

LCL axioms

LCL001-0.ax Wajsberg algebra

$\text{implies}(\text{truth}, x) = x$ $\text{cnf}(\text{wajsberg}_1, \text{axiom})$
 $\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z))) = \text{truth}$ $\text{cnf}(\text{wajsberg}_2, \text{axiom})$
 $\text{implies}(\text{implies}(x, y), y) = \text{implies}(\text{implies}(y, x), x)$ $\text{cnf}(\text{wajsberg}_3, \text{axiom})$
 $\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x)) = \text{truth}$ $\text{cnf}(\text{wajsberg}_4, \text{axiom})$

LCL001-1.ax Wajsberg algebra lattice structure definitions

$\text{big_V}(x, y) = \text{implies}(\text{implies}(x, y), y)$ $\text{cnf}(\text{big_V_definition}, \text{axiom})$
 $\text{big_hat}(x, y) = \text{not}(\text{big_V}(\text{not}(x), \text{not}(y)))$ $\text{cnf}(\text{big_hat_definition}, \text{axiom})$
 $\text{ordered}(x, y) \Rightarrow \text{implies}(x, y) = \text{truth}$ $\text{cnf}(\text{partial_order_definition}_1, \text{axiom})$
 $\text{implies}(x, y) = \text{truth} \Rightarrow \text{ordered}(x, y)$ $\text{cnf}(\text{partial_order_definition}_2, \text{axiom})$

LCL001-2.ax Wajsberg algebra AND and OR definitions

$\text{or}(x, y) = \text{implies}(\text{not}(x), y)$ $\text{cnf}(\text{or_definition}, \text{axiom})$
 $\text{or}(\text{or}(x, y), z) = \text{or}(x, \text{or}(y, z))$ $\text{cnf}(\text{or_associativity}, \text{axiom})$
 $\text{or}(x, y) = \text{or}(y, x)$ $\text{cnf}(\text{or_commutativity}, \text{axiom})$
 $\text{and}(x, y) = \text{not}(\text{or}(\text{not}(x), \text{not}(y)))$ $\text{cnf}(\text{and_definition}, \text{axiom})$
 $\text{and}(\text{and}(x, y), z) = \text{and}(x, \text{and}(y, z))$ $\text{cnf}(\text{and_associativity}, \text{axiom})$
 $\text{and}(x, y) = \text{and}(y, x)$ $\text{cnf}(\text{and_commutativity}, \text{axiom})$

LCL002-0.ax Alternative Wajsberg algebra

$\text{not}(x) = \text{xor}(x, \text{truth})$ $\text{cnf}(\text{axiom}_1, \text{axiom})$
 $\text{xor}(x, \text{falsehood}) = x$ $\text{cnf}(\text{axiom}_2, \text{axiom})$
 $\text{xor}(x, x) = \text{falsehood}$ $\text{cnf}(\text{axiom}_3, \text{axiom})$
 $\text{and_star}(x, \text{truth}) = x$ $\text{cnf}(\text{axiom}_4, \text{axiom})$
 $\text{and_star}(x, \text{falsehood}) = \text{falsehood}$ $\text{cnf}(\text{axiom}_5, \text{axiom})$
 $\text{and_star}(\text{xor}(\text{truth}, x), x) = \text{falsehood}$ $\text{cnf}(\text{axiom}_6, \text{axiom})$
 $\text{xor}(x, \text{xor}(\text{truth}, y)) = \text{xor}(\text{xor}(x, \text{truth}), y)$ $\text{cnf}(\text{axiom}_7, \text{axiom})$
 $\text{and_star}(\text{xor}(\text{and_star}(\text{xor}(\text{truth}, x), y), \text{truth}), y) = \text{and_star}(\text{xor}(\text{and_star}(\text{xor}(\text{truth}, y), x), \text{truth}), x)$ $\text{cnf}(\text{axiom}_8, \text{axiom})$

LCL002-1.ax Alternative Wajsberg algebra definitions

$\text{xor}(x, y) = \text{or}(\text{and}(x, \text{not}(y)), \text{and}(\text{not}(x), y))$ $\text{cnf}(\text{xor_definition}, \text{axiom})$
 $\text{xor}(x, y) = \text{xor}(y, x)$ $\text{cnf}(\text{xor_commutativity}, \text{axiom})$
 $\text{and_star}(x, y) = \text{not}(\text{or}(\text{not}(x), \text{not}(y)))$ $\text{cnf}(\text{and_star_definition}, \text{axiom})$
 $\text{and_star}(\text{and_star}(x, y), z) = \text{and_star}(x, \text{and_star}(y, z))$ $\text{cnf}(\text{and_star_associativity}, \text{axiom})$
 $\text{and_star}(x, y) = \text{and_star}(y, x)$ $\text{cnf}(\text{and_star_commutativity}, \text{axiom})$
 $\text{not}(\text{truth}) = \text{falsehood}$ $\text{cnf}(\text{false_definition}, \text{axiom})$

LCL003-0.ax Propositional logic deduction

$\text{axiom}(\text{or}(\text{not}(\text{or}(a, a)), a))$ $\text{cnf}(\text{axiom_l}_2, \text{axiom})$
 $\text{axiom}(\text{or}(\text{not}(a), \text{or}(b, a)))$ $\text{cnf}(\text{axiom_l}_3, \text{axiom})$
 $\text{axiom}(\text{or}(\text{not}(\text{or}(a, b)), \text{or}(b, a)))$ $\text{cnf}(\text{axiom_l}_4, \text{axiom})$
 $\text{axiom}(\text{or}(\text{not}(\text{or}(a, \text{or}(b, c))), \text{or}(b, \text{or}(a, c))))$ $\text{cnf}(\text{axiom_l}_5, \text{axiom})$
 $\text{axiom}(\text{or}(\text{not}(\text{or}(\text{not}(a), b)), \text{or}(\text{not}(\text{or}(c, a)), \text{or}(c, b))))$ $\text{cnf}(\text{axiom_l}_6, \text{axiom})$
 $\text{axiom}(x) \Rightarrow \text{theorem}(x)$ $\text{cnf}(\text{rule}_1, \text{axiom})$
 $(\text{axiom}(\text{or}(\text{not}(y), x)) \text{ and } \text{theorem}(y)) \Rightarrow \text{theorem}(x)$ $\text{cnf}(\text{rule}_2, \text{axiom})$
 $(\text{axiom}(\text{or}(\text{not}(x), y)) \text{ and } \text{theorem}(\text{or}(\text{not}(y), z))) \Rightarrow \text{theorem}(\text{or}(\text{not}(x), z))$ $\text{cnf}(\text{rule}_3, \text{axiom})$

LCL004-0.ax Propositional logic deduction

$\text{axiom}(\text{implies}(\text{or}(a, a), a))$ $\text{cnf}(\text{axiom_l}_2, \text{axiom})$
 $\text{axiom}(\text{implies}(a, \text{or}(b, a)))$ $\text{cnf}(\text{axiom_l}_3, \text{axiom})$
 $\text{axiom}(\text{implies}(\text{or}(a, b), \text{or}(b, a)))$ $\text{cnf}(\text{axiom_l}_4, \text{axiom})$
 $\text{axiom}(\text{implies}(\text{or}(a, \text{or}(b, c)), \text{or}(b, \text{or}(a, c))))$ $\text{cnf}(\text{axiom_l}_5, \text{axiom})$
 $\text{axiom}(\text{implies}(\text{implies}(a, b), \text{implies}(\text{or}(c, a), \text{or}(c, b))))$ $\text{cnf}(\text{axiom_l}_6, \text{axiom})$
 $\text{implies}(x, y) = \text{or}(\text{not}(x), y)$ $\text{cnf}(\text{implies_definition}, \text{axiom})$
 $\text{axiom}(x) \Rightarrow \text{theorem}(x)$ $\text{cnf}(\text{rule}_1, \text{axiom})$
 $(\text{theorem}(\text{implies}(y, x)) \text{ and } \text{theorem}(y)) \Rightarrow \text{theorem}(x)$ $\text{cnf}(\text{rule}_2, \text{axiom})$

LCL004-1.ax Propositional logic deduction axioms for AND

$\text{and}(p, q) = \text{not}(\text{or}(\text{not}(p), \text{not}(q)))$ $\text{cnf}(\text{and_defn}, \text{axiom})$

LCL004-2.ax Propositional logic deduction axioms for EQUIVALENT

$\text{equivalent}(p, q) = \text{and}(\text{implies}(p, q), \text{implies}(q, p)) \quad \text{cnf}(\text{equivalent_defn}, \text{axiom})$

LCL005-0.ax Propositional logic

$c_PropLog_Opl_Ofalse \neq c_PropLog_Opl_Ovar(v_a_H, t_a) \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_1_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Ovar(v_a_H, t_a) \neq c_PropLog_Opl_Ofalse \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_2_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Ofalse \neq c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_3_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \neq c_PropLog_Opl_Ofalse \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_4_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Ovar(v_a, t_a) \neq c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_5_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \neq c_PropLog_Opl_Ovar(v_a, t_a) \quad \text{cnf}(\text{cls_PropLog_Opl_Oddistinct_6_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Ovar(v_a, t_a) = c_PropLog_Opl_Ovar(v_a_H, t_a) \Rightarrow v_a = v_a_H \quad \text{cnf}(\text{cls_PropLog_Opl_Oinject_1_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Oop_A_N_{62}(v_pl1, v_pl2, t_a) = c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \Rightarrow v_pl1 = v_pl1_H \quad \text{cnf}(\text{cls_PropLog_Opl_Oinject_2_iff1}_0, \text{axiom})$
 $c_PropLog_Opl_Oop_A_N_{62}(v_pl1, v_pl2, t_a) = c_PropLog_Opl_Oop_A_N_{62}(v_pl1_H, v_pl2_H, t_a) \Rightarrow v_pl2 = v_pl2_H \quad \text{cnf}(\text{cls_PropLog_Opl_Oinject_3_iff1}_0, \text{axiom})$
 $c_in(v_p, v_H, tc_PropLog_Opl(t_a)) \Rightarrow c_in(v_p, c_PropLog_Othms(v_H, t_a), tc_PropLog_Opl(t_a)) \quad \text{cnf}(\text{cls_PropLog_Othms}, \text{axiom})$

LCL006+0.ax Propositional logic rules and axioms

$\text{modus_ponens} \iff \forall x, y: ((\text{is_a_theorem}(x) \text{ and } \text{is_a_theorem}(\text{implies}(x, y))) \Rightarrow \text{is_a_theorem}(y)) \quad \text{fof}(\text{modus_ponens}, \text{axiom})$
 $\text{substitution_of_equivalents} \iff \forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow x = y) \quad \text{fof}(\text{substitution_of_equivalents}, \text{axiom})$
 $\text{modus_tollens} \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(y), \text{not}(x)), \text{implies}(x, y))) \quad \text{fof}(\text{modus_tollens}, \text{axiom})$
 $\text{implies}_1 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x))) \quad \text{fof}(\text{implies}_1, \text{axiom})$
 $\text{implies}_2 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{implies}(x, \text{implies}(x, y)), \text{implies}(x, y))) \quad \text{fof}(\text{implies}_2, \text{axiom})$
 $\text{implies}_3 \iff \forall x, y, z: \text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{fof}(\text{implies}_3, \text{axiom})$
 $\text{and}_1 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{and}(x, y), x)) \quad \text{fof}(\text{and}_1, \text{axiom})$
 $\text{and}_2 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{and}(x, y), y)) \quad \text{fof}(\text{and}_2, \text{axiom})$
 $\text{and}_3 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(x, \text{implies}(y, \text{and}(x, y)))) \quad \text{fof}(\text{and}_3, \text{axiom})$
 $\text{or}_1 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(x, \text{or}(x, y))) \quad \text{fof}(\text{or}_1, \text{axiom})$
 $\text{or}_2 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(y, \text{or}(x, y))) \quad \text{fof}(\text{or}_2, \text{axiom})$
 $\text{or}_3 \iff \forall x, y, z: \text{is_a_theorem}(\text{implies}(\text{implies}(x, z), \text{implies}(\text{implies}(y, z), \text{implies}(\text{or}(x, y), z)))) \quad \text{fof}(\text{or}_3, \text{axiom})$
 $\text{equivalence}_1 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{equiv}(x, y), \text{implies}(x, y))) \quad \text{fof}(\text{equivalence}_1, \text{axiom})$
 $\text{equivalence}_2 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{equiv}(x, y), \text{implies}(y, x))) \quad \text{fof}(\text{equivalence}_2, \text{axiom})$
 $\text{equivalence}_3 \iff \forall x, y: \text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, x), \text{equiv}(x, y)))) \quad \text{fof}(\text{equivalence}_3, \text{axiom})$
 $\text{kn}_1 \iff \forall p: \text{is_a_theorem}(\text{implies}(p, \text{and}(p, p))) \quad \text{fof}(\text{kn}_1, \text{axiom})$
 $\text{kn}_2 \iff \forall p, q: \text{is_a_theorem}(\text{implies}(\text{and}(p, q), p)) \quad \text{fof}(\text{kn}_2, \text{axiom})$
 $\text{kn}_3 \iff \forall p, q, r: \text{is_a_theorem}(\text{implies}(\text{implies}(p, q), \text{implies}(\text{not}(\text{and}(q, r)), \text{not}(\text{and}(r, p)))) \quad \text{fof}(\text{kn}_3, \text{axiom})$
 $\text{cn}_1 \iff \forall p, q, r: \text{is_a_theorem}(\text{implies}(\text{implies}(p, q), \text{implies}(\text{implies}(q, r), \text{implies}(p, r)))) \quad \text{fof}(\text{cn}_1, \text{axiom})$
 $\text{cn}_2 \iff \forall p, q: \text{is_a_theorem}(\text{implies}(p, \text{implies}(\text{not}(p), q))) \quad \text{fof}(\text{cn}_2, \text{axiom})$
 $\text{cn}_3 \iff \forall p: \text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(p), p), p)) \quad \text{fof}(\text{cn}_3, \text{axiom})$
 $r_1 \iff \forall p: \text{is_a_theorem}(\text{implies}(\text{or}(p, p), p)) \quad \text{fof}(r_1, \text{axiom})$
 $r_2 \iff \forall p, q: \text{is_a_theorem}(\text{implies}(q, \text{or}(p, q))) \quad \text{fof}(r_2, \text{axiom})$
 $r_3 \iff \forall p, q: \text{is_a_theorem}(\text{implies}(\text{or}(p, q), \text{or}(q, p))) \quad \text{fof}(r_3, \text{axiom})$
 $r_4 \iff \forall p, q, r: \text{is_a_theorem}(\text{implies}(\text{or}(p, \text{or}(q, r)), \text{or}(q, \text{or}(p, r)))) \quad \text{fof}(r_4, \text{axiom})$
 $r_5 \iff \forall p, q, r: \text{is_a_theorem}(\text{implies}(\text{implies}(q, r), \text{implies}(\text{or}(p, q), \text{or}(p, r)))) \quad \text{fof}(r_5, \text{axiom})$

LCL006+1.ax Propositional logic definitions

$\text{op_or} \Rightarrow \forall x, y: \text{or}(x, y) = \text{not}(\text{and}(\text{not}(x), \text{not}(y))) \quad \text{fof}(\text{op_or}, \text{axiom})$
 $\text{op_and} \Rightarrow \forall x, y: \text{and}(x, y) = \text{not}(\text{or}(\text{not}(x), \text{not}(y))) \quad \text{fof}(\text{op_and}, \text{axiom})$
 $\text{op_implies_and} \Rightarrow \forall x, y: \text{implies}(x, y) = \text{not}(\text{and}(x, \text{not}(y))) \quad \text{fof}(\text{op_implies_and}, \text{axiom})$
 $\text{op_implies_or} \Rightarrow \forall x, y: \text{implies}(x, y) = \text{or}(\text{not}(x), y) \quad \text{fof}(\text{op_implies_or}, \text{axiom})$
 $\text{op_equiv} \Rightarrow \forall x, y: \text{equiv}(x, y) = \text{and}(\text{implies}(x, y), \text{implies}(y, x)) \quad \text{fof}(\text{op_equiv}, \text{axiom})$

LCL006+2.ax Hilbert's axiomatization of propositional logic

$\text{op_or} \quad \text{fof}(\text{hilbert_op_or}, \text{axiom})$
 $\text{op_implies_and} \quad \text{fof}(\text{hilbert_op_implies_and}, \text{axiom})$
 $\text{op_equiv} \quad \text{fof}(\text{hilbert_op_equiv}, \text{axiom})$
 $\text{modus_ponens} \quad \text{fof}(\text{hilbert_modus_ponens}, \text{axiom})$
 $\text{modus_tollens} \quad \text{fof}(\text{hilbert_modus_tollens}, \text{axiom})$
 $\text{implies}_1 \quad \text{fof}(\text{hilbert_implies}_1, \text{axiom})$
 $\text{implies}_2 \quad \text{fof}(\text{hilbert_implies}_2, \text{axiom})$
 $\text{implies}_3 \quad \text{fof}(\text{hilbert_implies}_3, \text{axiom})$
 $\text{and}_1 \quad \text{fof}(\text{hilbert_and}_1, \text{axiom})$
 $\text{and}_2 \quad \text{fof}(\text{hilbert_and}_2, \text{axiom})$
 $\text{and}_3 \quad \text{fof}(\text{hilbert_and}_3, \text{axiom})$

or_1 $fof(hilbert_or_1, axiom)$
 or_2 $fof(hilbert_or_2, axiom)$
 or_3 $fof(hilbert_or_3, axiom)$
 $equivalence_1$ $fof(hilbert_equivalence_1, axiom)$
 $equivalence_2$ $fof(hilbert_equivalence_2, axiom)$
 $equivalence_3$ $fof(hilbert_equivalence_3, axiom)$
 $substitution_of_equivalents$ $fof(substitution_of_equivalents, axiom)$

LCL006+3.ax Lukasiewicz's axiomatization of propositional logic

op_or $fof(luka_op_or, axiom)$
 $op_implies$ $fof(luka_op_implies, axiom)$
 op_equiv $fof(luka_op_equiv, axiom)$
 $modus_ponens$ $fof(luka_modus_ponens, axiom)$
 cn_1 $fof(luka_cn_1, axiom)$
 cn_2 $fof(luka_cn_2, axiom)$
 cn_3 $fof(luka_cn_3, axiom)$
 $substitution_of_equivalents$ $fof(substitution_of_equivalents, axiom)$

LCL006+4.ax Principia's axiomatization of propositional logic

$op_implies_or$ $fof(principia_op_implies_or, axiom)$
 op_and $fof(principia_op_and, axiom)$
 op_equiv $fof(principia_op_equiv, axiom)$
 $modus_ponens$ $fof(principia_modus_ponens, axiom)$
 r_1 $fof(principia_r_1, axiom)$
 r_2 $fof(principia_r_2, axiom)$
 r_3 $fof(principia_r_3, axiom)$
 r_4 $fof(principia_r_4, axiom)$
 r_5 $fof(principia_r_5, axiom)$
 $substitution_of_equivalents$ $fof(substitution_of_equivalents, axiom)$

LCL006+5.ax Rosser's axiomatization of propositional logic

op_or $fof(rosser_op_or, axiom)$
 $op_implies_and$ $fof(rosser_op_implies_and, axiom)$
 op_equiv $fof(rosser_op_equiv, axiom)$
 $modus_ponens$ $fof(rosser_modus_ponens, axiom)$
 kn_1 $fof(rosser_kn_1, axiom)$
 kn_2 $fof(rosser_kn_2, axiom)$
 kn_3 $fof(rosser_kn_3, axiom)$
 $substitution_of_equivalents$ $fof(substitution_of_equivalents, axiom)$

LCL007+0.ax Propositional modal logic rules and axioms

$necessitation$ $\iff \forall x: (is_a_theorem(x) \Rightarrow is_a_theorem(necessarily(x)))$ $fof(necessitation, axiom)$
 $modus_ponens_strict_implies$ $\iff \forall x, y: ((is_a_theorem(x) \text{ and } is_a_theorem(strict_implies(x, y))) \Rightarrow is_a_theorem(y))$ $fof(modus_ponens_strict_implies, axiom)$
 $adjunction$ $\iff \forall x, y: ((is_a_theorem(x) \text{ and } is_a_theorem(y)) \Rightarrow is_a_theorem(and(x, y)))$ $fof(adjunction, axiom)$
 $substitution_strict_equiv$ $\iff \forall x, y: (is_a_theorem(strict_equiv(x, y)) \Rightarrow x = y)$ $fof(substitution_strict_equiv, axiom)$
 $axiom_K$ $\iff \forall x, y: is_a_theorem(implies(necessarily(implies(x, y)), implies(necessarily(x), necessarily(y))))$ $fof(axiom_K, axiom)$
 $axiom_M$ $\iff \forall x: is_a_theorem(implies(necessarily(x), x))$ $fof(axiom_M, axiom)$
 $axiom_4$ $\iff \forall x: is_a_theorem(implies(necessarily(x), necessarily(necessarily(x))))$ $fof(axiom_4, axiom)$
 $axiom_B$ $\iff \forall x: is_a_theorem(implies(x, necessarily(possibly(x))))$ $fof(axiom_B, axiom)$
 $axiom_5$ $\iff \forall x: is_a_theorem(implies(possibly(x), necessarily(possibly(x))))$ $fof(axiom_5, axiom)$
 $axiom_s_1$ $\iff \forall x, y, z: is_a_theorem(implies(and(necessarily(implies(x, y)), necessarily(implies(y, z))), necessarily(implies(x, z))))$
 $axiom_s_2$ $\iff \forall p, q: is_a_theorem(strict_implies(possibly(and(p, q)), and(possibly(p), possibly(q))))$ $fof(axiom_s_2, axiom)$
 $axiom_s_3$ $\iff \forall x, y: is_a_theorem(strict_implies(strict_implies(x, y), strict_implies(not(possibly(y)), not(possibly(x)))))$ $fof(axiom_s_3, axiom)$
 $axiom_s_4$ $\iff \forall x: is_a_theorem(strict_implies(necessarily(x), necessarily(necessarily(x))))$ $fof(axiom_s_4, axiom)$
 $axiom_m_1$ $\iff \forall x, y: is_a_theorem(strict_implies(and(x, y), and(y, x)))$ $fof(axiom_m_1, axiom)$
 $axiom_m_2$ $\iff \forall x, y: is_a_theorem(strict_implies(and(x, y), x))$ $fof(axiom_m_2, axiom)$
 $axiom_m_3$ $\iff \forall x, y, z: is_a_theorem(strict_implies(and(and(x, y), z), and(x, and(y, z))))$ $fof(axiom_m_3, axiom)$
 $axiom_m_4$ $\iff \forall x: is_a_theorem(strict_implies(x, and(x, x)))$ $fof(axiom_m_4, axiom)$
 $axiom_m_5$ $\iff \forall x, y, z: is_a_theorem(strict_implies(and(strict_implies(x, y), strict_implies(y, z)), strict_implies(x, z)))$ $fof(axiom_m_5, axiom)$
 $axiom_m_6$ $\iff \forall x: is_a_theorem(strict_implies(x, possibly(x)))$ $fof(axiom_m_6, axiom)$
 $axiom_m_7$ $\iff \forall p, q: is_a_theorem(strict_implies(possibly(and(p, q)), p))$ $fof(axiom_m_7, axiom)$

$\text{axiom_m}_8 \iff \forall p, q: \text{is_a_theorem}(\text{strict_implies}(\text{strict_implies}(p, q), \text{strict_implies}(\text{possibly}(p), \text{possibly}(q)))) \quad \text{fof}(\text{axiom_m}_8, \text{axiom})$
 $\text{axiom_m}_9 \iff \forall x: \text{is_a_theorem}(\text{strict_implies}(\text{possibly}(\text{possibly}(x)), \text{possibly}(x))) \quad \text{fof}(\text{axiom_m}_9, \text{axiom})$
 $\text{axiom_m}_{10} \iff \forall x: \text{is_a_theorem}(\text{strict_implies}(\text{possibly}(x), \text{necessarily}(\text{possibly}(x)))) \quad \text{fof}(\text{axiom_m}_{10}, \text{axiom})$

LCL007+1.ax Propositional modal logic definitions

$\text{op_possibly} \Rightarrow \forall x: \text{possibly}(x) = \text{not}(\text{necessarily}(\text{not}(x))) \quad \text{fof}(\text{op_possibly}, \text{axiom})$
 $\text{op_necessarily} \Rightarrow \forall x: \text{necessarily}(x) = \text{not}(\text{possibly}(\text{not}(x))) \quad \text{fof}(\text{op_necessarily}, \text{axiom})$
 $\text{op_strict_implies} \Rightarrow \forall x, y: \text{strict_implies}(x, y) = \text{necessarily}(\text{implies}(x, y)) \quad \text{fof}(\text{op_strict_implies}, \text{axiom})$
 $\text{op_strict_equiv} \Rightarrow \forall x, y: \text{strict_equiv}(x, y) = \text{and}(\text{strict_implies}(x, y), \text{strict_implies}(y, x)) \quad \text{fof}(\text{op_strict_equiv}, \text{axiom})$

LCL007+2.ax KM5 axiomatization of S5 based on Hilbert's PC

$\text{op_possibly} \quad \text{fof}(\text{km5_op_possibly}, \text{axiom})$
 $\text{necessitation} \quad \text{fof}(\text{km5_necessitation}, \text{axiom})$
 $\text{axiom_K} \quad \text{fof}(\text{km5_axiom_K}, \text{axiom})$
 $\text{axiom_M} \quad \text{fof}(\text{km5_axiom_M}, \text{axiom})$
 $\text{axiom}_5 \quad \text{fof}(\text{km5_axiom}_5, \text{axiom})$

LCL007+3.ax KM4B axiomatization of S5 based on Hilbert's PC

$\text{op_possibly} \quad \text{fof}(\text{km4b_op_possibly}, \text{axiom})$
 $\text{necessitation} \quad \text{fof}(\text{km4b_necessitation}, \text{axiom})$
 $\text{axiom_K} \quad \text{fof}(\text{km4b_axiom_K}, \text{axiom})$
 $\text{axiom_M} \quad \text{fof}(\text{km4b_axiom_M}, \text{axiom})$
 $\text{axiom}_4 \quad \text{fof}(\text{km4b_axiom}_4, \text{axiom})$
 $\text{axiom_B} \quad \text{fof}(\text{km4b_axiom_B}, \text{axiom})$

LCL007+4.ax Axiomatization of S1-0

$\text{op_possibly} \quad \text{fof}(\text{s1_0_op_possibly}, \text{axiom})$
 $\text{op_or} \quad \text{fof}(\text{s1_0_op_or}, \text{axiom})$
 $\text{op_implies} \quad \text{fof}(\text{s1_0_op_implies}, \text{axiom})$
 $\text{op_strict_implies} \quad \text{fof}(\text{s1_0_op_strict_implies}, \text{axiom})$
 $\text{op_equiv} \quad \text{fof}(\text{s1_0_op_equiv}, \text{axiom})$
 $\text{op_strict_equiv} \quad \text{fof}(\text{s1_0_op_strict_equiv}, \text{axiom})$
 $\text{modus_ponens_strict_implies} \quad \text{fof}(\text{s1_0_modus_ponens_strict_implies}, \text{axiom})$
 $\text{substitution_strict_equiv} \quad \text{fof}(\text{s1_0_substitution_strict_equiv}, \text{axiom})$
 $\text{adjunction} \quad \text{fof}(\text{s1_0_adjunction}, \text{axiom})$
 $\text{axiom_m}_1 \quad \text{fof}(\text{s1_0_axiom_m}_1, \text{axiom})$
 $\text{axiom_m}_2 \quad \text{fof}(\text{s1_0_axiom_m}_2, \text{axiom})$
 $\text{axiom_m}_3 \quad \text{fof}(\text{s1_0_axiom_m}_3, \text{axiom})$
 $\text{axiom_m}_4 \quad \text{fof}(\text{s1_0_axiom_m}_4, \text{axiom})$
 $\text{axiom_m}_5 \quad \text{fof}(\text{s1_0_axiom_m}_5, \text{axiom})$

LCL007+5.ax M6S3M9B axiomatization of S5 based on S1-0

$\text{axiom_m}_6 \quad \text{fof}(\text{s1_0_m6s3m9b_axiom_m}_6, \text{axiom})$
 $\text{axiom_s}_3 \quad \text{fof}(\text{s1_0_m6s3m9b_axiom_s}_3, \text{axiom})$
 $\text{axiom_m}_9 \quad \text{fof}(\text{s1_0_m6s3m9b_axiom_m}_9, \text{axiom})$
 $\text{axiom_b} \quad \text{fof}(\text{s1_0_m6s3m9b_axiom_b}, \text{axiom})$

LCL007+6.ax M10 axiomatization of S5 based on S1-0

$\text{axiom_m}_{10} \quad \text{fof}(\text{s1_0_m10_axiom_m}_{10}, \text{axiom})$

LCL009^0.ax Translating constructive S4 (CS4) to bimodal classical S4 (BS4)

$\text{reli}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{reli}, \text{type})$
 $\text{relr}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{relr}, \text{type})$
 $\text{cs4_atom}: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_atom_decl}, \text{type})$
 $\text{cs4_and}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_and_decl}, \text{type})$
 $\text{cs4_or}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_or_decl}, \text{type})$
 $\text{cs4_impl}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_impl_decl}, \text{type})$
 $\text{cs4_true}: \$i \rightarrow \$o \quad \text{thf}(\text{cs4_true_decl}, \text{type})$
 $\text{cs4_false}: \$i \rightarrow \$o \quad \text{thf}(\text{cs4_false_decl}, \text{type})$
 $\text{cs4_all}: (\text{individuals} \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_all_decl}, \text{type})$
 $\text{cs4_box}: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cs4_box_decl}, \text{type})$
 $\text{cs4_atom} = (\lambda p: \$i \rightarrow \$o: (\text{mbox}@ \text{reli}@p)) \quad \text{thf}(\text{cs4_atom}, \text{definition})$
 $\text{cs4_and} = (\lambda a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: (\text{mand}@a@b)) \quad \text{thf}(\text{cs4_and}, \text{definition})$

$cs4_or = (\lambda a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: (mor@a@b)) \quad thf(cs4_or, definition)$
 $cs4_impl = (\lambda a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: (mbox@reli@(mimpl@a@b))) \quad thf(cs4_impl, definition)$
 $cs4_true = mtrue \quad thf(cs4_true, definition)$
 $cs4_false = mfalse \quad thf(cs4_false, definition)$
 $cs4_all = (\lambda a: individuals \rightarrow \$i \rightarrow \$o: (mbox@reli@(mall@a))) \quad thf(cs4_all, definition)$
 $cs4_box = (\lambda a: \$i \rightarrow \$o: (mbox@reli@(mbox@reli@a))) \quad thf(cs4_box, definition)$
 $cs4_valid: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cs4_valid_decl, type)$
 $cs4_valid = (\lambda a: \$i \rightarrow \$o: (mvalid@a)) \quad thf(cs4_valid_def, definition)$
 $\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@reli@a@a)) \quad thf(refl_axiom_i, axiom)$
 $\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@reli@a@a)) \quad thf(refl_axiom_r, axiom)$
 $\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@reli@a@(mbox@reli@(mbox@reli@a)))) \quad thf(trans_axiom_i, axiom)$
 $\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@reli@a@(mbox@reli@(mbox@reli@a)))) \quad thf(trans_axiom_r, axiom)$
 $\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@reli@(mbox@reli@a))@(mbox@reli@(mbox@reli@a)))) \quad thf(ax_i_r_commute, axiom)$

LCL013^1.ax Modal logic K

Embedding of monomodal logic K in simple type theory.

$rel_k: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_k_type, type)$
 $mbox_k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mbox_k_type, type)$
 $mbox_k = (\lambda phi: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg rel_k@w@v \text{ or } phi@v)) \quad thf(mbox_k, definition)$
 $mdia_k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mdia_k_type, type)$
 $mdia_k = (\lambda phi: \$i \rightarrow \$o: (mnot@(mbox_k@(mnot@phi)))) \quad thf(mdia_k, definition)$

LCL013^2.ax Modal logic D

Embedding of monomodal logic D in simple type theory

$rel_d: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_d_type, type)$
 $mbox_d: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mbox_d_type, type)$
 $mbox_d = (\lambda phi: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg rel_d@w@v \text{ or } phi@v)) \quad thf(mbox_d, definition)$
 $mdia_d: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mdia_d_type, type)$
 $mdia_d = (\lambda phi: \$i \rightarrow \$o: (mnot@(mbox_d@(mnot@phi)))) \quad thf(mdia_d, definition)$
 $mserial@rel_d \quad thf(a_1, axiom)$

LCL013^3.ax Modal logic M

Embedding of monomodal logic M in simple type theory.

$rel_m: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_m_type, type)$
 $mbox_m: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mbox_m_type, type)$
 $mbox_m = (\lambda phi: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg rel_m@w@v \text{ or } phi@v)) \quad thf(mbox_m, definition)$
 $mdia_m: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mdia_m_type, type)$
 $mdia_m = (\lambda phi: \$i \rightarrow \$o: (mnot@(mbox_m@(mnot@phi)))) \quad thf(mdia_m, definition)$
 $mreflexive@rel_m \quad thf(a_1, axiom)$

LCL013^4.ax Modal logic B

Embedding of monomodal logic B in simple type theory.

$rel_b: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_b_type, type)$
 $mbox_b: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mbox_b_type, type)$
 $mbox_b = (\lambda phi: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg rel_b@w@v \text{ or } phi@v)) \quad thf(mbox_b, definition)$
 $mdia_b: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mdia_b_type, type)$
 $mdia_b = (\lambda phi: \$i \rightarrow \$o: (mnot@(mbox_b@(mnot@phi)))) \quad thf(mdia_b, definition)$
 $mreflexive@rel_b \quad thf(a_1, axiom)$
 $msymmetric@rel_b \quad thf(a_2, axiom)$

LCL013^5.ax Modal logic S4

Embedding of monomodal logic S4 in simple type theory.

$rel_s4: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_s4_type, type)$
 $mbox_s4: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mbox_s4_type, type)$
 $mbox_s4 = (\lambda phi: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg rel_s4@w@v \text{ or } phi@v)) \quad thf(mbox_s4, definition)$
 $mdia_s4: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad thf(mdia_s4_type, type)$
 $mdia_s4 = (\lambda phi: \$i \rightarrow \$o: (mnot@(mbox_s4@(mnot@phi)))) \quad thf(mdia_s4, definition)$
 $mreflexive@rel_s4 \quad thf(a_1, axiom)$
 $mtransitive@rel_s4 \quad thf(a_2, axiom)$

LCL013^6.ax Modal logic S5

Embedding of monomodal logic S5 in simple type theory.

$rel_s5: \$i \rightarrow \$i \rightarrow \$o \quad thf(rel_s5_type, type)$

$\text{mbox_s5}: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \o $\text{thf}(\text{mbox_s5.type}, \text{type})$
 $\text{mbox_s5} = (\lambda\text{phi}: \$i \rightarrow \$o, w: \$i: \forall v: \$i: (\neg \text{rel_s5}@w@v \text{ or } \text{phi}@v))$ $\text{thf}(\text{mbox_s5}, \text{definition})$
 $\text{mdia_s5}: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \o $\text{thf}(\text{mdia_s5.type}, \text{type})$
 $\text{mdia_s5} = (\lambda\text{phi}: \$i \rightarrow \$o: (\text{mnot}@(\text{mbox_s5}@(\text{mnot}@\text{phi}))))$ $\text{thf}(\text{mdia_s5}, \text{definition})$
 $\text{mreflexive}@rel_s5$ $\text{thf}(a_1, \text{axiom})$
 $\text{mtransitive}@rel_s5$ $\text{thf}(a_2, \text{axiom})$
 $\text{msymmetric}@rel_s5$ $\text{thf}(a_3, \text{axiom})$

LCL014^0.ax Region Connection Calculus

$\text{reg}: \$t\text{Type}$ $\text{thf}(\text{reg.type}, \text{type})$
 $c: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(c.type, \text{type})$
 $dc: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(dc.type, \text{type})$
 $p: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(p.type, \text{type})$
 $eq: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(eq.type, \text{type})$
 $o: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(o.type, \text{type})$
 $po: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(po.type, \text{type})$
 $ec: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(ec.type, \text{type})$
 $pp: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(pp.type, \text{type})$
 $tpp: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(tpp.type, \text{type})$
 $ntpp: \text{reg} \rightarrow \text{reg} \rightarrow \o $\text{thf}(ntpp.type, \text{type})$
 $\forall x: \text{reg}: (c@x@x)$ $\text{thf}(c.\text{reflexive}, \text{axiom})$
 $\forall x: \text{reg}, y: \text{reg}: ((c@x@y) \Rightarrow (c@y@x))$ $\text{thf}(c.\text{symmetric}, \text{axiom})$
 $dc = (\lambda x: \text{reg}, y: \text{reg}: \neg c@x@y)$ $\text{thf}(dc, \text{definition})$
 $p = (\lambda x: \text{reg}, y: \text{reg}: \forall z: \text{reg}: ((c@z@x) \Rightarrow (c@z@y)))$ $\text{thf}(p, \text{definition})$
 $eq = (\lambda x: \text{reg}, y: \text{reg}: (p@x@y \text{ and } p@y@x))$ $\text{thf}(eq, \text{definition})$
 $o = (\lambda x: \text{reg}, y: \text{reg}: \exists z: \text{reg}: (p@z@x \text{ and } p@z@y))$ $\text{thf}(o, \text{definition})$
 $po = (\lambda x: \text{reg}, y: \text{reg}: (o@x@y \text{ and } \neg p@x@y \text{ and } \neg p@y@x))$ $\text{thf}(po, \text{definition})$
 $ec = (\lambda x: \text{reg}, y: \text{reg}: (c@x@y \text{ and } \neg o@x@y))$ $\text{thf}(ec, \text{definition})$
 $pp = (\lambda x: \text{reg}, y: \text{reg}: (p@x@y \text{ and } \neg p@y@x))$ $\text{thf}(pp, \text{definition})$
 $tpp = (\lambda x: \text{reg}, y: \text{reg}: (pp@x@y \text{ and } \exists z: \text{reg}: (ec@z@x \text{ and } ec@z@y)))$ $\text{thf}(tpp, \text{definition})$
 $ntpp = (\lambda x: \text{reg}, y: \text{reg}: (pp@x@y \text{ and } \neg \exists z: \text{reg}: (ec@z@x \text{ and } ec@z@y)))$ $\text{thf}(ntpp, \text{definition})$

LCL015^1.ax Cumulative domain specific axioms

$\forall x: \mu, v: \$i, w: \$i: ((\text{exists.in.world}@x@v \text{ and } \text{rel_s4}@v@w) \Rightarrow (\text{exists.in.world}@x@w))$ $\text{thf}(\text{cumulative_ax}, \text{axiom})$

LCL016^1.ax Embedding of second order modal logic in simple type theory

Extends K to KB by adding symmetric of rel.

$\text{msymmetric}: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \o $\text{thf}(\text{msymmetric.type}, \text{type})$
 $\text{msymmetric} = (\lambda r: \$i \rightarrow \$i \rightarrow \$o: \forall s, t: \$i: ((r@s@t) \Rightarrow (r@t@s)))$ $\text{thf}(\text{msymmetric}, \text{definition})$
 $\text{msymmetric}@rel$ $\text{thf}(\text{sym}, \text{axiom})$

LCL problems

LCL001-1.p The Whitehead-Russell system => the Meredith axiom

The Whitehead-Russell axiomatisation of the Disjunction/ Negation 2 valued sentential calculus is AN-1,AN-2,AN-3, AN-4. Show that the Meredith axiom can be derived from the Whitehead-Russell axiomatisation.

$(\text{is_a_theorem}(\text{or}(\text{not}(x), y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(y), z)), \text{or}(\text{not}(\text{or}(x, y)), \text{or}(x, z))))$ $\text{cnf}(\text{an}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{or}(\text{not}(\text{or}(x, y)), \text{or}(y, x)))$ $\text{cnf}(\text{an}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{or}(\text{not}(x), \text{or}(y, x)))$ $\text{cnf}(\text{an}_3, \text{axiom})$
 $\text{is_a_theorem}(\text{or}(\text{not}(\text{or}(x, x)), x))$ $\text{cnf}(\text{an}_4, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(\text{or}(\text{not}(a), b)), \text{or}(c, \text{or}(e, \text{falsehood})))), \text{or}(\text{not}(\text{or}(\text{not}(e), a)), \text{or}(c, \text{or}(\text{falsehood}, a))))))$ $\text{cnf}(\text{pro}, \text{axiom})$

LCL002-1.p AN-CAMerideth => AN-1

The Whitehead-Russell axiomatisation of the Disjunction/ Negation 2 valued sentential calculus is AN-1,AN-2,AN-3, AN-4. Show that AN-1 can be derived from the Meredith axiom.

$(\text{is_a_theorem}(\text{or}(\text{not}(x), y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(\text{or}(\text{not}(x), y)), \text{or}(z, \text{or}(u, v))), \text{or}(\text{not}(\text{or}(\text{not}(u), x)), \text{or}(z, \text{or}(v, x)))))$ $\text{cnf}(\text{an_CAMerideth}, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(b), c)), \text{or}(\text{not}(\text{or}(a, b)), \text{or}(a, c))))$ $\text{cnf}(\text{an}_1, \text{negated_conjecture})$

LCL003-1.p AN-CAMerideth => AN-2

The Whitehead-Russell axiomatisation of the Disjunction/ Negation 2 valued sentential calculus is AN-1,AN-2,AN-3, AN-4. Show that AN-2 can be derived from the Merideth axiom.

$$\begin{aligned} & (\text{is_a_theorem}(\text{or}(\text{not}(x), y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(\text{or}(\text{not}(x), y)), \text{or}(z, \text{or}(u, v)))), \text{or}(\text{not}(\text{or}(\text{not}(u), x)), \text{or}(z, \text{or}(v, x)))))) \quad \text{cnf}(\text{an_CAMerideth, axiom}) \\ & \neg \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(a, b)), \text{or}(b, a))) \quad \text{cnf}(\text{an}_2, \text{negated_conjecture}) \end{aligned}$$

LCL004-1.p AN-CAMerideth => AN-3

The Whitehead-Russell axiomatisation of the Disjunction/ Negation 2 valued sentential calculus is AN-1,AN-2,AN-3, AN-4. Show that AN-3 can be derived from the Merideth axiom.

$$\begin{aligned} & (\text{is_a_theorem}(\text{or}(\text{not}(x), y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(\text{or}(\text{not}(x), y)), \text{or}(z, \text{or}(u, v))), \text{or}(\text{not}(\text{or}(\text{not}(u), x)), \text{or}(z, \text{or}(v, x)))))) \quad \text{cnf}(\text{an_CAMerideth, axiom}) \\ & \neg \text{is_a_theorem}(\text{or}(\text{not}(a), \text{or}(b, a))) \quad \text{cnf}(\text{an}_3, \text{negated_conjecture}) \end{aligned}$$

LCL005-1.p AN-CAMerideth => AN-4

The Whitehead-Russell axiomatisation of the Disjunction/ Negation 2 valued sentential calculus is AN-1,AN-2,AN-3, AN-4. Show that AN-4 can be derived from the Merideth axiom.

$$\begin{aligned} & (\text{is_a_theorem}(\text{or}(\text{not}(x), y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(\text{not}(\text{or}(\text{not}(x), y)), \text{or}(z, \text{or}(u, v))), \text{or}(\text{not}(\text{or}(\text{not}(u), x)), \text{or}(z, \text{or}(v, x)))))) \quad \text{cnf}(\text{an_CAMerideth, axiom}) \\ & \neg \text{is_a_theorem}(\text{or}(\text{not}(\text{or}(a, a)), a)) \quad \text{cnf}(\text{an}_4, \text{negated_conjecture}) \end{aligned}$$

LCL006-1.p EC-1 depends on the Wajsberg system

Two axiomatisations of the equivalential calculus are EC-1,EC-2 by Lesniewski, and EC-4,EC-5 by Wajsburg. Show that EC-1 can be derived from the Wajsburg system.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(y, x))) \quad \text{cnf}(\text{ec}_4, \text{axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(\text{equivalent}(x, y), z), \text{equivalent}(x, \text{equivalent}(y, z)))) \quad \text{cnf}(\text{ec}_5, \text{axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(\text{equivalent}(a, b), \text{equivalent}(c, a)), \text{equivalent}(b, c))) \quad \text{cnf}(\text{prove_ec}_1, \text{negated_conjecture}) \end{aligned}$$

LCL007-1.p EC-2 depends on the Wajsberg system

Two axiomatisations of the equivalential calculus are EC-1,EC-2 by Lesniewski, and EC-4,EC-5 by Wajsburg. Show that EC-2 can be derived from the Wajsburg system.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(y, x))) \quad \text{cnf}(\text{ec}_4, \text{axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(\text{equivalent}(x, y), z), \text{equivalent}(x, \text{equivalent}(y, z)))) \quad \text{cnf}(\text{ec}_5, \text{axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(a, \text{equivalent}(b, c)), \text{equivalent}(\text{equivalent}(a, b), c))) \quad \text{cnf}(\text{prove_ec}_2, \text{negated_conjecture}) \end{aligned}$$

LCL008-1.p EC-4 depends on YQL

An axiomatisation of the equivalential calculus is EC-4, EC-5 by Wajsburg. Show that EC-4 can be derived from the single Lukasiewicz axiom YQL.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(\text{equivalent}(z, y), \text{equivalent}(x, z)))) \quad \text{cnf}(\text{yql, axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(a, b), \text{equivalent}(b, a))) \quad \text{cnf}(\text{prove_ec}_4, \text{negated_conjecture}) \end{aligned}$$

LCL009-1.p EC-5 depends on YQL

An axiomatisation of the equivalential calculus is EC-4, EC-5 by Wajsburg. Show that EC-5 can be derived from the single Lukasiewicz axiom YQL.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(\text{equivalent}(z, y), \text{equivalent}(x, z)))) \quad \text{cnf}(\text{yql, axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(\text{equivalent}(a, b), c), \text{equivalent}(a, \text{equivalent}(b, c)))) \quad \text{cnf}(\text{prove_ec}_5, \text{negated_conjecture}) \end{aligned}$$

LCL010-1.p YQL depends on YQF

Show that the single Lukasiewicz axiom YQL can be derived from the single Lukasiewicz axiom YQF.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(\text{equivalent}(x, z), \text{equivalent}(z, y)))) \quad \text{cnf}(\text{yqf, axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(a, b), \text{equivalent}(\text{equivalent}(c, b), \text{equivalent}(a, c)))) \quad \text{cnf}(\text{prove_yql, negated_conjecture}) \end{aligned}$$

LCL011-1.p YQF depends on YQJ

Show that the single Lukasiewicz axiom YQF can be derived from the single Lukasiewicz axiom YQJ.

$$\begin{aligned} & (\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom}) \\ & \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(x, y), \text{equivalent}(\text{equivalent}(z, x), \text{equivalent}(y, z)))) \quad \text{cnf}(\text{yqj, axiom}) \\ & \neg \text{is_a_theorem}(\text{equivalent}(\text{equivalent}(a, b), \text{equivalent}(\text{equivalent}(a, c), \text{equivalent}(c, b)))) \quad \text{cnf}(\text{prove_yqf, negated_conjecture}) \end{aligned}$$

LCL012-1.p YQJ depends on UM

Show that the single Lukasiewicz axiom YQJ can be derived from the single Meredith axiom UM.

$$(\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment, axiom})$$

is_a_theorem(equivalent(equivalent(equivalent(x, y), z), equivalent(y , equivalent(z, x)))) cnf(um, axiom)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(equivalent(c, a), equivalent(b, c)))) cnf(prove_yqj, negated_conjecture)

LCL013-1.p UM depends on XGF

Show that the single Meredith axiom UM can be derived from the single Meredith axiom XGF.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x , equivalent(equivalent(y , equivalent(x, z)), equivalent(z, y)))) cnf(xgf, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), c), equivalent(b , equivalent(c, a)))) cnf(prove_um, negated_conjecture)

LCL014-1.p XGF depends on WN

Show that the single Meredith axiom XGF can be derived from the single Meredith axiom WN.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x , equivalent(y, z)), equivalent(z , equivalent(x, y)))) cnf(wn, axiom)
 \neg is_a_theorem(equivalent(a , equivalent(equivalent(b , equivalent(a, c)), equivalent(c, b)))) cnf(prove_xgf, negated_conjecture)

LCL015-1.p WN depends on YRM

Show that the single Meredith axiom WN can be derived from the single Meredith axiom YRM.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, y), equivalent(z , equivalent(equivalent(y, z), x)))) cnf(yrm, axiom)
 \neg is_a_theorem(equivalent(equivalent(a , equivalent(b, c)), equivalent(c , equivalent(a, b)))) cnf(prove_wn, negated_conjecture)

LCL016-1.p YRM depends on YRO

Show that the single Meredith axiom YRM can be derived from the single Meredith axiom YRO.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, y), equivalent(z , equivalent(equivalent(z, y), x)))) cnf(yro, axiom)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(c , equivalent(equivalent(b, c), a)))) cnf(prove_yrm, negated_conjecture)

LCL017-1.p YRO depends on PYO

Show that the single Meredith axiom YRO can be derived from the single Meredith axiom PYO.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(equivalent(x , equivalent(y, z)), z), equivalent(y, x))) cnf(pyo, axiom)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(c , equivalent(equivalent(c, b), a)))) cnf(prove_yro, negated_conjecture)

LCL018-1.p PYO depends on PYM

Show that the single Meredith axiom PYO can be derived from the single Meredith axiom PYM.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(equivalent(x , equivalent(y, z)), y), equivalent(z, x))) cnf(pym, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a , equivalent(b, c)), c), equivalent(b, a))) cnf(prove_pyo, negated_conjecture)

LCL019-1.p PYM depends on XGK

Show that the single Meredith axiom PYM can be derived from the single Kalman axiom XGK.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x , equivalent(equivalent(y , equivalent(z, x)), equivalent(z, y)))) cnf(xgk, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a , equivalent(b, c)), b), equivalent(c, a))) cnf(prove_pym, negated_conjecture)

LCL020-1.p XGK depends on XHK

Show that the single Kalman axiom XGK can be derived from the single Winker axiom XHK.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x , equivalent(equivalent(y, z), equivalent(equivalent(x, z), y)))) cnf(xhk, axiom)
 \neg is_a_theorem(equivalent(a , equivalent(equivalent(b , equivalent(c, a)), equivalent(c, b)))) cnf(prove_xgk, negated_conjecture)

LCL021-1.p XHK depends on XHN

Show that the single Winker axiom XHK can be derived from the single Winker axiom XHN.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x , equivalent(equivalent(y, z), equivalent(equivalent(z, x), y)))) cnf(xhn, axiom)
 \neg is_a_theorem(equivalent(a , equivalent(equivalent(b, c), equivalent(equivalent(a, c), b)))) cnf(prove_xhk, negated_conjecture)

LCL022-1.p EC-1 depends on YQL

An axiomatisation of the equivalential calculus is EC-1, EC-2 by Lesniewski. Show that EC-1 can be derived from the single Lukasiewicz axiom YQL.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, y), equivalent(x, z)))) cnf(yql, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), equivalent(c, a)), equivalent(b, c))) cnf(prove_ec1, negated_conjecture)

LCL023-1.p EC-2 depends on YQL

An axiomatisation of the equivalential calculus is EC-1, EC-2 by Lesniewski. Show that EC-2 can be derived from the single Lukasiewicz axiom YQL.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, y), equivalent(x, z)))) cnf(yql, axiom)
 \neg is_a_theorem(equivalent(equivalent($a, equivalent(b, c)$), equivalent(equivalent(a, b), c))) cnf(prove_ec2, negated_conjecture)

LCL024-1.p PYO depends on XGK

Show that Kalman's shortest single axiom for the equivalential calculus, XGK, can be derived from the Meredith single axiom PYO.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(y , equivalent(z, x)), equivalent(z, y)))) cnf(xgk, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent($a, equivalent(b, c)$), c), equivalent(b, a))) cnf(prove_pyo, negated_conjecture)

LCL025-1.p C0-1 depends on the Church system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-1 can be derived from the Church system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x , implies(y, x))) cnf(c02, axiom)
 is_a_theorem(implies(implies(implies(x , falsehood), falsehood), x)) cnf(c05, axiom)
 is_a_theorem(implies(implies(x , implies(y, z)), implies(implies(x, y), implies(x, z)))) cnf(c06, axiom)
 \neg is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_c01, negated_conjecture)

LCL026-1.p C0-3 depends on the Church system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-3 can be derived from the Church system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x , implies(y, x))) cnf(c02, axiom)
 is_a_theorem(implies(implies(implies(x , falsehood), falsehood), x)) cnf(c05, axiom)
 is_a_theorem(implies(implies(x , implies(y, z)), implies(implies(x, y), implies(x, z)))) cnf(c06, axiom)
 \neg is_a_theorem(implies(implies(implies(a, b), a), a)) cnf(prove_c03, negated_conjecture)

LCL027-1.p C0-4 depends on the Church system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-4 can be derived from the Church system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x , implies(y, x))) cnf(c02, axiom)
 is_a_theorem(implies(implies(implies(x , falsehood), falsehood), x)) cnf(c05, axiom)
 is_a_theorem(implies(implies(x , implies(y, z)), implies(implies(x, y), implies(x, z)))) cnf(c06, axiom)
 \neg is_a_theorem(implies(falsehood, a)) cnf(prove_c04, negated_conjecture)

LCL028-1.p C0-CAMeredith depends on the Church system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that the Meredith axiom can be derived from the Church system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x , implies(y, x))) cnf(c02, axiom)
 is_a_theorem(implies(implies(implies(x , falsehood), falsehood), x)) cnf(c05, axiom)
 is_a_theorem(implies(implies(x , implies(y, z)), implies(implies(x, y), implies(x, z)))) cnf(c06, axiom)
 \neg is_a_theorem(implies(implies(implies(implies(implies(a, b), implies(c , falsehood)), e), falsehood), implies(implies(falsehood, a))))

LCL029-1.p C0-5 depends on the Tarski-Bernays system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-5 can be derived from the Tarski-Bernays system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(c01, axiom)
 is_a_theorem(implies(x , implies(y, x))) cnf(c02, axiom)
 is_a_theorem(implies(implies(implies(x, y), x), x)) cnf(c03, axiom)
 is_a_theorem(implies(falsehood, x)) cnf(c04, axiom)
 \neg is_a_theorem(implies(implies(implies(a , falsehood), falsehood), a)) cnf(prove_c05, negated_conjecture)

LCL030-1.p C0-6 depends on the Tarski-Bernays system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-6 can be derived from the Tarski-Bernays system.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z))))  cnf(c01, axiom)
is_a_theorem(implies(x, implies(y, x)))      cnf(c02, axiom)
is_a_theorem(implies(implies(implies(x, y), x), x))      cnf(c03, axiom)
is_a_theorem(implies(falsehood, x))      cnf(c04, axiom)
¬ is_a_theorem(implies(implies(a, implies(b, c)), implies(implies(a, b), implies(a, c))))  cnf(prove_c06, negated_conjecture)
```

LCL031-1.p C0-CAMerideth depends on the Tarski-Bernays system

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that the single Meredith axiom can be derived from the Tarski-Bernays system.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z))))  cnf(c01, axiom)
is_a_theorem(implies(x, implies(y, x)))      cnf(c02, axiom)
is_a_theorem(implies(implies(implies(x, y), x), x))      cnf(c03, axiom)
is_a_theorem(implies(falsehood, x))      cnf(c04, axiom)
¬ is_a_theorem(implies(implies(implies(implies(implies(a, b), implies(c, falsehood)), e), falsehood), implies(implies(falsehood, a), falsehood)))
```

LCL032-1.p C0-1 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-1 can be derived from the single Meredith axiom.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(implies(implies(x, y), implies(z, falsehood)), u), v), implies(implies(v, x), implies(z, x))))
¬ is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c))))  cnf(prove_c01, negated_conjecture)
```

LCL033-1.p C0-2 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-2 can be derived from the single Meredith axiom.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(implies(implies(x, y), implies(z, falsehood)), u), v), implies(implies(v, x), implies(z, x))))
¬ is_a_theorem(implies(a, implies(b, a)))      cnf(prove_c02, negated_conjecture)
```

LCL034-1.p C0-3 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-3 can be derived from the single Meredith axiom.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(implies(implies(x, y), implies(z, falsehood)), u), v), implies(implies(v, x), implies(z, x))))
¬ is_a_theorem(implies(implies(implies(a, b), a), a))      cnf(prove_c03, negated_conjecture)
```

LCL035-1.p C0-4 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-4 can be derived from the single Meredith axiom.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(implies(implies(x, y), implies(z, falsehood)), u), v), implies(implies(v, x), implies(z, x))))
¬ is_a_theorem(implies(falsehood, a))      cnf(prove_c04, negated_conjecture)
```

LCL036-1.p C0-5 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-5 can be derived from the single Meredith axiom.

```
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(implies(implies(x, y), implies(z, falsehood)), u), v), implies(implies(v, x), implies(z, x))))
¬ is_a_theorem(implies(implies(implies(a, falsehood), falsehood), a))      cnf(prove_c05, negated_conjecture)
```

LCL037-1.p C0-6 depends on the Merideth axiom

Axiomatisations for the Implication/Falsehood 2 valued sentential calculus are C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays, C0-2,C0-5,C0-6 by Church, and the single Meredith axioms. Show that C0-6 can be derived from the single Meredith axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(implies(implies(x,y), implies(z, falsehood)), u), v), implies(implies(v,x), implies(z,x))))
 \neg is_a_theorem(implies(implies(a, implies(b,c)), implies(implies(a,b), implies(a,c)))) cnf(prove_c06, negated_conjecture)

LCL038-1.p C0-1 depends on a single axiom

An axiomatisation for the Implication/Falsehood 2 valued sentential calculus is C0-1,C0-2,C0-3,C0-4 by Tarski-Bernays. Show that C0-1 can be derived from the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y), z), implies(implies(z,x), implies(u,x)))) cnf(ic_JLukasiewicz, axiom)
 \neg is_a_theorem(implies(implies(a,b), implies(implies(b,c), implies(a,c)))) cnf(prove_c01, negated_conjecture)

LCL039-1.p A theorem from Morgan

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-18,CN-35,CN-49 by Church. This can be extended to the modal logic T by the addition of three axioms for the modal operators. This problem proves a simple result of T.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(y,x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y,z)), implies(implies(x,y), implies(x,z)))) cnf(cn35, axiom)
 is_a_theorem(implies(implies(not(x), not(y)), implies(y,x))) cnf(cn49, axiom)
 is_a_theorem(implies(necessary(implies(x,y)), implies(necessary(x), necessary(y)))) cnf(necessitation1, axiom)
 is_a_theorem(implies(necessary(x), x)) cnf(necessitation2, axiom)
 is_a_theorem(x) \Rightarrow is_a_theorem(necessary(x)) cnf(axiom_of_necessitation, axiom)
 \neg is_a_theorem(implies(necessary(a), not(necessary(not(a)))) cnf(prove_this, negated_conjecture)

LCL040-1.p CN-21 depends on the rest of Frege's system

The first axiomatisation of Implication/Negation 2 valued sentential calculus was CN-18,CN-21,CN-35,CN-39,CN-39, CN-40,CN-46 by Frege. Show, like Lukasiewicz did, that CN-21 depends on the rest of this axiomatisation.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(y,x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y,z)), implies(implies(x,y), implies(x,z)))) cnf(cn35, axiom)
 is_a_theorem(implies(not(not(x)), x)) cnf(cn39, axiom)
 is_a_theorem(implies(x, not(not(x)))) cnf(cn40, axiom)
 is_a_theorem(implies(implies(x,y), implies(not(y), not(x)))) cnf(cn46, axiom)
 \neg is_a_theorem(implies(implies(a, implies(b,c)), implies(b, implies(a,c)))) cnf(prove_cn21, negated_conjecture)

LCL041-1.p CN-30 depends on the rest of Hilbert's system

An early axiomatisation of Implication/Negation 2 valued sentential calculus was CN-3,CN-18,CN-21,CN-22,CN-30, CN-54 by Hilbert. Show, like Lukasiewicz did, that CN-30 depends on the rest of this axiomatisation.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 is_a_theorem(implies(x, implies(y,x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y,z)), implies(y, implies(x,z)))) cnf(cn21, axiom)
 is_a_theorem(implies(implies(y,z), implies(implies(x,y), implies(x,z)))) cnf(cn22, axiom)
 is_a_theorem(implies(implies(x,y), implies(implies(not(x), y), y))) cnf(cn54, axiom)
 \neg is_a_theorem(implies(implies(a, implies(a,b)), implies(a,b))) cnf(prove_cn30, negated_conjecture)

LCL042-1.p CN-35 depends on Hilbert's system

Two axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-18,CN-21,CN-35,CN-39,CN-39, CN-40,CN-46 by Frege and CN-3,CN-18,CN-21,CN-22,CN-30, CN-54 by Hilbert. Show that CN-35 depends on the simplified Hilbert system (without CN-30).

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 is_a_theorem(implies(x, implies(y,x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y,z)), implies(y, implies(x,z)))) cnf(cn21, axiom)
 is_a_theorem(implies(implies(y,z), implies(implies(x,y), implies(x,z)))) cnf(cn22, axiom)
 is_a_theorem(implies(implies(x,y), implies(implies(not(x), y), y))) cnf(cn54, axiom)
 \neg is_a_theorem(implies(implies(a, implies(b,c)), implies(implies(a,b), implies(a,c)))) cnf(prove_cn35, negated_conjecture)

LCL043-1.p CN-39 depends on Hilbert's system

Two axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-18,CN-21,CN-35,CN-39,CN-39, CN-40,CN-46 by Frege and CN-3,CN-18,CN-21,CN-22,CN-30, CN-54 by Hilbert. Show that CN-39 depends on the simplified Hilbert system (without CN-30).

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 is_a_theorem(implies(x, implies(y, x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y, z)), implies(y, implies(x, z)))) cnf(cn21, axiom)
 is_a_theorem(implies(implies(y, z), implies(implies(x, y), implies(x, z)))) cnf(cn22, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(not(x), y), y))) cnf(cn54, axiom)
 \neg is_a_theorem(implies(not(not(a)), a)) cnf(prove_cn39, negated_conjecture)

LCL044-1.p CN-40 depends on Hilbert's system

Two axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-18,CN-21,CN-35,CN-39,CN-39, CN-40,CN-46 by Frege and CN-3,CN-18,CN-21,CN-22,CN-30, CN-54 by Hilbert. Show that CN-40 depends on the simplified Hilbert system (without CN-30).

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 is_a_theorem(implies(x, implies(y, x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y, z)), implies(y, implies(x, z)))) cnf(cn21, axiom)
 is_a_theorem(implies(implies(y, z), implies(implies(x, y), implies(x, z)))) cnf(cn22, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(not(x), y), y))) cnf(cn54, axiom)
 \neg is_a_theorem(implies(a, not(not(a)))) cnf(prove_cn40, negated_conjecture)

LCL045-1.p CN-46 depends on Hilbert's system

Two axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-18,CN-21,CN-35,CN-39,CN-39, CN-40,CN-46 by Frege and CN-3,CN-18,CN-21,CN-22,CN-30, CN-54 by Hilbert. Show that CN-46 depends on the simplified Hilbert system (without CN-30).

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 is_a_theorem(implies(x, implies(y, x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y, z)), implies(y, implies(x, z)))) cnf(cn21, axiom)
 is_a_theorem(implies(implies(y, z), implies(implies(x, y), implies(x, z)))) cnf(cn22, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(not(x), y), y))) cnf(cn54, axiom)
 \neg is_a_theorem(implies(implies(a, b), implies(not(b), not(a)))) cnf(prove_cn46, negated_conjecture)

LCL046-1.p CN-16 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-16 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 \neg is_a_theorem(implies(a, a)) cnf(prove_cn16, negated_conjecture)

LCL047-1.p CN-18 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-18 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 \neg is_a_theorem(implies(a, implies(b, a))) cnf(prove_cn18, negated_conjecture)

LCL048-1.p CN-19 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-19 depends on the short Lukasiewicz system.

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-35 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(a, implies(b, c)), implies(implies(a, b), implies(a, c)))) cnf(prove_cn₃₅, negated_conjecture)

LCL055-1.p CN-37 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-37 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(a, b), c), implies(not(a), c))) cnf(prove_cn₃₇, negated_conjecture)

LCL056-1.p CN-39 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-39 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(not(not(a)), a)) cnf(prove_cn₃₉, negated_conjecture)

LCL057-1.p CN-40 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-40 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(a, not(not(a)))) cnf(prove_cn₄₀, negated_conjecture)

LCL058-1.p CN-46 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-46 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(a, b), implies(not(b), not(a)))) cnf(prove_cn₄₆, negated_conjecture)

LCL059-1.p CN-49 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-49 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)

is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 ¬ is_a_theorem(implies(implies(not(a), not(b)), implies(b, a))) cnf(prove_cn49, negated_conjecture)

LCL060-1.p CN-54 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-54 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 ¬ is_a_theorem(implies(implies(a, b), implies(implies(not(a), b), b))) cnf(prove_cn54, negated_conjecture)

LCL061-1.p CN-59 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-59 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 ¬ is_a_theorem(implies(implies(not(a), c), implies(implies(b, c), implies(implies(a, b), c)))) cnf(prove_cn59, negated_conjecture)

LCL062-1.p CN-60 depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-60 depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 ¬ is_a_theorem(implies(implies(a, implies(not(b), c)), implies(a, implies(implies(e, c), implies(implies(b, e), c))))) cnf(prove...

LCL063-1.p CN-CAMerideth depends on the Lukasiewicz system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that the single Meredith axiom depends on the short Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn1, axiom)
 is_a_theorem(implies(implies(not(x), x), x)) cnf(cn2, axiom)
 is_a_theorem(implies(x, implies(not(x), y))) cnf(cn3, axiom)
 ¬ is_a_theorem(implies(implies(implies(implies(implies(a, b), implies(not(c), not(e))), c), falsehood), implies(implies(falsehood,

LCL064-1.p CN-1 depends on the Church system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-X depends on the Church system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x, implies(y, x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x, implies(y, z)), implies(implies(x, y), implies(x, z)))) cnf(cn35, axiom)
 is_a_theorem(implies(implies(not(x), not(y)), implies(y, x))) cnf(cn49, axiom)
 ¬ is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_cn1, negated_conjecture)

LCL064-2.p CN-1 depends on the Church system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-1 depends on a simplified Church system.

CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-1 depends on the Wos system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(y,z))) cnf(cn19, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(not(x),z))) cnf(cn37, axiom)
 is_a_theorem(implies(implies(x,implies(not(y),z)),implies(x,implies(implies(u,z),implies(implies(y,u),z))))) cnf(cn60, axiom)
 \neg is_a_theorem(implies(implies(a,b),implies(implies(b,c),implies(a,c)))) cnf(prove_cn1, negated_conjecture)

LCL071-1.p CN-2 depends on the Wos system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-2 depends on the Wos system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(y,z))) cnf(cn19, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(not(x),z))) cnf(cn37, axiom)
 is_a_theorem(implies(implies(x,implies(not(y),z)),implies(x,implies(implies(u,z),implies(implies(y,u),z))))) cnf(cn60, axiom)
 \neg is_a_theorem(implies(implies(not(a),a),a)) cnf(prove_cn2, negated_conjecture)

LCL072-1.p CN-3 depends on the Wos system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-3 depends on the Wos system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(y,z))) cnf(cn19, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(not(x),z))) cnf(cn37, axiom)
 is_a_theorem(implies(implies(x,implies(not(y),z)),implies(x,implies(implies(u,z),implies(implies(y,u),z))))) cnf(cn60, axiom)
 \neg is_a_theorem(implies(a,implies(not(a),b))) cnf(prove_cn3, negated_conjecture)

LCL073-1.p CN-1 depends on the single Meredith axiom

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-1 depends on the single Meredith axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(implies(implies(x,y),implies(not(z),not(u))),z),v),implies(implies(v,x),implies(u,x))))
 \neg is_a_theorem(implies(implies(a,b),implies(implies(b,c),implies(a,c)))) cnf(prove_cn1, negated_conjecture)

LCL074-1.p CN-2 depends on the single Meredith axiom

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-2 depends on the single Meredith axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(implies(implies(x,y),implies(not(z),not(u))),z),v),implies(implies(v,x),implies(u,x))))
 \neg is_a_theorem(implies(implies(not(a),a),a)) cnf(prove_cn2, negated_conjecture)

LCL075-1.p CN-3 depends on the single Meredith axiom

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-3 depends on the single Meredith axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(implies(implies(x,y),implies(not(z),not(u))),z),v),implies(implies(v,x),implies(u,x))))
 \neg is_a_theorem(implies(a,implies(not(a),b))) cnf(prove_cn3, negated_conjecture)

LCL076-1.p CN-40 depends on the Church system

Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18, CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith axiom. Show that CN-40 depends on the Church system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)

LCL079-1.p Transitivity can be derived from Church's system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-18,CN-35,CN-49 by Church. Show that transitivity of implies can be derived from the Church system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x,implies(y,x))) cnf(cn18, axiom)
 is_a_theorem(implies(implies(x,implies(y,z)),implies(implies(x,y),implies(x,z)))) cnf(cn35, axiom)
 is_a_theorem(implies(implies(y,x),implies(not(x),not(y)))) cnf(cn_49_reversed, axiom)
 is_a_theorem(implies(a,b)) cnf(a_implies_b, hypothesis)
 is_a_theorem(implies(b,c)) cnf(b_implies_c, hypothesis)
 \neg is_a_theorem(implies(a,c)) cnf(prove_transitivity, negated_conjecture)

LCL080-1.p The 1st Lukasiewicz axiom depends on Tarski-Bernays system

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that the 1st Lukasiewicz axiom depends on the Tarski-Bernays system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x,implies(y,x))) cnf(ic2, axiom)
 is_a_theorem(implies(implies(implies(x,y),x),x)) cnf(ic3, axiom)
 is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(ic4, axiom)
 \neg is_a_theorem(implies(implies(implies(a,b),c),implies(implies(c,a),implies(e,a)))) cnf(prove_ic_JLukasiewicz, negated_conjecture)

LCL080-2.p The 1st Lukasiewicz axiom depends on Tarski-Bernays system

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and Lukasiewicz axioms. Show that the 1st Lukasiewicz axiom depends on the Tarski-Bernays system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(x,x)) cnf(ic1, axiom)
 is_a_theorem(implies(x,implies(y,x))) cnf(ic2, axiom)
 is_a_theorem(implies(implies(implies(x,y),x),x)) cnf(ic3, axiom)
 is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(ic4, axiom)
 \neg is_a_theorem(implies(implies(implies(a,b),c),implies(implies(c,a),implies(e,a)))) cnf(prove_ic_JLukasiewicz, negated_conjecture)

LCL081-1.p IC-1 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-1 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(implies(z,x),implies(u,x)))) cnf(ic_JLukasiewicz, axiom)
 \neg is_a_theorem(implies(a,a)) cnf(prove_ic1, negated_conjecture)

LCL082-1.p IC-2 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-2 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(implies(z,x),implies(u,x)))) cnf(ic_JLukasiewicz, axiom)
 \neg is_a_theorem(implies(a,implies(b,a))) cnf(prove_ic2, negated_conjecture)

LCL083-1.p IC-3 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-3 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(implies(z,x),implies(u,x)))) cnf(ic_JLukasiewicz, axiom)
 \neg is_a_theorem(implies(implies(implies(a,b),a),a)) cnf(prove_ic3, negated_conjecture)

LCL083-2.p IC-3 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-3 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(x,y),z),implies(implies(z,x),implies(u,x)))) cnf(ic_JLukasiewicz, axiom)
 is_a_theorem(implies(x,x)) cnf(ic1, axiom)
 \neg is_a_theorem(implies(implies(implies(a,b),a),a)) cnf(prove_ic3, negated_conjecture)

LCL084-2.p IC-4 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-4 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)

is_a_theorem(implies(implies(implies(x, y), z), implies(implies(z, x), implies(u, x)))) cnf(ic_JLukasiewicz, axiom)
is_a_theorem(implies(x, x) cnf(ic₁, axiom)
¬ is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_ic₄, negated_conjecture)

LCL084-3.p IC-4 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-4 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(x, y), z), implies(implies(z, x), implies(u, x)))) cnf(ic_JLukasiewicz, axiom)
is_a_theorem(implies(x, x) cnf(ic₁, axiom)
is_a_theorem(implies(implies(implies(x, y), x), x)) cnf(ic₃, axiom)
¬ is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_ic₄, negated_conjecture)

LCL085-1.p IC-5 depends on the 1st Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-5 depends on the first Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(x, y), z), implies(implies(z, x), implies(u, x)))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies($a, implies(implies(a, b), b))) cnf(prove_ic₅, negated_conjecture)$

LCL086-1.p IC-1 depends on the 4th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-1 depends on the fourth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies($t, implies(implies(s, p), implies(r, p)))))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies(a, a)) cnf(prove_ic₁, negated_conjecture)$

LCL087-1.p IC-2 depends on the 4th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-2 depends on the fourth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies($t, implies(implies(s, p), implies(r, p)))))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies($a, implies(b, a)$)) cnf(prove_ic₂, negated_conjecture)$

LCL088-1.p IC-3 depends on the 4th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-3 depends on the fourth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies($t, implies(implies(s, p), implies(r, p)))))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies(implies(implies(a, b), a), a)) cnf(prove_ic₃, negated_conjecture)$

LCL089-1.p IC-4 depends on the 4th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-4 depends on the fourth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies($t, implies(implies(s, p), implies(r, p)))))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_ic₄, negated_conjecture)$

LCL090-1.p IC-1 depends on the 5th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-1 depends on the fifth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(implies(s, p), implies($t, implies(r, p)$)))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies(a, a)) cnf(prove_ic₁, negated_conjecture)

LCL091-1.p IC-2 depends on the 5th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-2 depends on the fifth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(implies(s, p), implies($t, implies(r, p)$)))) cnf(ic_JLukasiewicz, axiom)
¬ is_a_theorem(implies($a, implies(b, a)$)) cnf(prove_ic₂, negated_conjecture)

LCL092-1.p IC-3 depends on the 5th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2,IC-3,IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-3 depends on the fifth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(implies(s, p), implies(t, implies(r, p)))))) cnf(ic_JLukasiewicz)
 \neg is_a_theorem(implies(implies(implies(a, b), a), a)) cnf(prove_ic3, negated_conjecture)

LCL093-1.p IC-4 depends on the 5th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2, IC-3, IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-4 depends on the fifth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(implies(s, p), implies(t, implies(r, p)))))) cnf(ic_JLukasiewicz)
 \neg is_a_theorem(implies(implies(a, b), implies(implies(b, c), implies(a, c)))) cnf(prove_ic4, negated_conjecture)

LCL094-1.p IC-5 depends on the 4th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2, IC-3, IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-5 depends on the fourth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(t, implies(implies(s, p), implies(r, p)))))) cnf(ic_JLukasiewicz)
 \neg is_a_theorem(implies(a, implies(implies(a, b), b))) cnf(prove_ic5, negated_conjecture)

LCL095-1.p IC-5 depends on the 5th Lukasiewicz axiom

Axiomatisations of the Implicational propositional calculus are IC-2, IC-3, IC-4 by Tarski-Bernays and single Lukasiewicz axioms. Show that IC-5 depends on the fifth Lukasiewicz axiom.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(implies(implies(implies(p, q), implies(r, s)), implies(implies(s, p), implies(t, implies(r, p)))))) cnf(ic_JLukasiewicz)
 \neg is_a_theorem(implies(a, implies(implies(a, b), b))) cnf(prove_ic5, negated_conjecture)

LCL096-1.p LG-1 depends on LG-2, LG-3, LG-4

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-1 depends on a part of the Kalman system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), u), equivalent(equivalent(x, y), z), u))
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(x, y), z), u), equivalent(equivalent(equivalent(x, v), z), equivalent(equivalent(x, v), z), u))))
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, equivalent(equivalent(b, b), a)), c), c)) cnf(prove_lg1, negated_conjecture)

LCL097-1.p LG-4 depends on LG-2, LG-3

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-4 depends on a part of the Kalman system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), u), equivalent(equivalent(x, y), z), u))
 \neg is_a_theorem(equivalent(equivalent(equivalent(equivalent(a, b), c), e), equivalent(equivalent(equivalent(a, falsehood), c), equivalent(a, falsehood), c))))

LCL098-1.p LG-4 depends on LG-3

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-4 depends on LG-3.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), u), equivalent(equivalent(x, y), z), u))
 \neg is_a_theorem(equivalent(equivalent(equivalent(equivalent(a, b), c), e), equivalent(equivalent(equivalent(a, falsehood), c), equivalent(a, falsehood), c))))

LCL099-1.p LG-5 depends on the 1st McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-5 depends on the first McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), u), equivalent(equivalent(x, y), z), u))
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, equivalent(equivalent(b, a), c)), equivalent(equivalent(e, a), falsehood)), equivalent(a, falsehood), c))))

LCL100-1.p LG-3 depends on the 2nd McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-3 depends on the second McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg3, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), z), equivalent(equivalent(u, y), equivalent(equivalent(x, u), z)))) cnf(p1, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(equivalent(a, b), equivalent(a, c)), e), equivalent(equivalent(a, e), c)))))) cnf(p2, axiom)

LCL101-1.p P-1 depends on the 3rd McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that P-1 depends on the 3rd McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg3, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(equivalent(equivalent(y, z), equivalent(y, u)), equivalent(z, u)), x))) cnf(p1, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), c), equivalent(equivalent(e, b), equivalent(equivalent(a, e), c)))))) cnf(p2, axiom)

LCL102-1.p P-1 depends on the 4th McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that P-1 depends on the 4th McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z)), u), u)) cnf(lg3, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(y, z), equivalent(equivalent(z, y), x)))) cnf(q1, axiom)
 is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, x), equivalent(z, y)))) cnf(q2, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), c), equivalent(equivalent(e, b), equivalent(equivalent(a, e), c)))))) cnf(p1, axiom)

LCL103-1.p LG-2 depends on the 5th McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-2 depends on the 5th McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), z), equivalent(equivalent(u, y), equivalent(equivalent(x, u), z)))) cnf(p1, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(equivalent(y, x), z)), z)) cnf(q3, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(a, b), equivalent(a, c)), equivalent(b, c)), e), e)) cnf(p2, axiom)

LCL104-1.p P-1 depends on the 6th McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that P-1 depends on the sixth McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(equivalent(equivalent(y, z), equivalent(y, u)), equivalent(z, u)), x))) cnf(p1, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(equivalent(y, x), z)), z)) cnf(q3, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), c), equivalent(equivalent(e, b), equivalent(equivalent(a, e), c)))))) cnf(p2, axiom)

LCL105-1.p LG-2 depends on the 7th McCune system

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that LG-2 depends on the seventh McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(y, z), equivalent(equivalent(z, y), x)))) cnf(q1, axiom)
 is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, x), equivalent(z, y)))) cnf(q2, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(equivalent(y, x), z)), z)) cnf(q3, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(equivalent(equivalent(a, b), equivalent(a, c)), equivalent(b, c)), e), e)) cnf(p1, axiom)

LCL106-1.p Q-2 depends on Q-1, Q-4

Axiomatisations of the left group calculus are LG-1, LG-2, LG-3, LG-4, LG-5 by Kalman, LG-2, LG-3, LG-2, P-1, LG-2, P-4, LG-2, Q-1, Q-2, P-1, Q-3, P-4, Q-3, Q-1, Q-2, Q-3, Q-1, Q-3, Q-4, LG-27-1690 all by McCune. Show that Q-2 depends on Q-1 and Q-4.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(x , equivalent(equivalent(y, z), equivalent(equivalent(z, y), x)))) cnf(q1, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(y, z))) cnf(q4, axiom)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(equivalent(c, a), equivalent(c, b)))) cnf(prove_q2, negated_conjecture)

$\text{implies}(x, \text{big_hat}(y, z)) = \text{big_hat}(\text{implies}(x, y), \text{implies}(x, z))$ $\text{cnf}(\text{lemma7}, \text{axiom})$
 $\text{big_V}(\text{implies}(x, y), \text{implies}(y, x)) \neq \text{truth}$ $\text{cnf}(\text{prove_mv4}, \text{negated_conjecture})$

LCL109-6.p A theorem in the lattice structure of Wajsberg algebras

$\text{include}(\text{'Axioms/LCL002-0.ax'})$

$\text{xor}(x, y) = \text{xor}(y, x)$ $\text{cnf}(\text{xor_commutativity}, \text{axiom})$
 $\text{and_star}(\text{and_star}(x, y), z) = \text{and_star}(x, \text{and_star}(y, z))$ $\text{cnf}(\text{and_star_associativity}, \text{axiom})$
 $\text{and_star}(x, y) = \text{and_star}(y, x)$ $\text{cnf}(\text{and_star_commutativity}, \text{axiom})$
 $\text{not}(\text{truth}) = \text{falsehood}$ $\text{cnf}(\text{false_definition}, \text{axiom})$
 $\text{implies}(x, y) = \text{xor}(\text{truth}, \text{and_star}(x, \text{xor}(\text{truth}, y)))$ $\text{cnf}(\text{implies_definition}, \text{axiom})$
 $\text{implies}(\text{implies}(\text{implies}(a, b), \text{implies}(b, a)), \text{implies}(b, a)) \neq \text{truth}$ $\text{cnf}(\text{prove_wajsberg_mv4}, \text{negated_conjecture})$

LCL110-1.p MV-24 depends on the Merideth system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Show that MV24 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x)))$ $\text{cnf}(\text{mv1}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z))))$ $\text{cnf}(\text{mv2}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x)))$ $\text{cnf}(\text{mv3}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x)))$ $\text{cnf}(\text{mv5}, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{not}(\text{not}(a)), a))$ $\text{cnf}(\text{prove_mv24}, \text{negated_conjecture})$

LCL110-2.p MV-24 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-24 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$

$\text{implies}(\text{not}(\text{not}(x)), x) \neq \text{truth}$ $\text{cnf}(\text{prove_mv24}, \text{negated_conjecture})$

LCL111-1.p MV-25 depends on the Merideth system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Show that MV-25 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x)))$ $\text{cnf}(\text{mv1}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z))))$ $\text{cnf}(\text{mv2}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x)))$ $\text{cnf}(\text{mv3}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x)))$ $\text{cnf}(\text{mv5}, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(a, b), \text{implies}(\text{implies}(c, a), \text{implies}(c, b))))$ $\text{cnf}(\text{prove_mv25}, \text{negated_conjecture})$

LCL111-2.p MV-25 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-25 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$

$\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(z, x), \text{implies}(z, y))) \neq \text{truth}$ $\text{cnf}(\text{prove_mv25}, \text{negated_conjecture})$

LCL112-1.p MV-29 depends on the Merideth system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Show that 29 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x)))$ $\text{cnf}(\text{mv1}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z))))$ $\text{cnf}(\text{mv2}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x)))$ $\text{cnf}(\text{mv3}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x)))$ $\text{cnf}(\text{mv5}, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(a, \text{not}(\text{not}(a))))$ $\text{cnf}(\text{prove_mv29}, \text{negated_conjecture})$

LCL112-2.p MV-29 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-29 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$

$\text{implies}(x, \text{not}(\text{not}(x))) \neq \text{truth}$ $\text{cnf}(\text{prove_mv29}, \text{negated_conjecture})$

LCL113-1.p MV-33 depends on the Merideth system

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Show that 33 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y)$ $\text{cnf}(\text{condensed_detachment}, \text{axiom})$

$\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{mv}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x))) \quad \text{cnf}(\text{mv}_3, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_5, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(a), b), \text{implies}(\text{not}(b), a))) \quad \text{cnf}(\text{prove_mv}_{33}, \text{negated_conjecture})$

LCL113-2.p MV-33 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-33 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$
 $\text{implies}(\text{implies}(\text{not}(x), y), \text{implies}(\text{not}(y), x)) \neq \text{truth} \quad \text{cnf}(\text{prove_mv}_{33}, \text{negated_conjecture})$

LCL114-1.p MV-36 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Show that 36 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{mv}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x))) \quad \text{cnf}(\text{mv}_3, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_5, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(a, b), \text{implies}(\text{not}(b), \text{not}(a)))) \quad \text{cnf}(\text{prove_mv}_{36}, \text{negated_conjecture})$

LCL114-2.p MV-36 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-36 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$
 $\text{implies}(\text{implies}(x, y), \text{implies}(\text{not}(y), \text{not}(x))) \neq \text{truth} \quad \text{cnf}(\text{prove_mv}_{36}, \text{negated_conjecture})$

LCL115-1.p MV-39 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Show that 39 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{mv}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x))) \quad \text{cnf}(\text{mv}_3, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_5, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{not}(\text{implies}(a, b)), \text{not}(b))) \quad \text{cnf}(\text{prove_mv}_{39}, \text{negated_conjecture})$

LCL115-2.p MV-39 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-39 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$
 $\text{implies}(\text{not}(\text{implies}(a, b)), \text{not}(b)) \neq \text{truth} \quad \text{cnf}(\text{prove_mv}_{39}, \text{negated_conjecture})$

LCL116-1.p MV-50 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Show that 50 depends on the Meredith system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{mv}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(x, y), y), \text{implies}(\text{implies}(y, x), x))) \quad \text{cnf}(\text{mv}_3, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), \text{not}(y)), \text{implies}(y, x))) \quad \text{cnf}(\text{mv}_5, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{not}(a), \text{implies}(b, \text{not}(\text{implies}(b, a))))) \quad \text{cnf}(\text{prove_mv}_{50}, \text{negated_conjecture})$

LCL116-2.p MV-50 depends on the Meredith system

An axiomatisation of the many valued sentential calculus is MV-1, MV-2, MV-3, MV-5 by Meredith. Wajsberg presented an equality axiomatisation. Show that MV-50 depends on the Wajsberg axiomatisation.

$\text{include}(\text{'Axioms/LCL001-0.ax'})$
 $\text{implies}(\text{not}(a), \text{implies}(b, \text{not}(\text{implies}(b, a)))) \neq \text{truth} \quad \text{cnf}(\text{prove_mv}_{50}, \text{negated_conjecture})$

LCL117-1.p QYF depends on YQM

Single axioms for the R calculus are QYF, YQM, WO, all by Meredith and XGJ by Winker. Show that QYF depends on YQM.

$(\text{is_a_theorem}(\text{equivalent}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$

is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, y), equivalent(z, x)))) cnf(yqm, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent(a, b), equivalent(a, c)), equivalent(c, b))) cnf(prove_qyf, negated_conjecture)

LCL118-1.p YQM depends on WO

Single axioms for the R calculus are QYF, YQM, WO, all by Meredith and XGJ by Winker. Show that YQM depends on WO.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent($x, equivalent(y, z)$), equivalent($z, equivalent(y, x)$))) cnf(wo, axiom)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(equivalent(c, b), equivalent(c, a)))) cnf(prove_yqm, negated_conjecture)

LCL119-1.p WO depends on XGJ

Single axioms for the R calculus are QYF, YQM, WO, all by Meredith and XGJ by Winker. Show that WO depends on XGJ.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent($x, equivalent(equivalent(y, equivalent(z, x)), equivalent(y, z))$)) cnf(xgj, axiom)
 \neg is_a_theorem(equivalent(equivalent($a, equivalent(b, c)$), equivalent($c, equivalent(b, a)$))) cnf(prove_wo, negated_conjecture)

LCL120-1.p XGJ depends on QYF

Single axioms for the R calculus are QYF, YQM, WO, all by Meredith and XGJ by Winker. Show that XGJ depends on QYF.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(x, z)), equivalent(z, y))) cnf(qyf, axiom)
 \neg is_a_theorem(equivalent($a, equivalent(equivalent(b, equivalent(c, a)), equivalent(b, c))$)) cnf(prove_xgj, negated_conjecture)

LCL121-1.p LG-1 depends on LG-2

Kalman's axiomatisation of the right group calculus is LG-1, LG-2, LG-3, LG-4, LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2, Q-3, Q-3, Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-1 depends on LG-2.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent($x, equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(y, u), equivalent(z, u))))$)) cnf(lg1, axiom)
 \neg is_a_theorem(equivalent($a, equivalent(a, equivalent(equivalent(b, equivalent(c, c)), b))$)) cnf(prove_lg1, negated_conjecture)

LCL122-1.p LG-3 depends on LG-2

Kalman's axiomatisation of the right group calculus is LG-1, LG-2, LG-3, LG-4, LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2, Q-3, Q-3, Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-3 depends on LG-2.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent($x, equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(y, u), equivalent(z, u))))$)) cnf(lg3, axiom)
 \neg is_a_theorem(equivalent($a, equivalent(a, equivalent(equivalent(b, equivalent(c, e)), equivalent(b, equivalent(equivalent(c, f), e))$))

LCL123-1.p LG-4 depends on LG-2

Kalman's axiomatisation of the right group calculus is LG-1, LG-2, LG-3, LG-4, LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2, Q-3, Q-3, Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-4 depends on LG-2.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent($x, equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(y, u), equivalent(z, u))))$)) cnf(lg4, axiom)
 \neg is_a_theorem(equivalent(equivalent(equivalent($a, equivalent(b, c)$), equivalent($e, equivalent(b, f)$)), equivalent($a, equivalent(e, f)$))

LCL124-1.p LG-5 depends on LG-2

Kalman's axiomatisation of the right group calculus is LG-1, LG-2, LG-3, LG-4, LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2, Q-3, Q-3, Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-5 depends on LG-2.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent($x, equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(y, u), equivalent(z, u))))$)) cnf(lg5, axiom)
 \neg is_a_theorem(equivalent(equivalent($a, equivalent(b, equivalent(c, equivalent(e, f))$)), equivalent(equivalent($a, equivalent(f, c)$))

LCL125-1.p LG-2 depends on the 1st McCune system

Kalman's axiomatisation of the right group calculus is LG-1, LG-2, LG-3, LG-4, LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2, Q-3, Q-3, Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on the first McCune system.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
 is_a_theorem(equivalent(equivalent(equivalent(x, y), equivalent(z, y)), equivalent(x, z))) cnf(q_2 , axiom)
 is_a_theorem(equivalent($x, equivalent(equivalent(x, equivalent(y, z)), equivalent(z, y))$)) cnf(q_3 , axiom)
 \neg is_a_theorem(equivalent($a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))$)) cnf(p

LCL126-1.p Q-2 depends on the 2nd McCune system

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that Q-2 depends on the second McCune system.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x, equivalent(equivalent(x, equivalent(y, z)), equivalent(z, y))))      cnf(q3, axiom)
is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(x, z), equivalent(y, z))))      cnf(q4, axiom)
¬is_a_theorem(equivalent(equivalent(equivalent(a, b), equivalent(c, b)), equivalent(a, c)))      cnf(prove_q2, negated_conjecture)
```

LCL127-1.p LG-2 depends on S-2

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on S-2.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, equivalent(y, z)), equivalent(x, equivalent(equivalent(y, u), equivalent(z, u))))      cnf(s)
¬is_a_theorem(equivalent(a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))))      cnf(p)
```

LCL128-1.p LG-2 depends on S-3

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on S-3.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(x, equivalent(x, equivalent(equivalent(equivalent(y, z), equivalent(u, z)), equivalent(y, u))))      cnf(s)
¬is_a_theorem(equivalent(a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))))      cnf(p)
```

LCL129-1.p LG-2 depends on S-4

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on S-4.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, equivalent(y, z)), equivalent(equivalent(x, equivalent(u, z)), equivalent(y, u))))      cnf(s)
¬is_a_theorem(equivalent(a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))))      cnf(p)
```

LCL130-1.p LG-2 depends on P-4

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on P-4.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(y, u), equivalent(z, u))))), x)      cnf(p)
¬is_a_theorem(equivalent(a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))))      cnf(p)
```

LCL131-1.p LG-2 depends on S-6

Kalman's axiomatisation of the right group calculus is LG-1,LG-2,LG-3,LG-4,LG-5. McCune has shown that LG-2 is a single axiom. Other axiomatisations by McCune are Q-2,Q-3, Q-3,Q-4, S-2, S-3, S-4, P-4, S-6. Show that LG-2 depends on S-6.

```
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, equivalent(equivalent(equivalent(y, z), equivalent(u, z)), equivalent(y, u))), x)      cnf(s)
¬is_a_theorem(equivalent(a, equivalent(a, equivalent(equivalent(b, c), equivalent(equivalent(b, e), equivalent(c, e))))))      cnf(p)
```

LCL132-1.p A lemma in Wajsberg algebras

```
include('Axioms/LCL001-0.ax')
implies(x, x) ≠ truth      cnf(prove_wajsberg_lemma, negated_conjecture)
```

LCL133-1.p A lemma in Wajsberg algebras

```
include('Axioms/LCL001-0.ax')
implies(x, y) = implies(y, x)      cnf(lemma_antecedent, negated_conjecture)
x ≠ y      cnf(prove_wajsberg_lemma, negated_conjecture)
```

LCL134-1.p A lemma in Wajsberg algebras

```
include('Axioms/LCL001-0.ax')
implies(x, truth) ≠ truth      cnf(prove_wajsberg_lemma, negated_conjecture)
```

LCL135-1.p A lemma in Wajsberg algebras

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg provided a different axiomatisation. Show that MV-1 depends on the Wajsberg system.

include('Axioms/LCL001-0.ax')
 implies(x , implies(y , x)) \neq truth cnf(prove_wajsberg_lemma, negated_conjecture)

LCL136-1.p A lemma in Wajsberg algebras

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg provided a different axiomatisation. Show that a version of MV-2 depends on the Wajsberg system.

include('Axioms/LCL001-0.ax')
 implies(x , y) = implies(y , z) cnf(lemma_antecedent, negated_conjecture)
 implies(x , z) \neq truth cnf(prove_wajsberg_lemma, negated_conjecture)

LCL137-1.p A lemma in Wajsberg algebras

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg provided a different axiomatisation. Show that MV-3 depends on the Wajsberg system.

include('Axioms/LCL001-0.ax')
 implies(implies(implies(x , y), y), implies(implies(y , z), implies(x , z))) \neq truth cnf(prove_wajsberg_lemma, negated_conjecture)

LCL138-1.p A lemma in Wajsberg algebras

include('Axioms/LCL001-0.ax')
 implies(x , implies(y , z)) \neq implies(y , implies(x , z)) cnf(prove_wajsberg_lemma, negated_conjecture)

LCL139-1.p A lemma in Wajsberg algebras

include('Axioms/LCL001-0.ax')
 implies(x , not(truth)) \neq not(x) cnf(prove_wajsberg_lemma, negated_conjecture)

LCL140-1.p A lemma in Wajsberg algebras

include('Axioms/LCL001-0.ax')
 not(not(x)) \neq x cnf(prove_wajsberg_lemma, negated_conjecture)

LCL141-1.p A lemma in Wajsberg algebras

An axiomatisation of the many valued sentential calculus is MV-1,MV-2,MV-3,MV-5 by Meredith. Wajsberg provided a different axiomatisation. Show that MV-5 depends on the Wajsberg system.

include('Axioms/LCL001-0.ax')
 implies(not(x), not(y)) \neq implies(y , x) cnf(prove_wajsberg_lemma, negated_conjecture)

LCL142-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 ordered(x , y) cnf(antecedent, negated_conjecture)
 \neg ordered(implies(x , z), implies(y , z)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL143-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 ordered(x , y) cnf(antecedent, negated_conjecture)
 \neg ordered(implies(z , x), implies(z , y)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL144-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 ordered(x , implies(y , z)) or ordered(y , implies(x , z)) cnf(antecedent, negated_conjecture)
 ordered(x , implies(y , z)) \Rightarrow \neg ordered(y , implies(x , z)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL145-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 not(big-V(x , y)) \neq big-hat(not(x), not(y)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL146-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 not(big-hat(x , y)) \neq big-V(not(x), not(y)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL147-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')
 include('Axioms/LCL001-1.ax')
 implies(big-V(x , y), z) \neq big-hat(implies(x , z), implies(y , z)) cnf(prove_wajsberg_theorem, negated_conjecture)

LCL148-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-1.ax')

$\text{implies}(x, \text{big_hat}(y, z)) \neq \text{big_hat}(\text{implies}(x, y), \text{implies}(x, z))$ $\text{cnf}(\text{prove_wajsberg_theorem}, \text{negated_conjecture})$

LCL149-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-1.ax')

$\text{implies}(x, \text{big_V}(y, z)) \neq \text{big_V}(\text{implies}(x, y), \text{implies}(x, z))$ $\text{cnf}(\text{prove_wajsberg_theorem}, \text{negated_conjecture})$

LCL150-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-1.ax')

$\text{implies}(\text{big_hat}(x, y), z) \neq \text{big_V}(\text{implies}(x, z), \text{implies}(y, z))$ $\text{cnf}(\text{prove_wajsberg_theorem}, \text{negated_conjecture})$

LCL151-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-1.ax')

$\text{big_V}(\text{big_hat}(x, y), z) \neq \text{big_hat}(\text{big_V}(x, z), \text{big_V}(y, z))$ $\text{cnf}(\text{prove_wajsberg_theorem}, \text{negated_conjecture})$

LCL152-1.p A theorem in the lattice structure of Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-1.ax')

$\text{implies}(\text{big_hat}(x, y), z) \neq \text{implies}(\text{implies}(x, y), \text{implies}(x, z))$ $\text{cnf}(\text{prove_wajsberg_theorem}, \text{negated_conjecture})$

LCL153-1.p The 1st alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{not}(x) \neq \text{xor}(x, \text{truth})$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL154-1.p The 2nd alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{xor}(x, \text{falsehood}) \neq x$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL155-1.p The 3rd alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{xor}(x, x) \neq \text{falsehood}$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL156-1.p The 4th alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{and_star}(x, \text{truth}) \neq x$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL157-1.p The 5th alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{and_star}(x, \text{falsehood}) \neq \text{falsehood}$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL158-1.p The 6th alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{and_star}(\text{xor}(\text{truth}, x), x) \neq \text{falsehood}$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL159-1.p The 7th alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

$\text{xor}(x, \text{xor}(\text{truth}, y)) \neq \text{xor}(\text{xor}(x, \text{truth}), y)$ $\text{cnf}(\text{prove_alternative_wajsberg_axiom}, \text{negated_conjecture})$

LCL160-1.p The 8th alternative Wajsberg algebra axiom

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

include('Axioms/LCL002-1.ax')

and_star(xor(and_star(xor(truth, x), y), truth), y) ≠ and_star(xor(and_star(xor(truth, y), x), truth), x) cnf(prove_alternative_8, axiom)

LCL161-1.p The 1st Wajsberg algebra axiom, from the alternative axioms

include('Axioms/LCL002-0.ax')

xor(x, y) = xor(y, x) cnf(xor_commutativity, axiom)

and_star(and_star(x, y), z) = and_star(x, and_star(y, z)) cnf(and_star_associativity, axiom)

and_star(x, y) = and_star(y, x) cnf(and_star_commutativity, axiom)

not(truth) = falsehood cnf(false_definition, axiom)

implies(x, y) = xor(truth, and_star(x, xor(truth, y))) cnf(implies_definition, axiom)

implies(truth, x) ≠ x cnf(prove_wajsberg_axiom, negated_conjecture)

LCL162-1.p The 2nd Wajsberg algebra axiom, from the alternative axioms

include('Axioms/LCL002-0.ax')

xor(x, y) = xor(y, x) cnf(xor_commutativity, axiom)

and_star(and_star(x, y), z) = and_star(x, and_star(y, z)) cnf(and_star_associativity, axiom)

and_star(x, y) = and_star(y, x) cnf(and_star_commutativity, axiom)

not(truth) = falsehood cnf(false_definition, axiom)

implies(x, y) = xor(truth, and_star(x, xor(truth, y))) cnf(implies_definition, axiom)

implies(implies(x, y), implies(implies(y, z), implies(x, z))) ≠ truth cnf(prove_wajsberg_axiom, negated_conjecture)

LCL163-1.p The 3rd Wajsberg algebra axiom, from the alternative axioms

include('Axioms/LCL002-0.ax')

xor(x, y) = xor(y, x) cnf(xor_commutativity, axiom)

and_star(and_star(x, y), z) = and_star(x, and_star(y, z)) cnf(and_star_associativity, axiom)

and_star(x, y) = and_star(y, x) cnf(and_star_commutativity, axiom)

not(truth) = falsehood cnf(false_definition, axiom)

implies(x, y) = xor(truth, and_star(x, xor(truth, y))) cnf(implies_definition, axiom)

implies(implies(x, y), y) ≠ implies(implies(y, x), x) cnf(prove_wajsberg_axiom, negated_conjecture)

LCL164-1.p The 4th Wajsberg algebra axiom, from the alternative axioms

include('Axioms/LCL002-0.ax')

xor(x, y) = xor(y, x) cnf(xor_commutativity, axiom)

and_star(and_star(x, y), z) = and_star(x, and_star(y, z)) cnf(and_star_associativity, axiom)

and_star(x, y) = and_star(y, x) cnf(and_star_commutativity, axiom)

not(truth) = falsehood cnf(false_definition, axiom)

implies(x, y) = xor(truth, and_star(x, xor(truth, y))) cnf(implies_definition, axiom)

implies(implies(not(x), not(y)), implies(y, x)) ≠ truth cnf(prove_wajsberg_axiom, negated_conjecture)

LCL165-1.p A theorem in Wajsberg algebras

include('Axioms/LCL001-0.ax')

include('Axioms/LCL001-2.ax')

not(or(and(x, or(x, x)), and(x, x))) ≠ and(not(x), or(or(not(x), not(x)), and(not(x), not(x)))) cnf(prove_wajsberg_theorem, axiom)

LCL166-1.p UM depends on XHN

Show that the single Meredith axiom UM can be derived from the single Winker axiom XHN.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)

is_a_theorem(equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(z, x), y)))) cnf(xhn, axiom)

¬ is_a_theorem(equivalent(equivalent(equivalent(a, b), c), equivalent(b, equivalent(c, a)))) cnf(prove_um, negated_conjecture)

LCL167-1.p YRO depends on XHK

Show that the single Meredith axiom YRO can be derived from the single Winker axiom XHK.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)

is_a_theorem(equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(x, z), y)))) cnf(xhk, axiom)

¬ is_a_theorem(equivalent(equivalent(a, b), equivalent(c, equivalent(equivalent(c, b), a)))) cnf(prove_yro, negated_conjecture)

LCL168-1.p XEH is not a single axiom for the R-calculus

To show that XEH is not a single axiom, attempt to derive from it any one of YQM, WO, XGJ or QYF, which are known single axioms.

(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y) cnf(condensed_detachment, axiom)

is_a_theorem(equivalent(x, equivalent(equivalent(y, equivalent(equivalent(y, z), x)), z))) cnf(xeh, axiom)

\neg is_a_theorem(equivalent(equivalent(equivalent(a, b), equivalent(a, c)), equivalent(c, b))) cnf(try_prove_qyf, negated_conjecture)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(equivalent(c, b), equivalent(c, a)))) cnf(try_prove_yqm, negated_conjecture)
 \neg is_a_theorem(equivalent(equivalent(a, b), equivalent(c, a)), equivalent(c, b))) cnf(try_prove_wo, negated_conjecture)
 \neg is_a_theorem(equivalent(a, b), equivalent(equivalent(b, c), equivalent(c, a))) cnf(try_prove_xgj, negated_conjecture)

LCL169-1.p Principia Mathematica 2.01

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(p), not(p))), not(p))) cnf(prove_this, negated_conjecture)

LCL169-3.p Principia Mathematica 2.01

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(p , not(p)), not(p))) cnf(prove_this, negated_conjecture)

LCL170-1.p Principia Mathematica 2.02

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(q), or(not(p), q))) cnf(prove_this, negated_conjecture)

LCL170-3.p Principia Mathematica 2.02

include('Axioms/LCL004-0.ax')

\neg theorem(implies(q , implies(p , q))) cnf(prove_this, negated_conjecture)

LCL171-1.p Principia Mathematica 2.03

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(p), not(q))), or(not(q), not(p)))) cnf(prove_this, negated_conjecture)

LCL171-3.p Principia Mathematica 2.03

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(p , not(q)), implies(q , not(p)))) cnf(prove_this, negated_conjecture)

LCL172-1.p Principia Mathematica 2.04

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(p), or(not(q), r))), or(not(q), or(not(p), r)))) cnf(prove_this, negated_conjecture)

LCL172-3.p Principia Mathematica 2.04

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(p , implies(q , r)), implies(q , implies(p , r)))) cnf(prove_this, negated_conjecture)

LCL173-1.p Principia Mathematica 2.05

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(q), r)), or(not(or(not(p), q)), or(not(p), r)))) cnf(prove_this, negated_conjecture)

LCL173-3.p Principia Mathematica 2.05

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(q , r), implies(implies(p , q), implies(p , r)))) cnf(prove_this, negated_conjecture)

LCL174-1.p Principia Mathematica 2.06

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(p), q)), or(not(or(not(q), r)), or(not(p), r)))) cnf(prove_this, negated_conjecture)

LCL174-3.p Principia Mathematica 2.06

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(p , q), implies(implies(q , r), implies(p , r)))) cnf(prove_this, negated_conjecture)

LCL175-1.p Principia Mathematica 2.07

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(p), or(p , p))) cnf(prove_this, negated_conjecture)

LCL175-3.p Principia Mathematica 2.07

include('Axioms/LCL004-0.ax')

\neg theorem(implies(p , or(p , p))) cnf(prove_this, negated_conjecture)

LCL176-1.p Principia Mathematica 2.1 and 2.08

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(p), p)) cnf(prove_this, negated_conjecture)

LCL176-3.p Principia Mathematica 2.1 and 2.08

include('Axioms/LCL004-0.ax')

\neg theorem(implies(p , p)) cnf(prove_this, negated_conjecture)

LCL177-1.p Principia Mathematica 2.11

include('Axioms/LCL003-0.ax')
 \neg theorem(or(p , not(p))) cnf(prove_this, negated_conjecture)

LCL178-1.p Principia Mathematica 2.12

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(p), not(not(p)))) cnf(prove_this, negated_conjecture)

LCL178-3.p Principia Mathematica 2.12

include('Axioms/LCL004-0.ax')
 \neg theorem(implies(p , not(not(p)))) cnf(prove_this, negated_conjecture)

LCL179-1.p Principia Mathematica 2.13

include('Axioms/LCL003-0.ax')
 \neg theorem(or(p , not(not(not(p)))))) cnf(prove_this, negated_conjecture)

LCL180-1.p Principia Mathematica 2.14

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(not(not(p))), p)) cnf(prove_this, negated_conjecture)

LCL180-3.p Principia Mathematica 2.14

include('Axioms/LCL004-0.ax')
 \neg theorem(implies(not(not(p)), p)) cnf(prove_this, negated_conjecture)

LCL181+1.p Principia Mathematica 2.15

Judged by [SRM73] to be the 'hardest' of the first 52 theorems of [WR27].

$(\neg p \Rightarrow q) \iff (\neg q \Rightarrow p)$ fof(pel₄, conjecture)

LCL181-1.p Principia Mathematica 2.15

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(not(p)), q)), or(not(not(q)), p))) cnf(prove_this, negated_conjecture)

LCL181-2.p Principia Mathematica 2.15

Judged by [SRM73] to be the 'hardest' of the first 52 theorems of [WR27].

p or q cnf(clause₁, negated_conjecture)
 $q \Rightarrow \neg p$ cnf(clause₂, negated_conjecture)
 $\neg q$ cnf(clause₃, negated_conjecture)
 $\neg p$ cnf(clause₄, negated_conjecture)

LCL181-3.p Principia Mathematica 2.15

include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(not(p), q), implies(not(q), p))) cnf(prove_this, negated_conjecture)

LCL181^4.p Principia Mathematica 2.15

include('Axioms/LCL010^0.ax')

p : $\$i \rightarrow \o thf(p_type, type)

q : $\$i \rightarrow \o thf(q_type, type)

ivalid@(iequiv@(iimplies@(inot@(iatom@ p))@(iatom@ q))@(iimplies@(inot@(iatom@ q))@(iatom@ p))) thf(pel₄, conjecture)

LCL182-1.p Principia Mathematica 2.16

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(p), q)), or(not(not(q)), not(p)))) cnf(prove_this, negated_conjecture)

LCL182-3.p Principia Mathematica 2.16

include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(p , q), implies(not(q), not(p)))) cnf(prove_this, negated_conjecture)

LCL183-1.p Principia Mathematica 2.17

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(not(q)), not(p))), or(not(p), q))) cnf(prove_this, negated_conjecture)

LCL183-3.p Principia Mathematica 2.17

include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(not(q), not(p)), implies(p , q))) cnf(prove_this, negated_conjecture)

LCL184-1.p Principia Mathematica 2.18

include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(not(p)), p)), p)) cnf(prove_this, negated_conjecture)

LCL185-1.p Principia Mathematica 2.2

include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(p), or(p, q))) cnf(prove_this, negated_conjecture)

LCL185-3.p Principia Mathematica 2.2
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(p , or(p, q))) cnf(prove_this, negated_conjecture)

LCL186-1.p Principia Mathematica 2.21
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(not(p)), or(not(p, q)))) cnf(prove_this, negated_conjecture)

LCL186-3.p Principia Mathematica 2.21
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(not(p), implies(p, q))) cnf(prove_this, negated_conjecture)

LCL187-1.p Principia Mathematica 2.24
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(p), or(not(not(p), q)))) cnf(prove_this, negated_conjecture)

LCL187-3.p Principia Mathematica 2.24
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(p , implies(not(p), q))) cnf(prove_this, negated_conjecture)

LCL188-1.p Principia Mathematica 2.25
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(p , or(not(or(p, q), q)))) cnf(prove_this, negated_conjecture)

LCL188-3.p Principia Mathematica 2.25
 include('Axioms/LCL004-0.ax')
 ¬ theorem(or(p , implies(or(p, q), q))) cnf(prove_this, negated_conjecture)

LCL189-1.p Principia Mathematica 2.26 and 2.27
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(p), or(not(or(not(p), q), q)))) cnf(prove_this, negated_conjecture)

LCL189-3.p Principia Mathematica 2.26
 include('Axioms/LCL004-0.ax')
 ¬ theorem(or(not(p), implies(implies(p, q), q))) cnf(prove_this, negated_conjecture)

LCL190-1.p Principia Mathematica 2.3
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(or(p , or(q, r))), or(p , or(r, q)))) cnf(prove_this, negated_conjecture)

LCL190-3.p Principia Mathematica 2.3
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(or(p , or(q, r)), or(p , or(r, q)))) cnf(prove_this, negated_conjecture)

LCL191-1.p Principia Mathematica 2.31
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(or(p , or(q, r))), or(or(p, q), r))) cnf(prove_this, negated_conjecture)

LCL191-3.p Principia Mathematica 2.31
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(or(p , or(q, r)), or(or(p, q), r))) cnf(prove_this, negated_conjecture)

LCL192-1.p Principia Mathematica 2.32 and 2.33
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(or(or(p, q), r)), or(p , or(q, r)))) cnf(prove_this, negated_conjecture)

LCL192-3.p Principia Mathematica 2.32
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(or(or(p, q), r), or(p , or(q, r)))) cnf(prove_this, negated_conjecture)

LCL193-1.p Principia Mathematica 2.36
 include('Axioms/LCL003-0.ax')
 ¬ theorem(or(not(or(not(q), r)), or(not(or(p, q), or(r, p)))) cnf(prove_this, negated_conjecture)

LCL193-3.p Principia Mathematica 2.36
 include('Axioms/LCL004-0.ax')
 ¬ theorem(implies(implies(q, r), implies(or(p, q), or(r, p)))) cnf(prove_this, negated_conjecture)

LCL194-1.p Principia Mathematica 2.37
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(\text{not}(q), r)), \text{or}(\text{not}(\text{or}(q, p)), \text{or}(p, r)))$) cnf(prove_this, negated_conjecture)

LCL194-3.p Principia Mathematica 2.37
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{implies}(q, r), \text{implies}(\text{or}(q, p), \text{or}(p, r)))$) cnf(prove_this, negated_conjecture)

LCL195-1.p Principia Mathematica 2.38
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(\text{not}(q), r)), \text{or}(\text{not}(\text{or}(q, p)), \text{or}(r, p)))$) cnf(prove_this, negated_conjecture)

LCL195-3.p Principia Mathematica 2.38
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{implies}(q, r), \text{implies}(\text{or}(q, p), \text{or}(r, p)))$) cnf(prove_this, negated_conjecture)

LCL196-1.p Principia Mathematica 2.4
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(p, \text{or}(p, q))), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)

LCL196-3.p Principia Mathematica 2.4
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{or}(p, \text{or}(p, q)), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)

LCL197-1.p Principia Mathematica 2.41
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(q, \text{or}(p, q))), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)

LCL197-3.p Principia Mathematica 2.41
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{or}(q, \text{or}(p, q)), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)

LCL198-1.p Principia Mathematica 2.42 and 2.43
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(\text{not}(p), \text{or}(\text{not}(p), q))), \text{or}(\text{not}(p), q))$) cnf(prove_this, negated_conjecture)

LCL198-3.p Principia Mathematica 2.42
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{or}(\text{not}(p), \text{implies}(p, q)), \text{implies}(p, q))$) cnf(prove_this, negated_conjecture)

LCL199-1.p Principia Mathematica 2.45
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(p, q))), \text{not}(p))$) cnf(prove_this, negated_conjecture)

LCL199-3.p Principia Mathematica 2.45
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{or}(p, q)), \text{not}(p))$) cnf(prove_this, negated_conjecture)

LCL200-1.p Principia Mathematica 2.46
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(p, q))), \text{not}(q))$) cnf(prove_this, negated_conjecture)

LCL201-1.p Principia Mathematica 2.47
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(p, q))), \text{or}(\text{not}(p), q))$) cnf(prove_this, negated_conjecture)

LCL201-3.p Principia Mathematica 2.47
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{or}(p, q)), \text{or}(\text{not}(p), q))$) cnf(prove_this, negated_conjecture)

LCL202-1.p Principia Mathematica 2.48
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(p, q))), \text{or}(p, \text{not}(q)))$) cnf(prove_this, negated_conjecture)

LCL202-3.p Principia Mathematica 2.48
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{or}(p, q)), \text{or}(p, \text{not}(q)))$) cnf(prove_this, negated_conjecture)

LCL203-1.p Principia Mathematica 2.49
include('Axioms/LCL003-0.ax')

\neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(p, q))), \text{or}(\text{not}(p), \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL203-3.p Principia Mathematica 2.49
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{or}(p, q)), \text{or}(\text{not}(p), \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL204-1.p Principia Mathematica 2.5
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(\text{not}(p), q))), \text{or}(\text{not}(\text{not}(p)), q))$) cnf(prove_this, negated_conjecture)
LCL205-1.p Principia Mathematica 2.51
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(\text{not}(p), q))), \text{or}(\text{not}(p), \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL205-3.p Principia Mathematica 2.51
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{implies}(p, q)), \text{implies}(p, \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL206-1.p Principia Mathematica 2.52
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(\text{not}(p), q))), \text{or}(\text{not}(\text{not}(p)), \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL206-3.p Principia Mathematica 2.52
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{implies}(p, q)), \text{implies}(\text{not}(p), \text{not}(q)))$) cnf(prove_this, negated_conjecture)
LCL207-1.p Principia Mathematica 2.521
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(\text{or}(\text{not}(p), q))), \text{or}(\text{not}(q), p))$) cnf(prove_this, negated_conjecture)
LCL207-3.p Principia Mathematica 2.521
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(\text{implies}(p, q)), \text{implies}(q, p))$) cnf(prove_this, negated_conjecture)
LCL208-1.p Principia Mathematica 2.53
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(p, q)), \text{or}(\text{not}(\text{not}(p)), q))$) cnf(prove_this, negated_conjecture)
LCL208-3.p Principia Mathematica 2.53
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{or}(p, q), \text{implies}(\text{not}(p), q))$) cnf(prove_this, negated_conjecture)
LCL209-1.p Principia Mathematica 2.54
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(\text{not}(\text{not}(p)), q)), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)
LCL209-3.p Principia Mathematica 2.54
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{implies}(\text{not}(p), q), \text{or}(p, q))$) cnf(prove_this, negated_conjecture)
LCL210-1.p Principia Mathematica 2.55
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(p)), \text{or}(\text{not}(\text{or}(p, q)), q))$) cnf(prove_this, negated_conjecture)
LCL210-3.p Principia Mathematica 2.55
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(p), \text{implies}(\text{or}(p, q), q))$) cnf(prove_this, negated_conjecture)
LCL211-1.p Principia Mathematica 2.56
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{not}(q)), \text{or}(\text{not}(\text{or}(p, q)), p))$) cnf(prove_this, negated_conjecture)
LCL211-3.p Principia Mathematica 2.56
include('Axioms/LCL004-0.ax')
 \neg theorem($\text{implies}(\text{not}(q), \text{implies}(\text{or}(p, q), p))$) cnf(prove_this, negated_conjecture)
LCL212-1.p Principia Mathematica 2.6
include('Axioms/LCL003-0.ax')
 \neg theorem($\text{or}(\text{not}(\text{or}(\text{not}(\text{not}(p)), q)), \text{or}(\text{not}(\text{or}(\text{not}(p), q)), q))$) cnf(prove_this, negated_conjecture)
LCL212-3.p Principia Mathematica 2.6

```

include('Axioms/LCL004-0.ax')
¬ theorem(implies(not(p), implies(q, implies(implies(p, q), q))))    cnf(prove_this, negated_conjecture)

LCL213-1.p Principia Mathematica 2.61
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(p), q)), or(not(or(not(not(p)), q)), q)))    cnf(prove_this, negated_conjecture)

LCL213-3.p Principia Mathematica 2.61
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(p, q), implies(implies(not(p), q), q)))    cnf(prove_this, negated_conjecture)

LCL214-1.p Principia Mathematica 2.61
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(p, q)), or(not(or(not(p), q)), q)))    cnf(prove_this, negated_conjecture)

LCL214-3.p Principia Mathematica 2.61
include('Axioms/LCL004-0.ax')
¬ theorem(implies(or(p, q), implies(implies(p, q), q)))    cnf(prove_this, negated_conjecture)

LCL215-1.p Principia Mathematica 2.62 and 2.63
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(p), q)), or(not(or(p, q), q)))    cnf(prove_this, negated_conjecture)

LCL215-3.p Principia Mathematica 2.621
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(p, q), implies(or(p, q), q)))    cnf(prove_this, negated_conjecture)

LCL216-1.p Principia Mathematica 2.64
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(p, q)), or(not(or(p, not(q))), p)))    cnf(prove_this, negated_conjecture)

LCL216-3.p Principia Mathematica 2.64
include('Axioms/LCL004-0.ax')
¬ theorem(implies(or(p, q), implies(or(p, not(q)), p)))    cnf(prove_this, negated_conjecture)

LCL217-1.p Principia Mathematica 2.65
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(p), q)), or(not(or(not(p), not(q))), not(p))))    cnf(prove_this, negated_conjecture)

LCL217-3.p Principia Mathematica 2.65
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(p, q), implies(implies(p, not(q)), not(p))))    cnf(prove_this, negated_conjecture)

LCL218-1.p Principia Mathematica 2.67
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(or(p, q)), q)), or(not(p), q)))    cnf(prove_this, negated_conjecture)

LCL218-3.p Principia Mathematica 2.67
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(or(p, q), q), implies(p, q)))    cnf(prove_this, negated_conjecture)

LCL219-1.p Principia Mathematica 2.68
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(or(not(p), q)), q)), or(p, q)))    cnf(prove_this, negated_conjecture)

LCL219-3.p Principia Mathematica 2.68
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(implies(p, q), q), or(p, q)))    cnf(prove_this, negated_conjecture)

LCL220-1.p Principia Mathematica 2.69
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(or(not(p), q)), q)), or(not(or(not(q), p)), p)))    cnf(prove_this, negated_conjecture)

LCL220-3.p Principia Mathematica 2.69
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(implies(p, q), q), implies(implies(q, p), p)))    cnf(prove_this, negated_conjecture)

LCL221-1.p Principia Mathematica 2.73
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(p), q)), or(not(or(or(p, q), r)), or(q, r))))    cnf(prove_this, negated_conjecture)

```

LCL221-3.p Principia Mathematica 2.73
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(p, q), implies(or(or(p, q), r), or(q, r)))) cnf(prove_this, negated_conjecture)

LCL222-1.p Principia Mathematica 2.74
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(q, p)), or(not(or(or(p, q, r)), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL222-3.p Principia Mathematica 2.74
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(q, p), implies(or(or(p, q, r), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL223-1.p Principia Mathematica 2.75
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(p, q)), or(not(or(p, r), or(not(q, r))), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL223-3.p Principia Mathematica 2.75
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(or(p, q), implies(or(p, r), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL224-1.p Principia Mathematica 2.76
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(p, r), or(not(q, r))), or(not(or(p, q), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL224-3.p Principia Mathematica 2.76
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(or(p, r), implies(or(p, q), or(p, r)))))) cnf(prove_this, negated_conjecture)

LCL225-1.p Principia Mathematica 2.77
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(p), or(not(q, r))), or(not(or(not(p, q), or(not(p, r)))))))) cnf(prove_this, negated_conjecture)

LCL225-3.p Principia Mathematica 2.77
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(p, r), implies(implies(p, q), implies(p, r)))))) cnf(prove_this, negated_conjecture)

LCL226-1.p Principia Mathematica 2.8
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(q, r)), or(not(or(not(r, s)), or(q, s)))))) cnf(prove_this, negated_conjecture)

LCL226-3.p Principia Mathematica 2.8
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(or(q, r), implies(or(not(r, s), or(q, s)))))) cnf(prove_this, negated_conjecture)

LCL227-1.p Principia Mathematica 2.81
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(q), or(not(r, s))), or(not(or(p, q), or(not(or(p, r), or(p, s)))))))) cnf(prove_this, negated_conjecture)

LCL227-3.p Principia Mathematica 2.81
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(q, r), implies(or(p, q), implies(or(p, r), or(p, s)))))) cnf(prove_this, negated_conjecture)

LCL228-1.p Principia Mathematica 2.82
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(or(p, q, r)), or(not(or(or($p, not(r), s$)), or(or(p, q, s)))))))) cnf(prove_this, negated_conjecture)

LCL228-3.p Principia Mathematica 2.82
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(or(or(p, q, r), implies(or(or($p, not(r), s$), or(or(p, q, s)))))))) cnf(prove_this, negated_conjecture)

LCL229-1.p Principia Mathematica 2.83
include('Axioms/LCL003-0.ax')
 \neg theorem(or(not(or(not(p), or(not(q, r))), or(not(or(not(p), or(not(r, s))), or(not(p), or(not(q, s)))))))) cnf(prove_this, negated_conjecture)

LCL229-3.p Principia Mathematica 2.83
include('Axioms/LCL004-0.ax')
 \neg theorem(implies(implies(p, r), implies(implies(p, r), implies(p, r)))))) cnf(prove_this, negated_conjecture)

LCL230+1.p Principia Mathematica 2.85
Judged by [SRM73] to be the 'hardest' of the first 67 theorems of [WR27].

$((p \text{ or } q) \Rightarrow (p \text{ or } r)) \Rightarrow (p \text{ or } (q \Rightarrow r))$ fof(pel₅, conjecture)

LCL230-1.p Principia Mathematica 2.85

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(or(p, q)), or(p, r))), or(p, or(not(q, r)))) cnf(prove_this, negated_conjecture)

LCL230-2.p Principia Mathematica 2.85

Judged by [SRM73] to be the 'hardest' of the first 67 theorems of [WR27].

$q \Rightarrow (p \text{ or } r)$ cnf(clause₁, negated_conjecture)

$\neg p$ cnf(clause₂, negated_conjecture)

q cnf(clause₃, negated_conjecture)

$\neg r$ cnf(clause₄, negated_conjecture)

LCL230-3.p Principia Mathematica 2.85

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(or(p, q), or(p, r)), or(p, implies(q, r)))) cnf(prove_this, negated_conjecture)

LCL230^4.p Principia Mathematica 2.85

include('Axioms/LCL010^0.ax')

$p: \$i \rightarrow \o thf(p_type, type)

$q: \$i \rightarrow \o thf(q_type, type)

$r: \$i \rightarrow \o thf(r_type, type)

ivalid@(iimplies@(iimplies@(ior@(iatom@p)@(iatom@q)@(ior@(iatom@p)@(iatom@r))@(ior@(iatom@p)@(iimplies@(iatom

LCL231-1.p Principia Mathematica 2.86

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(or(not(p), q)), or(not(p), r))), or(not(p), or(not(q), r)))) cnf(prove_this, negated_conjecture)

LCL231-3.p Principia Mathematica 2.86

include('Axioms/LCL004-0.ax')

\neg theorem(implies(implies(implies(p, q), implies(p, r)), implies(p, implies(q, r)))) cnf(prove_this, negated_conjecture)

LCL234-1.p Principia Mathematica 3.2 and 3.12

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(p), or(not(q), not(or(not(p), not(q)))))) cnf(prove_this, negated_conjecture)

LCL234-3.p Principia Mathematica 3.2

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

\neg theorem(implies(p, implies(q, and(p, q)))) cnf(prove_this, negated_conjecture)

LCL235-1.p Principia Mathematica 3.13

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(not(not(or(not(p), not(q))))), or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL235-3.p Principia Mathematica 3.13

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

\neg theorem(implies(not(and(p, q)), or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL236-1.p Principia Mathematica 3.14

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(or(not(p), not(q)), not(not(or(not(p), not(q)))))) cnf(prove_this, negated_conjecture)

LCL236-3.p Principia Mathematica 3.14

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

\neg theorem(implies(or(not(p), not(q)), not(and(p, q)))) cnf(prove_this, negated_conjecture)

LCL237-1.p Principia Mathematica 3.21

include('Axioms/LCL003-0.ax')

\neg theorem(or(not(q), or(not(p), not(or(not(p), not(q)))))) cnf(prove_this, negated_conjecture)

LCL237-3.p Principia Mathematica 3.21

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

\neg theorem(implies(q, implies(p, and(p, q)))) cnf(prove_this, negated_conjecture)

LCL238-1.p Principia Mathematica 3.22

```

include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(p), not(q))), not(or(not(q), not(p))))))    cnf(prove_this, negated_conjecture)

LCL238-3.p Principia Mathematica 3.22
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(p, q), and(q, p)))    cnf(prove_this, negated_conjecture)

LCL239-1.p Principia Mathematica 3.24
include('Axioms/LCL003-0.ax')
¬ theorem(not(not(or(not(p), not(not(p))))))    cnf(prove_this, negated_conjecture)

LCL239-3.p Principia Mathematica 3.24
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(not(and(p, not(p))))    cnf(prove_this, negated_conjecture)

LCL240-1.p Principia Mathematica 3.26
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(p), not(q))), p))    cnf(prove_this, negated_conjecture)

LCL240-3.p Principia Mathematica 3.26
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(p, q), p))    cnf(prove_this, negated_conjecture)

LCL241-1.p Principia Mathematica 3.27
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(p), not(q))), q))    cnf(prove_this, negated_conjecture)

LCL241-3.p Principia Mathematica 3.27
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(p, q), q))    cnf(prove_this, negated_conjecture)

LCL242-1.p Principia Mathematica 3.3
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(not(or(not(p), not(q))), r)), or(not(p), or(not(q), r))))    cnf(prove_this, negated_conjecture)

LCL242-3.p Principia Mathematica 3.3
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies(and(p, q), r), implies(p, implies(q, r))))    cnf(prove_this, negated_conjecture)

LCL243-1.p Principia Mathematica 3.31
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(p), or(not(q), r))), or(not(not(or(not(p), not(q))), r))))    cnf(prove_this, negated_conjecture)

LCL243-3.p Principia Mathematica 3.31
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies(p, implies(q, r)), implies(and(p, q), r)))    cnf(prove_this, negated_conjecture)

LCL244-1.p Principia Mathematica 3.33
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not(p), q)), not(or(not(q), r))))), or(not(p), r)))    cnf(prove_this, negated_conjecture)

LCL245-1.p Principia Mathematica 3.34
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not(q), r)), not(or(not(p), q))))), or(not(p), r)))    cnf(prove_this, negated_conjecture)

LCL245-3.p Principia Mathematica 3.34
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(implies(q, r), implies(p, q)), implies(p, r)))    cnf(prove_this, negated_conjecture)

LCL246-1.p Principia Mathematica 3.35
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(p), not(or(not(p), q))))), q))    cnf(prove_this, negated_conjecture)

```

LCL246-3.p Principia Mathematica 3.35

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and( $p$ , implies( $p$ ,  $q$ )),  $q$ ))      cnf(prove_this, negated_conjecture)
```

LCL247-1.p Principia Mathematica 3.37

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not(not(or(not( $p$ ), not( $q$ ))),  $r$ )), or(not(not(or(not( $p$ ), not(not( $r$ ))))), not( $q$ ))))      cnf(prove_this, negated_conjecture)
```

LCL247-3.p Principia Mathematica 3.37

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies(and( $p$ ,  $q$ ),  $r$ ), implies(and( $p$ , not( $r$ )), not( $q$ ))))      cnf(prove_this, negated_conjecture)
```

LCL248-1.p Principia Mathematica 3.4

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not( $p$ ), not( $q$ ))), or(not( $p$ ),  $q$ )))      cnf(prove_this, negated_conjecture)
```

LCL248-3.p Principia Mathematica 3.4

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and( $p$ ,  $q$ ), implies( $p$ ,  $q$ )))      cnf(prove_this, negated_conjecture)
```

LCL249-1.p Principia Mathematica 3.41

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not( $p$ ),  $r$ )), or(not(not(or(not( $p$ ), not( $q$ ))),  $r$ )))      cnf(prove_this, negated_conjecture)
```

LCL249-3.p Principia Mathematica 3.41

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies( $p$ ,  $r$ ), implies(and( $p$ ,  $q$ ),  $r$ )))      cnf(prove_this, negated_conjecture)
```

LCL250-1.p Principia Mathematica 3.42

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not( $q$ ),  $r$ )), or(not(not(or(not( $p$ ), not( $q$ ))),  $r$ )))      cnf(prove_this, negated_conjecture)
```

LCL250-3.p Principia Mathematica 3.42

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies( $q$ ,  $r$ ), implies(and( $p$ ,  $q$ ),  $r$ )))      cnf(prove_this, negated_conjecture)
```

LCL251-1.p Principia Mathematica 3.43

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not( $p$ ),  $q$ )), not(or(not( $p$ ),  $r$ ))))), or(not( $p$ ), not(or(not( $q$ ), not( $r$ ))))))      cnf(prove_this, negated_conjecture)
```

LCL251-3.p Principia Mathematica 3.43

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(implies( $p$ ,  $q$ ), implies( $p$ ,  $r$ )), implies( $p$ , and( $q$ ,  $r$ ))))      cnf(prove_this, negated_conjecture)
```

LCL252-1.p Principia Mathematica 3.44

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not( $q$ ),  $p$ )), not(or(not( $r$ ),  $p$ ))))), or(not(or( $q$ ,  $r$ ),  $p$ )))      cnf(prove_this, negated_conjecture)
```

LCL252-3.p Principia Mathematica 3.44

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(implies( $q$ ,  $p$ ), implies( $r$ ,  $p$ )), implies(or( $q$ ,  $r$ ),  $p$ )))      cnf(prove_this, negated_conjecture)
```

LCL253-1.p Principia Mathematica 3.45

```
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(or(not( $p$ ),  $q$ )), or(not(not(or(not( $p$ ), not( $r$ ))), not(or(not( $q$ ), not( $r$ ))))))      cnf(prove_this, negated_conjecture)
```

LCL253-3.p Principia Mathematica 3.45

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(implies( $p$ ,  $q$ ), implies(and( $p$ ,  $r$ ), and( $q$ ,  $r$ ))))      cnf(prove_this, negated_conjecture)
```

LCL254-1.p Principia Mathematica 3.47


```

include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not(p), r)), not(or(not(q), s))))) , or(not(not(or(not(p), not(q)))) , not(or(not(r), not(s))))) )

LCL254-3.p Principia Mathematica 3.47
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(implies(p, r), implies(q, s)), implies(and(p, q), and(r, s))))    cnf(prove_this, negated_conjecture)

LCL255-1.p Principia Mathematica 3.48
include('Axioms/LCL003-0.ax')
¬ theorem(or(not(not(or(not(or(not(p), r)), not(or(not(q), s))))) , or(not(or(p, q), or(r, s))))    cnf(prove_this, negated_conjec

LCL255-3.p Principia Mathematica 3.48
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(implies(p, r), implies(q, s)), implies(or(p, q), or(r, s))))    cnf(prove_this, negated_conjecture)

LCL256-1.p A formula that can be derived from the Lukasiewicz system
Axiomatisations of the Implication/Negation 2 valued sentential calculus are CN-1,CN-2,CN-3 by Lukasiewicz, CN-
18,CN-21,CN-35,CN-39,CN-39,CN-40,CN-46 by Frege, CN-3,CN-18,CN-21,CN-22,CN-30,CN-54 by Hilbert, CN-18,
CN-35,CN-49 by Church, CN-19,CN-37,CN-59 by Lukasiewicz, CN-19,CN-37,CN-60 by Wos, and the single Meredith
axiom. Show that not(not(p → p)) can be derived from the short Lukasiewicz system.
(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y)    cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z))))    cnf(cn1, axiom)
is_a_theorem(implies(implies(not(x), x), x))    cnf(cn2, axiom)
is_a_theorem(implies(x, implies(not(x), y)))    cnf(cn3, axiom)
¬ is_a_theorem(not(not(implies(p, p))))    cnf(prove_not_not_implies, negated_conjecture)

LCL257-1.p XHN depends on YQL
Show that XHN can be derived from the single Lukasiewicz axiom YQL.
(is_a_theorem(equivalent(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y)    cnf(condensed_detachment, axiom)
is_a_theorem(equivalent(equivalent(x, y), equivalent(equivalent(z, y), equivalent(x, z))))    cnf(yql, axiom)
¬ is_a_theorem(equivalent(x, equivalent(equivalent(y, z), equivalent(equivalent(z, x), y))))    cnf(prove_xhn, negated_conjecture)

LCL258-3.p Principia Mathematica 2.27
include('Axioms/LCL004-0.ax')
¬ theorem(implies(p, implies(implies(p, q), q)))    cnf(prove_this, negated_conjecture)

LCL259-3.p Principia Mathematica 2.43
include('Axioms/LCL004-0.ax')
¬ theorem(implies(implies(p, implies(p, q)), implies(p, q)))    cnf(prove_this, negated_conjecture)

LCL260-3.p Principia Mathematica 2.63
include('Axioms/LCL004-0.ax')
¬ theorem(implies(or(p, q), implies(or(not(p), q), q)))    cnf(prove_this, negated_conjecture)

LCL261-3.p Principia Mathematica 3.12
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(or(not(p), or(not(q), and(p, q))))    cnf(prove_this, negated_conjecture)

LCL262-3.p Principia Mathematica 4.10
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(implies(p, q), implies(not(q), not(p))))    cnf(prove_this, negated_conjecture)

LCL263-3.p Principia Mathematica 4.11
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(equivalent(p, q), equivalent(not(p), not(q))))    cnf(prove_this, negated_conjecture)

LCL264-3.p Principia Mathematica 4.12
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')

```

\neg theorem(equivalent(equivalent(p, q), equivalent(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL265-3.p Principia Mathematica 4.13

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($p, \text{not}(\text{not}(p))$)) cnf(prove_this, negated_conjecture)

LCL266-3.p Principia Mathematica 4.14

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and($p, \text{implies}(q, r)$), and($p, \text{implies}(\text{not}(r), \text{not}(q))$)))) cnf(prove_this, negated_conjecture)

LCL267-3.p Principia Mathematica 4.15

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and($p, \text{implies}(q, \text{not}(r))$), and($q, \text{implies}(r, \text{not}(p))$)))) cnf(prove_this, negated_conjecture)

LCL268-3.p Principia Mathematica 4.2

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(p, p)) cnf(prove_this, negated_conjecture)

LCL269-3.p Principia Mathematica 4.21

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(equivalent(p, q), equivalent(q, p))) cnf(prove_this, negated_conjecture)

LCL270-3.p Principia Mathematica 4.22

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(and(equivalent(p, q), equivalent(q, r)), equivalent(p, r)))) cnf(prove_this, negated_conjecture)

LCL271-3.p Principia Mathematica 4.24

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($p, \text{and}(p, p)$)) cnf(prove_this, negated_conjecture)

LCL272-3.p Principia Mathematica 4.25

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($p, \text{or}(p, p)$)) cnf(prove_this, negated_conjecture)

LCL273-3.p Principia Mathematica 4.3

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and(p, q), and(q, p))) cnf(prove_this, negated_conjecture)

LCL274-3.p Principia Mathematica 4.31

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(or(p, q), or(q, p))) cnf(prove_this, negated_conjecture)

LCL275-3.p Principia Mathematica 4.32

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($\text{and}(r, \text{and}(p, q))$, $\text{and}(p, \text{and}(q, r))$)) cnf(prove_this, negated_conjecture)

LCL276-3.p Principia Mathematica 4.33

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($\text{or}(r, \text{or}(p, q))$, $\text{or}(p, \text{or}(q, r))$)) cnf(prove_this, negated_conjecture)

LCL277-3.p Principia Mathematica 4.36

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(equivalent(p, q), equivalent($\text{and}(p, r)$, $\text{and}(q, r)$))) cnf(prove_this, negated_conjecture)

LCL278-3.p Principia Mathematica 4.37

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(equivalent(p, q), equivalent($\text{or}(p, r)$, $\text{or}(q, r)$))) cnf(prove_this, negated_conjecture)

LCL279-3.p Principia Mathematica 4.38

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies($\text{and}(\text{equivalent}(p, r)$, $\text{equivalent}(q, s))$, $\text{equivalent}(\text{and}(p, q)$, $\text{and}(r, s))$)) cnf(prove_this, negated_conjecture)

LCL280-3.p Principia Mathematica 4.39

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies($\text{and}(\text{equivalent}(p, r)$, $\text{equivalent}(q, s)$), $\text{equivalent}(\text{or}(p, r)$, $\text{or}(r, s))$)) cnf(prove_this, negated_conjecture)

LCL281-3.p Principia Mathematica 4.4

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($\text{and}(p, \text{or}(q, r))$, $\text{or}(\text{and}(p, q)$, $\text{and}(p, r))$)) cnf(prove_this, negated_conjecture)

LCL282-3.p Principia Mathematica 4.41

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent($\text{or}(p, \text{and}(q, r))$, $\text{and}(\text{or}(p, q)$, $\text{or}(p, r))$)) cnf(prove_this, negated_conjecture)

LCL283-3.p Principia Mathematica 4.42

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(p , $\text{or}(\text{and}(p, q)$, $\text{and}(p, \text{not}(q))$)) cnf(prove_this, negated_conjecture)

LCL284-3.p Principia Mathematica 4.43

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(p , $\text{and}(\text{or}(p, q)$, $\text{or}(p, \text{not}(q))$)) cnf(prove_this, negated_conjecture)

LCL285-3.p Principia Mathematica 4.44

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(p , $\text{or}(p, \text{and}(p, q))$)) cnf(prove_this, negated_conjecture)

LCL286-3.p Principia Mathematica 4.45

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(p , and(p , or(p , q)))) cnf(prove_this, negated_conjecture)

LCL287-3.p Principia Mathematica 4.5

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(and(p , q), not(or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL288-3.p Principia Mathematica 4.52

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(and(p , not(q)), or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL289-3.p Principia Mathematica 4.53

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(and(p , not(q))), or(not(p), q))) cnf(prove_this, negated_conjecture)

LCL290-3.p Principia Mathematica 4.54

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(and(p , q)), not(or(p , not(q)))) cnf(prove_this, negated_conjecture)

LCL291-3.p Principia Mathematica 4.55

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(not(and(p , q))), or(not(p), q))) cnf(prove_this, negated_conjecture)

LCL292-3.p Principia Mathematica 4.56

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(and(not(p), not(q)), or(p , q))) cnf(prove_this, negated_conjecture)

LCL293-3.p Principia Mathematica 4.57

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(and(not(p), not(q))), or(p , q))) cnf(prove_this, negated_conjecture)

LCL294-3.p Principia Mathematica 4.6

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(implies(p , q), or(not(p), q))) cnf(prove_this, negated_conjecture)

LCL295-3.p Principia Mathematica 4.61

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(implies(p , q)), and(p , not(q)))) cnf(prove_this, negated_conjecture)

LCL296-3.p Principia Mathematica 4.62

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(implies(p , not(q)), or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL297-3.p Principia Mathematica 4.63

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(not(implies(p , not(q))), and(p , q))) cnf(prove_this, negated_conjecture)

LCL298-3.p Principia Mathematica 4.51

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(not(and(p , q)), or(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL299-3.p Principia Mathematica 4.65

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(not(implies(not(p), q)), and(not(p), not(q)))) cnf(prove_this, negated_conjecture)

LCL300-3.p Principia Mathematica 4.66

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(not(p), not(q)), or(p , not(q)))) cnf(prove_this, negated_conjecture)

LCL301-3.p Principia Mathematica 4.67

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(not(implies(not(p), not(q))), and(not(p), q))) cnf(prove_this, negated_conjecture)

LCL302-3.p Principia Mathematica 4.7

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(p , q), implies(p , and(p , q)))) cnf(prove_this, negated_conjecture)

LCL303-3.p Principia Mathematica 4.71

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(p , q), equivalent(q , or(p , q)))) cnf(prove_this, negated_conjecture)

LCL304-3.p Principia Mathematica 4.72

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(p , q), equivalent(q , or(p , q)))) cnf(prove_this, negated_conjecture)

LCL305-3.p Principia Mathematica 4.73

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(q , equivalent(p , and(p , q)))) cnf(prove_this, negated_conjecture)

LCL306-3.p Principia Mathematica 4.74

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(not(p), equivalent(q , or(p , q)))) cnf(prove_this, negated_conjecture)

LCL307-3.p Principia Mathematica 4.76

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and(implies(p , q), implies(p , r)), implies(p , and(q , r)))) cnf(prove_this, negated_conjecture)

LCL308-3.p Principia Mathematica 4.77

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and(implies(q, p), implies(r, p)), implies(or(q, r), p))) cnf(prove_this, negated_conjecture)

LCL309-3.p Principia Mathematica 4.78

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(or(implies(p, q), implies(p, r)), implies($p, \text{or}(q, r)$))) cnf(prove_this, negated_conjecture)

LCL310-3.p Principia Mathematica 4.79

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(or(implies(q, p), implies(r, p)), implies(and(q, r), p))) cnf(prove_this, negated_conjecture)

LCL311-3.p Principia Mathematica 4.8

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies($p, \text{not}(p)$), not(p))) cnf(prove_this, negated_conjecture)

LCL312-3.p Principia Mathematica 4.81

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(not(p), p), p)) cnf(prove_this, negated_conjecture)

LCL313-3.p Principia Mathematica 4.82

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and(implies(p, q), implies($p, \text{not}(q)$)), not(p))) cnf(prove_this, negated_conjecture)

LCL314-3.p Principia Mathematica 4.83

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(and(implies(p, q), implies(not(p), q)), q)) cnf(prove_this, negated_conjecture)

LCL315-3.p Principia Mathematica 4.84

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(equivalent(p, q), equivalent(implies(p, r), implies(q, r)))) cnf(prove_this, negated_conjecture)

LCL316-3.p Principia Mathematica 4.85

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(equivalent(p, q), equivalent(implies(r, p), implies(r, q)))) cnf(prove_this, negated_conjecture)

LCL317-3.p Principia Mathematica 4.86

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(implies(equivalent(p, q), equivalent(equivalent(p, r), equivalent(q, r)))) cnf(prove_this, negated_conjecture)

LCL318-3.p Principia Mathematica 4.87

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

\neg theorem(equivalent(implies(and(p, q), r), equivalent(implies($p, \text{implies}(q, r)$), implies($q, \text{equivalent}(\text{implies}(p, r), \text{implies}(\text{and}(\text{implies}(p, r), \text{implies}(q, r))$))))))

LCL319-3.p Principia Mathematica 5.1

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(implies(and(p, q), equivalent(p, q))) cnf(prove_this, negated_conjecture)

LCL320-3.p Principia Mathematica 5.11

include('Axioms/LCL004-0.ax')

– theorem(or(implies(p, q), implies(not(p), q))) cnf(prove_this, negated_conjecture)

LCL321-3.p Principia Mathematica 5.12

include('Axioms/LCL004-0.ax')

– theorem(or(implies(p, q), implies(p , not(q)))) cnf(prove_this, negated_conjecture)

LCL322-3.p Principia Mathematica 5.13

include('Axioms/LCL004-0.ax')

– theorem(or(implies(p, q), implies(q, p))) cnf(prove_this, negated_conjecture)

LCL323-3.p Principia Mathematica 5.14

include('Axioms/LCL004-0.ax')

– theorem(or(implies(p, q), implies(q, r))) cnf(prove_this, negated_conjecture)

LCL324-3.p Principia Mathematica 5.16

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(not(and(equivalent(p, q), equivalent(p , not(q)))))) cnf(prove_this, negated_conjecture)

LCL325-3.p Principia Mathematica 5.17

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(and(or(p, q), not(and(p, q))), equivalent(p , not(q)))) cnf(prove_this, negated_conjecture)

LCL326-3.p Principia Mathematica 5.19

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(not(equivalent(p , not(p)))) cnf(prove_this, negated_conjecture)

LCL327-3.p Principia Mathematica 5.21

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(implies(and(not(p), not(q)), equivalent(p, q))) cnf(prove_this, negated_conjecture)

LCL328-3.p Principia Mathematica 5.23

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(equivalent(p, q), or(and(p, q), and(not(p), not(q)))))) cnf(prove_this, negated_conjecture)

LCL329-3.p Principia Mathematica 5.24

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(not(or(and(p, q), and(not(p), not(q))), or(and(p , not(q)), and(q , not(p)))))) cnf(prove_this, negated_conjecture)

LCL330-3.p Principia Mathematica 5.25

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(or(p, q), implies(implies(p, q), q))) cnf(prove_this, negated_conjecture)

LCL331-3.p Principia Mathematica 5.3

include('Axioms/LCL004-0.ax')

include('Axioms/LCL004-1.ax')

include('Axioms/LCL004-2.ax')

– theorem(equivalent(implies(and(p, q), r), implies(and(p, q), and(p, r)))) cnf(prove_this, negated_conjecture)

LCL332-3.p Principia Mathematica 5.31

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
¬ theorem(implies(and(r, implies(p, q)), implies(p, and(q, r))))    cnf(prove_this, negated_conjecture)
```

LCL333-3.p Principia Mathematica 5.32

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(implies(p, equivalent(q, r)), equivalent(and(p, q), and(p, r))))    cnf(prove_this, negated_conjecture)
```

LCL334-3.p Principia Mathematica 5.33

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(and(p, implies(q, r)), and(p, implies(and(p, q), r))))    cnf(prove_this, negated_conjecture)
```

LCL335-3.p Principia Mathematica 5.35

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(implies(and(implies(p, q), implies(q, r)), implies(p, equivalent(q, r))))    cnf(prove_this, negated_conjecture)
```

LCL336-3.p Principia Mathematica 5.36

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(and(p, equivalent(p, q)), and(q, equivalent(p, q))))    cnf(prove_this, negated_conjecture)
```

LCL337-3.p Principia Mathematica 5.4

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(implies(p, implies(p, q)), implies(p, q)))    cnf(prove_this, negated_conjecture)
```

LCL338-3.p Principia Mathematica 5.41

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(implies(implies(p, q), implies(q, r)), implies(p, implies(q, r))))    cnf(prove_this, negated_conjecture)
```

LCL339-3.p Principia Mathematica 5.42

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(equivalent(implies(p, implies(q, r)), implies(p, implies(q, and(p, r)))))    cnf(prove_this, negated_conjecture)
```

LCL340-3.p Principia Mathematica 5.44

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(implies(implies(p, q), equivalent(implies(p, r), implies(p, and(q, r)))))    cnf(prove_this, negated_conjecture)
```

LCL341-3.p Principia Mathematica 5.5

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(implies(p, equivalent(implies(p, q), q)))    cnf(prove_this, negated_conjecture)
```

LCL342-3.p Principia Mathematica 5.501

```
include('Axioms/LCL004-0.ax')
include('Axioms/LCL004-1.ax')
include('Axioms/LCL004-2.ax')
¬ theorem(implies(p, equivalent(q, equivalent(p, q))))    cnf(prove_this, negated_conjecture)
```

LCL343-3.p Principia Mathematica 5.53

```
include('Axioms/LCL004-0.ax')
```


include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(implies(or(or(p, q), r), s), and(and(implies(p, s), implies(q, s), implies(r, s)))))) cnf(prove_this, negated_conjecture)

LCL344-3.p Principia Mathematica 5.54
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(or(equivalent(and(p, q), p), equivalent(and(p, q), q))) cnf(prove_this, negated_conjecture)

LCL345-3.p Principia Mathematica 5.55
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(or(equivalent(or(p, q), p), equivalent(or(p, q), q))) cnf(prove_this, negated_conjecture)

LCL346-3.p Principia Mathematica 5.6
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(implies(and($p, \text{not}(q)$), r), implies($p, \text{or}(q, r)$))) cnf(prove_this, negated_conjecture)

LCL347-3.p Principia Mathematica 5.61
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(and(or(p, q), $\text{not}(q)$), and($p, \text{not}(q)$))) cnf(prove_this, negated_conjecture)

LCL348-3.p Principia Mathematica 5.62
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(or(and(p, q), $\text{not}(q)$), or($p, \text{not}(q)$))) cnf(prove_this, negated_conjecture)

LCL349-3.p Principia Mathematica 5.63
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(or(p, q), or($p, \text{and}(\text{not}(p), q)$))) cnf(prove_this, negated_conjecture)

LCL350-3.p Principia Mathematica 5.7
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(equivalent(or(p, r), or(q, r)), or($r, \text{equivalent}(p, q)$))) cnf(prove_this, negated_conjecture)

LCL351-3.p Principia Mathematica 5.71
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(implies(implies($q, \text{not}(r)$), equivalent(and(or(p, q), r), and(p, r)))) cnf(prove_this, negated_conjecture)

LCL352-3.p Principia Mathematica 5.74
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(equivalent(implies($p, \text{equivalent}(q, r)$), equivalent(implies(p, q), implies(p, r)))) cnf(prove_this, negated_conjecture)

LCL353-3.p Principia Mathematica 5.75
 include('Axioms/LCL004-0.ax')
 include('Axioms/LCL004-1.ax')
 include('Axioms/LCL004-2.ax')
 \neg theorem(implies(equivalent(and(implies($r, \text{not}(q)$), p), or(q, r)), equivalent(and($p, \text{not}(q)$), r))) cnf(prove_this, negated_conjecture)

LCL354+1.p Independence of an Axiom for Temporal Intervals

Shows that the 5th axiom of temporal intervals is not dependant on the first three by building a model of the first three and the negation of the 5th.

$\forall p, q, r, s: ((\text{meets}(p, q) \text{ and } \text{meets}(p, s) \text{ and } \text{meets}(r, q)) \Rightarrow \text{meets}(r, s)) \quad \text{fof}(m_1, \text{axiom})$
 $\forall p, q, r, s: ((\text{meets}(p, q) \text{ and } \text{meets}(r, s)) \Rightarrow \text{meets}(p, s) < > \exists t: (\text{meets}(p, t) \text{ and } \text{meets}(t, s)) < > \exists t: (\text{meets}(r, t) \text{ and } \text{meets}(t, s))) \quad \text{fof}(m_2, \text{axiom})$
 $\forall p: \exists q, r: (\text{meets}(q, p) \text{ and } \text{meets}(p, r)) \quad \text{fof}(m_3, \text{axiom})$
 $\neg \forall p, q: (\text{meets}(p, q) \Rightarrow \exists r, s, t: (\text{meets}(r, p) \text{ and } \text{meets}(q, s) \text{ and } \text{meets}(r, t) \text{ and } \text{meets}(t, s))) \quad \text{fof}(\text{not_m}_5, \text{axiom})$

LCL355-1.p CN-04 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-04 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(\text{implies}(x, y), \text{implies}(z, y)), u), \text{implies}(\text{implies}(z, x), u))) \quad \text{cnf}(\text{prove_cn}_{04}, \text{negated_conjecture})$

LCL356-1.p CN-05 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-05 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(x, \text{implies}(y, z)), \text{implies}(\text{implies}(u, y), \text{implies}(x, \text{implies}(u, z))))) \quad \text{cnf}(\text{prove_cn}_{05}, \text{negated_conjecture})$

LCL357-1.p CN-06 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-06 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(\text{implies}(x, z), u), \text{implies}(\text{implies}(y, z), u)))) \quad \text{cnf}(\text{prove_cn}_{06}, \text{negated_conjecture})$

LCL358-1.p CN-07 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-07 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(x, \text{implies}(\text{implies}(y, z), u)), \text{implies}(\text{implies}(y, v), \text{implies}(x, \text{implies}(\text{implies}(v, z), u))))) \quad \text{cnf}(\text{prove_cn}_{07}, \text{negated_conjecture})$

LCL359-1.p CN-08 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-08 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(z, x), \text{implies}(\text{implies}(y, u), \text{implies}(z, u))))) \quad \text{cnf}(\text{prove_cn}_{08}, \text{negated_conjecture})$

LCL360-1.p CN-09 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-09 depends on the Lukasiewicz system.

$(\text{is_a_theorem}(\text{implies}(x, y)) \text{ and } \text{is_a_theorem}(x)) \Rightarrow \text{is_a_theorem}(y) \quad \text{cnf}(\text{condensed_detachment}, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(x, y), \text{implies}(\text{implies}(y, z), \text{implies}(x, z)))) \quad \text{cnf}(\text{cn}_1, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(\text{implies}(\text{not}(x), x), x)) \quad \text{cnf}(\text{cn}_2, \text{axiom})$
 $\text{is_a_theorem}(\text{implies}(x, \text{implies}(\text{not}(x), y))) \quad \text{cnf}(\text{cn}_3, \text{axiom})$
 $\neg \text{is_a_theorem}(\text{implies}(\text{implies}(\text{implies}(\text{not}(x), y), z), \text{implies}(x, z))) \quad \text{cnf}(\text{prove_cn}_{09}, \text{negated_conjecture})$

LCL361-1.p CN-10 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-23 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,implies(y,z)),u),implies(implies(y,implies(x,z)),u))) cnf(prove_cn₂₃, negated.c)

LCL369-1.p CN-25 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-25 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,y),z),implies(implies(x,u),implies(implies(u,y),z)))) cnf(prove_cn₂₅, negated.c)

LCL370-1.p CN-26 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-26 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,y),z),implies(implies(z,x),x))) cnf(prove_cn₂₆, negated_conjecture)

LCL371-1.p CN-27 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-27 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,y),y),implies(implies(y,x),x))) cnf(prove_cn₂₇, negated_conjecture)

LCL372-1.p CN-28 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-28 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(implies(x,y),y),z),implies(implies(implies(y,u),x),z))) cnf(prove_cn₂₈, negated.c)

LCL373-1.p CN-29 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-29 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,y),z),implies(implies(x,z),z))) cnf(prove_cn₂₉, negated_conjecture)

LCL374-1.p CN-31 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-31 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x,y),implies(implies(implies(x,z),u),implies(implies(y,u),u)))) cnf(prove_cn₃₁, negated.c)

LCL375-1.p CN-32 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-32 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(x,y),z),implies(implies(x,u),implies(implies(u,z),z)))) cnf(prove_cn₃₂, negated.conjecture)

LCL376-1.p CN-33 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-33 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x,y),implies(implies(y,implies(z,implies(x,u))),implies(z,implies(x,u))))) cnf(prove_cn₃₃, negated.conjecture)

LCL377-1.p CN-34 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-34 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x,implies(y,implies(z,u))),implies(implies(z,x),implies(y,implies(z,u))))) cnf(prove_cn₃₄, negated.conjecture)

LCL378-1.p CN-36 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-36 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(not(x),implies(x,y))) cnf(prove_cn₃₆, negated.conjecture)

LCL379-1.p CN-38 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-38 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x,not(x)),not(x))) cnf(prove_cn₃₈, negated.conjecture)

LCL380-1.p CN-41 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-41 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x,y),implies(not(not(x)),y))) cnf(prove_cn₄₁, negated.conjecture)

LCL381-1.p CN-42 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-42 depends on the Lukasiewicz system.

(is_a_theorem(implies(x,y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x),x),x)) cnf(cn₂, axiom)
is_a_theorem(implies(x,implies(not(x),y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(not(not(x)),y),z),implies(implies(x,y),z))) cnf(prove_cn₄₂, negated.conjecture)

LCL382-1.p CN-43 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-52 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x, implies(y, not(z))), implies(x, implies(z, not(y))))) cnf(prove_cn₅₂, negated_conjecture)

LCL390-1.p CN-53 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-53 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(not(x), y), implies(implies(x, y), y))) cnf(prove_cn₅₃, negated_conjecture)

LCL391-1.p CN-55 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-55 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x, y), implies(implies(x, not(y)), not(x)))) cnf(prove_cn₅₅, negated_conjecture)

LCL392-1.p CN-56 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-56 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(implies(x, y), y), z), implies(implies(not(x), y), z))) cnf(prove_cn₅₆, negated_conjecture)

LCL393-1.p CN-57 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-57 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(not(x), y), implies(implies(x, z), implies(implies(z, y), y)))) cnf(prove_cn₅₇, negated_conjecture)

LCL394-1.p CN-58 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-58 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(implies(implies(x, y), implies(implies(y, z), z)), u), implies(implies(not(x), z), u))) cnf(prove_cn₅₈, negated_conjecture)

LCL395-1.p CN-61 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz.
Show that CN-61 depends on the Lukasiewicz system.

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) \Rightarrow is_a_theorem(y) cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z)))) cnf(cn₁, axiom)
is_a_theorem(implies(implies(not(x), x), x)) cnf(cn₂, axiom)
is_a_theorem(implies(x, implies(not(x), y))) cnf(cn₃, axiom)
 \neg is_a_theorem(implies(implies(x, y), implies(implies(z, y), implies(implies(not(x), z), y)))) cnf(prove_cn₆₁, negated_conjecture)

LCL396-1.p CN-62 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-69 depends on the Lukasiewicz system.

```
(is_a_theorem(implies(x,y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z))))    cnf(cn1, axiom)
is_a_theorem(implies(implies(not(x),x),x))                                cnf(cn2, axiom)
is_a_theorem(implies(x,implies(not(x),y)))                                cnf(cn3, axiom)
¬is_a_theorem(implies(x,implies(not(y),not(implies(x,y))))))            cnf(prove_cn69, negated_conjecture)
```

LCL404-1.p CN-70 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-70 depends on the Lukasiewicz system.

```
(is_a_theorem(implies(x,y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z))))    cnf(cn1, axiom)
is_a_theorem(implies(implies(not(x),x),x))                                cnf(cn2, axiom)
is_a_theorem(implies(x,implies(not(x),y)))                                cnf(cn3, axiom)
¬is_a_theorem(implies(x,implies(y,not(implies(x,not(y))))))            cnf(prove_cn70, negated_conjecture)
```

LCL405-1.p CN-71 depends on the Lukasiewicz system

An axiomatisation of the Implication/Negation 2 valued sentential calculus is CN-1,CN-2,CN-3 by Lukasiewicz. Show that CN-71 depends on the Lukasiewicz system.

```
(is_a_theorem(implies(x,y)) and is_a_theorem(x)) => is_a_theorem(y)      cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(x,y),implies(implies(y,z),implies(x,z))))    cnf(cn1, axiom)
is_a_theorem(implies(implies(not(x),x),x))                                cnf(cn2, axiom)
is_a_theorem(implies(x,implies(not(x),y)))                                cnf(cn3, axiom)
¬is_a_theorem(not(implies(implies(x,x),not(implies(y,y))))))            cnf(prove_cn71, negated_conjecture)
```

LCL406-1.p Generate LTL structures of size 4

```
x ≤ x      cnf(reflexivity_of_less_or_equal, axiom)
(x ≤ y and y ≤ z) => x ≤ z    cnf(transitivity_of_less_or_equal, axiom)
x ≤ y or y ≤ x    cnf(completeness_of_less_or_equal, axiom)
successor(x) = y => x ≤ y     cnf(predecessor_less_or_equal, axiom)
x ≤ y => (y = x or y = successor(x) or y = successor(successor(x)) or y = successor(successor(successor(x))))  cnf(four_s
```

LCL407-1.p Wajsberg algebra axioms

```
include('Axioms/LCL001-0.ax')
```

LCL407-2.p Alternative Wajsberg algebra axioms

```
include('Axioms/LCL002-0.ax')
```

LCL408-1.p Wajsberg algebra lattice structure definitions

```
include('Axioms/LCL001-0.ax')
```

```
include('Axioms/LCL001-1.ax')
```

LCL409-1.p Wajsberg algebra AND and OR definitions

```
include('Axioms/LCL001-0.ax')
```

```
include('Axioms/LCL001-2.ax')
```

LCL410-1.p Alternative Wajsberg algebra definitions

```
include('Axioms/LCL002-0.ax')
```

```
include('Axioms/LCL001-2.ax')
```

```
include('Axioms/LCL002-1.ax')
```

LCL411-1.p Propositional logic deduction axioms

```
include('Axioms/LCL003-0.ax')
```

LCL411-2.p Propositional logic deduction axioms

```
include('Axioms/LCL004-0.ax')
```

LCL412-1.p Propositional logic deduction axioms for AND

```
include('Axioms/LCL004-0.ax')
```

```
include('Axioms/LCL004-1.ax')
```

LCL413-1.p Propositional logic deduction axioms for EQUIVALENT

```
include('Axioms/LCL004-0.ax')
```

```
include('Axioms/LCL004-1.ax')
```

```
include('Axioms/LCL004-2.ax')
```


include('Axioms/MSC001-0.ax')

c.in(v_q, c.PropLog_Othms(c.insert(v_p, v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a)) \Rightarrow c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), v_H, tc.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_H_MP_0, axiom)
 c.in(c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), t_a), tc.PropLog_Opl(t_a)), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 c.in(v_q, c.PropLog_Othms(c.insert(v_p, v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 c.in(v_q, c.PropLog_Othms(c.insert(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Ofalse, t_a), v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a))
 \neg c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_2, negated_conjecture)

LCL445-2.p Problem about propositional logic

c.in(v_q, c.PropLog_Othms(c.insert(v_p, v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 \neg c.in(v_q, c.PropLog_Othms(c.insert(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Ofalse, t_a), v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a))
 \neg c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_2, negated_conjecture)
 c.in(v_q, c.PropLog_Othms(c.insert(v_p, v_H, tc.PropLog_Opl(t_a)), t_a), tc.PropLog_Opl(t_a)) \Rightarrow c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)
 c.in(c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), t_a), tc.PropLog_Opl(t_a)), tc.PropLog_Opl(t_a))

LCL446-1.p Problem about propositional logic

include('Axioms/LCL005-0.ax')

include('Axioms/MSC001-2.ax')

include('Axioms/MSC001-0.ax')

c.in(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_p, t_a), t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)
 c.in(c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_r, t_a), t_a), c.PropLog_Opl(t_a)), tc.PropLog_Opl(t_a))
 \neg c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_p, t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)

LCL446-2.p Problem about propositional logic

\neg c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_p, t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 c.in(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_p, t_a), t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)
 c.in(c.PropLog_Opl_Oop_A_N62(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_r, t_a), t_a), c.PropLog_Opl(t_a)), tc.PropLog_Opl(t_a))

LCL447-1.p Problem about propositional logic

include('Axioms/LCL005-0.ax')

include('Axioms/MSC001-2.ax')

include('Axioms/MSC001-0.ax')

c.in(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_p, t_a), t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 \neg c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_1, negated_conjecture)

LCL447-2.p Problem about propositional logic

c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_0, negated_conjecture)
 \neg c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.conjecture_1, negated_conjecture)
 c.in(c.PropLog_Opl_Oop_A_N62(v_p, c.PropLog_Opl_Oop_A_N62(v_q, v_p, t_a), t_a), c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a))
 (c.in(v_p, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) and c.in(c.PropLog_Opl_Oop_A_N62(v_p, v_q, t_a), c.PropLog_Opl(t_a)))
 c.in(v_q, c.PropLog_Othms(v_H, t_a), tc.PropLog_Opl(t_a)) cnf(cls.PropLog_Othms_OMP_0, axiom)

LCL448+1.p Redundant axiom in Principia axiomatization

include('Axioms/LCL006+0.ax')

include('Axioms/LCL006+1.ax')

op_implies_or fof(principia_op_implies_or, axiom)

op_and fof(principia_op_and, axiom)

op_equiv fof(principia_op_equiv, axiom)

modus_ponens fof(principia_modus_ponens, axiom)

r1 fof(principia_r1, axiom)

r2 fof(principia_r2, axiom)

r_3 fof(principia_r3, axiom)
 r_5 fof(principia_r5, axiom)
substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 r_4 fof(principia_r4, conjecture)

LCL449+1.p Congruence of equiv, to admit substitution of equivalents

include('Axioms/LCL006+0.ax')

include('Axioms/LCL006+1.ax')

op_or fof(hilbert_op_or, axiom)

op_implies_and fof(hilbert_op_implies_and, axiom)

op_equiv fof(hilbert_op_equiv, axiom)

modus_ponens fof(hilbert_modus_ponens, axiom)

modus_tollens fof(hilbert_modus_tollens, axiom)

implies₁ fof(hilbert_implies₁, axiom)

implies₂ fof(hilbert_implies₂, axiom)

implies₃ fof(hilbert_implies₃, axiom)

and₁ fof(hilbert_and₁, axiom)

and₂ fof(hilbert_and₂, axiom)

and₃ fof(hilbert_and₃, axiom)

or₁ fof(hilbert_or₁, axiom)

or₂ fof(hilbert_or₂, axiom)

or₃ fof(hilbert_or₃, axiom)

equivalence₁ fof(hilbert_equivalence₁, axiom)

equivalence₂ fof(hilbert_equivalence₂, axiom)

equivalence₃ fof(hilbert_equivalence₃, axiom)

$(\forall x: \text{is_a_theorem}(\text{equiv}(x, x)) \text{ and } \forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow \text{is_a_theorem}(\text{equiv}(\text{not}(x), \text{not}(y)))) \text{ and } \forall x_1, x_2, y_1, y_2: (\text{is_a_theorem}(\text{equiv}(\text{and}(x_1, y_1), \text{and}(x_2, y_2)))) \text{ and } \forall x, y: ((\text{is_a_theorem}(x) \text{ and } \text{is_a_theorem}(\text{equiv}(x, y))) \Rightarrow \text{is_a_theorem}(y)))$

$\forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow x = y)$ fof(make_subs_of_equiv, axiom)

$\forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow x = y)$ fof(subs_of_equiv, conjecture)

LCL450+1.p Congruence of equiv lemmas, to admit substitution of equivalents

include('Axioms/LCL006+0.ax')

include('Axioms/LCL006+1.ax')

op_or fof(hilbert_op_or, axiom)

op_implies_and fof(hilbert_op_implies_and, axiom)

op_equiv fof(hilbert_op_equiv, axiom)

modus_ponens fof(hilbert_modus_ponens, axiom)

modus_tollens fof(hilbert_modus_tollens, axiom)

implies₁ fof(hilbert_implies₁, axiom)

implies₂ fof(hilbert_implies₂, axiom)

implies₃ fof(hilbert_implies₃, axiom)

and₁ fof(hilbert_and₁, axiom)

and₂ fof(hilbert_and₂, axiom)

and₃ fof(hilbert_and₃, axiom)

or₁ fof(hilbert_or₁, axiom)

or₂ fof(hilbert_or₂, axiom)

or₃ fof(hilbert_or₃, axiom)

equivalence₁ fof(hilbert_equivalence₁, axiom)

equivalence₂ fof(hilbert_equivalence₂, axiom)

equivalence₃ fof(hilbert_equivalence₃, axiom)

$\forall x: \text{is_a_theorem}(\text{equiv}(x, x)) \text{ and } \forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow \text{is_a_theorem}(\text{equiv}(\text{not}(x), \text{not}(y)))) \text{ and } \forall x_1, x_2, y_1, y_2: (\text{is_a_theorem}(\text{equiv}(\text{and}(x_1, y_1), \text{and}(x_2, y_2)))) \text{ and } \forall x, y: ((\text{is_a_theorem}(x) \text{ and } \text{is_a_theorem}(\text{equiv}(x, y))) \Rightarrow \text{is_a_theorem}(y)))$

$\forall x, y: (\text{is_a_theorem}(\text{equiv}(x, y)) \Rightarrow x = y)$ fof(make_subs_of_equiv, axiom)

LCL450+2.p Congruence of equiv lemmas, to admit substitution of equivalents

include('Axioms/LCL006+0.ax')

include('Axioms/LCL006+1.ax')

op_or fof(hilbert_op_or, axiom)

op_implies_and fof(hilbert_op_implies_and, axiom)

op_equiv fof(hilbert_op_equiv, axiom)

modus_ponens fof(hilbert_modus_ponens, axiom)

modus_tollens fof(hilbert_modus_tollens, axiom)

```

implies1    fof(hilbert_implies1, axiom)
implies2    fof(hilbert_implies2, axiom)
implies3    fof(hilbert_implies3, axiom)
and1       fof(hilbert_and1, axiom)
and2       fof(hilbert_and2, axiom)
and3       fof(hilbert_and3, axiom)
or1        fof(hilbert_or1, axiom)
or2        fof(hilbert_or2, axiom)
or3        fof(hilbert_or3, axiom)
equivalence1  fof(hilbert_equivalence1, axiom)
equivalence2  fof(hilbert_equivalence2, axiom)
equivalence3  fof(hilbert_equivalence3, axiom)
∀x: is_a_theorem(equiv(x, x)) and ∀x, y: (is_a_theorem(equiv(x, y)) ⇒ is_a_theorem(equiv(not(x), not(y)))) and ∀x1, x2, y: (is_a_theorem(equiv(and(x1, y), and(x2, y)))) and ∀x1, x2, y: (is_a_theorem(equiv(x1, x2)) ⇒ is_a_theorem(equiv(and(y, x1), and(y, x2)))) and
is_a_theorem(y)    fof(equiv_congruence, conjecture)

```

LCL451+1.p Prove Lukasiewicz's cn1 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or    fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv  fof(luka_op_equiv, axiom)
cn1    fof(luka_cn1, conjecture)

```

LCL452+1.p Prove Lukasiewicz's cn2 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or    fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv  fof(luka_op_equiv, axiom)
cn2    fof(luka_cn2, conjecture)

```

LCL453+1.p Prove Lukasiewicz's cn3 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or    fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv  fof(luka_op_equiv, axiom)
cn3    fof(luka_cn3, conjecture)

```

LCL454+1.p Prove Principia's r1 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_implies_or  fof(principia_op_implies_or, axiom)
op_and    fof(principia_op_and, axiom)
op_equiv  fof(principia_op_equiv, axiom)
r1    fof(principia_r1, conjecture)

```

LCL455+1.p Prove Principia's r2 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_implies_or  fof(principia_op_implies_or, axiom)
op_and    fof(principia_op_and, axiom)
op_equiv  fof(principia_op_equiv, axiom)
r2    fof(principia_r2, conjecture)

```

LCL456+1.p Prove Principia's r3 axiom from Hilbert's axiomatization

```

include('Axioms/LCL006+0.ax')

```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r3              fof(principia_r3, conjecture)
```

LCL457+1.p Prove Principia's r4 axiom from Hilbert's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r4              fof(principia_r4, conjecture)
```

LCL458+1.p Prove Principia's r5 axiom from Hilbert's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r5              fof(principia_r5, conjecture)
```

LCL459+1.p Prove Rosser's kn1 axiom from Hilbert's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or           fof(rosser_op_or, axiom)
op_implies_and  fof(rosser_op_implies_and, axiom)
op_equiv        fof(rosser_op_equiv, axiom)
kn1             fof(rosser_kn1, conjecture)
```

LCL460+1.p Prove Rosser's kn2 axiom from Hilbert's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or           fof(rosser_op_or, axiom)
op_implies_and  fof(rosser_op_implies_and, axiom)
op_equiv        fof(rosser_op_equiv, axiom)
kn2             fof(rosser_kn2, conjecture)
```

LCL461+1.p Prove Rosser's kn3 axiom from Hilbert's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
op_or           fof(rosser_op_or, axiom)
op_implies_and  fof(rosser_op_implies_and, axiom)
op_equiv        fof(rosser_op_equiv, axiom)
kn3             fof(rosser_kn3, conjecture)
```

LCL462+1.p Prove Hilbert's modus_tollens axiom from Lukasiewicz's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or           fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv        fof(hilbert_op_equiv, axiom)
modus_tollens   fof(hilbert_modus_tollens, conjecture)
```

LCL463+1.p Prove Hilbert's implies_1 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_1  fof(hilbert_implies_1, conjecture)

```

LCL464+1.p Prove Hilbert's implies_2 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_2  fof(hilbert_implies_2, conjecture)

```

LCL465+1.p Prove Hilbert's implies_3 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_3  fof(hilbert_implies_3, conjecture)

```

LCL466+1.p Prove Hilbert's and_1 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_1      fof(hilbert_and_1, conjecture)

```

LCL467+1.p Prove Hilbert's and_2 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_2      fof(hilbert_and_2, conjecture)

```

LCL468+1.p Prove Hilbert's and_3 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_3      fof(hilbert_and_3, conjecture)

```

LCL469+1.p Prove Hilbert's or_1 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_1       fof(hilbert_or_1, conjecture)

```

LCL470+1.p Prove Hilbert's or_2 axiom from Lukasiewicz's axiomatization

```

include('Axioms/LCL006+0.ax')

```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_2      fof(hilbert_or_2, conjecture)
```

LCL471+1.p Prove Hilbert's or_3 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_3      fof(hilbert_or_3, conjecture)
```

LCL472+1.p Prove Hilbert's equivalence_1 axiom from Lukasiewicz's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_1  fof(hilbert_equivalence_1, conjecture)
```

LCL473+1.p Prove Hilbert's equivalence_2 axiom from Lukasiewicz's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_2  fof(hilbert_equivalence_2, conjecture)
```

LCL474+1.p Prove Hilbert's equivalence_3 axiom from Lukasiewicz's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_3  fof(hilbert_equivalence_3, conjecture)
```

LCL475+1.p Prove Principia's r1 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_implies_or  fof(principia_op_implies_or, axiom)
op_and         fof(principia_op_and, axiom)
op_equiv       fof(principia_op_equiv, axiom)
r_1           fof(principia_r_1, conjecture)
```

LCL476+1.p Prove Principia's r2 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_implies_or  fof(principia_op_implies_or, axiom)
op_and         fof(principia_op_and, axiom)
op_equiv       fof(principia_op_equiv, axiom)
r_2           fof(principia_r_2, conjecture)
```

LCL477+1.p Prove Principia's r3 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv       fof(principia_op_equiv, axiom)
r3             fof(principia_r3, conjecture)
```

LCL478+1.p Prove Principia's r4 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv       fof(principia_op_equiv, axiom)
r4             fof(principia_r4, conjecture)
```

LCL479+1.p Prove Principia's r5 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv       fof(principia_op_equiv, axiom)
r5             fof(principia_r5, conjecture)
```

LCL480+1.p Prove Rosser's kn1 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or          fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv       fof(rosser_op_equiv, axiom)
kn1           fof(rosser_kn1, conjecture)
```

LCL481+1.p Prove Rosser's kn2 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or          fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv       fof(rosser_op_equiv, axiom)
kn2           fof(rosser_kn2, conjecture)
```

LCL482+1.p Prove Rosser's kn3 axiom from Lukasiewicz's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')
op_or          fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv       fof(rosser_op_equiv, axiom)
kn3           fof(rosser_kn3, conjecture)
```

LCL483+1.p Prove Hilbert's modus_tollens axiom from Principia's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or          fof(hilbert_op_or, axiom)
op_implies_and fof(hilbert_op_implies_and, axiom)
op_equiv       fof(hilbert_op_equiv, axiom)
modus_tollens  fof(hilbert_modus_tollens, conjecture)
```

LCL484+1.p Prove Hilbert's implies_1 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_1  fof(hilbert_implies_1, conjecture)

```

LCL485+1.p Prove Hilbert's implies_2 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_2  fof(hilbert_implies_2, conjecture)

```

LCL486+1.p Prove Hilbert's implies_3 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_3  fof(hilbert_implies_3, conjecture)

```

LCL487+1.p Prove Hilbert's and_1 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_1      fof(hilbert_and_1, conjecture)

```

LCL488+1.p Prove Hilbert's and_2 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_2      fof(hilbert_and_2, conjecture)

```

LCL489+1.p Prove Hilbert's and_3 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_3      fof(hilbert_and_3, conjecture)

```

LCL490+1.p Prove Hilbert's or_1 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_1       fof(hilbert_or_1, conjecture)

```

LCL491+1.p Prove Hilbert's or_2 axiom from Principia's axiomatization

```

include('Axioms/LCL006+0.ax')

```



```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_2      fof(hilbert_or_2, conjecture)
```

LCL492+1.p Prove Hilbert's or_3 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_3      fof(hilbert_or_3, conjecture)
```

LCL493+1.p Prove Hilbert's equivalence_1 axiom from Principia's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_1  fof(hilbert_equivalence_1, conjecture)
```

LCL494+1.p Prove Hilbert's equivalence_2 axiom from Principia's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_2  fof(hilbert_equivalence_2, conjecture)
```

LCL495+1.p Prove Hilbert's equivalence_3 axiom from Principia's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_3  fof(hilbert_equivalence_3, conjecture)
```

LCL496+1.p Prove Lukasiewicz's cn1 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn_1      fof(luka_cn_1, conjecture)
```

LCL497+1.p Prove Lukasiewicz's cn2 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn_2      fof(luka_cn_2, conjecture)
```

LCL498+1.p Prove Lukasiewicz's cn3 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(luka_op_or, axiom)
op_implies fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn3        fof(luka_cn3, conjecture)
```

LCL499+1.p Prove Rosser's kn1 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv   fof(rosser_op_equiv, axiom)
kn1        fof(rosser_kn1, conjecture)
```

LCL500+1.p Prove Rosser's kn2 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv   fof(rosser_op_equiv, axiom)
kn2        fof(rosser_kn2, conjecture)
```

LCL501+1.p Prove Rosser's kn3 axiom from Principia's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')
op_or      fof(rosser_op_or, axiom)
op_implies_and fof(rosser_op_implies_and, axiom)
op_equiv   fof(rosser_op_equiv, axiom)
kn3        fof(rosser_kn3, conjecture)
```

LCL502+1.p Prove Hilbert's modus_tollens axiom from Rosser's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
modus_tollens fof(hilbert_modus_tollens, conjecture)
```

LCL503+1.p Prove Hilbert's implies_1 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_1  fof(hilbert_implies_1, conjecture)
```

LCL504+1.p Prove Hilbert's implies_2 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_2  fof(hilbert_implies_2, conjecture)
```

LCL505+1.p Prove Hilbert's implies_3 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
implies_3  fof(hilbert_implies_3, conjecture)
```

LCL506+1.p Prove Hilbert's and₁ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_1      fof(hilbert_and_1, conjecture)
```

LCL507+1.p Prove Hilbert's and₂ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_2      fof(hilbert_and_2, conjecture)
```

LCL508+1.p Prove Hilbert's and₃ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
and_3      fof(hilbert_and_3, conjecture)
```

LCL509+1.p Prove Hilbert's or₁ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_1       fof(hilbert_or_1, conjecture)
```

LCL510+1.p Prove Hilbert's or₂ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_2       fof(hilbert_or_2, conjecture)
```

LCL511+1.p Prove Hilbert's or₃ axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
or_3       fof(hilbert_or_3, conjecture)
```

LCL512+1.p Prove Hilbert's equivalence₁ axiom from Rosser's system

```
include('Axioms/LCL006+0.ax')
```

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_1  fof(hilbert_equivalence_1, conjecture)
```

LCL513+1.p Prove Hilbert's equivalence_2 axiom from Rosser's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_2  fof(hilbert_equivalence_2, conjecture)
```

LCL514+1.p Prove Hilbert's equivalence_3 axiom from Rosser's system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
equivalence_3  fof(hilbert_equivalence_3, conjecture)
```

LCL515+1.p Prove Lukasiewicz's cn1 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn1       fof(luka_cn1, conjecture)
```

LCL516+1.p Prove Lukasiewicz's cn2 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn2       fof(luka_cn2, conjecture)
```

LCL517+1.p Prove Lukasiewicz's cn3 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_or      fof(luka_op_or, axiom)
op_implies  fof(luka_op_implies, axiom)
op_equiv   fof(luka_op_equiv, axiom)
cn3       fof(luka_cn3, conjecture)
```

LCL518+1.p Prove Principia's r1 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_implies_or  fof(principia_op_implies_or, axiom)
op_and         fof(principia_op_and, axiom)
op_equiv      fof(principia_op_equiv, axiom)
r1           fof(principia_r1, conjecture)
```

LCL519+1.p Prove Principia's r2 axiom from Rosser's axiomatization

```
include('Axioms/LCL006+0.ax')
```

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r2              fof(principia_r2, conjecture)

```

LCL520+1.p Prove Principia's r3 axiom from Rosser's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r3              fof(principia_r3, conjecture)

```

LCL521+1.p Prove Principia's r4 axiom from Rosser's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r4              fof(principia_r4, conjecture)

```

LCL522+1.p Prove Principia's r5 axiom from Rosser's axiomatization

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')
op_implies_or   fof(principia_op_implies_or, axiom)
op_and          fof(principia_op_and, axiom)
op_equiv        fof(principia_op_equiv, axiom)
r5              fof(principia_r5, conjecture)

```

LCL523+1.p Prove axiom 4 from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
axiom4          fof(km4b_axiom4, conjecture)

```

LCL524+1.p Prove axiom B from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
axiom_B        fof(km4b_axiom_B, conjecture)

```

LCL525+1.p Prove strict implies modus ponens from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly     fof(s1_0_op_possibly, axiom)
op_or           fof(s1_0_op_or, axiom)
op_implies      fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)

```

```

op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
modus_ponens_strict_implies  fof(s1_0_modus_ponens_strict_implies, conjecture)

```

LCL526+1.p Prove SSE from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or         fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
substitution_strict_equiv  fof(s1_0_substitution_strict_equiv, conjecture)

```

LCL527+1.p Prove adjunction from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or         fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
adjunction    fof(s1_0_adjunction, conjecture)

```

LCL528+1.p Prove axiom m1 from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or         fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
axiom_m1      fof(s1_0_axiom_m1, conjecture)

```

LCL529+1.p Prove axiom m2 from KM5 axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or         fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)

```

op_strict_equiv fof(s1.0_op_strict_equiv, axiom)
 axiom_m2 fof(s1.0_axiom_m2, conjecture)

LCL530+1.p Prove axiom m3 from KM5 axiomatization of S5

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL006+2.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+2.ax')
 op_possibly fof(s1.0_op_possibly, axiom)
 op_or fof(s1.0_op_or, axiom)
 op_implies fof(s1.0_op_implies, axiom)
 op_strict_implies fof(s1.0_op_strict_implies, axiom)
 op_equiv fof(s1.0_op_equiv, axiom)
 op_strict_equiv fof(s1.0_op_strict_equiv, axiom)
 axiom_m3 fof(s1.0_axiom_m3, conjecture)

LCL531+1.p Prove axiom m4 from KM5 axiomatization of S5

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL006+2.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+2.ax')
 op_possibly fof(s1.0_op_possibly, axiom)
 op_or fof(s1.0_op_or, axiom)
 op_implies fof(s1.0_op_implies, axiom)
 op_strict_implies fof(s1.0_op_strict_implies, axiom)
 op_equiv fof(s1.0_op_equiv, axiom)
 op_strict_equiv fof(s1.0_op_strict_equiv, axiom)
 axiom_m4 fof(s1.0_axiom_m4, conjecture)

LCL532+1.p Prove axiom m5 from KM5 axiomatization of S5

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL006+2.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+2.ax')
 op_possibly fof(s1.0_op_possibly, axiom)
 op_or fof(s1.0_op_or, axiom)
 op_implies fof(s1.0_op_implies, axiom)
 op_strict_implies fof(s1.0_op_strict_implies, axiom)
 op_equiv fof(s1.0_op_equiv, axiom)
 op_strict_equiv fof(s1.0_op_strict_equiv, axiom)
 axiom_m5 fof(s1.0_axiom_m5, conjecture)

LCL533+1.p Prove axiom m6 from KM5 axiomatization of S5

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL006+2.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+2.ax')
 op_possibly fof(s1.0_op_possibly, axiom)
 op_or fof(s1.0_op_or, axiom)
 op_implies fof(s1.0_op_implies, axiom)
 op_strict_implies fof(s1.0_op_strict_implies, axiom)
 op_equiv fof(s1.0_op_equiv, axiom)
 op_strict_equiv fof(s1.0_op_strict_equiv, axiom)

axiom_m6 fof(s1_0_m6s3m9b_axiom_m6, conjecture)

LCL534+1.p Prove axiom s3 from KM5 axiomatization of S5

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies     fof(s1_0_op_implies, axiom)
op_strict_implies   fof(s1_0_op_strict_implies, axiom)
op_equiv       fof(s1_0_op_equiv, axiom)
op_strict_equiv   fof(s1_0_op_strict_equiv, axiom)
axiom_s3       fof(s1_0_m6s3m9b_axiom_s3, conjecture)
```

LCL535+1.p Prove axiom m9 from KM5 axiomatization of S5

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies     fof(s1_0_op_implies, axiom)
op_strict_implies   fof(s1_0_op_strict_implies, axiom)
op_equiv       fof(s1_0_op_equiv, axiom)
op_strict_equiv   fof(s1_0_op_strict_equiv, axiom)
axiom_m9       fof(s1_0_m6s3m9b_axiom_m9, conjecture)
```

LCL536+1.p Prove axiom m10 from KM5 axiomatization of S5

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies     fof(s1_0_op_implies, axiom)
op_strict_implies   fof(s1_0_op_strict_implies, axiom)
op_equiv       fof(s1_0_op_equiv, axiom)
op_strict_equiv   fof(s1_0_op_strict_equiv, axiom)
axiom_m10      fof(s1_0_m10_axiom_m10, conjecture)
```

LCL537+1.p Prove axiom 5 from KM4B axiomatization of S5

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
axiom_5       fof(km5_axiom_5, conjecture)
```

LCL538+1.p Prove strict implies modus ponens from KM4B axiomatization of S5

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
```



```

include('Axioms/LCL007+3.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or        fof(s1_0_op_or, axiom)
op_implies   fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv     fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
modus_ponens_strict_implies fof(s1_0_modus_ponens_strict_implies, conjecture)

```

LCL539+1.p Prove SSE from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or        fof(s1_0_op_or, axiom)
op_implies   fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv     fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
substitution_strict_equiv fof(s1_0_substitution_strict_equiv, conjecture)

```

LCL540+1.p Prove adjunction from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or        fof(s1_0_op_or, axiom)
op_implies   fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv     fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
adjunction   fof(s1_0_adjunction, conjecture)

```

LCL541+1.p Prove axiom m1 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly   fof(s1_0_op_possibly, axiom)
op_or        fof(s1_0_op_or, axiom)
op_implies   fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv     fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
axiom_m1     fof(s1_0_axiom_m1, conjecture)

```

LCL542+1.p Prove axiom m2 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')

```

```

op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
axiom_m2       fof(s1_0_axiom_m2, conjecture)

```

LCL543+1.p Prove axiom m3 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
axiom_m3       fof(s1_0_axiom_m3, conjecture)

```

LCL544+1.p Prove axiom m4 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
axiom_m4       fof(s1_0_axiom_m4, conjecture)

```

LCL545+1.p Prove axiom m5 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly    fof(s1_0_op_possibly, axiom)
op_or          fof(s1_0_op_or, axiom)
op_implies    fof(s1_0_op_implies, axiom)
op_strict_implies  fof(s1_0_op_strict_implies, axiom)
op_equiv      fof(s1_0_op_equiv, axiom)
op_strict_equiv  fof(s1_0_op_strict_equiv, axiom)
axiom_m5       fof(s1_0_axiom_m5, conjecture)

```

LCL546+1.p Prove axiom m6 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly    fof(s1_0_op_possibly, axiom)

```

```

op_or      fof(s1_0_op_or, axiom)
op_implies fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv   fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
axiom_m6   fof(s1_0_m6s3m9b_axiom_m6, conjecture)

```

LCL547+1.p Prove axiom s3 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly fof(s1_0_op_possibly, axiom)
op_or      fof(s1_0_op_or, axiom)
op_implies fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv   fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
axiom_s3   fof(s1_0_m6s3m9b_axiom_s3, conjecture)

```

LCL548+1.p Prove axiom m9 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly fof(s1_0_op_possibly, axiom)
op_or      fof(s1_0_op_or, axiom)
op_implies fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv   fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
axiom_m9   fof(s1_0_m6s3m9b_axiom_m9, conjecture)

```

LCL549+1.p Prove axiom m10 from KM4B axiomatization of S5

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
op_possibly fof(s1_0_op_possibly, axiom)
op_or      fof(s1_0_op_or, axiom)
op_implies fof(s1_0_op_implies, axiom)
op_strict_implies fof(s1_0_op_strict_implies, axiom)
op_equiv   fof(s1_0_op_equiv, axiom)
op_strict_equiv fof(s1_0_op_strict_equiv, axiom)
axiom_m10  fof(s1_0_m10_axiom_m10, conjecture)

```

LCL550+1.p Prove Hilbert's modus ponens rule from the S1-0 system

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)

```

substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 modus_ponens fof(hilbert_modus_ponens, conjecture)

LCL551+1.p Prove Hilbert's modus_tollens axiom from the S1-0 system

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 op_or fof(hilbert_op_or, axiom)
 op_implies_and fof(hilbert_op_implies_and, axiom)
 op_equiv fof(hilbert_op_equiv, axiom)
 substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 modus_tollens fof(hilbert_modus_tollens, conjecture)

LCL552+1.p Prove Hilbert's implies_1 axiom from the S1-0 system

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 op_or fof(hilbert_op_or, axiom)
 op_implies_and fof(hilbert_op_implies_and, axiom)
 op_equiv fof(hilbert_op_equiv, axiom)
 substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 implies_1 fof(hilbert_implies_1, conjecture)

LCL553+1.p Prove Hilbert's implies_2 axiom from the S1-0 system

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 op_or fof(hilbert_op_or, axiom)
 op_implies_and fof(hilbert_op_implies_and, axiom)
 op_equiv fof(hilbert_op_equiv, axiom)
 substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 implies_2 fof(hilbert_implies_2, conjecture)

LCL554+1.p Prove Hilbert's implies_3 axiom from the S1-0 system

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 op_or fof(hilbert_op_or, axiom)
 op_implies_and fof(hilbert_op_implies_and, axiom)
 op_equiv fof(hilbert_op_equiv, axiom)
 substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 implies_3 fof(hilbert_implies_3, conjecture)

LCL555+1.p Prove Hilbert's and_1 axiom from the S1-0 system

include('Axioms/LCL006+0.ax')
 include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 op_or fof(hilbert_op_or, axiom)
 op_implies_and fof(hilbert_op_implies_and, axiom)
 op_equiv fof(hilbert_op_equiv, axiom)
 substitution_of_equivalents fof(substitution_of_equivalents, axiom)
 and_1 fof(hilbert_and_1, conjecture)

LCL556+1.p Prove Hilbert's and_2 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
and_2      fof(hilbert_and_2, conjecture)
```

LCL557+1.p Prove Hilbert's and_3 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
and_3      fof(hilbert_and_3, conjecture)
```

LCL558+1.p Prove Hilbert's or_1 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
or_1       fof(hilbert_or_1, conjecture)
```

LCL559+1.p Prove Hilbert's or_2 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
or_2       fof(hilbert_or_2, conjecture)
```

LCL560+1.p Prove Hilbert's or_3 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
or_3       fof(hilbert_or_3, conjecture)
```

LCL561+1.p Prove Hilbert's equivalence_1 axiom from the S1-0 system

```
include('Axioms/LCL006+0.ax')
```

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
equivalence_1  fof(hilbert_equivalence_1, conjecture)

```

LCL562+1.p Prove Hilbert's equivalence_2 axiom from the S1-0 system

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
equivalence_2  fof(hilbert_equivalence_2, conjecture)

```

LCL563+1.p Prove Hilbert's equivalence_3 axiom from the S1-0 system

```

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
equivalence_3  fof(hilbert_equivalence_3, conjecture)

```

LCL564+1.p Prove axiom K from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
axiom_K    fof(km5_axiom_K, conjecture)

```

LCL565+1.p Prove necessitation from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
necessitation  fof(km5_necessitation, conjecture)

```

LCL566+1.p Prove axiom M from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')

```

```

include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
axiom_M    fof(km5_axiom_M, conjecture)

```

LCL567+1.p Prove axiom 5 from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
axiom_5    fof(km5_axiom_5, conjecture)

```

LCL568+1.p Prove axiom 4 from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
axiom_4    fof(km4b_axiom_4, conjecture)

```

LCL569+1.p Prove axiom m10 from the S1-0M6S3M9B axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
axiom_m10  fof(s1_0_m10_axiom_m10, conjecture)

```

LCL570+1.p Prove axiom K from the S1-0M10 axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)
axiom_K    fof(km5_axiom_K, conjecture)

```

LCL571+1.p Prove necessitation from the S1-0M10 axiomatization of S5

```

include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or      fof(hilbert_op_or, axiom)
op_implies_and  fof(hilbert_op_implies_and, axiom)
op_equiv   fof(hilbert_op_equiv, axiom)
substitution_of_equivalents  fof(substitution_of_equivalents, axiom)

```

necessitation fof(km5_necessitation, conjecture)

LCL572+1.p Prove axiom M from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or    fof(hilbert_op_or, axiom)
op_implies_and    fof(hilbert_op_implies_and, axiom)
op_equiv    fof(hilbert_op_equiv, axiom)
substitution_of_equivalents    fof(substitution_of_equivalents, axiom)
axiom_M    fof(km5_axiom_M, conjecture)
```

LCL573+1.p Prove axiom 5 from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or    fof(hilbert_op_or, axiom)
op_implies_and    fof(hilbert_op_implies_and, axiom)
op_equiv    fof(hilbert_op_equiv, axiom)
substitution_of_equivalents    fof(substitution_of_equivalents, axiom)
axiom_5    fof(km5_axiom_5, conjecture)
```

LCL574+1.p Prove axiom 4 from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or    fof(hilbert_op_or, axiom)
op_implies_and    fof(hilbert_op_implies_and, axiom)
op_equiv    fof(hilbert_op_equiv, axiom)
substitution_of_equivalents    fof(substitution_of_equivalents, axiom)
axiom_4    fof(km4b_axiom_4, conjecture)
```

LCL575+1.p Prove axiom B from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
op_or    fof(hilbert_op_or, axiom)
op_implies_and    fof(hilbert_op_implies_and, axiom)
op_equiv    fof(hilbert_op_equiv, axiom)
substitution_of_equivalents    fof(substitution_of_equivalents, axiom)
axiom_B    fof(km4b_axiom_B, conjecture)
```

LCL576+1.p Prove axiom m6 from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
axiom_m6    fof(s1_0_m6s3m9b_axiom_m6, conjecture)
```

LCL577+1.p Prove axiom s3 from the S1-0M10 axiomatization of S5

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
```


include('Axioms/LCL007+6.ax')
 axiom_s3 fof(s1.0_m6s3m9b_axiom_s3, conjecture)

LCL578+1.p Prove axiom m9 from the S1-0M10 axiomatization of S5

include('Axioms/LCL006+1.ax')
 include('Axioms/LCL007+0.ax')
 include('Axioms/LCL007+1.ax')
 include('Axioms/LCL007+4.ax')
 include('Axioms/LCL007+6.ax')
 axiom_m9 fof(s1.0_m6s3m9b_axiom_m9, conjecture)

LCL579^1.p Leibniz-equality definition means it's an equivalence

$\forall x: \$i, y: \$i: (\forall p: \$i \rightarrow \$o: ((p@x) \Rightarrow (p@y)) \Rightarrow \forall p: \$i \rightarrow \$o: ((p@y) \Rightarrow (p@x)))$ thf(a, conjecture)

LCL579^2.p Leibniz-equality definition means it's an equivalence

cY: \$i thf(cY, type)
 cX: \$i thf(cX, type)
 $\forall p: \$i \rightarrow \$o: ((p@cY) \Rightarrow (p@cX)) \Rightarrow \forall r: \$i \rightarrow \$o: ((r@cX) \Rightarrow (r@cY))$ thf(cTHM76A, conjecture)

LCL580^1.p Popkorn problem 1

include('Axioms/LCL008^0.ax')
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 mvalid@(mbox@r@mtrue) thf(thm, conjecture)

LCL581^1.p Popkorn problem 2

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 mvalid@(mimpl@(mbox@r@a)@(mbox@r@a)) thf(thm, conjecture)

LCL582^1.p Popkorn problem 3

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 s: \$i \rightarrow \$i \rightarrow \$o thf(s, type)
 mvalid@(mimpl@(mbox@r@a)@(mbox@s@a)) thf(thm, conjecture)

LCL583^1.p Popkorn problem 4

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 s: \$i \rightarrow \$i \rightarrow \$o thf(s, type)
 mvalid@(mbox@s@(mimpl@(mbox@r@a)@(mbox@r@a))) thf(thm, conjecture)

LCL584^1.p Popkorn problem 5

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 b: \$i \rightarrow \$o thf(b, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 s: \$i \rightarrow \$i \rightarrow \$o thf(s, type)
 mvalid@(miff@(mbox@r@(mand@a@b))@(mand@(mbox@r@a)@(mbox@r@b))) thf(thm, conjecture)

LCL585^1.p Popkorn problem 6

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 b: \$i \rightarrow \$o thf(b, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)
 s: \$i \rightarrow \$i \rightarrow \$o thf(s, type)
 mvalid@(mimpl@(mdia@r@(mimpl@a@b))@(mimpl@(mbox@r@a)@(mdia@r@b))) thf(thm, conjecture)

LCL586^1.p Popkorn problem 7

include('Axioms/LCL008^0.ax')
 a: \$i \rightarrow \$o thf(a, type)
 b: \$i \rightarrow \$o thf(b, type)
 r: \$i \rightarrow \$i \rightarrow \$o thf(r, type)

$mvalid@(mimpl@(mnot@(mdia@r@a))@(mbox@r@(mimpl@a@b)))$ $thf(thm, conjecture)$

LCL587 \wedge **1.p** Popkorn problem 8

$include('Axioms/LCL008^0.ax')$

$a: \$i \rightarrow \o $thf(a, type)$

$b: \$i \rightarrow \o $thf(b, type)$

$r: \$i \rightarrow \$i \rightarrow \$o$ $thf(r, type)$

$mvalid@(mimpl@(mbox@r@b))@(mbox@r@(mimpl@a@b)))$ $thf(thm, conjecture)$

LCL588 \wedge **1.p** Popkorn problem 9

$include('Axioms/LCL008^0.ax')$

$a: \$i \rightarrow \o $thf(a, type)$

$b: \$i \rightarrow \o $thf(b, type)$

$r: \$i \rightarrow \$i \rightarrow \$o$ $thf(r, type)$

$mvalid@(mimpl@(mimpl@(mdia@r@a))@(mbox@r@b))@(mbox@r@(mimpl@a@b)))$ $thf(thm, conjecture)$

LCL589 \wedge **1.p** Popkorn problem 10

$include('Axioms/LCL008^0.ax')$

$a: \$i \rightarrow \o $thf(a, type)$

$b: \$i \rightarrow \o $thf(b, type)$

$r: \$i \rightarrow \$i \rightarrow \$o$ $thf(r, type)$

$mvalid@(mimpl@(mimpl@(mdia@r@a))@(mbox@r@b))@(mimpl@(mbox@r@a))@(mbox@r@b)))$ $thf(thm, conjecture)$

LCL590 \wedge **1.p** Popkorn problem 11

$include('Axioms/LCL008^0.ax')$

$a: \$i \rightarrow \o $thf(a, type)$

$b: \$i \rightarrow \o $thf(b, type)$

$r: \$i \rightarrow \$i \rightarrow \$o$ $thf(r, type)$

$mvalid@(mimpl@(mimpl@(mdia@r@a))@(mbox@r@b))@(mimpl@(mdia@r@a))@(mdia@r@b)))$ $thf(thm, conjecture)$

LCL591 \wedge **1.p** Axiom N is valid

$include('Axioms/LCL008^0.ax')$

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o: (mvalid@a) \Rightarrow (mvalid@(mbox@r@a))$ $thf(thm, conjecture)$

LCL592 \wedge **1.p** Axiom D is valid

$include('Axioms/LCL008^0.ax')$

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@a@b))@(mimpl@(mbox@r@a))@(mbox@r@b)))$

LCL593 \wedge **1.p** Is axiom T valid in K?

$include('Axioms/LCL008^0.ax')$

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a@a))$ $thf(thm, conjecture)$

LCL594 \wedge **1.p** Relation for all propositions making T valid in K

Is there a relation R such that for all modal propositions A, axiom T is valid in K

$include('Axioms/LCL008^0.ax')$

$\exists r: \$i \rightarrow \$i \rightarrow \$o: \forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a@a))$ $thf(thm, conjecture)$

LCL595 \wedge **1.p** Is axiom T equivalent to reflexivity of R in K

$include('Axioms/LCL008^0.ax')$

$include('Axioms/SET008^2.ax')$

$r: \$i \rightarrow \$i \rightarrow \$o$ $thf(r, type)$

$\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a@a)) \iff (reflexive@r))$ $thf(thm, conjecture)$

LCL596 \wedge **1.p** Is axiom 4 valid in K?

$include('Axioms/LCL008^0.ax')$

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a))@(mbox@r@(mbox@r@a)))$ $thf(thm, conjecture)$

LCL597 \wedge **1.p** Relation and proposition for 4 in K

Is there a relation R and a modal proposition A for which axiom 4 is valid in K?

$include('Axioms/LCL008^0.ax')$

$\exists r: \$i \rightarrow \$i \rightarrow \$o: \forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a))@(mbox@r@(mbox@r@a)))$ $thf(thm, conjecture)$

LCL598 \wedge **1.p** Is axiom 4 equivalent to irreflexivity?

$include('Axioms/LCL008^0.ax')$

$include('Axioms/SET008^2.ax')$

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a))@(mbox@r@(mbox@r@a))) \iff (irreflexive@r))$ $thf(thm, conjecture)$

LCL599 \wedge **1.p** Is axiom 4 equivalent to symmetry?

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \iff (symmetric@r))$ thf(thm, conjecture)

LCL600 \wedge **1.p** Is axiom 4 equivalent to transitivity of R in K?

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \iff (transitive@r))$ thf(thm, conjecture)

LCL601 \wedge **1.p** Axiom 4 for all R means all R are valid

If axiom 4 is valid for all relations R then all relations R are transitive.

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \iff \forall r: \$i \rightarrow \$i \rightarrow \$o: (transitive@r)$ thf(thm, conjecture)

LCL602 \wedge **1.p** T and 4 equivalent to reflexivity and transitivity of R in K

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \text{ and } mvalid@(mimpl@(mbox@r@a)@(mbox@r@a))) \iff (reflexive@r \text{ and } transitive@r)$ thf(thm, conjecture)

LCL603 \wedge **1.p** T and 4 imply reflexivity and transitivity of R in K

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \text{ and } mvalid@(mimpl@(mbox@r@a)@(mbox@r@a))) \iff (reflexive@r \text{ and } transitive@r)$ thf(thm, conjecture)

LCL604 \wedge **1.p** T and 4 implied by reflexivity and transitivity of R in K

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall a: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@a)@(mbox@r@(mbox@r@a)))) \text{ and } mvalid@(mimpl@(mbox@r@a)@(mbox@r@a))) \iff (reflexive@r \text{ and } transitive@r)$ thf(thm, conjecture)

LCL606 \wedge **1.p** LAMBDA \wedge mm_1 validates the Barcan formula axioms

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \text{individuals} \rightarrow \$i \rightarrow \$o: (mvalid@(mimpl@(mall@\lambda x: \text{individuals}: (mbox@r@(a@x)))@(mbox@r@(mall@\lambda x: \text{individuals}: (mbox@r@(a@x))))))$

LCL607 \wedge **1.p** LAMBDA \wedge mm_1 validates the axioms defining possibility

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i \rightarrow \$o: (mvalid@(miff@(mdia@r@a)@(mnot@(mbox@r@(mnot@a))))$ thf(thm, conjecture)

LCL608 \wedge **1.p** LAMBDA \wedge mm_1 validates the modus ponens rule

include('Axioms/LCL008^0.ax')

$\forall a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: ((mvalid@a \text{ and } mvalid@(mimpl@a@b)) \Rightarrow (mvalid@b))$ thf(thm, conjecture)

LCL609 \wedge **1.p** LAMBDA \wedge mm_1 validates the generalization rule

include('Axioms/LCL008^0.ax')

$\forall p: \text{individuals} \rightarrow \$i \rightarrow \$o: (\forall x: \text{individuals}: (mvalid@(p@x)) \Rightarrow (mvalid@(mall@\lambda x: \text{individuals}: (p@x))))$ thf(thm, conjecture)

LCL611 \wedge **1.p** LAMBDA \wedge mm_1 validates the converse Barcan formula

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \text{individuals} \rightarrow \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mall@\lambda x: \text{individuals}: (a@x)))@(mall@\lambda x: \text{individuals}: (mbox@r@(a@x))))$

LCL612 \wedge **1.p** Modus Ponens holds in K

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((mvalid@x \text{ and } mvalid@(mimpl@x@y)) \Rightarrow (mvalid@y))$ thf(modus_ponens, conjecture)

LCL613 \wedge **1.p** Simple theorem of K

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mand@x@y))@(mbox@r@x)))$ thf(thm, conjecture)

LCL614 \wedge **1.p** Regularity is a derived rule in K

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((mvalid@(mimpl@x@y)) \Rightarrow (mvalid@(mimpl@(mbox@r@x)@(mbox@r@x))))$ thf(thm, conjecture)

LCL615 \wedge **1.p** Axiom KB

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mdia@r@x))))$ thf(kb, conjecture)

LCL617^1.p Axiom GL - the Loeb formula

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@(mbox@r@x)@x)@(mbox@r@x)))$ thf(thm, conjecture)

LCL618^1.p Axiom GL implies Axiom K4 in K

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@(mbox@r@x)@x)@(mbox@r@x)))$ thf(gl, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@(mbox@r@x))))$ thf(k4, conjecture)

LCL619^1.p A simple theorem of K4

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))$ thf(k4, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@x@y)@(mimpl@(mbox@r@x)@(mbox@r@x))))$

LCL620^1.p A simple theorem of propositional logic

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (mvalid@(mimpl@(mnot@(mand@x@y)@(mor@(mnot@x)@(mnot@y))))$ thf(thm)

LCL621^1.p A simple theorem of K4

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))$ thf(k4, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (mvalid@(mimpl@(mand@(mbox@r@(mdia@r@x)@(mbox@r@y)@(mbox@r@mdia@r@x))))$

LCL623^1.p The Loeb formula is a theorem in GL

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@(mbox@r@x)@x)@(mbox@r@x)))$ thf(gl, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, p: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@(mbox@r@p)@p)@(mbox@r@p)))$ thf(loeb, conjecture)

LCL624^1.p A simple theorem of K

include('Axioms/LCL008^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o, p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: (mvalid@(mimpl@(mand@(mdia@r@(mbox@r@p)@(mbox@r@(mdia@r@q)@(mdia@r@p))))$

LCL625^1.p GL/K4 axiom is valid in this frame

In a frame that is transitive and upwards well-founded, the GL/K4 axiom is valid.

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (transitive@r \text{ and } upwards_well_founded@r)$ thf(upwf_trans, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, p: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@(mimpl@(mbox@r@p)@p)@(mbox@r@p)))$ thf(loeb, conjecture)

LCL626^1.p Loeb axiom is valid in this frame

In a frame that is transitive and upwards well-founded, the Loeb axiom is valid.

include('Axioms/LCL008^0.ax')

include('Axioms/SET008^2.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: (transitive@r \text{ and } upwards_well_founded@r)$ thf(upwf_trans, axiom)

$\forall r: \$i \rightarrow \$i \rightarrow \$o, x: \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@(mbox@r@x))))$ thf(k4, conjecture)

LCL629^1.p Simple theorem about knowledge

include('Axioms/LCL008^0.ax')

$a: \$i \rightarrow \$i \rightarrow \$o$ thf(a, type)

$b: \$i \rightarrow \$i \rightarrow \$o$ thf(b, type)

$c: \$i \rightarrow \$i \rightarrow \$o$ thf(c, type)

$\forall x: \$i \rightarrow \$o, r: \$i \rightarrow \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@x))$ thf(knowledge_implies_truth, axiom)

$\forall x: \$i \rightarrow \$o, r: \$i \rightarrow \$i \rightarrow \$o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))$ thf(positive_introspection, axiom)

$\forall x: \$i \rightarrow \$o, r: \$i \rightarrow \$i \rightarrow \$o: (mvalid@(mimpl@(mnot@(mbox@r@x)@(mbox@r@(mnot@(mbox@r@x))))))$ thf(negative_introspection, axiom)

$\forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (mvalid@(mimpl@(mand@(mbox@a@(mnot@(mbox@b@(mnot@(mbox@b@(mnot@y))))))@(mbox@a@y))))$

LCL630^1.p The muddy forehead puzzle

include('Axioms/LCL008^0.ax')

$a: \$i \rightarrow \$i \rightarrow \$o$ thf(a, type)

$b: \$i \rightarrow \$i \rightarrow \$o$ thf(b, type)

$c: \$i \rightarrow \$i \rightarrow \$o$ thf(c, type)

$mfa: \$i \rightarrow \o thf(mfa, type)

$mfb: \$i \rightarrow \o thf(mfb, type)

```

mfc: $i → $o    thf(mfc, type)
ck: ($i → $o) → $i → $o    thf(ck, type)
s: $i → $o    thf(s, type)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@x))    thf(knowledge_implies_truth, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))    thf(positive_introspection, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mnot@(mbox@r@x)@(mbox@r@(mnot@(mbox@r@x))))))    thf(negative_introspection, axiom)
ck = (λx: $i → $o, w: $i: ∀r: $i → $i → $o: (mbox@r@x@w))    thf(common_knowledge, definition)
mvalid@(ck@(mor@(mbox@a@mfb)@(mbox@a@(mnot@mfb))))    thf(what_a_knows_about_b, axiom)
mvalid@(ck@(mor@(mbox@a@mfc)@(mbox@a@(mnot@mfc))))    thf(what_a_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@b@mfa)@(mbox@b@(mnot@mfa))))    thf(what_b_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@b@mfc)@(mbox@b@(mnot@mfc))))    thf(what_b_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@c@mfa)@(mbox@c@(mnot@mfa))))    thf(what_c_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@c@mfb)@(mbox@c@(mnot@mfb))))    thf(what_c_knows_about_b, axiom)
s = (mor@(mbox@a@mfa)@(mor@(mbox@a@(mnot@mfa)@(mor@(mbox@b@mfb)@(mor@(mbox@b@(mnot@mfb))@(mor@
mvalid@(mnot@(mimpl@(ck@(mor@mfa@(mor@mfb@mfc))@s))    thf(thm, conjecture)

```

LCL631^1.p The muddy forehead puzzle

```

include('Axioms/LCL008^0.ax')
a: $i → $i → $o    thf(a, type)
b: $i → $i → $o    thf(b, type)
c: $i → $i → $o    thf(c, type)
mfa: $i → $o    thf(mfa, type)
mfb: $i → $o    thf(mfb, type)
mfc: $i → $o    thf(mfc, type)
ck: ($i → $o) → $i → $o    thf(ck, type)
s: $i → $o    thf(s, type)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@x))    thf(knowledge_implies_truth, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))    thf(positive_introspection, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mnot@(mbox@r@x)@(mbox@r@(mnot@(mbox@r@x))))))    thf(negative_introspection, axiom)
ck = (λx: $i → $o, w: $i: ∀r: $i → $i → $o: (mbox@r@x@w))    thf(common_knowledge, definition)
mvalid@(ck@(mor@(mbox@a@mfb)@(mbox@a@(mnot@mfb))))    thf(what_a_knows_about_b, axiom)
mvalid@(ck@(mor@(mbox@a@mfc)@(mbox@a@(mnot@mfc))))    thf(what_a_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@b@mfa)@(mbox@b@(mnot@mfa))))    thf(what_b_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@b@mfc)@(mbox@b@(mnot@mfc))))    thf(what_b_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@c@mfa)@(mbox@c@(mnot@mfa))))    thf(what_c_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@c@mfb)@(mbox@c@(mnot@mfb))))    thf(what_c_knows_about_b, axiom)
s = (mor@(mbox@a@mfa)@(mor@(mbox@a@(mnot@mfa)@(mor@(mbox@b@mfb)@(mor@(mbox@b@(mnot@mfb))@(mor@
mvalid@(mnot@(mimpl@(ck@(mnot@(mimpl@(ck@(mor@mfa@(mor@mfb@mfc))@s))@s))    thf(thm, conjecture)

```

LCL632^1.p The muddy forehead puzzle

```

include('Axioms/LCL008^0.ax')
a: $i → $i → $o    thf(a, type)
b: $i → $i → $o    thf(b, type)
c: $i → $i → $o    thf(c, type)
mfa: $i → $o    thf(mfa, type)
mfb: $i → $o    thf(mfb, type)
mfc: $i → $o    thf(mfc, type)
ck: ($i → $o) → $i → $o    thf(ck, type)
s: $i → $o    thf(s, type)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@x))    thf(knowledge_implies_truth, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mbox@r@x)@(mbox@r@(mbox@r@x))))    thf(positive_introspection, axiom)
∀x: $i → $o, r: $i → $i → $o: (mvalid@(mimpl@(mnot@(mbox@r@x)@(mbox@r@(mnot@(mbox@r@x))))))    thf(negative_introspection, axiom)
ck = (λx: $i → $o, w: $i: ∀r: $i → $i → $o: (mbox@r@x@w))    thf(common_knowledge, definition)
mvalid@(ck@(mor@(mbox@a@mfb)@(mbox@a@(mnot@mfb))))    thf(what_a_knows_about_b, axiom)
mvalid@(ck@(mor@(mbox@a@mfc)@(mbox@a@(mnot@mfc))))    thf(what_a_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@b@mfa)@(mbox@b@(mnot@mfa))))    thf(what_b_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@b@mfc)@(mbox@b@(mnot@mfc))))    thf(what_b_knows_about_c, axiom)
mvalid@(ck@(mor@(mbox@c@mfa)@(mbox@c@(mnot@mfa))))    thf(what_c_knows_about_a, axiom)
mvalid@(ck@(mor@(mbox@c@mfb)@(mbox@c@(mnot@mfb))))    thf(what_c_knows_about_b, axiom)
s = (mor@(mbox@a@mfa)@(mor@(mbox@a@(mnot@mfa)@(mor@(mbox@b@mfb)@(mor@(mbox@b@(mnot@mfb))@(mor@

```

mvalid@(mimpl@(ck@(mnot@(mimpl@(ck@(mnot@(mimpl@(ck@(mor@mfa@(mor@mfb@mfc)))@s)))@s)))@s) thf(thm,

LCL633+1.p Goedel's ontological argument on the existence of God

include('Axioms/LCL008^0.ax')

a: \$tType thf(a_type, type)

p: (a → \$i → \$o) → \$i → \$o thf(p_type, type)

g: a → \$i → \$o thf(g_type, type)

e: (a → \$i → \$o) → a → \$i → \$o thf(e_type, type)

r: \$i → \$i → \$o thf(r_type, type)

∀x: a → \$i → \$o: (mvalid@(mimpl@(mnot@(p@x))@(p@λz: a: (mnot@(x@z)))) thf(positiveness, axiom)

g = (λz: a, w: \$i: ∀x: a → \$i → \$o: (mimpl@(p@x)@(x@z)@w)) thf(g, definition)

e = (λx: a → \$i → \$o, z: a, p: \$i: ∀y: a → \$i → \$o: (mimpl@(y@z)@(mbox@r@λq: \$i: ∀w: a: (mimpl@(x@w)@(y@w)@q))@p))

mvalid@λw: \$i: ∀z: a: (mimpl@(g@z)@(e@g@z)@w) thf(thm, conjecture)

LCL636+1.001.p In K, the branching formula made provable, size 1

The branching formula plus a negation symbol in front and an additional subformula to make the formula provable.

¬∃x: ¬¬∀y: (¬r₁(x, y) or p₂(y)) or ¬∀y: (¬r₁(x, y) or ((¬∀x: (¬r₁(y, x) or ¬¬p₂(x) and ¬p₁₀₂(x) and p₁₀₁(x)) and ¬∀x

LCL637+1.001.p In K, the branching formula, size 1

¬∃x: (∀y: (¬r₁(x, y) or ((¬∀x: (¬r₁(y, x) or ¬¬p₂(x) and ¬p₁₀₂(x) and p₁₀₁(x)) and ¬∀x: (¬r₁(y, x) or ¬p₂(x) and ¬p₁

LCL638+1.001.p In K, D & A4 & B p0/p0 → T, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬¬∀x: (¬r₁(y, x) or ¬∀y: (¬r₁(x, y) or p₁(y))) and ¬p₁(y) or ¬∀y: (¬r₁(x, y) or ¬¬∀x: (¬r₁

LCL639+1.001.p In K, A5 not provable with instances of D, A4, and T, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬¬∀x: (¬r₁(y, x) or ∀y: (¬r₁(x, y) or ¬¬p₁(y) and ∀x: (¬r₁(y, x) or p₁(x)))) and ∀x: (¬r₁(y, x)

LCL644+1.001.p In K, H2 → L, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬p₂(y) and ∀x: (¬r₁(y, x) or p₂(x)) or p₁(y) or ∀y: (¬r₁(x, y) or p₂(y) or ¬p₁(y) and ∀x: (¬r₁

LCL645+1.001.p In K, L+ is not provable with instances of L, size 1

¬∃x: ¬∀y: (¬r₁(x, y) or p₁(y) or ¬∀x: (¬r₁(y, x) or p₁(x))) or ∀y: (¬r₁(x, y) or p₁(y) or ¬∀x: (¬r₁(y, x) or p₁(x))) fof

LCL646+1.001.p In K, path through a labyrinth, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or p₆(y)) or ¬∀y: (¬r₁(x, y) or p₂(y)) or ¬∀y: (¬r₁(x, y) or p₄(y)) or ¬∀y: (¬r₁(x, y) or p₂(y)) or

LCL648+1.001.p In K, pigeonhole formulae, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬p₂₀₁(y) and p₁₀₁(y)) or ∀y: (¬r₁(x, y) or ¬p₂₀₁(y) and p₁₀₁(y)) fof(main, conjecture)

LCL649+1.001.p In K, pigeonhole formulae missing a conjunct, size 1

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬p₂₀₁(y) and ¬p₁₀₁(y)) or ∀y: (¬r₁(x, y) or ¬p₂₀₁(y) and p₁₀₁(y)) fof(main, conjecture)

LCL650+1.001.p In K, black and white polygon with odd number of vertices, size 1

If we have a polygon with n vertices, and all the vertices are either black or white, then two adjacent vertices have the same colour.

¬∃x: ¬∀y: (¬r₁(x, y) or ∀x: (¬r₁(y, x) or ∀y: (¬r₁(x, y) or ∀x: (¬r₁(y, x) or (¬p₈(x) and ¬p₆(x) and ¬p₄(x) and ¬p₂(x))))

LCL654+1.001.p In KT, A5box p0/p0 & box A5 p0/p0 → A4, size 1

∀x: r₁(x, x) fof(reflexivity, axiom)

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬¬∀x: (¬r₁(y, x) or ¬∀y: (¬r₁(x, y) or ¬p₁(y))) and p₁(y) or ¬∀y: (¬r₁(x, y) or ∀x: (¬r₁(y, x)

LCL655+1.001.p In KT, A5 not provable with instances of A4, size 1

∀x: r₁(x, x) fof(reflexivity, axiom)

¬∃x: ¬¬∀y: (¬r₁(x, y) or ¬¬∀x: (¬r₁(y, x) or ∀y: (¬r₁(x, y) or ¬¬p₁(y) and ∀x: (¬r₁(y, x) or p₁(x)))) and ∀x: (¬r₁(y, x)

LCL656+1.001.p In KT, the branching formula made provable, size 1

The branching formula plus a negation symbol in front and an additional subformula to make the formula provable.

∀x: r₁(x, x) fof(reflexivity, axiom)

¬∃x: ¬¬∀y: (¬r₁(x, y) or p₂(y)) or ¬∀y: (¬r₁(x, y) or ((¬∀x: (¬r₁(y, x) or ¬¬p₂(x) and ¬p₁₀₂(x) and p₁₀₁(x)) and ¬∀x

LCL657+1.001.p In KT, the branching formula, size 1

∀x: r₁(x, x) fof(reflexivity, axiom)

¬∃x: (∀y: (¬r₁(x, y) or ((¬∀x: (¬r₁(y, x) or ¬¬p₂(x) and ¬p₁₀₂(x) and p₁₀₁(x)) and ¬∀x: (¬r₁(y, x) or ¬p₂(x) and ¬p₁

LCL662+1.001.p In KT, in backward search find a way through box and dia, size 1

¬∃x: ¬¬p₁(x) or p₁(x) fof(main, conjecture)

LCL663+1.001.p In KT, in backwards search no way through box and dia, size 1

¬∃x: ¬p₁(x) fof(main, conjecture)

LCL664+1.001.p In KT, path through a labyrinth, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } p_{16}(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_{12}(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_{14}(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_{12}(y))$

LCL666+1.001.p In KT, pigeonhole formulae, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_{201}(y) \text{ and } p_{101}(y)) \text{ or } \neg p_{201}(x) \text{ and } p_{101}(x)$ fof(main, conjecture)

LCL667+1.001.p In KT, pigeonhole formulae missing a conjunct, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_{201}(y) \text{ and } \neg p_{101}(y)) \text{ or } \neg p_{201}(x) \text{ and } p_{101}(x)$ fof(main, conjecture)

LCL672+1.001.p In S4, A5box p0/p0 & box A5 p0/p0 \rightarrow A5, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg (\neg \forall y: (\neg r_1(x, y) \text{ or } p_2(y)) \text{ and } \neg \forall y: (\neg r_1(x, y) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_1(y)))) \text{ and } p_1(y)) \text{ and } \neg \forall y: (\neg r_1(x, y) \text{ or } p_2(y))$

LCL674+1.001.p In S4, the branching formula made provable, size 1

The branching formula plus a negation symbol in front and an additional subformula to make the formula provable.

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } p_2(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } ((\neg \forall x: (\neg r_1(y, x) \text{ or } \neg \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x)) \text{ and } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x)))) \text{ and } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x))$

LCL675+1.001.p In S4, the branching formula, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: (\forall y: (\neg r_1(x, y) \text{ or } ((\neg \forall x: (\neg r_1(y, x) \text{ or } \neg \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x)) \text{ and } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x)))) \text{ and } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg p_2(x) \text{ and } \neg p_{102}(x) \text{ and } p_{101}(x))$

LCL678+1.001.p In S4, formula provable in intuitionistic logic, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \text{\$} \text{false or } \neg \forall y: (\neg r_1(x, y) \text{ or } \text{\$} \text{false or } \neg \forall x: (\neg r_1(y, x) \text{ or } \forall y: (\neg r_1(x, y) \text{ or } p_1(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_1(y))))$ f

LCL679+1.001.p In S4, formula not provable in intuitionistic logic, size 1

$\neg \exists x: \neg \text{\$} \text{false or } \text{\$} \text{false}$ fof(main, conjecture)

LCL680+1.001.p In S4, in backward search find a way through box and dia, size 1

$\neg \exists x: \neg \neg p_1(x) \text{ or } p_1(x)$ fof(main, conjecture)

LCL681+1.001.p In S4, in backwards search no way through box and dia, size 1

$\neg \exists x: \neg p_1(x)$ fof(main, conjecture)

LCL682+1.001.p In S4, path through a labyrinth, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \neg \neg \forall x: (\neg r_1(y, x) \text{ or } p_{16}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{12}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{14}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{12}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{14}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{12}(x)) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } p_{14}(x))$

LCL684+1.001.p In S4, pigeonhole formulae, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \forall x: (\neg r_1(y, x) \text{ or } \neg p_{201}(x) \text{ and } p_{101}(x))) \text{ or } \neg p_{201}(x) \text{ and } p_{101}(x)$ fof(main, conjecture)

LCL685+1.001.p In S4, pigeonhole formulae missing a conjunct, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \forall x: (\neg r_1(y, x) \text{ or } \neg p_{201}(x) \text{ and } \neg p_{101}(x))) \text{ or } \neg p_{201}(x) \text{ and } p_{101}(x)$ fof(main, conjecture)

LCL686+1.001.p In S4, formula provable in S5 embedding, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_3(y) \text{ or } \forall x: (\neg r_1(y, x) \text{ or } \neg p_1(x))) \text{ or } \forall y: (\neg r_1(x, y) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } \text{\$} \text{false or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_1(y))))$

LCL687+1.001.p In S4, formula not provable in S5 embedding, size 1

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)

$\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)

$\neg \exists x: \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_6(y) \text{ or } \forall x: (\neg r_1(y, x) \text{ or } \neg p_1(x))) \text{ or } \forall y: (\neg r_1(x, y) \text{ or } \neg \forall x: (\neg r_1(y, x) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } p_6(y))))$

LCL688+1.001.p In S4, formula with T and A4, size 1

Tdia p0/p0 & box T box dia p0/p0 & A4dia p0/p0 & box(dia box dia p0 \rightarrow (p0 \rightarrow box p0)) \rightarrow dia box p0 \rightarrow dia box p0.

$\forall x: r_1(x, x)$ fof(reflexivity, axiom)
 $\forall x, y, z: ((r_1(x, y) \text{ and } r_1(y, z)) \Rightarrow r_1(x, z))$ fof(transitivity, axiom)
 $\neg \exists x: \neg \neg \forall y: (\neg r_1(x, y) \text{ or } \neg p_4(y)) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } \neg \neg \forall x: (\neg r_1(y, x) \text{ or } \neg \forall y: (\neg r_1(x, y) \text{ or } \forall x: (\neg r_1(y, x) \text{ or } p_3(x)) \text{ or } \dots)) \text{ or } \dots))$

LCL690 \wedge **1.p** Prove K in the CS4 translation
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
 $\forall a: \$i \rightarrow \$o, b: \$i \rightarrow \$o: (\text{cs4_valid}@\text{(cs4_impl}@\text{(cs4_box}@\text{(cs4_impl}@\text{(cs4_atom}@\text{a})}@\text{(cs4_atom}@\text{b}))))@(\text{cs4_impl}@\text{(cs4_box}@\text{(cs4_atom}@\text{a}))))$

LCL691 \wedge **1.p** Prove reflexivity in the CS4 translation
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
 $\forall a: \$i \rightarrow \$o: (\text{cs4_valid}@\text{(cs4_impl}@\text{(cs4_box}@\text{(cs4_atom}@\text{a})}@\text{(cs4_atom}@\text{a}))))$ thf(cs4_refl, conjecture)

LCL692 \wedge **1.p** Prove transitivity in the CS4 translation
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
 $\forall a: \$i \rightarrow \$o: (\text{cs4_valid}@\text{(cs4_impl}@\text{(cs4_box}@\text{(cs4_atom}@\text{a})}@\text{(cs4_box}@\text{(cs4_box}@\text{(cs4_atom}@\text{a}))))))$ thf(cs4_trans, conjecture)

LCL693 \wedge **1.p** Prove the Barcan formula in the CS4 translation
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
 $\forall a: \text{individuals} \rightarrow \$i \rightarrow \$o: (\text{cs4_valid}@\text{(cs4_impl}@\text{(cs4_all}@\text{\lambda i: individuals: (cs4_box}@\text{(cs4_atom}@\text{(a@i))))}@\text{(cs4_box}@\text{(cs4_all}@\text{\lambda i: individuals: (cs4_impl}@\text{(cs4_box}@\text{(cs4_atom}@\text{(a@i))))}@\text{(cs4_atom}@\text{a}))))))$

LCL694 \wedge **1.p** Prove the converse Barcan formula in the CS4 translation
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
 $\forall a: \text{individuals} \rightarrow \$i \rightarrow \$o: (\text{cs4_valid}@\text{(cs4_impl}@\text{(cs4_box}@\text{(cs4_all}@\text{\lambda i: individuals: (cs4_atom}@\text{(a@i))))}@\text{(cs4_all}@\text{\lambda i: individuals: (cs4_impl}@\text{(cs4_box}@\text{(cs4_atom}@\text{(a@i))))}@\text{(cs4_atom}@\text{a}))))))$

LCL695 \wedge **1.p** Propositional intuitionistic logic in HOL
An embedding of propositional intuitionistic logic in HOL based on Goedel's first translation of propositional intuitionistic logic to modal logic S4.
include('Axioms/LCL010^0.ax')

LCL696 \wedge **1.p** Propositional intuitionistic logic in HOL
An embedding of propositional intuitionistic logic in HOL based on Goedel's second translation of propositional intuitionistic logic to modal logic S4.
include('Axioms/LCL011^0.ax')

LCL697 \wedge **1.p** Propositional intuitionistic logic in HOL
An embedding of propositional intuitionistic logic in HOL based on the McKinsey/Tarski translation of propositional intuitionistic logic to modal logic S4.
include('Axioms/LCL012^0.ax')

LCL698 \wedge **1.p** Embedding of quantified multimodal logic in simple type theory
include('Axioms/LCL013^0.ax')

LCL699 \wedge **1.p** Accessibility relation implies axiom for reflexivity
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mreflexive}@\text{r}) \Rightarrow (\text{mvalid}@\text{(mforall_prop}@\text{\lambda a: \$i} \rightarrow \$o: (\text{mimplies}@\text{(mbox}@\text{r}@\text{a})}@\text{a}))))$ thf(conj, conj)

LCL700 \wedge **1.p** Accessibility relation implies axiom for symmetry
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{msymmetric}@\text{r}) \Rightarrow (\text{mvalid}@\text{(mforall_prop}@\text{\lambda a: \$i} \rightarrow \$o: (\text{mimplies}@\text{a}@\text{(mbox}@\text{r}@\text{(mdia}@\text{r}@\text{a}))))))$

LCL701 \wedge **1.p** Accessibility relation implies axiom for seriality
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mserial}@\text{r}) \Rightarrow (\text{mvalid}@\text{(mforall_prop}@\text{\lambda a: \$i} \rightarrow \$o: (\text{mimplies}@\text{(mbox}@\text{r}@\text{a})}@\text{(mdia}@\text{r}@\text{a}))))$ thf(c

LCL702 \wedge **1.p** Accessibility relation implies axiom for transitivity
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mtransitive}@\text{r}) \Rightarrow (\text{mvalid}@\text{(mforall_prop}@\text{\lambda a: \$i} \rightarrow \$o: (\text{mimplies}@\text{(mbox}@\text{r}@\text{a})}@\text{(mbox}@\text{r}@\text{(mbox}@\text{r}@\text{a}))))$

LCL703 \wedge **1.p** Accessibility relation implies axiom for Euclidianity
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{meuclidean}@\text{r}) \Rightarrow (\text{mvalid}@\text{(mforall_prop}@\text{\lambda a: \$i} \rightarrow \$o: (\text{mimplies}@\text{(mdia}@\text{r}@\text{a})}@\text{(mbox}@\text{r}@\text{(mdia}@\text{r}@\text{a}))))$

LCL704 \wedge **1.p** Accessibility relation implies axiom for partial functionality
include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mpartially_functional}@r) \Rightarrow (\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@r@a)@(\text{mbox}@r@a))))))$

LCL705 \wedge **1.p** Accessibility relation implies axiom for functionality

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mfunctional}@r) \Rightarrow (\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mequiv}@(\text{mdia}@r@a)@(\text{mbox}@r@a))))))$ thf

LCL706 \wedge **1.p** Accessibility relation implies axiom for weak density

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mweakly_dense}@r) \Rightarrow (\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@r@(\text{mbox}@r@a))@(\text{mbox}@r@a))))))$

LCL707 \wedge **1.p** Accessibility relation implies axiom for weak connectedness

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mweakly_connected}@r) \Rightarrow (\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mforall_prop}@ \lambda b: \$i \rightarrow \$o: (\text{mor}@(\text{mbox}@r@(\text{mimplies}@(\text{mand}@a@(\text{mbox}@r@a))@b))@(\text{mbox}@r@(\text{mimplies}@(\text{mand}@b@(\text{mbox}@r@b))@a))))))))))$

LCL708 \wedge **1.p** Accessibility relation implies axiom for weak directedness

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mweakly_directed}@r) \Rightarrow (\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@r@(\text{mbox}@r@a))@(\text{mbox}@r@a))))))$

LCL709 \wedge **1.p** Axiom implies accessibility relation for reflexivity

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@r@a)@a))) \Rightarrow (\text{mreflexive}@r))$ thf(conj, conj)

LCL710 \wedge **1.p** Axiom implies accessibility relation for symmetry

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@a@(\text{mbox}@r@(\text{mdia}@r@a)))))) \Rightarrow (\text{msymmetric}@r))$

LCL711 \wedge **1.p** Axiom implies accessibility relation for seriality

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@r@a)@(\text{mdia}@r@a)))))) \Rightarrow (\text{mserial}@r))$ thf(c

LCL712 \wedge **1.p** Axiom implies accessibility relation for transitivity

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@r@a)@(\text{mbox}@r@(\text{mbox}@r@a)))))) \Rightarrow (\text{mtransitive}@r))$ thf(conj, conjecture)

LCL713 \wedge **1.p** Axiom implies accessibility relation for Euclidianity

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@r@a)@(\text{mbox}@r@(\text{mdia}@r@a)))))) \Rightarrow (\text{meuclidean}@r))$ thf(conj, conjecture)

LCL714 \wedge **1.p** Axiom implies accessibility relation for partial functionality

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@r@a)@(\text{mbox}@r@a)))))) \Rightarrow (\text{mpartially_functional}@r))$

LCL715 \wedge **1.p** Axiom implies accessibility relation for functionality

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mequiv}@(\text{mdia}@r@a)@(\text{mbox}@r@a)))))) \Rightarrow (\text{mfunctional}@r))$ thf

LCL716 \wedge **1.p** Axiom implies accessibility relation for weak density

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@r@(\text{mbox}@r@a))@(\text{mbox}@r@a)))))) \Rightarrow (\text{mweakly_dense}@r))$ thf(conj, conjecture)

LCL717 \wedge **1.p** Axiom implies accessibility relation for weak connectedness

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mforall_prop}@ \lambda b: \$i \rightarrow \$o: (\text{mor}@(\text{mbox}@r@(\text{mimplies}@(\text{mand}@a@a)@b))@(\text{mbox}@r@(\text{mimplies}@(\text{mand}@a@a)@b)))))))))) \Rightarrow (\text{mweakly_connected}@r))$ thf(conj, conjecture)

LCL718 \wedge **1.p** Axiom implies accessibility relation for weak directedness

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@r@(\text{mbox}@r@a))@(\text{mbox}@r@(\text{mdia}@r@a)))))) \Rightarrow (\text{mweakly_directed}@r))$ thf(conj, conjecture)

LCL719 \wedge **1.p** Necessitation rule holds in monomodal logic K

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

phi: $\$i \rightarrow \o thf(phi, type)

$(\text{mvalid}@phi) \Rightarrow (\text{mvalid}@(\text{mbox}_k@phi))$ thf(conj, conjecture)

LCL720 \wedge **1.p** Distribution axiom holds in monomodal logic K

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

phi: $\$i \rightarrow \o thf(phi, type)

psi: $\$i \rightarrow \o thf(psi, type)

mvalid@(mimplies@(mbox_k@(mimplies@phi@psi))@(mimplies@(mbox_k@phi))@(mbox_k@psi))) thf(conj, conjecture)

LCL721 \wedge **1.p** Axiom D holds in monomodal logic D

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

phi: $\$i \rightarrow \o thf(phi, type)

mvalid@(mimplies@(mbox_d@phi))@(mdia_d@phi)) thf(conj, conjecture)

LCL722 \wedge **1.p** Axiom M holds in monomodal logic M

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

phi: $\$i \rightarrow \o thf(phi, type)

mvalid@(mimplies@(mbox_m@phi))@phi) thf(conj, conjecture)

LCL723 \wedge **1.p** Axioms M and B hold in monomodal logic B

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^4.ax')

phi: $\$i \rightarrow \o thf(phi, type)

mvalid@(mimplies@(mbox_b@phi))@phi) and mvalid@(mimplies@phi@(mbox_b@(mdia_b@phi))) thf(conj, conjecture)

LCL724 \wedge **1.p** Axioms M and 4 hold in monomodal logic S4

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^5.ax')

phi: $\$i \rightarrow \o thf(phi, type)

mvalid@(mimplies@(mbox_s4@phi))@phi) and mvalid@(mimplies@(mbox_s4@phi))@(mbox_s4@(mbox_s4@phi))) thf(conj,

LCL725 \wedge **1.p** Axioms M, 4, and B hold in monomodal logic S5

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

phi: $\$i \rightarrow \o thf(phi, type)

mvalid@(mimplies@(mbox_s5@phi))@phi) and mvalid@(mimplies@(mbox_s5@phi))@(mbox_s5@(mbox_s5@phi))) and mvalid@

LCL726 \wedge **5.p** TPS problem THM534

AC1 \Rightarrow AC17 from [RR93].

a: $\$tType$ thf(a_type, type)

$\forall xs: (a \rightarrow \$o) \rightarrow \$o: (\forall x: a \rightarrow \$o: ((xs@x) \Rightarrow \exists xt: a: (x@xt)) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x)))) \Rightarrow \forall xg: ((a \rightarrow \$o) \rightarrow a) \rightarrow a \rightarrow \$o: (\forall xh: (a \rightarrow \$o) \rightarrow a: \exists xu: a: (xg@xh@xu) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: (xg@xf@(xf@(xg@xf))))$ thf(cTHM₅₃₄, conjecture)

LCL727 \wedge **5.p** TPS problem THM533

AC3 \Rightarrow AC1 from [RR93].

a: $\$tType$ thf(a_type, type)

$\forall xr: (a \rightarrow \$o) \rightarrow a \rightarrow \$o: \exists xg: (a \rightarrow \$o) \rightarrow a: \forall xx: a \rightarrow \$o: (\exists xy: a: (xr@xx@xy) \Rightarrow (xr@xx@(xg@xx))) \Rightarrow \forall xs: (a \rightarrow \$o) \rightarrow \$o: (\forall x: a \rightarrow \$o: ((xs@x) \Rightarrow \exists xt: a: (x@xt)) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x))))$ thf(cTHM₅₃₃, conjecture)

LCL728 \wedge **5.p** TPS problem THM532

AC1 \Rightarrow AC3 from [RR93].

b: $\$tType$ thf(b_type, type)

a: $\$tType$ thf(a_type, type)

$\forall xs: (b \rightarrow \$o) \rightarrow \$o: (\forall x: b \rightarrow \$o: ((xs@x) \Rightarrow \exists xy: b: (x@xy)) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall x: b \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x)))) \Rightarrow \forall xr: a \rightarrow b \rightarrow \$o: \exists xg: a \rightarrow b: \forall xx: a: (\exists xy: b: (xr@xx@xy) \Rightarrow (xr@xx@(xg@xx)))$ thf(cTHM₅₃₂, conjecture)

LCL729 \wedge **5.p** TPS problem THM560

AC1(A) equiv AC3(OA,A) from [RR93]. Note that AC3 usually refers to 'relations' where here we are using it for relations on OA x A.

a: $\$tType$ thf(a_type, type)

$\forall xr: (a \rightarrow \$o) \rightarrow a \rightarrow \$o: \exists xg: (a \rightarrow \$o) \rightarrow a: \forall xx: a \rightarrow \$o: (\exists xy: a: (xr@xx@xy) \Rightarrow (xr@xx@(xg@xx))) \iff \forall xs: (a \rightarrow \$o) \rightarrow \$o: (\forall x: a \rightarrow \$o: ((xs@x) \Rightarrow \exists xt: a: (x@xt)) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x))))$ thf(cTHM₅₆₀, conjecture)

LCL730 \wedge **5.p** TPS problem X5310

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xr: (b \rightarrow \$o) \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: (xr@xx@xy) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o: (xr@xx@(xf@xx))) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310, conjecture)

LCL731 \wedge **5.p** TPS problem THM541

Equivalence of global choice at type A (usual way of expressing AC in type theory).

a : \$tType thf(a_type, type)

$\exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt) \Rightarrow (x@(xf@x))) \iff \forall xs: (a \rightarrow \$o) \rightarrow \$o: (\forall x: a \rightarrow \$o: ((xs@x) \Rightarrow \exists xt: a: (x@xt)) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x))))$ thf(cTHM541, conjecture)

LCL732 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

y : \$i thf(y, type)

p : \$i \rightarrow \$o thf(p, type)

$\forall xx: b \rightarrow \$o: \exists xy_0: b: (\exists xx_0: $i: (p@xx_0) \Rightarrow (p@y)) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o: (\exists xx_0: $i: (p@xx_0) \Rightarrow (p@y))$ thf(cX5310_SUB3, conjecture)

LCL733 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xr: (b \rightarrow \$o) \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: (\exists xx_0: b: (xx@xx_0) \Rightarrow (xx@xy)) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o: (\exists xx_0: b: (xx@xx_0) \Rightarrow (xx@(xf@xx)))) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310_SUB4, conjecture)

LCL734 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xr_3: (b \rightarrow \$o) \rightarrow b \rightarrow \$o, xr_4: (b \rightarrow \$o) \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: (xr_3@xx@xy \text{ or } xr_4@xx@xy) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o: (xr_3@xx@(xf@xx) \text{ or } xr_4@xx@(xf@xx)) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310B, conjecture)

LCL735 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xa: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, xb: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: \forall xw: b: (xa@xx@xy@xw \text{ or } xb@xx@xy@xw) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o, xw: b: (xa@xx@(xf@xx)@xw \text{ or } xb@xx@(xf@xx)@xw)) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310_SUB5, conjecture)

LCL736 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xr_{41}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, xr_{42}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b, xw_{11}: b: (xr_{41}@xx@xy@xw_{11} \text{ or } xr_{42}@xx@xy@xw_{11})) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o: \exists xw_{11}: b: (xr_{41}@xx@(xf@xx)@xw_{11} \text{ or } xr_{42}@xx@(xf@xx)@xw_{11})) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310D, conjecture)

LCL738 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\exists xr_{24}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, xr_{23}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: \forall xw_6: b: (xr_{23}@xx@xy@xw_6 \text{ or } xr_{24}@xx@xy@xw_6) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o, xw_6: b: (xr_{23}@xx@(xf@xx)@xw_6 \text{ or } xr_{24}@xx@(xf@xx)@xw_6)) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310_SUB, conjecture)

LCL739 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall xr_{24}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, xr_{23}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\forall xx: b \rightarrow \$o: \exists xy: b: \forall xw_6: b: (xr_{23}@xx@xy@xw_6 \text{ or } xr_{24}@xx@xy@xw_6) \Rightarrow \exists xf: (b \rightarrow \$o) \rightarrow b: \forall xx: b \rightarrow \$o, xw_6: b: (xr_{23}@xx@(xf@xx)@xw_6 \text{ or } xr_{24}@xx@(xf@xx)@xw_6)) \Rightarrow \exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xz: b: (xp@xz) \Rightarrow (xp@(xj@xp)))$ thf(cX5310_SUB2, conjecture)

LCL740 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

b : \$tType thf(b_type, type)

$\forall x_{r12}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, x_{r13}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\exists x_f: (b \rightarrow \$o) \rightarrow b: \forall x_x: b \rightarrow \$o, x_{w4}: b: (x_{r12}@x_x@(x_f@x_x)@x_w4) \rightarrow \exists x_y: b: \forall x_{w4}: b: (x_{r12}@x_x@x_y@x_{w4} \text{ or } x_{r13}@x_x@x_y@x_{w4})) \Rightarrow \exists x_j: (b \rightarrow \$o) \rightarrow b: \forall x_p: b \rightarrow \$o: (\exists x_z: b: (x_p@x_z) \Rightarrow (x_p@(x_j@x_p))))$ thf(cX5310_SUB_R, conjecture)

LCL741 \wedge **5.p** TPS problem from AC-THMS

Related to the axiom of choice.

$b: \$tType$ thf(b_type, type)

$\forall x_r: (b \rightarrow \$o) \rightarrow b \rightarrow \$o, x_{r27}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o, x_{r28}: (b \rightarrow \$o) \rightarrow b \rightarrow b \rightarrow \$o: (\forall x_x: b \rightarrow \$o: \exists x_y: b: \forall x_{w11}: b: (x_{r27}@x_x@x_y@x_{w11} \text{ or } x_{r28}@x_x@x_y@x_{w11})) \Rightarrow \exists x_f: (b \rightarrow \$o) \rightarrow b: \forall x_x: b \rightarrow \$o, x_{w11}: b: (x_{r27}@x_x@(x_f@x_w11) \Rightarrow \exists x_j: (b \rightarrow \$o) \rightarrow b: \forall x_p: b \rightarrow \$o: (\exists x_z: b: (x_p@x_z) \Rightarrow (x_p@(x_j@x_p))))$ thf(cX5310X_pme, conjecture)

LCL742 \wedge **5.p** TPS problem from AC-FUNS-THMS

$f: \$i \rightarrow \i thf(f, type)

$g: \$i \rightarrow \i thf(g, type)

$\exists x_j: (\$i \rightarrow \$o) \rightarrow \$i: \forall x_p: \$i \rightarrow \$o: (\exists x_x: \$i: (x_p@x_x) \Rightarrow (x_p@(x_j@x_p))) \Rightarrow (\forall x_x: \$i, x_y: \$i: ((g@x_x) = (g@x_y) \Rightarrow (f@x_x) = (f@x_y))) \Rightarrow \exists x_h: \$i \rightarrow \$i: (\lambda x_x: \$i: (x_h@(g@x_x))) = f$ thf(cTHM588AC_pme, conjecture)

LCL743 \wedge **5.p** TPS problem from AXIOMOFDESCR

Related to the axiom of description.

$a: \$tType$ thf(a_type, type)

$cF: a \rightarrow \$o$ thf(cF, type)

$cJ: (a \rightarrow \$o) \rightarrow a$ thf(cJ, type)

$cX: a$ thf(cX, type)

$(cF@cX) \Rightarrow (\forall y: a: ((cF@y) \Rightarrow cX = y) \Rightarrow (cF@(cJ@cF)))$ thf(cDESCR_CHURCH, conjecture)

LCL744-1.p Strong normalization of typed lambda calculus 019_3

$c_Lambda_Obeta(v_s, v_t) \Rightarrow c_Lambda_Obeta(c_Lambda_OdB_OAbs(v_s), c_Lambda_OdB_OAbs(v_t))$ cnf(cls_abs0, axiom)

$c_Lambda_Obeta(c_Lambda_OdB_OAbs(v_r), v_s) \Rightarrow v_s = c_Lambda_OdB_OAbs(c_Lambda_Osiko_Lambda_Xbeta_cases_2_1(v_r, v_s))$ cnf(cls_obeta_beta, axiom)

$c_Lambda_Obeta(v_s, v_t) \Rightarrow c_Lambda_Obeta(c_Lambda_OdB_OApp(v_s, v_u), c_Lambda_OdB_OApp(v_t, v_u))$ cnf(cls_obeta_app, axiom)

$c_Lambda_Obeta(v_s, v_t) \Rightarrow c_Lambda_Obeta(c_Lambda_OdB_OApp(v_u, v_s), c_Lambda_OdB_OApp(v_u, v_t))$ cnf(cls_obeta_app, axiom)

$c_Lambda_Obeta(v_r, v_s) \Rightarrow c_Lambda_Obeta(c_Lambda_Osubst(v_r, v_t, v_i), c_Lambda_Osubst(v_s, v_t, v_i))$ cnf(cls_obeta_subst, axiom)

$c_Lambda_OdB_OAbs(v_dB) = c_Lambda_OdB_OAbs(v_dB_H) \Rightarrow v_dB = v_dB_H$ cnf(cls_dB_Osimsps_I3_J0, axiom)

$c_Lambda_Obeta(c_Lambda_OdB_OApp(c_Lambda_OdB_OAbs(v_s), v_t), c_Lambda_Osubst(v_s, v_t, c_HOL_Ozero_class_Ozero)) \Rightarrow c_Lambda_OdB_OApp(v_dB1, v_dB2) = c_Lambda_OdB_OApp(v_dB1_H, v_dB2_H) \Rightarrow v_dB2 = v_dB2_H$ cnf(cls_dB_Osimsps_I9_J0, axiom)

$c_Lambda_OdB_OApp(v_dB1, v_dB2) = c_Lambda_OdB_OApp(v_dB1_H, v_dB2_H) \Rightarrow v_dB1 = v_dB1_H$ cnf(cls_dB_Osimsps_I8_J0, axiom)

$c_Lambda_Obeta(c_Lambda_OdB_OAbs(v_r), v_s) \Rightarrow c_Lambda_Obeta(v_r, c_Lambda_Osiko_Lambda_Xbeta_cases_2_1(v_r, v_s))$ cnf(cls_obeta_beta, axiom)

$c_Lambda_OdB_OAbs(v_dB_H) \neq c_Lambda_OdB_OApp(v_dB1, v_dB2)$ cnf(cls_dB_Osimsps_I9_J0, axiom)

$c_Lambda_OdB_OApp(v_dB1, v_dB2) \neq c_Lambda_OdB_OAbs(v_dB_H)$ cnf(cls_dB_Osimsps_I8_J0, axiom)

$c_Lambda_Osubst(c_Lambda_OdB_OApp(v_t, v_u), v_s, v_k) = c_Lambda_OdB_OApp(c_Lambda_Osubst(v_t, v_s, v_k), c_Lambda_Osubst(v_u, v_s, v_k))$ cnf(cls_obeta_subst, axiom)

$c_Lambda_Olift(c_Lambda_OdB_OApp(v_s, v_t), v_k) = c_Lambda_OdB_OApp(c_Lambda_Olift(v_s, v_k), c_Lambda_Olift(v_t, v_k))$ cnf(cls_obeta_lift, axiom)

$c_InductTermi_OIT(v_r) \Rightarrow c_InductTermi_OIT(c_Lambda_OdB_OAbs(v_r))$ cnf(cls_Lambda0, axiom)

$c_Lambda_Osubst(c_Lambda_Olift(v_t, v_k), v_s, v_k) = v_t$ cnf(cls_subst_lift0, axiom)

$c_Lambda_Obeta(v_r, v_s) \Rightarrow c_Lambda_Obeta(c_Lambda_Olift(v_r, v_i), c_Lambda_Olift(v_s, v_i))$ cnf(cls_lift_preserves, axiom)

$c_InductTermi_OIT(v_t)$ cnf(cls_conjecture0, negated_conjecture)

$\neg c_InductTermi_OIT(c_Lambda_Olift(v_t, v_i))$ cnf(cls_conjecture1, negated_conjecture)

LCL859 \wedge **1.p** Modal logic S5(=M5) coincides with MB5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((mreflexive@r \text{ and } meuclidean@r) \iff (mreflexive@r \text{ and } msymmetric@r \text{ and } meuclidean@r))$ thf(cLCL859, axiom)

LCL860 \wedge **1.p** Modal logic S5(=M5) coincides with M4B5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((mreflexive@r \text{ and } meuclidean@r) \iff (mreflexive@r \text{ and } mtransitive@r \text{ and } msymmetric@r \text{ and } meuclidean@r))$ thf(cLCL860, axiom)

LCL861 \wedge **1.p** Modal logic S5(=M5) coincides with M45

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((mreflexive@r \text{ and } meuclidean@r) \iff (mreflexive@r \text{ and } mtransitive@r \text{ and } meuclidean@r))$ thf(cLCL861, axiom)

LCL862 \wedge **1.p** Modal logic S5(=M5) coincides with M4B

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((mreflexive@r \text{ and } meuclidean@r) \iff (mreflexive@r \text{ and } mtransitive@r \text{ and } msymmetric@r))$ thf(cLCL862, axiom)

LCL863 \wedge **1.p** Modal logic S5(=M5) coincides with D4B

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((mreflexive@r \text{ and } meuclidean@r) \iff (mserial@r \text{ and } mtransitive@r \text{ and } msymmetric@r))$ thf(cLCL863, axiom)

LCL864 \wedge **1.p** Modal logic S5(=M5) coincides with D4B5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mreflexive}@r \text{ and } \text{meuclidean}@r) \iff (\text{mserial}@r \text{ and } \text{mtransitive}@r \text{ and } \text{msymmetric}@r \text{ and } \text{meuclidean}@r))$

LCL865 \wedge **1.p** Modal logic S5(=M5) coincides with DB5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mreflexive}@r \text{ and } \text{meuclidean}@r) \iff (\text{mserial}@r \text{ and } \text{msymmetric}@r \text{ and } \text{meuclidean}@r))$ thf(conj)

LCL866 \wedge **1.p** Modal logic KB5 coincides with K4B5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{msymmetric}@r \text{ and } \text{meuclidean}@r) \iff (\text{mtransitive}@r \text{ and } \text{msymmetric}@r \text{ and } \text{meuclidean}@r))$ thf(conj)

LCL867 \wedge **1.p** Modal logic KB5 coincides with K4B

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{msymmetric}@r \text{ and } \text{meuclidean}@r) \iff (\text{mtransitive}@r \text{ and } \text{msymmetric}@r))$ thf(conj, conjecture)

LCL868 \wedge **1.p** Modal logic D45 'includes' modal logic M5

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mreflexive}@r \text{ and } \text{meuclidean}@r) \Rightarrow (\text{mserial}@r \text{ and } \text{mtransitive}@r \text{ and } \text{meuclidean}@r))$ thf(conj, co

LCL869 \wedge **1.p** Modal logic M5 'includes' modal logic D45

include('Axioms/LCL013^0.ax')

$\forall r: \$i \rightarrow \$i \rightarrow \$o: ((\text{mserial}@r \text{ and } \text{mtransitive}@r \text{ and } \text{meuclidean}@r) \Rightarrow (\text{mreflexive}@r \text{ and } \text{meuclidean}@r))$ thf(conj, co

LCL870 \wedge **1.p** The Barcan formula is valid in quantified modal logic K

include('Axioms/LCL013^0.ax')

$r: \$i \rightarrow \$i \rightarrow \$o$ thf(r , type)

$p: \mu \rightarrow \$i \rightarrow \o thf(p , type)

$\text{mvalid}@(\text{mimplies}@(\text{mforall_ind}@ \lambda x: \mu: (\text{mbox}@r@(p@x))@(\text{mbox}@r@(\text{mforall_ind}@ \lambda x: \mu: (p@x))))))$ thf(ex₁, conjecture)

LCL871 \wedge **1.p** The converse Barcan formula is valid in quantified modal logic K

include('Axioms/LCL013^0.ax')

$r: \$i \rightarrow \$i \rightarrow \$o$ thf(r , type)

$p: \mu \rightarrow \$i \rightarrow \o thf(p , type)

$\text{mvalid}@(\text{mimplies}@(\text{mbox}@r@(\text{mforall_ind}@ \lambda x: \mu: (p@x))@(\text{mforall_ind}@ \lambda x: \mu: (\text{mbox}@r@(p@x))))))$ thf(ex2b, conjecture)

LCL872 \wedge **1.p** Correspondence between box and diamond and a confluence property

include('Axioms/LCL013^0.ax')

$i: \$i \rightarrow \$i \rightarrow \$o$ thf(i , type)

$j: \$i \rightarrow \$i \rightarrow \$o$ thf(j , type)

$k: \$i \rightarrow \$i \rightarrow \$o$ thf(k , type)

$l: \$i \rightarrow \$i \rightarrow \$o$ thf(l , type)

confluence: $(\$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$ thf(confluences_type, type)

confluence = $(\lambda i: \$i \rightarrow \$i \rightarrow \$o, j: \$i \rightarrow \$i \rightarrow \$o, k: \$i \rightarrow \$i \rightarrow \$o, l: \$i \rightarrow \$i \rightarrow \$o: \forall a: \$i, b: \$i, c: \$i: ((i@a@b \text{ and } k@a@c) \Rightarrow \exists d: \$i: (j@b@d \text{ and } l@c@d)))$ thf(confluence, definition)

$(\text{mvalid}@(\text{mforall_prop}@ \lambda p: \$i \rightarrow \$o: (\text{mimplies}@(\text{mdia}@i@(\text{mbox}@j@p))@(\text{mbox}@k@(\text{mdia}@l@p)))))) \iff (\text{confluence}@i@j@k@l@p)$

LCL873 \wedge **1.p** Commutativity implies orthogonality in 2-D modal logic S5

include('Axioms/LCL013^0.ax')

$ra: \$i \rightarrow \$i \rightarrow \$o$ thf(ra , type)

$rb: \$i \rightarrow \$i \rightarrow \$o$ thf(rb , type)

$\text{mreflexive}@ra$ thf(ax₁, axiom)

$\text{mreflexive}@rb$ thf(ax₂, axiom)

$\text{mtransitive}@ra$ thf(ax₃, axiom)

$\text{mtransitive}@rb$ thf(ax₄, axiom)

$\text{meuclidean}@ra$ thf(ax₅, axiom)

$\text{meuclidean}@rb$ thf(ax₆, axiom)

$\text{mvalid}@(\text{mforall_prop}@ \lambda a: \$i \rightarrow \$o: (\text{mforall_prop}@ \lambda b: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@ra@(\text{mor}@(\text{mbox}@ra@a))@(\text{mbox}@rb@b)), \text{mforall_prop}@ \lambda b: \$i \rightarrow \$o: (\text{mimplies}@(\text{mbox}@rb@(\text{mor}@(\text{mbox}@ra@a))@(\text{mbox}@rb@b))))@(\text{mor}@(\text{mbox}@rb@a))@(\text{mbox}@ra@b))))))$

LCL874 \wedge **1.p** Inclusion statement in a 2-D logic of knowledge and belief

Suppose we want to work with a 2-dimensional logic combining a modality box_k of knowledge with a modality box_b of belief. Moreover, suppose we model box_k as an S5 modality and box_b as an D45 modality and let us furthermore add two axioms characterizing their relationship. We may then want to check whether or not box_k and box_b coincide, i.e., whether box_k includes box_b

```

include('Axioms/LCL013^0.ax')
rk: $i → $i → $o    thf(rk, type)
rb: $i → $i → $o    thf(rb, type)
mreflexive@rk    thf(ax1, axiom)
mserial@rb    thf(ax2, axiom)
mtransitive@rk    thf(ax3, axiom)
mtransitive@rb    thf(ax4, axiom)
meuclidean@rk    thf(ax5, axiom)
meuclidean@rb    thf(ax6, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@rk@a)@(mbox@rb@a)))    thf(ax7, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@rb@a)@(mbox@rb@(mbox@rk@a))))    thf(ax8, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@rb@a)@(mbox@rk@a)))    thf(conj, conjecture)

```

LCL875-1.p The Rezus formula

```

(is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y)    cnf(condensed_detachment, axiom)
is_a_theorem(implies(implies(a, b), implies(implies(c, a), implies(c, b))))    cnf(b, axiom)
is_a_theorem(implies(implies(a, implies(b, c)), implies(b, implies(a, c))))    cnf(c, axiom)
is_a_theorem(implies(a, a))    cnf(i, axiom)
is_a_theorem(implies(implies(u, implies(u, v)), implies(u, v)))    cnf(w, axiom)
¬ is_a_theorem(implies(implies(implies(x, implies(implies(implies(y, y), implies(implies(z, z), implies(implies(u, u), implies(im

```

LCL876+1.p Prove Mv5 from MV1–MV4

```

∀y, x: ((is_a_theorem(implies(x, y)) and is_a_theorem(x)) ⇒ is_a_theorem(y))    fof(cd, axiom)
∀y, x: is_a_theorem(implies(x, implies(y, x)))    fof(mv1, axiom)
∀z, y, x: is_a_theorem(implies(implies(x, y), implies(implies(y, z), implies(x, z))))    fof(mv2, axiom)
∀y, x: is_a_theorem(implies(implies(implies(x, y), y), implies(implies(y, x), x)))    fof(mv3, axiom)
∀y, x: is_a_theorem(implies(implies(not(x), not(y)), implies(y, x)))    fof(mv4, axiom)
∀y, x: is_a_theorem(implies(implies(implies(x, y), implies(y, x)), implies(y, x)))    fof(mv5, conjecture)

```

LCL877∧1.p Variants of axiom 5

```

include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: ((mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mnot@(mbox@r@phi))@(mbox@r@(mnot@(mbox@r@phi))))))
(mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mdia@r@phi)@(mbox@r@(mdia@r@phi))))))    thf(conj, conjecture)

```

LCL877∧2.p Variants of axiom 5

```

include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: ((mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mnot@(mbox@r@phi))@(mbox@r@(mnot@(mbox@r@phi))))))
(meuclidean@r))    thf(conj, conjecture)

```

LCL878∧1.p Correspondence for axiom I

```

include('Axioms/LCL013^0.ax')
∀i: $i → $i → $o, j: $i → $i → $o: ((mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mbox@i@phi)@(mbox@j@phi)))) ←
∀u: $i, v: $i: ((j@u@v) ⇒ (i@u@v))    thf(conj, conjecture)

```

LCL879∧1.p Correspondence for axiom 4s

```

include('Axioms/LCL013^0.ax')
∀i: $i → $i → $o, j: $i → $i → $o: ((mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mbox@i@phi)@(mbox@j@(mbox@i@phi)))) ←
∀u: $i, v: $i, w: $i: ((j@u@v and i@v@w) ⇒ (i@u@w))    thf(conj, conjecture)

```

LCL880∧1.p Correspondence for axiom 5s

```

include('Axioms/LCL013^0.ax')
∀i: $i → $i → $o, j: $i → $i → $o: ((mvalid@(mforall_prop@λphi: $i → $o: (mimplies@(mnot@(mbox@i@phi))@(mbox@j@(mbox@i@phi)))) ←
∀u: $i, v: $i, w: $i: ((j@u@v and i@u@w) ⇒ (i@v@w))    thf(conj, conjecture)

```

LCL882+1.p An involutive pocrim that does not have unique halving

```

∀a, b, c: a' +' b' +' c = a' +' b' +' c    fof(sos01, axiom)
∀a, b: a' +' b = b' +' a    fof(sos02, axiom)
∀a: a' +' '0' = a    fof(sos03, axiom)
∀a: a' >= ' a    fof(sos04, axiom)
∀x0, x1, x2: ((x'0 >= ' x1 and x'1 >= ' x2) ⇒ x'0 >= ' x2)    fof(sos05, axiom)
∀x3, x4: ((x'3 >= ' x4 and x'4 >= ' x3) ⇒ x3 = x4)    fof(sos06, axiom)
∀x5, x6, x7: (x'5 +' x'6 >= ' x7 ⇔ x'6 >= ' x'5 ==> ' x7)    fof(sos07, axiom)
∀a: a' >= ' '0'    fof(sos08, axiom)
∀x8, x9, x10: (x'8 >= ' x9 ⇒ x'8 +' x'10 >= ' x'9 +' x10)    fof(sos09, axiom)

```

$\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= x_{12} \Rightarrow x'_{12} ==> x'_{13} >= x'_{11} ==> x_{13})$ fof(sos₁₀, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= x_{15} \Rightarrow x'_{16} ==> x'_{14} >= x'_{16} ==> x_{15})$ fof(sos₁₁, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos₁₂, axiom)
 $\forall a: a' ==> '1' = a$ fof(sos₁₃, axiom)
 $\forall x_{17}, x_{18}, x_{19}: ((x_{17} = x'_{17} ==> x_{18} \text{ and } x_{19} = x'_{19} ==> x_{18}) \Rightarrow x_{17} = x_{19})$ fof(goals₁₄, conjecture)

LCL883+1.p An involutive pocrim that is not a hoop

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos₀₁, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos₀₂, axiom)
 $\forall a: a' + '0' = a$ fof(sos₀₃, axiom)
 $\forall a: a' >= a$ fof(sos₀₄, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= x_1 \text{ and } x'_1 >= x_2) \Rightarrow x'_0 >= x_2)$ fof(sos₀₅, axiom)
 $\forall x_3, x_4: ((x'_3 >= x_4 \text{ and } x'_4 >= x_3) \Rightarrow x_3 = x_4)$ fof(sos₀₆, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 >= x_7 \iff x'_6 >= x'_5 ==> x_7)$ fof(sos₀₇, axiom)
 $\forall a: a' >= '0'$ fof(sos₀₈, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= x_9 \Rightarrow x'_8 + x'_{10} >= x'_9 + x_{10})$ fof(sos₀₉, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= x_{12} \Rightarrow x'_{12} ==> x'_{13} >= x'_{11} ==> x_{13})$ fof(sos₁₀, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= x_{15} \Rightarrow x'_{16} ==> x'_{14} >= x'_{16} ==> x_{15})$ fof(sos₁₁, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos₁₂, axiom)
 $\forall a: a' ==> '1' = a$ fof(sos₁₃, axiom)
 $\forall x_{17}, x_{18}: x'_{17} + x'_{17} ==> x_{18} = x'_{18} + x'_{18} ==> x_{17}$ fof(goals₁₄, conjecture)

LCL884+1.p A bounded hoop that is not an involutive pocrim

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos₀₁, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos₀₂, axiom)
 $\forall a: a' + '0' = a$ fof(sos₀₃, axiom)
 $\forall a: a' >= a$ fof(sos₀₄, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= x_1 \text{ and } x'_1 >= x_2) \Rightarrow x'_0 >= x_2)$ fof(sos₀₅, axiom)
 $\forall x_3, x_4: ((x'_3 >= x_4 \text{ and } x'_4 >= x_3) \Rightarrow x_3 = x_4)$ fof(sos₀₆, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 >= x_7 \iff x'_6 >= x'_5 ==> x_7)$ fof(sos₀₇, axiom)
 $\forall a: a' >= '0'$ fof(sos₀₈, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= x_9 \Rightarrow x'_8 + x'_{10} >= x'_9 + x_{10})$ fof(sos₀₉, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= x_{12} \Rightarrow x'_{12} ==> x'_{13} >= x'_{11} ==> x_{13})$ fof(sos₁₀, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= x_{15} \Rightarrow x'_{16} ==> x'_{14} >= x'_{16} ==> x_{15})$ fof(sos₁₁, axiom)
 $\forall a, b: a' + a' ==> b = b' + b' ==> a$ fof(sos₁₂, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos₁₃, axiom)
 $\forall x_{17}: x'_{17} ==> '1' = x_{17}$ fof(goals₁₄, conjecture)

LCL885+1.p An involutive pocrim that is not a coop

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos₀₁, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos₀₂, axiom)
 $\forall a: a' + '0' = a$ fof(sos₀₃, axiom)
 $\forall a: a' >= a$ fof(sos₀₄, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= x_1 \text{ and } x'_1 >= x_2) \Rightarrow x'_0 >= x_2)$ fof(sos₀₅, axiom)
 $\forall x_3, x_4: ((x'_3 >= x_4 \text{ and } x'_4 >= x_3) \Rightarrow x_3 = x_4)$ fof(sos₀₆, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 >= x_7 \iff x'_6 >= x'_5 ==> x_7)$ fof(sos₀₇, axiom)
 $\forall a: a' >= '0'$ fof(sos₀₈, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= x_9 \Rightarrow x'_8 + x'_{10} >= x'_9 + x_{10})$ fof(sos₀₉, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= x_{12} \Rightarrow x'_{12} ==> x'_{13} >= x'_{11} ==> x_{13})$ fof(sos₁₀, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= x_{15} \Rightarrow x'_{16} ==> x'_{14} >= x'_{16} ==> x_{15})$ fof(sos₁₁, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos₁₂, axiom)
 $\forall a: a' ==> '1' = a$ fof(sos₁₃, axiom)
 $\forall x_{17}: \exists x_{18}: x_{18} = x'_{18} ==> x_{17}$ fof(goals₁₄, conjecture)

LCL886+1.p An involutive pocrim that is not idempotent

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos₀₁, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos₀₂, axiom)
 $\forall a: a' + '0' = a$ fof(sos₀₃, axiom)
 $\forall a: a' >= a$ fof(sos₀₄, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= x_1 \text{ and } x'_1 >= x_2) \Rightarrow x'_0 >= x_2)$ fof(sos₀₅, axiom)
 $\forall x_3, x_4: ((x'_3 >= x_4 \text{ and } x'_4 >= x_3) \Rightarrow x_3 = x_4)$ fof(sos₀₆, axiom)

$\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq 0$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \implies x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \implies x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \implies x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a: a' + 1 = 1$ fof(sos12, axiom)
 $\forall a: a' \implies 1' \implies 1' = a$ fof(sos13, axiom)
 $\forall x_{17}: x'_{17} + x_{17} = x_{17}$ fof(goals14, conjecture)

LCL887+1.p An idempotent hoop that is not an involutive pocrim

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos01, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos02, axiom)
 $\forall a: a' + 0 = a$ fof(sos03, axiom)
 $\forall a: a' \geq a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 \geq x_1 \text{ and } x'_1 \geq x_2) \implies x'_0 \geq x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 \geq x_4 \text{ and } x'_4 \geq x_3) \implies x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq 0$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \implies x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \implies x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \implies x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a: a' + 1 = 1$ fof(sos12, axiom)
 $\forall a: a' + a = a$ fof(sos13, axiom)
 $\forall x_{17}: x'_{17} \implies 1' \implies 1' = x_{17}$ fof(goals14, conjecture)

LCL888+1.p Halving is unique in a hoop

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos01, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos02, axiom)
 $\forall a: a' + 0 = a$ fof(sos03, axiom)
 $\forall a: a' \geq a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 \geq x_1 \text{ and } x'_1 \geq x_2) \implies x'_0 \geq x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 \geq x_4 \text{ and } x'_4 \geq x_3) \implies x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq 0$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \implies x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \implies x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \implies x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a, b: a' + a' \implies b = b' + b' \implies a$ fof(sos12, axiom)
 $\forall x_{17}, x_{18}, x_{19}: ((x_{17} = x'_{17} \implies x_{18} \text{ and } x_{19} = x'_{19} \implies x_{18}) \implies x_{17} = x_{19})$ fof(goals13, conjecture)

LCL889+1.p Halving is unique in a hoop, rule for $x \geq a/2$

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos01, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos02, axiom)
 $\forall a: a' + 0 = a$ fof(sos03, axiom)
 $\forall a: a' \geq a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 \geq x_1 \text{ and } x'_1 \geq x_2) \implies x'_0 \geq x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 \geq x_4 \text{ and } x'_4 \geq x_3) \implies x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq 0$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \implies x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \implies x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \implies x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a, b: a' + a' \implies b = b' + b' \implies a$ fof(sos12, axiom)
 $\forall x_{17}, x_{18}, x_{19}: ((x'_{17} \geq x'_{17} \implies x_{18} \text{ and } x_{19} = x'_{19} \implies x_{18}) \implies x'_{17} \geq x_{19})$ fof(goals13, conjecture)

LCL890+1.p Halving is unique in a hoop, rule for $a/2 \geq x$

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos01, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos02, axiom)
 $\forall a: a' + 0 = a$ fof(sos03, axiom)
 $\forall a: a' + 1 = 1$ fof(sos04, axiom)
 $\forall a: a' \geq a$ fof(sos05, axiom)

$\forall x_0, x_1, x_2: ((x'_0 \geq' x_1 \text{ and } x'_1 \geq' x_2) \Rightarrow x'_0 \geq' x_2) \quad \text{fof}(\text{sos}_{06}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 \geq' x_4 \text{ and } x'_4 \geq' x_3) \Rightarrow x_3 = x_4) \quad \text{fof}(\text{sos}_{07}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 + ' x'_6 \geq' x_7 \iff x'_6 \geq' x'_5 \implies' x_7) \quad \text{fof}(\text{sos}_{08}, \text{axiom})$
 $\forall a: a' \geq' '0' \quad \text{fof}(\text{sos}_{09}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 \geq' x_9 \Rightarrow x'_8 + ' x'_{10} \geq' x'_9 + ' x_{10}) \quad \text{fof}(\text{sos}_{10}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq' x_{12} \Rightarrow x'_{12} \implies' x'_{13} \geq' x'_{11} \implies' x_{13}) \quad \text{fof}(\text{sos}_{11}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq' x_{15} \Rightarrow x'_{16} \implies' x'_{14} \geq' x'_{16} \implies' x_{15}) \quad \text{fof}(\text{sos}_{12}, \text{axiom})$
 $\forall a, b: a' + ' a' \implies' b = b' + ' b' \implies' a \quad \text{fof}(\text{sos}_{13}, \text{axiom})$
 $\forall x_{17}, x_{18}, x_{19}: ((x'_{17} \implies' x'_{18} \geq' x_{17} \text{ and } x_{19} = x'_{19} \implies' x_{18}) \Rightarrow x'_{19} \geq' x_{17}) \quad \text{fof}(\text{goals}_{14}, \text{conjecture})$

LCL891+1.p Halving is unique in a hoop, rule for $a/2 \geq x$

$\forall a, b, c: a' + ' b' + ' c = a' + ' b' + ' c \quad \text{fof}(\text{sos}_{01}, \text{axiom})$
 $\forall a, b: a' + ' b = b' + ' a \quad \text{fof}(\text{sos}_{02}, \text{axiom})$
 $\forall a: a' + ' '0' = a \quad \text{fof}(\text{sos}_{03}, \text{axiom})$
 $\forall a: a' \geq' a \quad \text{fof}(\text{sos}_{04}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 \geq' x_1 \text{ and } x'_1 \geq' x_2) \Rightarrow x'_0 \geq' x_2) \quad \text{fof}(\text{sos}_{05}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 \geq' x_4 \text{ and } x'_4 \geq' x_3) \Rightarrow x_3 = x_4) \quad \text{fof}(\text{sos}_{06}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 + ' x'_6 \geq' x_7 \iff x'_6 \geq' x'_5 \implies' x_7) \quad \text{fof}(\text{sos}_{07}, \text{axiom})$
 $\forall a: a' \geq' '0' \quad \text{fof}(\text{sos}_{08}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 \geq' x_9 \Rightarrow x'_8 + ' x'_{10} \geq' x'_9 + ' x_{10}) \quad \text{fof}(\text{sos}_{09}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq' x_{12} \Rightarrow x'_{12} \implies' x'_{13} \geq' x'_{11} \implies' x_{13}) \quad \text{fof}(\text{sos}_{10}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq' x_{15} \Rightarrow x'_{16} \implies' x'_{14} \geq' x'_{16} \implies' x_{15}) \quad \text{fof}(\text{sos}_{11}, \text{axiom})$
 $\forall a, b: a' + ' a' \implies' b = b' + ' b' \implies' a \quad \text{fof}(\text{sos}_{12}, \text{axiom})$
 $\forall x_{17}, x_{18}, x_{19}: ((x'_{17} \implies' x'_{18} \geq' x_{17} \text{ and } x_{19} = x'_{19} \implies' x_{18}) \Rightarrow x'_{19} \geq' x_{17}) \quad \text{fof}(\text{goals}_{13}, \text{conjecture})$

LCL892+1.p Halving is unique in a hoop, rule for $a/2 \geq x$

$\forall a, b, c: a' + ' b' + ' c = a' + ' b' + ' c \quad \text{fof}(\text{sos}_{01}, \text{axiom})$
 $\forall a, b: a' + ' b = b' + ' a \quad \text{fof}(\text{sos}_{02}, \text{axiom})$
 $\forall a: a' + ' '0' = a \quad \text{fof}(\text{sos}_{03}, \text{axiom})$
 $\forall a: a' \geq' a \quad \text{fof}(\text{sos}_{04}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 \geq' x_1 \text{ and } x'_1 \geq' x_2) \Rightarrow x'_0 \geq' x_2) \quad \text{fof}(\text{sos}_{05}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 \geq' x_4 \text{ and } x'_4 \geq' x_3) \Rightarrow x_3 = x_4) \quad \text{fof}(\text{sos}_{06}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 + ' x'_6 \geq' x_7 \iff x'_6 \geq' x'_5 \implies' x_7) \quad \text{fof}(\text{sos}_{07}, \text{axiom})$
 $\forall a: a' \geq' '0' \quad \text{fof}(\text{sos}_{08}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 \geq' x_9 \Rightarrow x'_8 + ' x'_{10} \geq' x'_9 + ' x_{10}) \quad \text{fof}(\text{sos}_{09}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq' x_{12} \Rightarrow x'_{12} \implies' x'_{13} \geq' x'_{11} \implies' x_{13}) \quad \text{fof}(\text{sos}_{10}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq' x_{15} \Rightarrow x'_{16} \implies' x'_{14} \geq' x'_{16} \implies' x_{15}) \quad \text{fof}(\text{sos}_{11}, \text{axiom})$
 $\forall a, b: a' + ' a' \implies' b = b' + ' b' \implies' a \quad \text{fof}(\text{sos}_{12}, \text{axiom})$
 $\forall a: a' + ' '1' = '1' \quad \text{fof}(\text{sos}_{13}, \text{axiom})$
 $\forall x_{17}, x_{18}, x_{19}: ((x'_{17} \implies' x'_{18} \geq' x_{17} \text{ and } x_{19} = x'_{19} \implies' x_{18}) \Rightarrow x'_{19} \geq' x_{17}) \quad \text{fof}(\text{goals}_{14}, \text{conjecture})$

LCL893+1.p In a coop, $x/2 = x$ implies $x = 0$

$\forall a, b, c: a' + ' b' + ' c = a' + ' b' + ' c \quad \text{fof}(\text{sos}_{01}, \text{axiom})$
 $\forall a, b: a' + ' b = b' + ' a \quad \text{fof}(\text{sos}_{02}, \text{axiom})$
 $\forall a: a' + ' '0' = a \quad \text{fof}(\text{sos}_{03}, \text{axiom})$
 $\forall a: a' + ' '1' = '1' \quad \text{fof}(\text{sos}_{04}, \text{axiom})$
 $\forall a: a' \geq' a \quad \text{fof}(\text{sos}_{05}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 \geq' x_1 \text{ and } x'_1 \geq' x_2) \Rightarrow x'_0 \geq' x_2) \quad \text{fof}(\text{sos}_{06}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 \geq' x_4 \text{ and } x'_4 \geq' x_3) \Rightarrow x_3 = x_4) \quad \text{fof}(\text{sos}_{07}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 + ' x'_6 \geq' x_7 \iff x'_6 \geq' x'_5 \implies' x_7) \quad \text{fof}(\text{sos}_{08}, \text{axiom})$
 $\forall a: a' \geq' '0' \quad \text{fof}(\text{sos}_{09}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 \geq' x_9 \Rightarrow x'_8 + ' x'_{10} \geq' x'_9 + ' x_{10}) \quad \text{fof}(\text{sos}_{10}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq' x_{12} \Rightarrow x'_{12} \implies' x'_{13} \geq' x'_{11} \implies' x_{13}) \quad \text{fof}(\text{sos}_{11}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq' x_{15} \Rightarrow x'_{16} \implies' x'_{14} \geq' x'_{16} \implies' x_{15}) \quad \text{fof}(\text{sos}_{12}, \text{axiom})$
 $\forall a, b: a' + ' a' \implies' b = b' + ' b' \implies' a \quad \text{fof}(\text{sos}_{13}, \text{axiom})$
 $\forall a: h(a) = h(a)' \implies' a \quad \text{fof}(\text{sos}_{14}, \text{axiom})$
 $\forall x_{17}: (h(x_{17}) = x_{17} \Rightarrow x_{17} = '0') \quad \text{fof}(\text{goals}_{15}, \text{conjecture})$

LCL894+1.p Weak conjunction is lub in a hoop using horn axioms

$\forall a, b, c: a' + ' b' + ' c = a' + ' b' + ' c \quad \text{fof}(\text{sos}_{01}, \text{axiom})$
 $\forall a, b: a' + ' b = b' + ' a \quad \text{fof}(\text{sos}_{02}, \text{axiom})$

$\forall a: a' +' '0' = a$ fof(sos03, axiom)
 $\forall a: a' >= ' a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7)$ fof(sos07, axiom)
 $\forall a: a' >= ' '0'$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15})$ fof(sos11, axiom)
 $\forall a, b: a' +' a' ==>' b = b' +' b' ==>' a$ fof(sos12, axiom)
 $(c' >= ' a \text{ and } c' >= ' b) \iff c' >= ' a' +' a' ==>' b$ fof(goals13, conjecture)

LCL895+1.p Weak conjunction is lub in a hoop using equational axioms

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c$ fof(sos01, axiom)
 $\forall a, b: a' +' b = b' +' a$ fof(sos02, axiom)
 $\forall a: a' +' '0' = a$ fof(sos03, axiom)
 $\forall a: a' ==>' a = '0'$ fof(sos04, axiom)
 $\forall a: a' ==>' '0' = '0'$ fof(sos05, axiom)
 $\forall a: '0' ==>' a = a$ fof(sos06, axiom)
 $\forall a, b, c: a' +' b' ==>' c = a' ==>' b' ==>' c$ fof(sos07, axiom)
 $\forall a, b: a' +' a' ==>' b = b' +' b' ==>' a$ fof(sos08, axiom)
 $(c' ==>' a = '0' \text{ and } c' ==>' b = '0') \iff c' ==>' a' +' a' ==>' b = '0'$ fof(goals09, conjecture)

LCL896+1.p Associativity of weak conjunction implies commutativity

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c$ fof(sos01, axiom)
 $\forall a, b: a' +' b = b' +' a$ fof(sos02, axiom)
 $\forall a: a' +' '0' = a$ fof(sos03, axiom)
 $\forall a: a' >= ' a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7)$ fof(sos07, axiom)
 $\forall a: a' >= ' '0'$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15})$ fof(sos11, axiom)
 $\forall a, b, c: a' +' a' ==>' b' +' a' +' a' ==>' b' ==>' c = a' +' a' ==>' b' +' b' ==>' c$ fof(sos12, axiom)
 $\forall x_{17}, x_{18}: x'_{17} +' x'_{17} ==>' x_{18} = x'_{18} +' x'_{18} ==>' x_{17}$ fof(goals13, conjecture)

LCL897+1.p Weak conjunction is associative in a hoop

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c$ fof(sos01, axiom)
 $\forall a, b: a' +' b = b' +' a$ fof(sos02, axiom)
 $\forall a: a' +' '0' = a$ fof(sos03, axiom)
 $\forall a: a' ==>' a = '0'$ fof(sos04, axiom)
 $\forall a: a' ==>' '0' = '0'$ fof(sos05, axiom)
 $\forall a: '0' ==>' a = a$ fof(sos06, axiom)
 $\forall a, b, c: a' +' b' ==>' c = a' ==>' b' ==>' c$ fof(sos07, axiom)
 $\forall a, b: a' +' a' ==>' b = b' +' b' ==>' a$ fof(sos08, axiom)
 $a' +' a' ==>' b' +' a' +' a' ==>' b' ==>' c = a' +' a' ==>' b' +' b' ==>' c$ fof(goals09, conjecture)

LCL898+1.p Strong disjunction is commutative in an involutive hoop

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c$ fof(sos01, axiom)
 $\forall a, b: a' +' b = b' +' a$ fof(sos02, axiom)
 $\forall a: a' +' '0' = a$ fof(sos03, axiom)
 $\forall a: a' >= ' a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7)$ fof(sos07, axiom)
 $\forall a: a' >= ' '0'$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15})$ fof(sos11, axiom)

$\forall a, b: a' +' a' ==>' b = b' +' b' ==>' a \quad \text{fof(sos}_{12}, \text{axiom})$
 $\forall a: a' ==>' '1' ==>' '1' = a \quad \text{fof(sos}_{13}, \text{axiom})$
 $\forall a: a' +' '1' = '1' \quad \text{fof(sos}_{14}, \text{axiom})$
 $\forall x_{17}, x_{18}: x'_{17} ==>' x'_{18} ==>' x_{18} = x'_{18} ==>' x'_{17} ==>' x_{17} \quad \text{fof(goals}_{15}, \text{conjecture})$

LCL899+1.p A bounded pocrim property

A bounded pocrim with commutative strong disjunction is involutive.

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c \quad \text{fof(sos}_{01}, \text{axiom})$
 $\forall a, b: a' +' b = b' +' a \quad \text{fof(sos}_{02}, \text{axiom})$
 $\forall a: a' +' '0' = a \quad \text{fof(sos}_{03}, \text{axiom})$
 $\forall a: a' >= ' a \quad \text{fof(sos}_{04}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2) \quad \text{fof(sos}_{05}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4) \quad \text{fof(sos}_{06}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7) \quad \text{fof(sos}_{07}, \text{axiom})$
 $\forall a: a' >= ' '0' \quad \text{fof(sos}_{08}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10}) \quad \text{fof(sos}_{09}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13}) \quad \text{fof(sos}_{10}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15}) \quad \text{fof(sos}_{11}, \text{axiom})$
 $\forall a: a' +' '1' = '1' \quad \text{fof(sos}_{12}, \text{axiom})$
 $\forall a, b: a' ==>' b' ==>' b = b' ==>' a' ==>' a \quad \text{fof(sos}_{13}, \text{axiom})$
 $\forall x_{17}: x'_{17} ==>' '1' ==>' '1' = x_{17} \quad \text{fof(goals}_{14}, \text{conjecture})$

LCL900+1.p A bounded pocrim with commutative strong disjunction is a hoop

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c \quad \text{fof(sos}_{01}, \text{axiom})$
 $\forall a, b: a' +' b = b' +' a \quad \text{fof(sos}_{02}, \text{axiom})$
 $\forall a: a' +' '0' = a \quad \text{fof(sos}_{03}, \text{axiom})$
 $\forall a: a' >= ' a \quad \text{fof(sos}_{04}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2) \quad \text{fof(sos}_{05}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4) \quad \text{fof(sos}_{06}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7) \quad \text{fof(sos}_{07}, \text{axiom})$
 $\forall a: a' >= ' '0' \quad \text{fof(sos}_{08}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10}) \quad \text{fof(sos}_{09}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13}) \quad \text{fof(sos}_{10}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15}) \quad \text{fof(sos}_{11}, \text{axiom})$
 $\forall a: a' +' '1' = '1' \quad \text{fof(sos}_{12}, \text{axiom})$
 $\forall a, b: a' ==>' b' ==>' b = b' ==>' a' ==>' a \quad \text{fof(sos}_{13}, \text{axiom})$
 $\forall x_{17}, x_{18}: x'_{17} +' x'_{17} ==>' x_{18} = x'_{18} +' x'_{18} ==>' x_{17} \quad \text{fof(goals}_{14}, \text{conjecture})$

LCL901+1.p An idempotent pocrim property

An idempotent pocrim with commutative strong disjunction is boolean.

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c \quad \text{fof(sos}_{01}, \text{axiom})$
 $\forall a, b: a' +' b = b' +' a \quad \text{fof(sos}_{02}, \text{axiom})$
 $\forall a: a' +' '0' = a \quad \text{fof(sos}_{03}, \text{axiom})$
 $\forall a: a' >= ' a \quad \text{fof(sos}_{04}, \text{axiom})$
 $\forall x_0, x_1, x_2: ((x'_0 >= ' x_1 \text{ and } x'_1 >= ' x_2) \Rightarrow x'_0 >= ' x_2) \quad \text{fof(sos}_{05}, \text{axiom})$
 $\forall x_3, x_4: ((x'_3 >= ' x_4 \text{ and } x'_4 >= ' x_3) \Rightarrow x_3 = x_4) \quad \text{fof(sos}_{06}, \text{axiom})$
 $\forall x_5, x_6, x_7: (x'_5 +' x'_6 >= ' x_7 \iff x'_6 >= ' x'_5 ==>' x_7) \quad \text{fof(sos}_{07}, \text{axiom})$
 $\forall a: a' >= ' '0' \quad \text{fof(sos}_{08}, \text{axiom})$
 $\forall x_8, x_9, x_{10}: (x'_8 >= ' x_9 \Rightarrow x'_8 +' x'_{10} >= ' x'_9 +' x_{10}) \quad \text{fof(sos}_{09}, \text{axiom})$
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} >= ' x_{12} \Rightarrow x'_{12} ==>' x'_{13} >= ' x'_{11} ==>' x_{13}) \quad \text{fof(sos}_{10}, \text{axiom})$
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} >= ' x_{15} \Rightarrow x'_{16} ==>' x'_{14} >= ' x'_{16} ==>' x_{15}) \quad \text{fof(sos}_{11}, \text{axiom})$
 $\forall a: a' +' '1' = '1' \quad \text{fof(sos}_{12}, \text{axiom})$
 $\forall a, b: a' ==>' b' ==>' b = b' ==>' a' ==>' a \quad \text{fof(sos}_{13}, \text{axiom})$
 $\forall a: a' +' a = a \quad \text{fof(sos}_{14}, \text{axiom})$
 $\forall x_{17}: x'_{17} ==>' '1' ==>' x'_{17} ==>' x_{17} = '0' \quad \text{fof(goals}_{15}, \text{conjecture})$

LCL902+1.p A boolean pocrim is involutive

$\forall a, b, c: a' +' b' +' c = a' +' b' +' c \quad \text{fof(sos}_{01}, \text{axiom})$
 $\forall a, b: a' +' b = b' +' a \quad \text{fof(sos}_{02}, \text{axiom})$
 $\forall a: a' +' '0' = a \quad \text{fof(sos}_{03}, \text{axiom})$
 $\forall a: a' >= ' a \quad \text{fof(sos}_{04}, \text{axiom})$

$\forall x_0, x_1, x_2: ((x'_0 \geq x_1 \text{ and } x'_1 \geq x_2) \Rightarrow x'_0 \geq x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 \geq x_4 \text{ and } x'_4 \geq x_3) \Rightarrow x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq '0'$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \Rightarrow x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \Rightarrow x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \Rightarrow x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos12, axiom)
 $\forall a: a' \implies '1' \implies a' \implies a = '0'$ fof(sos13, axiom)
 $\forall x_{17}: x'_{17} \implies '1' \implies '1' = x_{17}$ fof(goals14, conjecture)

LCL903+1.p A boolean pocrim is idempotent

$\forall a, b, c: a' + b' + c = a' + b' + c$ fof(sos01, axiom)
 $\forall a, b: a' + b = b' + a$ fof(sos02, axiom)
 $\forall a: a' + '0' = a$ fof(sos03, axiom)
 $\forall a: a' \geq a$ fof(sos04, axiom)
 $\forall x_0, x_1, x_2: ((x'_0 \geq x_1 \text{ and } x'_1 \geq x_2) \Rightarrow x'_0 \geq x_2)$ fof(sos05, axiom)
 $\forall x_3, x_4: ((x'_3 \geq x_4 \text{ and } x'_4 \geq x_3) \Rightarrow x_3 = x_4)$ fof(sos06, axiom)
 $\forall x_5, x_6, x_7: (x'_5 + x'_6 \geq x_7 \iff x'_6 \geq x'_5 \implies x_7)$ fof(sos07, axiom)
 $\forall a: a' \geq '0'$ fof(sos08, axiom)
 $\forall x_8, x_9, x_{10}: (x'_8 \geq x_9 \Rightarrow x'_8 + x'_{10} \geq x'_9 + x_{10})$ fof(sos09, axiom)
 $\forall x_{11}, x_{12}, x_{13}: (x'_{11} \geq x_{12} \Rightarrow x'_{12} \implies x'_{13} \geq x'_{11} \implies x_{13})$ fof(sos10, axiom)
 $\forall x_{14}, x_{15}, x_{16}: (x'_{14} \geq x_{15} \Rightarrow x'_{16} \implies x'_{14} \geq x'_{16} \implies x_{15})$ fof(sos11, axiom)
 $\forall a: a' + '1' = '1'$ fof(sos12, axiom)
 $\forall a: a' \implies '1' \implies a' \implies a = '0'$ fof(sos13, axiom)
 $\forall x_{17}: x'_{17} + x_{17} = x_{17}$ fof(goals14, conjecture)

LCL904 \wedge 1.p Axioms for Modal logic S4 under cumulative domains

include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^0.ax')
include('Axioms/LCL015^1.ax')

LCL905-1.p Alternative Wajsberg algebra

include('Axioms/LCL002-0.ax')
include('Axioms/LCL002-0.ax')

LCL906-1.p Lattice theory (equality) axioms

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
include('Axioms/LAT001-3.ax')

LCL907+1.p Hilbert's axiomatization of propositional logic

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')

LCL908+1.p Lukasiewicz's axiomatization of propositional logic

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+3.ax')

LCL909+1.p Principia's axiomatization of propositional logic

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+4.ax')

LCL910+1.p Rosser's axiomatization of propositional logic

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+5.ax')

LCL911+1.p KM5 axiomatization of S5 based on Hilbert's PC

include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')

```
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+2.ax')
```

LCL912+1.p KM4B axiomatization of S5 based on Hilbert's PC

```
include('Axioms/LCL006+0.ax')
include('Axioms/LCL006+1.ax')
include('Axioms/LCL006+2.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+3.ax')
```

LCL913+1.p Axiomatization of S1-0

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
```

LCL914+1.p M6S3M9B axiomatization of S5 based on S1-0

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+5.ax')
```

LCL915+1.p M10 axiomatization of S5 based on S1-0

```
include('Axioms/LCL006+1.ax')
include('Axioms/LCL007+0.ax')
include('Axioms/LCL007+1.ax')
include('Axioms/LCL007+4.ax')
include('Axioms/LCL007+6.ax')
```

LCL916 \wedge 1.p Multi-Modal Logic

```
include('Axioms/LCL008^0.ax')
```

LCL917 \wedge 1.p Translating constructive S4 (CS4) to bimodal classical S4 (BS4)

```
include('Axioms/LCL008^0.ax')
include('Axioms/LCL009^0.ax')
```

LCL918 \wedge 1.p Modal logic K

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
```

LCL919 \wedge 1.p Modal logic D

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
```

LCL920 \wedge 1.p Modal logic M

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
```

LCL921 \wedge 1.p Modal logic B

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
```

LCL922 \wedge 1.p Modal logic S4

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
```

LCL923 \wedge 1.p Modal logic S5

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
```

LCL924 \wedge 1.p Region Connection Calculus

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL014^0.ax')
```

LCL925 \wedge **1.p** Embedding of quantified multimodal logic in simple type theory

include('Axioms/LCL015^0.ax')

include('Axioms/LCL015^1.ax')

include('Axioms/LCL013^5.ax')

LCL926-1.p IO in TW+ [AxL,AxTO] $(p(i(a, b)) \text{ and } p(a)) \Rightarrow p(b)$ cnf(modus_ponens, axiom) $p(i(i(a, b), i(i(b, c), i(a, c))))$ cnf(axBp, axiom) $p(i(i(i(x, y), y), i(i(y, x), x)))$ cnf(axL, axiom) $p(i(i(i(x, y), i(y, x)), i(y, x)))$ cnf(axTO, axiom) $p(i(x, \text{or}(x, y)))$ cnf(axorI₁, axiom) $p(i(y, \text{or}(x, y)))$ cnf(axorI₂, axiom) $\neg p(i(i(i(x, y), y), \text{or}(x, y)))$ cnf(io, negated_conjecture)**LCL927-1.p** AxK and AxC in TW+ [AxL,AxTO] + (Resid) $(p(i(a, b)) \text{ and } p(a)) \Rightarrow p(b)$ cnf(modus_ponens, axiom) $p(i(f(a, b), c)) \Rightarrow p(i(a, i(b, c)))$ cnf(resid₁, axiom) $p(i(a, i(b, c))) \Rightarrow p(i(f(a, b), c))$ cnf(resid₂, axiom) $p(i(i(a, b), i(i(b, c), i(a, c))))$ cnf(axBp, axiom) $p(i(i(i(x, y), y), i(i(y, x), x)))$ cnf(axL, axiom) $p(i(i(i(x, y), i(y, x)), i(y, x)))$ cnf(axTO, axiom) $p(i(c_1, i(c_2, c_1))) \Rightarrow \neg p(i(i(a, i(b, c)), i(b, i(a, c))))$ cnf(axK_axC, negated_conjecture)**LCL928-1.p** AxTO in BCK \rightarrow [AxL] + (Resid) $(p(i(a, b)) \text{ and } p(a)) \Rightarrow p(b)$ cnf(modus_ponens, axiom) $p(i(f(a, b), c)) \Rightarrow p(i(a, i(b, c)))$ cnf(resid₁, axiom) $p(i(a, i(b, c))) \Rightarrow p(i(f(a, b), c))$ cnf(resid₂, axiom) $p(i(i(a, b), i(i(b, c), i(a, c))))$ cnf(axBp, axiom) $p(i(i(i(a, b), b), i(i(b, a), a)))$ cnf(axL, axiom) $p(i(i(a, i(b, c)), i(b, i(a, c))))$ cnf(axC, axiom) $\neg p(i(i(i(c_1, c_2), i(c_2, c_1)), i(c_2, c_1)))$ cnf(axTO, negated_conjecture)**LCL929-1.p** AxK in TW \rightarrow [AxL] + (Resid) $(p(i(a, b)) \text{ and } p(a)) \Rightarrow p(b)$ cnf(modus_ponens, axiom) $p(i(f(a, b), c)) \Rightarrow p(i(a, i(b, c)))$ cnf(resid₁, axiom) $p(i(a, i(b, c))) \Rightarrow p(i(f(a, b), c))$ cnf(resid₂, axiom) $p(i(a, a))$ cnf(axI, axiom) $p(i(i(a, b), i(i(b, c), i(a, c))))$ cnf(axBp, axiom) $p(i(i(i(a, b), b), i(i(b, a), a)))$ cnf(axL, axiom) $\neg p(i(a, i(b, a)))$ cnf(axK, negated_conjecture)**LCL930** \wedge **1.p** Embedding of second order modal logic S5 with universal access

include('Axioms/LCL017^0.ax')