goals, conjecture)

KLE149+1.p Strong version of unfold

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: \text{strong_iteration}(x_0 + x_1) = (x_1^* \cdot x_0) \cdot \text{strong_iteration}(x_0 + x_1) + \text{strong_iteration}(x_1)$ fof(goals, conjecture)

KLE149+2.p Strong version of unfold

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (\text{strong_iteration}(x_0 + x_1) \leq (x_1^* \cdot x_0) \cdot \text{strong_iteration}(x_0 + x_1) + \text{strong_iteration}(x_1) \text{ and } (x_1^* \cdot x_0) \cdot \text{strong_iteration}(x_0 + x_1) + \text{strong_iteration}(x_1) \leq \text{strong_iteration}(x_0 + x_1))$ fof(goals, conjecture)

KLE150+1.p Iterating non-terminating elements reduces to the element itself

include('Axioms/KLE004+0.ax')

$\forall x_0: \text{strong_iteration}(x_0 \cdot 0) = 1 + x_0 \cdot 0$ fof(goals, conjecture)

KLE150+2.p Iterating non-terminating elements reduces to the element itself

include('Axioms/KLE004+0.ax')

$\forall x_0: (\text{strong_iteration}(x_0 \cdot 0) \leq 1 + x_0 \cdot 0 \text{ and } 1 + x_0 \cdot 0 \leq \text{strong_iteration}(x_0 \cdot 0))$ fof(goals, conjecture)

KLE151+1.p Sliding of strong iteration

Two ways of grouping an alternation of x's and y's that starts and ends with x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: x_0 \cdot \text{strong_iteration}(x_1 \cdot x_0) \leq \text{strong_iteration}(x_0 \cdot x_1) \cdot x_0$ fof(goals, conjecture)

KLE152+1.p Sliding of strong iteration

Two ways of grouping an alternation of x's and y's that starts and ends with x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: \text{strong_iteration}(x_0 \cdot x_1) \cdot x_0 \leq x_0 \cdot \text{strong_iteration}(x_1 \cdot x_0)$ fof(goals, conjecture)

KLE153+1.p Sliding of strong iteration

Two ways of grouping an alternation of x's and y's that starts and ends with x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: x_0 \cdot \text{strong_iteration}(x_1 \cdot x_0) = \text{strong_iteration}(x_0 \cdot x_1) \cdot x_0$ fof(goals, conjecture)

KLE154+1.p Denesting of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: \text{strong_iteration}(x_0 + x_1) = \text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_1 \cdot \text{strong_iteration}(x_0))$ fof(goals, conjecture)

KLE154+2.p Denesting of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (\text{strong_iteration}(x_0+x_1) \leq \text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_1 \cdot \text{strong_iteration}(x_0)))$ and $\text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_1 \cdot \text{strong_iteration}(x_0)) \leq \text{strong_iteration}(x_0 + x_1))$ fof(goals, conjecture)

KLE155+1.p Denesting of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: \text{strong_iteration}(x_0^* \cdot x_1) \cdot \text{strong_iteration}(x_0) = \text{strong_iteration}(x_0 + x_1)$ fof(goals, conjecture)

KLE155+2.p Denesting of strong iteration

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (\text{strong_iteration}(x_0 + x_1) \leq \text{strong_iteration}(x_0^* \cdot x_1) \cdot \text{strong_iteration}(x_0))$ and $\text{strong_iteration}(x_0^* \cdot x_1) \cdot \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_0 + x_1))$ fof(goals, conjecture)

KLE156+1.p Semicommuation law of finite iteration

If x semicommutates over y every finite sequence of x's and y's can be rearranged to a finite sequence of x's followed by finite sequence of y's.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow (x_1 + x_0)^* = x_1^* \cdot x_0^*)$ fof(goals, conjecture)

KLE156+2.p Semicommuation law of finite iteration

If x semicommutates over y every finite sequence of x's and y's can be rearranged to a finite sequence of x's followed by finite sequence of y's.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow (x_1 + x_0)^* \leq x_1^* \cdot x_0^*) \text{ and } x_1^* \cdot x_0^* \leq (x_1 + x_0)^*)$ fof(goals, conjecture)

KLE157+1.p Semicommuation law of finite iteration

If x semicommutates over y every sequence of x's and y's can be rearranged to a sequence of x's followed by sequence of y's.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow \text{strong_iteration}(x_1+x_0) = \text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0))$ fof(goals, conjecture)

KLE157+2.p Semicommuation law of finite iteration

If x semicommutates over y every sequence of x's and y's can be rearranged to a sequence of x's followed by sequence of y's.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow \text{strong_iteration}(x_1+x_0) \leq \text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0))$ and $\text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_1 + x_0))$ fof(goals, conjecture)

KLE158+1.p Simulation law for data refinement

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2: (x_0 \cdot x_1 \leq x_2 \cdot x_0 \Rightarrow x_0 \cdot \text{strong_iteration}(x_1) \leq \text{strong_iteration}(x_2) \cdot x_0)$ fof(goals, conjecture)

KLE159+1.p Simulation law for data refinement

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2: (x_0 \cdot x_1 \leq x_2 \cdot x_0 \Rightarrow x_0 \cdot x_1^* \leq x_2^* \cdot x_0)$ fof(goals, conjecture)

KLE160+1.p Simulation law for data refinement

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2: (x_0 \cdot x_1 \leq x_1 \cdot x_2 \Rightarrow x_0^* \cdot x_1 \leq x_1 \cdot x_2^*)$ fof(goals, conjecture)

KLE161+1.p Data refinement

The first hypothesis says that b cannot loop infinitely. The second hypothesis says that a is data refined by aa w.r.t. upward simulation z. By the third hypothesis, 1 is data refined by b. The fourth and fifth condition expresses the standard data refinement of initialisations and finalisations.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7: ((\text{strong_iteration}(x_7) = x_7^* \text{ and } x_0 \cdot x_6 \leq x_5 \cdot x_0 \text{ and } x_0 \cdot x_7 \leq x_0 \text{ and } x_2 \leq x_1 \cdot x_0 \text{ and } x_0 \cdot x_4 \leq x_3) \Rightarrow x_2 \cdot (\text{strong_iteration}(x_6 + x_7) \cdot x_4) \leq x_1 \cdot (\text{strong_iteration}(x_5) \cdot x_3))$ fof(goals, conjecture)

KLE162+1.p Part 1 of Back's atomicity refinement theorem

Back's atomicity refinement theorem is proved up to the reconstruction of concurrency.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5: ((x_0 \leq x_3 \cdot x_0 \text{ and } x_4 \cdot x_1 \leq x_1 \cdot x_4 \text{ and } (x_0 + (x_4 + x_1)) \cdot x_2 \leq x_2 \cdot (x_0 + (x_4 + x_1)) \text{ and } x_4 \cdot x_3 \leq x_3 \cdot x_4 \text{ and } x_4^* = \text{strong_iteration}(x_4) \text{ and } x_3 \cdot x_2 \leq x_2 \cdot x_3 \text{ and } x_3 \cdot x_1 = 0 \text{ and } x_5 \leq x_5 \cdot x_3) \Rightarrow x_5 \cdot (\text{strong_iteration}((x_0 + (x_4 + x_1)) + x_2) \cdot x_3) \leq (x_5 \cdot (\text{strong_iteration}(x_2) \cdot x_3)) \cdot ((\text{strong_iteration}(x_4) \cdot x_3) \cdot \text{strong_iteration}((x_0 \cdot \text{strong_iteration}(x_1)) \cdot (x_3 \cdot \text{strong_iteration}(x_4))))))$ fof(goals, conjecture)

KLE163+1.p Part 2 of Back's atomicity refinement theorem

The concurrency bit of Back's atomicity refinement theorem.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5: ((x_0 \leq x_3 \cdot x_0 \text{ and } x_4 \cdot x_1 \leq x_1 \cdot x_4 \text{ and } (x_0 + (x_4 + x_1)) \cdot x_2 \leq x_2 \cdot (x_0 + (x_4 + x_1)) \text{ and } x_4 \cdot x_3 \leq x_3 \cdot x_4 \text{ and } x_4^* = \text{strong_iteration}(x_4) \text{ and } x_3 \cdot x_2 \leq x_2 \cdot x_3 \text{ and } x_3 \cdot x_1 = 0 \text{ and } x_5 \leq x_5 \cdot x_3 \text{ and } x_3 \leq 1) \Rightarrow (x_5 \cdot (\text{strong_iteration}(x_2) \cdot x_3)) \cdot ((\text{strong_iteration}(x_4) \cdot x_3) \cdot \text{strong_iteration}((x_0 \cdot \text{strong_iteration}(x_1)) \cdot (x_3 \cdot \text{strong_iteration}(x_4)))) \leq x_5 \cdot \text{strong_iteration}(((x_0 \cdot \text{strong_iteration}(x_1)) \cdot x_3 + x_4) + x_2)) \quad \text{fof}(\text{goals, conjecture})$

KLE164+1.p Back's atomicity refinement theorem

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5: ((x_0 \leq x_3 \cdot x_0 \text{ and } x_4 \cdot x_1 \leq x_1 \cdot x_4 \text{ and } (x_0 + (x_4 + x_1)) \cdot x_2 \leq x_2 \cdot (x_0 + (x_4 + x_1)) \text{ and } x_4 \cdot x_3 \leq x_3 \cdot x_4 \text{ and } x_4^* = \text{strong_iteration}(x_4) \text{ and } x_3 \cdot x_2 \leq x_2 \cdot x_3 \text{ and } x_3 \cdot x_1 = 0 \text{ and } x_5 \leq x_5 \cdot x_3 \text{ and } x_3 \leq 1) \Rightarrow x_5 \cdot (\text{strong_iteration}((x_0 + (x_4 + x_1)) + x_2) \cdot x_3) \leq x_5 \cdot \text{strong_iteration}(((x_0 \cdot \text{strong_iteration}(x_1)) \cdot x_3 + x_4) + x_2)) \quad \text{fof}(\text{goals, conjecture})$

KLE165+1.p Denest for weakly quasi commutation

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 \leq x_1 \cdot \text{strong_iteration}(x_1 + x_0) \Rightarrow \text{strong_iteration}(x_1 + x_0) = \text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0)) \quad \text{fof}(\text{goals, conjecture})$

KLE165+2.p Denest for weakly quasi commutation

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1 \leq x_1 \cdot \text{strong_iteration}(x_1 + x_0) \Rightarrow \text{strong_iteration}(x_1 + x_0) \leq \text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0)) \text{ and } \text{strong_iteration}(x_0) \leq \text{strong_iteration}(x_1 + x_0)) \quad \text{fof}(\text{goals, conjecture})$

KLE166+1.p Strong iteration

If x weakly quasicommutates over y, x and y is strongly terminating iff x + y strongly terminates.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 \leq x_1 \cdot \text{strong_iteration}(x_1 + x_0) \Rightarrow (\text{strong_iteration}(x_1 + x_0) \cdot 0 = 0 \iff \text{strong_iteration}(x_1) \cdot 0 + \text{strong_iteration}(x_0) \cdot 0 = 0)) \quad \text{fof}(\text{goals, conjecture})$

KLE167+1.p Blocking law

If y is blocked by x then x before finite iteration of y reduces to x.

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: ((x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot x_1^* \leq x_0) \text{ and } x_0 \leq x_0 \cdot x_1^*) \quad \text{fof}(\text{goals, conjecture})$

KLE168+1.p If x quasicommutates over y then $x \wedge \infty$ quasicommutates over $y \wedge \infty$

include('Axioms/KLE004+0.ax')

$\forall x_0, x_1: (x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow \text{strong_iteration}(x_0) \cdot \text{strong_iteration}(x_1) \leq \text{strong_iteration}(x_1) \cdot \text{strong_iteration}(x_0)) \quad \text{fof}(\text{goals, conjecture})$

KLE169+1.p Exponential automata

Automata grows exponentially with N in $\text{sigma}^*.a.\text{sigma}^{\wedge N}.a$

include('Axioms/KLE002+0.ax')

$\text{sigma} = a + b \quad \text{fof}(\text{an, axiom})$

$a \cdot (b \cdot a) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a)) \quad \text{fof}(a, \text{conjecture})$

KLE170+1.002.p $a^{\wedge N} \leq a^*$, N=2

include('Axioms/KLE002+0.ax')

$a \cdot a \leq a^* \quad \text{fof}(a, \text{conjecture})$

KLE170+1.004.p $a^{\wedge N} \leq a^*$, N=4

include('Axioms/KLE002+0.ax')

$a \cdot (a \cdot (a \cdot a)) \leq a^* \quad \text{fof}(a, \text{conjecture})$

KLE170+1.006.p $a^{\wedge N} \leq a^*$, N=6

include('Axioms/KLE002+0.ax')

$a \cdot (a \cdot (a \cdot (a \cdot (a \cdot a)))) \leq a^* \quad \text{fof}(a, \text{conjecture})$

KLE171+1.p Ben's problem 1

include('Axioms/KLE002+0.ax')

$\text{sigma} = a + b \quad \text{fof}(\text{an, axiom})$

$a \cdot (b \cdot (b \cdot a)) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a)) \quad \text{fof}(a, \text{conjecture})$

KLE172+1.p Ben's problem 2

include('Axioms/KLE002+0.ax')

$\text{sigma} = a + b \quad \text{fof}(\text{an, axiom})$

$a \cdot (b \cdot (b \cdot (a \cdot b))) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a)) \quad \text{fof}(a, \text{conjecture})$

KLE173+1.p Idempotent semirings with tests

include('Axioms/KLE001+0.ax')

```
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')
```

KLE174+1.p Idempotent semirings with domain/codomain, modal

```
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
```

KLE175+1.p Idempotent semirings with tests

```
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
```

KLE176+1.p Kleene algebra

```
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')
```

KLE177+1.p Kleene algebra

```
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
```

KLE178+1.p Kleene Algebra

```
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+5.ax')
```

KLE179+1.p Omega algebra

```
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')
```

KLE180+1.p Omega algebra

```
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
```

KLE181+1.p Omega algebra

```
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+5.ax')
```

KLE182+1.p Demonic Refinement Algebra

```
include('Axioms/KLE004+0.ax')
```