

LDA axioms

LDA001-0.ax Embedding algebra

LD-algebras are related to large cardinals. Under a very strong large cardinal assumption, the free-monogenic LD-algebra can be represented by an algebra of elementary embeddings. Theorems about this algebra can be proved from a small number of properties, suggesting the definition of an embedding algebra.

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a_1 , axiom)
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(a1a, axiom)
critical_point($f(u, v)$) = $a(u, \text{critical_point}(v))$ cnf(a_2 , axiom)
 $\neg \text{less}(x, x)$ cnf(b_1 , axiom)
 $\text{less}(x, y)$ or $\text{less}(y, x)$ or $x = y$ cnf(b_2 , axiom)
 $(\text{less}(x, y)$ and $\text{less}(y, z)) \Rightarrow \text{less}(x, z)$ cnf(b_3 , axiom)
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ cnf(b_4 , axiom)
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ cnf(c_2 , axiom)
 $\text{less}(x, \text{critical_point}(u))$ or $\text{less}(x, a(u, x))$ cnf(c_3 , axiom)

LDA problems

LDA001-1.p Verify $3^*2^*U = UUU$, where $U = 2^*2$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a_1 , axiom)
 $n_2 = f(n_1, n_1)$ cnf(clause_2 , axiom)
 $n_3 = f(n_2, n_1)$ cnf(clause_3 , axiom)
 $u = f(n_2, n_2)$ cnf(clause_4 , axiom)
 $f(f(n_3, n_2), u) \neq f(f(u, u), u)$ cnf(prove_equation , negated_conjecture)

LDA002-1.p Verify $3^*2(U2)(UU(UU)) = U1(U3)(UU(UU))$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a_1 , axiom)
 $n_2 = f(n_1, n_1)$ cnf(clause_2 , axiom)
 $n_3 = f(n_2, n_1)$ cnf(clause_3 , axiom)
 $u = f(n_2, n_2)$ cnf(clause_4 , axiom)
 $u_1 = f(u, n_1)$ cnf(clause_5 , axiom)
 $u_2 = f(u, n_2)$ cnf(clause_6 , axiom)
 $u_3 = f(u, n_3)$ cnf(clause_7 , axiom)
 $uu = f(u, u)$ cnf(clause_8 , axiom)
 $a = f(f(n_3, n_2), u_2)$ cnf(clause_9 , axiom)
 $b = f(u_1, u_3)$ cnf(clause_{10} , axiom)
 $v = f(uu, uu)$ cnf(clause_{11} , axiom)
 $f(a, v) \neq f(b, v)$ cnf(prove_equation , negated_conjecture)

LDA003-1.p Show that 3 is a left segment of $U = 2^*2$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a_1 , axiom)
 $\text{left}(x, f(x, y))$ cnf(a_2 , axiom)
 $(\text{left}(x, y)$ and $\text{left}(y, z)) \Rightarrow \text{left}(x, z)$ cnf(a_3 , axiom)
 $n_2 = f(n_1, n_1)$ cnf(clause_4 , axiom)
 $n_3 = f(n_2, n_1)$ cnf(clause_5 , axiom)
 $u = f(n_2, n_2)$ cnf(clause_6 , axiom)
 $\neg \text{left}(n_3, u)$ cnf(prove_equation , negated_conjecture)

LDA004-1.p Show that $3^*2(U2)$ is a left segment of $U1(U3)$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a_1 , axiom)
 $\text{left}(x, f(x, y))$ cnf(a_2 , axiom)
 $(\text{left}(x, y)$ and $\text{left}(y, z)) \Rightarrow \text{left}(x, z)$ cnf(a_3 , axiom)
 $n_2 = f(n_1, n_1)$ cnf(clause_4 , axiom)
 $n_3 = f(n_2, n_1)$ cnf(clause_5 , axiom)
 $u = f(n_2, n_2)$ cnf(clause_6 , axiom)
 $u_1 = f(u, n_1)$ cnf(clause_7 , axiom)
 $u_2 = f(u, n_2)$ cnf(clause_8 , axiom)
 $u_3 = f(u, n_3)$ cnf(clause_9 , axiom)
 $a = f(f(n_3, n_2), u_2)$ cnf(clause_{10} , axiom)
 $b = f(u_1, u_3)$ cnf(clause_{11} , axiom)
 $\neg \text{left}(a, b)$ cnf(prove_equation , negated_conjecture)

LDA005-1.p Let $g=cr(t)$. Show that $tt(tsg) < t(tsg)$ (for any s)

include('Axioms/LDA001-0.ax')

$tt = f(t, t)$ cnf(*clause*₁, axiom)

$st = f(s, t)$ cnf(*clause*₂, axiom)

$ts = f(t, s)$ cnf(*clause*₃, axiom)

$k = critical_point(t)$ cnf(*clause*₄, axiom)

$sk = f(s, k)$ cnf(*clause*₅, axiom)

$tk = f(t, k)$ cnf(*clause*₆, axiom)

$stk = f(st, k)$ cnf(*clause*₇, axiom)

$tsk = f(ts, k)$ cnf(*clause*₈, axiom)

$\neg less(f(tt, tsk), f(t, tsk))$ cnf(*prove.equation*, *negated.conjecture*)

LDA005-2.p Let $g=cr(t)$. Show that $tt(tsg) < t(tsg)$ (for any s)

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(*a*₁, axiom)

$a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(*a1a*, axiom)

$critical_point(f(u, v)) = a(u, critical_point(v))$ cnf(*a*₂, axiom)

$\neg less(x, x)$ cnf(*b*₁, axiom)

$less(x, y) \Rightarrow less(a(u, x), a(u, y))$ cnf(*b*₄, axiom)

$x = a(u, x)$ or $less(x, a(u, x))$ cnf(*c*₁, axiom)

$less(x, critical_point(u)) \Rightarrow a(u, x) = x$ cnf(*c*₂, axiom)

$less(x, critical_point(u))$ or $less(x, a(u, x))$ cnf(*c*₃, axiom)

$a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(*d*₂, axiom)

$tt = f(t, t)$ cnf(*clause*₁, axiom)

$st = f(s, t)$ cnf(*clause*₂, axiom)

$ts = f(t, s)$ cnf(*clause*₃, axiom)

$k = critical_point(t)$ cnf(*clause*₄, axiom)

$sk = f(s, k)$ cnf(*clause*₅, axiom)

$tk = f(t, k)$ cnf(*clause*₆, axiom)

$stk = f(st, k)$ cnf(*clause*₇, axiom)

$tsk = f(ts, k)$ cnf(*clause*₈, axiom)

$\neg less(f(tt, tsk), f(t, tsk))$ cnf(*prove.equation*, *negated.conjecture*)

LDA006-1.p Let $g = cr(t)$. Show that tsg is not in the range of t

Showing that tsg is not in the range of t is the same as showing that $tsg <> ta$ for any a .

include('Axioms/LDA001-0.ax')

$tt = f(t, t)$ cnf(*clause*₁, axiom)

$st = f(s, t)$ cnf(*clause*₂, axiom)

$ts = f(t, s)$ cnf(*clause*₃, axiom)

$k = critical_point(t)$ cnf(*clause*₄, axiom)

$sk = f(s, k)$ cnf(*clause*₅, axiom)

$tk = f(t, k)$ cnf(*clause*₆, axiom)

$stk = f(st, k)$ cnf(*clause*₇, axiom)

$tsk = f(ts, k)$ cnf(*clause*₈, axiom)

$tsk = f(t, skolem)$ cnf(*prove.equation*, *negated.conjecture*)

LDA006-2.p Let $g = cr(t)$. Show that tsg is not in the range of t

Showing that tsg is not in the range of t is the same as showing that $tsg <> ta$ for any a .

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(*a*₁, axiom)

$a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(*a1a*, axiom)

$critical_point(f(u, v)) = a(u, critical_point(v))$ cnf(*a*₂, axiom)

$\neg less(x, x)$ cnf(*b*₁, axiom)

$less(x, y) \Rightarrow less(a(u, x), a(u, y))$ cnf(*b*₄, axiom)

$x = a(u, x)$ or $less(x, a(u, x))$ cnf(*c*₁, axiom)

$less(x, critical_point(u)) \Rightarrow a(u, x) = x$ cnf(*c*₂, axiom)

$less(x, critical_point(u))$ or $less(x, a(u, x))$ cnf(*c*₃, axiom)

$a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(*d*₂, axiom)

$tt = f(t, t)$ cnf(*clause*₁, axiom)

$st = f(s, t)$ cnf(*clause*₂, axiom)

$ts = f(t, s)$ cnf(*clause*₃, axiom)

$k = critical_point(t)$ cnf(*clause*₄, axiom)

$sk = f(s, k)$ cnf(*clause*₅, axiom)

$tk = f(t, k)$ $\text{cnf}(\text{clause}_6, \text{axiom})$
 $stk = f(st, k)$ $\text{cnf}(\text{clause}_7, \text{axiom})$
 $tsk = f(ts, k)$ $\text{cnf}(\text{clause}_8, \text{axiom})$
 $tsk = f(t, \text{skolem})$ $\text{cnf}(\text{prove_equation}, \text{negated_conjecture})$

LDA007-1.p Let $g = \text{cr}(t)$. Show that $t(\text{tsg}) = \text{tt}(\text{ts})(\text{tg})$

$\text{include}(\text{'Axioms/LDA001-0.ax'})$
 $tt = f(t, t)$ $\text{cnf}(\text{clause}_1, \text{axiom})$
 $st = f(s, t)$ $\text{cnf}(\text{clause}_2, \text{axiom})$
 $ts = f(t, s)$ $\text{cnf}(\text{clause}_3, \text{axiom})$
 $tt_ts = f(tt, ts)$ $\text{cnf}(\text{clause}_4, \text{axiom})$
 $k = \text{critical_point}(t)$ $\text{cnf}(\text{clause}_5, \text{axiom})$
 $sk = f(s, k)$ $\text{cnf}(\text{clause}_6, \text{axiom})$
 $tk = f(t, k)$ $\text{cnf}(\text{clause}_7, \text{axiom})$
 $stk = f(st, k)$ $\text{cnf}(\text{clause}_8, \text{axiom})$
 $tsk = f(ts, k)$ $\text{cnf}(\text{clause}_9, \text{axiom})$
 $ttk = f(tt, k)$ $\text{cnf}(\text{clause}_{10}, \text{axiom})$
 $f(t, \text{tsk}) \neq f(tt_ts, tk)$ $\text{cnf}(\text{prove_equation}, \text{negated_conjecture})$

LDA007-2.p Let $g = \text{cr}(t)$. Show that $t(\text{tsg}) = \text{tt}(\text{ts})(\text{tg})$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ $\text{cnf}(a_1, \text{axiom})$
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ $\text{cnf}(a_{1a}, \text{axiom})$
 $\text{critical_point}(f(u, v)) = a(u, \text{critical_point}(v))$ $\text{cnf}(a_2, \text{axiom})$
 $\neg \text{less}(x, x)$ $\text{cnf}(b_1, \text{axiom})$
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ $\text{cnf}(b_4, \text{axiom})$
 $x = a(u, x) \text{ or } \text{less}(x, a(u, x))$ $\text{cnf}(c_1, \text{axiom})$
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ $\text{cnf}(c_2, \text{axiom})$
 $\text{less}(x, \text{critical_point}(u)) \text{ or } \text{less}(x, a(u, x))$ $\text{cnf}(c_3, \text{axiom})$
 $a(u, x) = x \Rightarrow a(f(v, u), x) = x$ $\text{cnf}(d_2, \text{axiom})$
 $tt = f(t, t)$ $\text{cnf}(\text{clause}_1, \text{axiom})$
 $st = f(s, t)$ $\text{cnf}(\text{clause}_2, \text{axiom})$
 $ts = f(t, s)$ $\text{cnf}(\text{clause}_3, \text{axiom})$
 $tt_ts = f(tt, ts)$ $\text{cnf}(\text{clause}_4, \text{axiom})$
 $k = \text{critical_point}(t)$ $\text{cnf}(\text{clause}_5, \text{axiom})$
 $sk = f(s, k)$ $\text{cnf}(\text{clause}_6, \text{axiom})$
 $tk = f(t, k)$ $\text{cnf}(\text{clause}_7, \text{axiom})$
 $stk = f(st, k)$ $\text{cnf}(\text{clause}_8, \text{axiom})$
 $tsk = f(ts, k)$ $\text{cnf}(\text{clause}_9, \text{axiom})$
 $ttk = f(tt, k)$ $\text{cnf}(\text{clause}_{10}, \text{axiom})$
 $f(t, \text{tsk}) \neq f(tt_ts, tk)$ $\text{cnf}(\text{prove_equation}, \text{negated_conjecture})$

LDA007-3.p Let $g = \text{cr}(t)$. Show that $t(\text{tsg}) = \text{tt}(\text{ts})(\text{tg})$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ $\text{cnf}(a_1, \text{axiom})$
 $tt = f(t, t)$ $\text{cnf}(\text{clause}_1, \text{axiom})$
 $ts = f(t, s)$ $\text{cnf}(\text{clause}_2, \text{axiom})$
 $tt_ts = f(tt, ts)$ $\text{cnf}(\text{clause}_3, \text{axiom})$
 $tk = f(t, k)$ $\text{cnf}(\text{clause}_4, \text{axiom})$
 $tsk = f(ts, k)$ $\text{cnf}(\text{clause}_5, \text{axiom})$
 $f(t, \text{tsk}) \neq f(tt_ts, tk)$ $\text{cnf}(\text{prove_equation}, \text{negated_conjecture})$

LDA008-1.p Let $g = \text{cr}(t) = \text{cr}(T)$. If $Ta < Tsg$, then $ta < tsg$

$\text{include}(\text{'Axioms/LDA001-0.ax'})$
 $ts = f(t, s)$ $\text{cnf}(\text{clause}_1, \text{axiom})$
 $bts = f(bt, s)$ $\text{cnf}(\text{clause}_2, \text{axiom})$
 $ta = f(t, \text{aconst})$ $\text{cnf}(\text{clause}_3, \text{axiom})$
 $bta = f(bt, \text{aconst})$ $\text{cnf}(\text{clause}_4, \text{axiom})$
 $k = \text{critical_point}(t)$ $\text{cnf}(\text{clause}_5, \text{axiom})$
 $\text{critical_point}(bt) = k$ $\text{cnf}(\text{clause}_6, \text{axiom})$
 $tsk = f(ts, k)$ $\text{cnf}(\text{clause}_7, \text{axiom})$
 $btsk = f(bts, k)$ $\text{cnf}(\text{clause}_8, \text{axiom})$
 $\text{less}(bta, btsk)$ $\text{cnf}(\text{clause}_9, \text{axiom})$

\neg less(ta, tsk) cnf(prove_equation, negated_conjecture)

LDA008-2.p Let $g = \text{cr}(t) = \text{cr}(T)$. If $Ta < Tsg$, then $ta < tsg$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a₁, axiom)
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(a1a, axiom)
 $\text{critical_point}(f(u, v)) = a(u, \text{critical_point}(v))$ cnf(a₂, axiom)
 \neg less(x, x) cnf(b₁, axiom)
 $(\text{less}(x, y) \text{ and } \text{less}(y, z)) \Rightarrow \text{less}(x, z)$ cnf(b₃, axiom)
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ cnf(b₄, axiom)
 $x = a(u, x) \text{ or } \text{less}(x, a(u, x))$ cnf(c₁, axiom)
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ cnf(c₂, axiom)
 $\text{less}(x, \text{critical_point}(u)) \text{ or } \text{less}(x, a(u, x))$ cnf(c₃, axiom)
 $a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(d₂, axiom)
 $\text{bts} = f(\text{bt}, s)$ cnf(clause₂, axiom)
 $\text{ta} = f(t, \text{aconst})$ cnf(clause₃, axiom)
 $\text{bta} = f(\text{bt}, \text{aconst})$ cnf(clause₄, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₅, axiom)
 $\text{critical_point}(\text{bt}) = k$ cnf(clause₆, axiom)
 $\text{tsk} = f(\text{ts}, k)$ cnf(clause₇, axiom)
 $\text{btsk} = f(\text{bts}, k)$ cnf(clause₈, axiom)
 $\text{less}(\text{bta}, \text{btsk})$ cnf(clause₉, axiom)
 \neg less(ta, tsk) cnf(prove_equation, negated_conjecture)

LDA009-1.p Let $g = \text{cr}(t)$. If $g < sg$, then $\text{st}(\text{ts})g < \text{stt}(sg)$

include('Axioms/LDA001-0.ax')
 $\text{st} = f(s, t)$ cnf(clause₁, axiom)
 $\text{stt} = f(\text{st}, t)$ cnf(clause₂, axiom)
 $\text{sttt} = f(\text{stt}, t)$ cnf(clause₃, axiom)
 $\text{stts} = f(\text{stt}, s)$ cnf(clause₄, axiom)
 $\text{ts} = f(t, s)$ cnf(clause₅, axiom)
 $\text{sts} = f(\text{st}, s)$ cnf(clause₆, axiom)
 $\text{st_ts} = f(\text{st}, \text{ts})$ cnf(clause₇, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₈, axiom)
 $\text{sk} = f(s, k)$ cnf(clause₉, axiom)
 $\text{stk} = f(\text{st}, k)$ cnf(clause₁₀, axiom)
 $\text{less}(k, \text{sk})$ cnf(clause₁₁, hypothesis)
 \neg less($f(\text{st_ts}, k), f(\text{stt}, \text{sk})$) cnf(prove_equation, negated_conjecture)

LDA009-2.p Let $g = \text{cr}(t)$. If $g < sg$, then $\text{st}(\text{ts})g < \text{stt}(sg)$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a₁, axiom)
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(a1a, axiom)
 $\text{critical_point}(f(u, v)) = a(u, \text{critical_point}(v))$ cnf(a₂, axiom)
 \neg less(x, x) cnf(b₁, axiom)
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ cnf(b₄, axiom)
 $x = a(u, x) \text{ or } \text{less}(x, a(u, x))$ cnf(c₁, axiom)
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ cnf(c₂, axiom)
 $\text{less}(x, \text{critical_point}(u)) \text{ or } \text{less}(x, a(u, x))$ cnf(c₃, axiom)
 $a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(d₂, axiom)
 $\text{stt} = f(\text{st}, t)$ cnf(clause₂, axiom)
 $\text{sttt} = f(\text{stt}, t)$ cnf(clause₃, axiom)
 $\text{stts} = f(\text{stt}, s)$ cnf(clause₄, axiom)
 $\text{ts} = f(t, s)$ cnf(clause₅, axiom)
 $\text{sts} = f(\text{st}, s)$ cnf(clause₆, axiom)
 $\text{st_ts} = f(\text{st}, \text{ts})$ cnf(clause₇, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₈, axiom)
 $\text{sk} = f(s, k)$ cnf(clause₉, axiom)
 $\text{stk} = f(\text{st}, k)$ cnf(clause₁₀, axiom)
 $\text{less}(k, \text{sk})$ cnf(clause₁₁, hypothesis)
 \neg less($f(\text{st_ts}, k), f(\text{stt}, \text{sk})$) cnf(prove_equation, negated_conjecture)

LDA010-1.p Let $g = \text{cr}(t)$. Show that $\text{stts}(\text{sttt})(\text{stts})g < \text{stt}(sg)$

include('Axioms/LDA001-0.ax')
 $st = f(s, t)$ cnf(clause₁, axiom)
 $stt = f(st, t)$ cnf(clause₂, axiom)
 $sttt = f(stt, t)$ cnf(clause₃, axiom)
 $stts = f(stt, s)$ cnf(clause₄, axiom)
 $ts = f(t, s)$ cnf(clause₅, axiom)
 $sts = f(st, s)$ cnf(clause₆, axiom)
 $st_ts = f(st, ts)$ cnf(clause₇, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₈, axiom)
 $sk = f(s, k)$ cnf(clause₉, axiom)
 $stk = f(st, k)$ cnf(clause₁₀, axiom)
 $\text{less}(k, sk)$ cnf(clause₁₁, hypothesis)
 $\neg \text{less}(f(f(f(stts, sttt), stts), k), f(stt, sk))$ cnf(prove_equation, negated_conjecture)

LDA010-2.p Let $g = \text{cr}(t)$. Show that $\text{stts}(sttt)(stts)g < stt(\text{sg})$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a₁, axiom)
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(a1a, axiom)
 $\text{critical_point}(f(u, v)) = a(u, \text{critical_point}(v))$ cnf(a₂, axiom)
 $\neg \text{less}(x, x)$ cnf(b₁, axiom)
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ cnf(b₄, axiom)
 $x = a(u, x)$ or $\text{less}(x, a(u, x))$ cnf(c₁, axiom)
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ cnf(c₂, axiom)
 $\text{less}(x, \text{critical_point}(u))$ or $\text{less}(x, a(u, x))$ cnf(c₃, axiom)
 $a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(d₂, axiom)
 $stt = f(st, t)$ cnf(clause₂, axiom)
 $sttt = f(stt, t)$ cnf(clause₃, axiom)
 $stts = f(stt, s)$ cnf(clause₄, axiom)
 $ts = f(t, s)$ cnf(clause₅, axiom)
 $sts = f(st, s)$ cnf(clause₆, axiom)
 $st_ts = f(st, ts)$ cnf(clause₇, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₈, axiom)
 $sk = f(s, k)$ cnf(clause₉, axiom)
 $stk = f(st, k)$ cnf(clause₁₀, axiom)
 $\text{less}(k, sk)$ cnf(clause₁₁, hypothesis)
 $\neg \text{less}(f(f(f(stts, sttt), stts), k), f(stt, sk))$ cnf(prove_equation, negated_conjecture)

LDA011-1.p Let $g = \text{cr}(t)$. Show that $\text{stts}(sttt)(stts)stts(sttt)g < stt(\text{sg})$

include('Axioms/LDA001-0.ax')
 $st = f(s, t)$ cnf(clause₁, axiom)
 $stt = f(st, t)$ cnf(clause₂, axiom)
 $sttt = f(stt, t)$ cnf(clause₃, axiom)
 $stts = f(stt, s)$ cnf(clause₄, axiom)
 $ts = f(t, s)$ cnf(clause₅, axiom)
 $sts = f(st, s)$ cnf(clause₆, axiom)
 $st_ts = f(st, ts)$ cnf(clause₇, axiom)
 $k = \text{critical_point}(t)$ cnf(clause₈, axiom)
 $sk = f(s, k)$ cnf(clause₉, axiom)
 $stk = f(st, k)$ cnf(clause₁₀, axiom)
 $\text{less}(k, sk)$ cnf(clause₁₁, hypothesis)
 $\neg \text{less}(f(f(f(stts, sttt), stts), f(f(stts, sttt), k)), f(stt, sk))$ cnf(prove_equation, negated_conjecture)

LDA011-2.p Let $g = \text{cr}(t)$. Show that $\text{stts}(sttt)(stts)stts(sttt)g < stt(\text{sg})$

$f(x, f(y, z)) = f(f(x, y), f(x, z))$ cnf(a₁, axiom)
 $a(x, a(y, z)) = a(f(x, y), a(x, z))$ cnf(a1a, axiom)
 $\text{critical_point}(f(u, v)) = a(u, \text{critical_point}(v))$ cnf(a₂, axiom)
 $\neg \text{less}(x, x)$ cnf(b₁, axiom)
 $\text{less}(x, y) \Rightarrow \text{less}(a(u, x), a(u, y))$ cnf(b₄, axiom)
 $x = a(u, x)$ or $\text{less}(x, a(u, x))$ cnf(c₁, axiom)
 $\text{less}(x, \text{critical_point}(u)) \Rightarrow a(u, x) = x$ cnf(c₂, axiom)
 $\text{less}(x, \text{critical_point}(u))$ or $\text{less}(x, a(u, x))$ cnf(c₃, axiom)
 $a(u, x) = x \Rightarrow a(f(v, u), x) = x$ cnf(d₂, axiom)

```

stt = f(st, t)    cnf clause2, axiom
sttt = f(stt, t)  cnf clause3, axiom
stts = f(stt, s)  cnf clause4, axiom
ts = f(t, s)     cnf clause5, axiom
sts = f(st, s)   cnf clause6, axiom
st_ts = f(st, ts) cnf clause7, axiom
k = critical_point(t)  cnf clause8, axiom
sk = f(s, k)     cnf clause9, axiom
stk = f(st, k)   cnf clause10, axiom
less(k, sk)     cnf clause11, hypothesis
¬less(f(f(f(stts, sttt), stts), f(f(stts, sttt), k)), f(stt, sk))  cnf(prove_equation, negated_conjecture)

```

LDA012-1.p Let $g = cr(t)$. Show that $stts(sttt)g = g$
include('Axioms/LDA001-0.ax')

```

st = f(s, t)    cnf clause1, axiom
stt = f(st, t)  cnf clause2, axiom
sttt = f(stt, t)  cnf clause3, axiom
stts = f(stt, s)  cnf clause4, axiom
ts = f(t, s)    cnf clause5, axiom
sts = f(st, s)  cnf clause6, axiom
st_ts = f(st, ts)  cnf clause7, axiom
k = critical_point(t)  cnf clause8, axiom
sk = f(s, k)    cnf clause9, axiom
stk = f(st, k)  cnf clause10, axiom
less(k, sk)    cnf clause11, hypothesis
f(f(stts, sttt), k) ≠ k  cnf(prove_equation, negated_conjecture)

```

LDA012-2.p Let $g = cr(t)$. Show that $stts(sttt)g = g$

```

f(x, f(y, z)) = f(f(x, y), f(x, z))  cnf(a1, axiom)
a(x, a(y, z)) = a(f(x, y), a(x, z))  cnf(a1a, axiom)
critical_point(f(u, v)) = a(u, critical_point(v))  cnf(a2, axiom)
¬less(x, x)  cnf(b1, axiom)
less(x, y) ⇒ less(a(u, x), a(u, y))  cnf(b4, axiom)
x = a(u, x) or less(x, a(u, x))  cnf(c1, axiom)
less(x, critical_point(u)) ⇒ a(u, x) = x  cnf(c2, axiom)
less(x, critical_point(u)) or less(x, a(u, x))  cnf(c3, axiom)
a(u, x) = x ⇒ a(f(v, u), x) = x  cnf(d2, axiom)
stt = f(st, t)    cnf clause2, axiom
sttt = f(stt, t)  cnf clause3, axiom
stts = f(stt, s)  cnf clause4, axiom
ts = f(t, s)     cnf clause5, axiom
sts = f(st, s)   cnf clause6, axiom
st_ts = f(st, ts)  cnf clause7, axiom
k = critical_point(t)  cnf clause8, axiom
sk = f(s, k)     cnf clause9, axiom
stk = f(st, k)   cnf clause10, axiom
less(k, sk)     cnf clause11, hypothesis
f(f(stts, sttt), k) ≠ k  cnf(prove_equation, negated_conjecture)

```

LDA013-1.p Let $g = cr(t)$. Show that $aag \leq ag$, $t=a$

This is the base step of an induction proof.

```

include('Axioms/LDA001-0.ax')
k = critical_point(aconst)  cnf clause1, axiom
aa = f(aconst, aconst)  cnf clause2, axiom
aak = f(aa, k)  cnf clause3, axiom
ak = f(aconst, k)  cnf clause4, axiom
less(ak, aak)  cnf(prove_equation, negated_conjecture)

```

LDA014-1.p Let $g = cr(t)$. Show that $aag \leq ag$, $t=a$

This is the induction step of an induction proof.

```

include('Axioms/LDA001-0.ax')

```


$a \cdot a = a$ cnf(sos₀₁, axiom)

$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$ cnf(sos₀₂, axiom)

$((a \cdot b) \cdot ((c \cdot a) \cdot a)) \cdot (((a \cdot b) \cdot c) \cdot ((c \cdot a) \cdot d)) = ((a \cdot b) \cdot (c \cdot a)) \cdot (((a \cdot b) \cdot (a \cdot c)) \cdot ((c \cdot a) \cdot d))$ cnf(sos₀₃, axiom)

$((x_0 \cdot (x_1 \cdot x_0)) \cdot ((x_1 \cdot x_0) \cdot x_0)) \cdot (x_0 \cdot x_1) \cdot x_2 \neq ((x_0 \cdot (x_1 \cdot x_0)) \cdot (x_1 \cdot x_0)) \cdot ((x_0 \cdot (x_0 \cdot x_1)) \cdot x_2)$ cnf(goals, negated_conjecture)

LDA041-1.p Embedding algebra

include('Axioms/LDA001-0.ax')