NUM axioms

NUM001-0.ax Number theory axioms
$a + n_0 = a$ cnf(adding_zero, axiom)
$a + \text{successor}(b) = \text{successor}(a + b)$ cnf(addition, axiom)
$a \cdot n_0 = n_0$ cnf(times_zero, axiom)
$a \cdot \text{successor}(b) = a \cdot b + a$ cnf(times, axiom)
$\text{successor}(a) = \text{successor}(b) \Rightarrow a = b$ cnf($\text{successor\_equality}_1$, axiom)
$a = b \Rightarrow \text{successor}(a) = \text{successor}(b)$ cnf(successor_substitution, axiom)

NUM001-1.ax Number theory less axioms
$(\text{less}(a, b) \text{ and } \text{less}(c, a)) \Rightarrow \text{less}(c, b)$ cnf(transitivity_of_less, axiom)
$\text{successor}(a) + b = c \Rightarrow \text{less}(b, c)$ cnf(smaller_number, axiom)
$\text{less}(a, b) \Rightarrow \text{successor}(\text{predecessor\_of\_1st\_minus\_2nd}(b, a)) + a = b$ cnf(less_lemma, axiom)

NUM001-2.ax Number theory div axioms
$\text{divides}(a, b) \Rightarrow (\text{less}(a, b) \text{ or } a = b)$ cnf(divides_only_less_or_equal, axiom)
$\text{less}(a, b) \Rightarrow \text{divides}(a, b)$ cnf(divides_if_less, axiom)
$a = b \Rightarrow \text{divides}(a, b)$ cnf(divides_if_equal, axiom)

NUM002-0.ax Number theory (equality) axioms
$a = a$ cnf(reflexivity, axiom)
$(a = b \text{ and } b = c) \Rightarrow a = c$ cnf(transitivity, axiom)
$a + b = b + a$ cnf(commutativity_of_addition, axiom)
$a + (b + c) = (a + b) + c$ cnf(associativity_of_addition, axiom)
$\text{subtract}(a + b, b) = a$ cnf($\text{addition\_inverts\_subtraction}_1$, axiom)
$a = \text{subtract}(a + b, b)$ cnf($\text{addition\_inverts\_subtraction}_2$, axiom)
$\text{subtract}(a, b) + c = \text{subtract}(a + c, b)$ cnf(commutativity_1, axiom)
$\text{subtract}(a + b, c) = \text{subtract}(a, c) + b$ cnf(commutativity_2, axiom)
$(a = b \text{ and } c = a + d) \Rightarrow c = b + d$ cnf($\text{add\_substitution}_1$, axiom)
$(a = b \text{ and } c = d + a) \Rightarrow c = d + b$ cnf($\text{add\_substitution}_2$, axiom)
$(a = b \text{ and } c = \text{subtract}(a, d)) \Rightarrow c = \text{subtract}(b, d)$ cnf($\text{subtract\_substitution}_1$, axiom)
$(a = b \text{ and } c = \text{subtract}(d, a)) \Rightarrow c = \text{subtract}(d, b)$ cnf($\text{subtract\_substitution}_2$, axiom)

NUM005+1.ax Less in RDN format
Impements a "human style" less using RDN format.
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_1))$ fof(rdn_digit_1, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_2))$ fof(rdn_digit_2, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_3))$ fof(rdn_digit_3, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_4))$ fof(rdn_digit_4, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_5))$ fof(rdn_digit_5, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_6))$ fof(rdn_digit_6, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_7))$ fof(rdn_digit_7, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_8))$ fof(rdn_digit_8, axiom)
$\text{rdn\_non\_zero\_digit}(\text{rdnn}(n_9))$ fof(rdn_digit_9, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_0), \text{rdnn}(n_1))$ fof($\text{rdn\_positive\_less}_{01}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_1), \text{rdnn}(n_2))$ fof($\text{rdn\_positive\_less}_{12}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_2), \text{rdnn}(n_3))$ fof($\text{rdn\_positive\_less}_{23}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_3), \text{rdnn}(n_4))$ fof($\text{rdn\_positive\_less}_{34}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_4), \text{rdnn}(n_5))$ fof($\text{rdn\_positive\_less}_{45}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_5), \text{rdnn}(n_6))$ fof($\text{rdn\_positive\_less}_{56}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_6), \text{rdnn}(n_7))$ fof($\text{rdn\_positive\_less}_{67}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_7), \text{rdnn}(n_8))$ fof($\text{rdn\_positive\_less}_{78}$, axiom)
$\text{rdn\_positive\_less}(\text{rdnn}(n_8), \text{rdnn}(n_9))$ fof($\text{rdn\_positive\_less}_{89}$, axiom)
$\forall x, y, z$: $((\text{rdn\_positive\_less}(\text{rdnn}(x), \text{rdnn}(y)) \text{ and } \text{rdn\_positive\_less}(\text{rdnn}(y), \text{rdnn}(z))) \Rightarrow \text{rdn\_positive\_less}(\text{rdnn}(x), \text{rdnn}(z))$
$\forall \text{ds}, \text{os}, \text{db}, \text{ob}$: $(\text{rdn\_positive\_less}(\text{os}, \text{ob}) \Rightarrow \text{rdn\_positive\_less}(\text{rdn}(\text{rdnn}(\text{ds}), \text{os}), \text{rdn}(\text{rdnn}(\text{db}), \text{ob})))$ fof(rdn_positive_less_m
$\forall \text{ds}, o, \text{db}$: $((\text{rdn\_positive\_less}(\text{rdnn}(\text{ds}), \text{rdnn}(\text{db})) \text{ and } \text{rdn\_non\_zero}(o)) \Rightarrow \text{rdn\_positive\_less}(\text{rdn}(\text{rdnn}(\text{ds}), o), \text{rdn}(\text{rdnn}(\text{db}),$
$\forall d, \text{db}, \text{ob}$: $(\text{rdn\_non\_zero}(\text{ob}) \Rightarrow \text{rdn\_positive\_less}(\text{rdnn}(d), \text{rdn}(\text{rdnn}(\text{db}), \text{ob})))$ fof(rdn_extra_digits_positive_less, axiom)
$\forall x$: $(\text{rdn\_non\_zero\_digit}(\text{rdnn}(x)) \Rightarrow \text{rdn\_non\_zero}(\text{rdnn}(x)))$ fof(rdn_non_zero_by_digit, axiom)
$\forall d, o$: $(\text{rdn\_non\_zero}(o) \Rightarrow \text{rdn\_non\_zero}(\text{rdn}(\text{rdnn}(d), o)))$ fof(rdn_non_zero_by_structure, axiom)

$\forall x, y,$ rDN_X, rDN_Y: ((rdn_translate(x, rdn_pos(rDN_X)) and rdn_translate(y, rdn_pos(rDN_Y)) and rdn_positive_less(rDN_X
less(x, y))   fof(less_entry_point_pos_pos, axiom)
$\forall x, y,$ rDN_X, rDN_Y: ((rdn_translate(x, rdn_neg(rDN_X)) and rdn_translate(y, rdn_pos(rDN_Y))) $\Rightarrow$ less(x, y))   fof(less_e
$\forall x, y,$ rDN_X, rDN_Y: ((rdn_translate(x, rdn_neg(rDN_X)) and rdn_translate(y, rdn_neg(rDN_Y)) and rdn_positive_less(rDN_Y
less(x, y))   fof(less_entry_point_neg_neg, axiom)
$\forall x, y$: (less(x, y) $\iff$ ($\neg$ less(y, x) and $y \neq x$))   fof(less_property, axiom)
$\forall x, y$: ($x \leq y \iff$ (less(x, y) or $x = y$))   fof(less_or_equal, axiom)
$\forall x, y, z$: (($x + n_1 = y$ and less(z, y)) $\Rightarrow$ $z \leq x$)   fof(less_successor, axiom)

NUM006∧0.ax Church Numerals in Simple Type Theory
0: (\$i → \$i) → \$i → \$i   thf(zero, type)
1: (\$i → \$i) → \$i → \$i   thf(one, type)
two: (\$i → \$i) → \$i → \$i   thf(two, type)
three: (\$i → \$i) → \$i → \$i   thf(three, type)
four: (\$i → \$i) → \$i → \$i   thf(four, type)
five: (\$i → \$i) → \$i → \$i   thf(five, type)
six: (\$i → \$i) → \$i → \$i   thf(six, type)
seven: (\$i → \$i) → \$i → \$i   thf(seven, type)
eight: (\$i → \$i) → \$i → \$i   thf(eight, type)
nine: (\$i → \$i) → \$i → \$i   thf(nine, type)
ten: (\$i → \$i) → \$i → \$i   thf(ten, type)
succ: ((\$i → \$i) → \$i → \$i) → (\$i → \$i) → \$i → \$i   thf(succ, type)
+: ((\$i → \$i) → \$i → \$i) → ((\$i → \$i) → \$i → \$i) → (\$i → \$i) → \$i → \$i   thf(plus, type)
·: ((\$i → \$i) → \$i → \$i) → ((\$i → \$i) → \$i → \$i) → (\$i → \$i) → \$i → \$i   thf(mult, type)
0 = (λx: \$i → \$i, y: \$i: y)   thf(zero_ax, definition)
1 = (λx: \$i → \$i, y: \$i: (x@y))   thf(one_ax, definition)
two = (λx: \$i → \$i, y: \$i: (x@(x@y)))   thf(two_ax, definition)
three = (λx: \$i → \$i, y: \$i: (x@(x@(x@y))))   thf(three_ax, definition)
four = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@y)))))   thf(four_ax, definition)
five = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@y))))))   thf(five_ax, definition)
six = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@(x@y)))))))   thf(six_ax, definition)
seven = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@(x@(x@y))))))))   thf(seven_ax, definition)
eight = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@(x@(x@(x@y)))))))))   thf(eight_ax, definition)
nine = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@(x@(x@(x@(x@y))))))))))   thf(nine_ax, definition)
ten = (λx: \$i → \$i, y: \$i: (x@(x@(x@(x@(x@(x@(x@(x@(x@(x@y)))))))))))   thf(ten_ax, definition)
succ = (λn: (\$i → \$i) → \$i → \$i, x: \$i → \$i, y: \$i: (x@(n@x@y)))   thf(succ_ax, definition)
+ = (λm: (\$i → \$i) → \$i → \$i, n: (\$i → \$i) → \$i → \$i, x: \$i → \$i, y: \$i: (m@x@(n@x@y)))   thf(plus_ax, definition)
· = (λm: (\$i → \$i) → \$i → \$i, n: (\$i → \$i) → \$i → \$i, x: \$i → \$i, y: \$i: (m@(n@x)@y))   thf(mult_ax, definition)

NUM problems

NUM001-1.p (A + B) + C = A + (B + C)
include('Axioms/NUM002-0.ax')
$\neg\,(a + b) + c = a + (b + c)$ cnf(prove_equation, negated_conjecture)

NUM002-1.p (X - Y) + Z = X + (Z - Y)
include('Axioms/NUM002-0.ax')
$\neg$ subtract(a, b) + c = a + subtract(c, b)   cnf(prove_equation, negated_conjecture)

NUM003-1.p A + (B - C) = (A - C) + B
include('Axioms/NUM002-0.ax')
$\neg$ a + subtract(b, c) = subtract(a, c) + b cnf(prove_equation, negated_conjecture)

NUM004-1.p (A + B) - C = A + (B - C)
include('Axioms/NUM002-0.ax')
$\neg$ subtract($a + b, c$) = a + subtract(b, c)   cnf(prove_equation, negated_conjecture)

NUM005-1.p Greatest Common Divisor
If GCD(a,b) is the greatest common divisor of two positive a, b, then for any positive integer d, GCD(a*d,b*d) = GCD(a,b)*d.
divides(x, x)   cnf(reflexivity_of_divides, axiom)
(divides(x, y) and divides(y, z)) $\Rightarrow$ divides(x, z)   cnf(transitivity_of_divides, axiom)

$\text{divides}(x, x \cdot y)$ $\quad$ cnf(operand_divides_product, axiom)
$\text{divides}(y, z) \Rightarrow \text{divides}(x \cdot y, x \cdot z)$ $\quad$ cnf(divides_and_multiply, axiom)
$\text{divides}(\text{quotient}(x, x), y)$ $\quad$ cnf(one_divides_everything, axiom)
$(\text{divides}(y, x) \text{ and } \text{divides}(\text{quotient}(x, y), z)) \Rightarrow \text{divides}(x, y \cdot z)$ $\quad$ cnf(divides_quotient_multiply$_1$, axiom)
$(\text{divides}(z, y) \text{ and } \text{divides}(x \cdot z, y)) \Rightarrow \text{divides}(x, \text{quotient}(y, z))$ $\quad$ cnf(divides_quotient_multiply$_2$, axiom)
$(\text{divides}(y, x) \text{ and } \text{divides}(x, y \cdot z)) \Rightarrow \text{divides}(\text{quotient}(x, y), z)$ $\quad$ cnf(divides_quotient_multiply$_3$, axiom)
$\text{gcd}(x, y, u) \Rightarrow \text{divides}(u, y)$ $\quad$ cnf(gcd_divides$_1$, axiom)
$\text{gcd}(x, y, u) \Rightarrow \text{divides}(u, x)$ $\quad$ cnf(gcd_divides$_2$, axiom)
$\text{divides}(k(y, x), x)$ $\quad$ cnf(divides_k$_1$, axiom)
$\text{divides}(k(y, x), y)$ $\quad$ cnf(divides_k$_2$, axiom)
$(\text{divides}(v, x) \text{ and } \text{divides}(v, y)) \Rightarrow \text{divides}(v, k(y, x))$ $\quad$ cnf(divides_k$_3$, axiom)
$(\text{divides}(v, x) \text{ and } \text{divides}(v, y) \text{ and } \text{gcd}(x, y, u)) \Rightarrow \text{divides}(v, u)$ $\quad$ cnf(gcd$_1$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y)) \Rightarrow (\text{gcd}(x, y, u) \text{ or } \text{divides}(h(y, x, u), x))$ $\quad$ cnf(gcd$_2$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y)) \Rightarrow (\text{gcd}(x, y, u) \text{ or } \text{divides}(h(y, x, u), y))$ $\quad$ cnf(gcd$_3$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y) \text{ and } \text{divides}(h(y, x, u), u)) \Rightarrow \text{gcd}(x, y, u)$ $\quad$ cnf(gcd$_4$, axiom)
$h(x, y, z) = h(y, x, z)$ $\quad$ cnf(commutativity_of_h, axiom)
$k(x, y) = k(y, x)$ $\quad$ cnf(commutativity_of_k, axiom)
$x \cdot y = y \cdot x$ $\quad$ cnf(commutativity_of_multiply, axiom)
$\text{gcd}(x, y, z) \Rightarrow \text{gcd}(y, x, z)$ $\quad$ cnf(commutativity_of_gcd, axiom)
$\text{gcd}(a, b, e)$ $\quad$ cnf(e_is_gcd_of_a_and_b, hypothesis)
$\neg \text{gcd}(a \cdot c, b \cdot c, e \cdot c)$ $\quad$ cnf(prove_gcd, negated_conjecture)

NUM006-1.p Goldbach conjecture
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
$f_{79} \in \text{even\_numbers}$ $\quad$ cnf(an_even_number, hypothesis)
$\neg f_{79} \in \text{non\_ordered\_pair}(\text{empty\_set}, \text{successor}(\text{successor}(\text{empty\_set})))$ $\quad$ cnf(its_not_0_or$_2$, hypothesis)
$(x \in \text{prime\_numbers} \text{ and } y \in \text{prime\_numbers}) \Rightarrow \text{apply\_to\_two\_arguments}(+, x, y) \neq f_{79}$ $\quad$ cnf(prove_its_not_the_sum_of_tw

NUM007-1.p Least Common Multiple
If LCM(a,b) is the least common multiple of two positive integers a, b, then LCM(a,b) = a*b/GCD(a,b).
$\text{divides}(x, x)$ $\quad$ cnf(reflexivity_of_divides, axiom)
$(\text{divides}(x, y) \text{ and } \text{divides}(y, z)) \Rightarrow \text{divides}(x, z)$ $\quad$ cnf(transitivity_of_divides, axiom)
$\text{divides}(x, x \cdot y)$ $\quad$ cnf(operand_divides_product, axiom)
$\text{divides}(y, z) \Rightarrow \text{divides}(x \cdot y, x \cdot z)$ $\quad$ cnf(divides_and_multiply, axiom)
$\text{divides}(\text{quotient}(x, x), y)$ $\quad$ cnf(one_divides_everything, axiom)
$(\text{divides}(y, x) \text{ and } \text{divides}(\text{quotient}(x, y), z)) \Rightarrow \text{divides}(x, y \cdot z)$ $\quad$ cnf(divides_quotient_multiply$_1$, axiom)
$(\text{divides}(z, y) \text{ and } \text{divides}(x \cdot z, y)) \Rightarrow \text{divides}(x, \text{quotient}(y, z))$ $\quad$ cnf(divides_quotient_multiply$_2$, axiom)
$(\text{divides}(y, x) \text{ and } \text{divides}(x, y \cdot z)) \Rightarrow \text{divides}(\text{quotient}(x, y), z)$ $\quad$ cnf(divides_quotient_multiply$_3$, axiom)
$\text{gcd}(x, y, u) \Rightarrow \text{divides}(u, y)$ $\quad$ cnf(gcd_divides$_1$, axiom)
$\text{gcd}(x, y, u) \Rightarrow \text{divides}(u, x)$ $\quad$ cnf(gcd_divides$_2$, axiom)
$(\text{divides}(v, x) \text{ and } \text{divides}(v, y) \text{ and } \text{gcd}(x, y, u)) \Rightarrow \text{divides}(v, u)$ $\quad$ cnf(gcd$_1$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y)) \Rightarrow (\text{gcd}(x, y, u) \text{ or } \text{divides}(h(y, x, u), x))$ $\quad$ cnf(gcd$_2$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y)) \Rightarrow (\text{gcd}(x, y, u) \text{ or } \text{divides}(h(y, x, u), y))$ $\quad$ cnf(gcd$_3$, axiom)
$(\text{divides}(u, x) \text{ and } \text{divides}(u, y) \text{ and } \text{divides}(h(y, x, u), u)) \Rightarrow \text{gcd}(x, y, u)$ $\quad$ cnf(gcd$_4$, axiom)
$\text{gcd}(x, y, u) \Rightarrow \text{gcd}(z \cdot x, z \cdot y, z \cdot u)$ $\quad$ cnf(property_of_gcd, axiom)
$(\text{divides}(x, u) \text{ and } \text{divides}(y, u)) \Rightarrow (\text{lcm}(x, y, u) \text{ or } \text{divides}(x, k(y, x, u)))$ $\quad$ cnf(lcm$_1$, axiom)
$(\text{divides}(x, u) \text{ and } \text{divides}(y, u)) \Rightarrow (\text{lcm}(x, y, u) \text{ or } \text{divides}(y, k(y, x, u)))$ $\quad$ cnf(lcm$_2$, axiom)
$(\text{divides}(x, u) \text{ and } \text{divides}(y, u) \text{ and } \text{divides}(u, k(y, x, u))) \Rightarrow \text{lcm}(x, y, u)$ $\quad$ cnf(lcm$_3$, axiom)
$k(x, y, z) = k(y, x, z)$ $\quad$ cnf(commutativity_of_k, axiom)
$h(x, y, z) = h(y, x, z)$ $\quad$ cnf(commutativity_of_h, axiom)
$x \cdot y = y \cdot x$ $\quad$ cnf(commutativity_of_multiply, axiom)
$\text{lcm}(x, y, z) \Rightarrow \text{lcm}(y, x, z)$ $\quad$ cnf(commutativity_of_lcm, axiom)
$\text{gcd}(x, y, z) \Rightarrow \text{gcd}(y, x, z)$ $\quad$ cnf(commutativity_of_gcd, axiom)
$\text{gcd}(a, b, c)$ $\quad$ cnf(c_is_gcd_of_a_and_b, negated_conjecture)
$\neg \text{lcm}(a, b, \text{quotient}(a \cdot b, c))$ $\quad$ cnf(prove_lcm, negated_conjecture)

NUM008-1.p Peano axiom 0
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')

include('Axioms/NUM003-0.ax')
$\neg$ little_set(natural_numbers)   cnf(prove_naturals_are_a_set, negated_conjecture)

NUM009-1.p Peano axiom 1
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
$\neg$ empty_set $\in$ natural_numbers   cnf(prove_zero_is_a_natural, negated_conjecture)

NUM010-1.p Peano axiom 2
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
f_{74} $\in$ natural_numbers   cnf(a_natural_number, hypothesis)
$\neg$ successor(f_{74}) $\in$ natural_numbers   cnf(prove_it_has_a_successor, negated_conjecture)

NUM011-1.p Peano axiom 3
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
f_{75} $\in$ natural_numbers   cnf(a_natural_number, hypothesis)
empty_set = successor(f_{75})   cnf(prove_zero_is_first, negated_conjecture)

NUM012-1.p Peano axiom 4
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
f_{76} $\in$ natural_numbers   cnf(a_natural_number, hypothesis)
f_{77} $\in$ natural_numbers   cnf(another_natural_number, hypothesis)
successor(f_{76}) = successor(f_{77})   cnf(successors_are_equal, hypothesis)
$f_{76} \neq f_{77}$ cnf(prove_well_definedness_of_successor, negated_conjecture)

NUM013-1.p Peano axiom 5
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
empty_set $\in f_{78}$ cnf(zero_in_set, hypothesis)
xk $\in f_{78}$ $\Rightarrow$ successor(xk) $\in f_{78}$ cnf(successor_in_set, hypothesis)
$\neg$ natural_numbers $\subseteq f_{78}$ cnf(prove_set_is_in_naturals, negated_conjecture)

NUM014-1.p If a is a prime and a = b∧2/c∧2 then a divides b
$x \cdot x$=square(x)   cnf(square, axiom)
$x \cdot y$=z $\Rightarrow$ $y \cdot x$=z cnf(commutativity, axiom)
$x \cdot y$=z $\Rightarrow$ divides(x, z)   cnf(divides, axiom)
(prime(x) and $y \cdot z$=u and divides(x, u)) $\Rightarrow$ (divides(x, y) or divides(x, z))   cnf(remainder, axiom)
prime(a)   cnf(a_is_prime, hypothesis)
$a \cdot$ square(c)=square(b)   cnf(a_equals_b_squared_by_c_squared, hypothesis)
$\neg$ divides(a, b)   cnf(prove_a_divides_b, negated_conjecture)

NUM015-1.p Any number greater than 1 has a prime divisor
divides(x, x)   cnf(divide_self, axiom)
(divides(x, y) and divides(y, z)) $\Rightarrow$ divides(x, z)   cnf(transitive_divide, axiom)
prime(x) or divides(divisor(x), x)   cnf(prime, axiom)
prime(x) or less(n_1, divisor(x))   cnf(divisor$_1$, axiom)
prime(x) or less(divisor(x), x)   cnf(divisor$_2$, axiom)
(less(n_1, x) and less(x, a)) $\Rightarrow$ prime(factor_of(x))   cnf(factor$_1$, axiom)
(less(n_1, x) and less(x, a)) $\Rightarrow$ divides(factor_of(x), x)   cnf(factor$_2$, axiom)
less(n_1, a)   cnf(a_is_greater_than$_1$, hypothesis)
prime(x) $\Rightarrow$ $\neg$ divides(x, a)   cnf(prove_a_has_prime_divisor, negated_conjecture)

NUM016-1.p There exist infinitely many primes
$\neg$ less(x, x)   cnf(nothing_is_less_than_itself, axiom)
less(x, y) $\Rightarrow$ $\neg$ less(y, x)   cnf(numbers_are_different, axiom)
divides(x, x)   cnf(everything_divides_itself, axiom)

(divides(x, y) and divides(y, z)) $\Rightarrow$ divides(x, z) cnf(transitivity_of_divides, axiom)
divides(x, y) $\Rightarrow$ $\neg$ less(y, x) cnf(small_divides_large, axiom)
less(x, factorial_plus_one(x)) cnf(a_prime_is_less_than_the_next_one, axiom)
divides(x, factorial_plus_one(y)) $\Rightarrow$ less(y, x) cnf(divisor_is_smaller, axiom)
prime(x) or divides(prime_divisor(x), x) cnf(division_by_prime_divisor, axiom)
prime(x) or prime(prime_divisor(x)) cnf(prime_divsiors, axiom)
prime(x) or less(prime_divisor(x), x) cnf(smaller_prime_divisors, axiom)
prime(a) cnf(a_is_prime, hypothesis)
(prime(x) and less(a, x)) $\Rightarrow$ less(factorial_plus_one(a), x) cnf(prove_there_is_another_prime, negated_conjecture)

NUM016-2.p There exist infinitely many primes
$\neg$ less(x, x) cnf(nothing_is_less_than_itself, axiom)
less(x, y) $\Rightarrow$ $\neg$ less(y, x) cnf(numbers_are_different, axiom)
less(x, factorial_plus_one(x)) cnf(a_prime_is_less_than_the_next_one, axiom)
divides(x, factorial_plus_one(y)) $\Rightarrow$ less(y, x) cnf(divisor_is_smaller, axiom)
prime(x) or divides(prime_divisor(x), x) cnf(division_by_prime_divisor, axiom)
prime(x) or prime(prime_divisor(x)) cnf(prime_divsiors, axiom)
prime(x) or less(prime_divisor(x), x) cnf(smaller_prime_divisors, axiom)
(prime(x) and less(a, x)) $\Rightarrow$ less(factorial_plus_one(a), x) cnf(prove_there_is_another_prime, negated_conjecture)

NUM016∧5.p TPS problem NUM016-1
There exist infinitely many primes.
a: $i thf($a$, type)
factorial_plus_one: $i → $i thf(factorial_plus_one, type)
less: $i → $i → $o thf(less, type)
prime: $i → $o thf(prime, type)
prime_divisor: $i → $i thf(prime_divisor, type)
divides: $i → $i → $o thf(divides, type)
¬ ∀x: $i: ¬ less@$x$@$x$ and ∀x: $i, y: $i: (¬ less@$x$@$y$ or ¬ less@y@x) and ∀x: $i: (divides@$x$@$x$) and ∀$x$: $i, y: $i, z: $i: (¬ divide

NUM017-1.p Square root of this prime is irrational
If a is prime, and a is not b∧2/c∧2, then the square root of a is irrational.
x=x cnf(reflexivity, axiom)
x=y $\Rightarrow$ y=x cnf(symmetry, axiom)
(x=y and y=z) $\Rightarrow$ x=z cnf(transitivity, axiom)
(d=b and $c \cdot a$=d) $\Rightarrow$ $c \cdot a$=b cnf(product_substitution$_1$, axiom)
(d=b and $c \cdot d$=a) $\Rightarrow$ $c \cdot b$=a cnf(product_substitution$_2$, axiom)
(c=b and $c \cdot d$=a) $\Rightarrow$ $b \cdot d$=a cnf(product_substitution$_3$, axiom)
(b=a and divides(c, b)) $\Rightarrow$ divides(c, a) cnf(divides_substitution$_1$, axiom)
(a=b and divides(a, c)) $\Rightarrow$ divides(b, c) cnf(divides_substitution$_2$, axiom)
(a=b and prime(a)) $\Rightarrow$ prime(b) cnf(prime_substitution, axiom)
$a \cdot b$=$a \cdot b$ cnf(closure_of_product, axiom)
($a \cdot b$=c and $d \cdot e$=b and $a \cdot d$=f) $\Rightarrow$ $f \cdot e$=c cnf(product_associativity$_1$, axiom)
($a \cdot b$=c and $d \cdot b$=e and $f \cdot d$=a) $\Rightarrow$ $f \cdot e$=c cnf(product_associativity$_2$, axiom)
$a \cdot b$=c $\Rightarrow$ $b \cdot a$=c cnf(product_commutativity, axiom)
($a \cdot b$=c and $a \cdot d$=c) $\Rightarrow$ b=d cnf(product_left_cancellation, axiom)
(divides(a, b) and divides(c, a)) $\Rightarrow$ divides(c, b) cnf(transitivity_of_divides, axiom)
($a \cdot b$=c and $a \cdot b$=d) $\Rightarrow$ d=c cnf(well_defined_product, axiom)
divides(a, b) $\Rightarrow$ $a \cdot$ second_divided_by_1st(a, b)=b cnf(divides_implies_product, axiom)
$a \cdot b$=c $\Rightarrow$ divides(a, c) cnf(product_divisible_by_operand, axiom)
(divides(a, b) and $c \cdot c$=b and prime(a)) $\Rightarrow$ divides(a, c) cnf(primes_lemma$_1$, axiom)
prime(a) cnf(a_is_prime, hypothesis)
$b \cdot b$=d cnf(b_squared, hypothesis)
$c \cdot c$=e cnf(c_squared, hypothesis)
$\neg$ $a \cdot e$=d cnf(a_times_c_squared_is_not_b_squared, hypothesis)
divides(a, c) $\Rightarrow$ $\neg$ divides(a, b) cnf(prove_there_is_no_common_divisor, negated_conjecture)

NUM017-2.p Square root of this prime is irrational
If a is prime, and a is not b∧2/c∧2, then the square root of a is irrational.
$a \cdot b$=$a \cdot b$ cnf(closure_of_product, axiom)
($a \cdot b$=c and $d \cdot e$=b and $a \cdot d$=f) $\Rightarrow$ $f \cdot e$=c cnf(product_associativity$_1$, axiom)

$(a \cdot b{=}c$ and $d \cdot b{=}e$ and $f \cdot d{=}a) \;\Rightarrow\; f \cdot e{=}c$ cnf(product_associativity$_2$, axiom)
$a \cdot b{=}c \;\Rightarrow\; b \cdot a{=}c$ cnf(product_commutativity, axiom)
$(a \cdot b{=}c$ and $a \cdot d{=}c) \;\Rightarrow\; b = d$ cnf(product_left_cancellation, axiom)
(divides(a, b) and divides(c, a)) $\Rightarrow$ divides(c, b) cnf(transitivity_of_divides, axiom)
$(a \cdot b{=}c$ and $a \cdot b{=}d) \;\Rightarrow\; d = c$ cnf(well_defined_product, axiom)
divides(a, b) $\Rightarrow$ $a \cdot$ second_divided_by_1st$(a, b){=}b$ cnf(divides_implies_product, axiom)
$a \cdot b{=}c \;\Rightarrow$ divides(a, c) cnf(product_divisible_by_operand, axiom)
(divides(a, b) and $c \cdot c{=}b$ and prime(a)) $\Rightarrow$ divides(a, c) cnf(primes_lemma$_1$, axiom)
prime(a) cnf(a_is_prime, hypothesis)
$b \cdot b{=}d$ cnf(b_squared, hypothesis)
$c \cdot c{=}e$ cnf(c_squared, hypothesis)
$\neg\, a \cdot e{=}d$ cnf(a_times_c_squared_is_not_b_squared, hypothesis)
divides(a, c) $\Rightarrow$ $\neg$ divides(a, b) cnf(prove_there_is_no_common_divisor, negated_conjecture)

NUM018-1.p There is an infinite number of twin prime numbers
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')
finite(twin_prime_numbers) cnf(prove_infinite_number_of_twin_primes, negated_conjecture)

NUM019-1.p Symmetry of equality can be derived
include('Axioms/NUM001-0.ax')
$x{=}x$ cnf(reflexivity, hypothesis)
$(x{=}y$ and $x{=}z) \;\Rightarrow\; y{=}z$ cnf(transitivity, hypothesis)
$\neg$ successor$(a){=}n_0$ cnf(zero_is_the_first_number, hypothesis)
$a{=}$aa cnf(a_equals_aa, hypothesis)
$\neg$ aa$=a$ cnf(prove_b_equals_a, negated_conjecture)

NUM020-1.p a + 1 = successor(a)
include('Axioms/NUM001-0.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$n_1{=}$successor(n_0) cnf(one_succeeds_zero, axiom)
$\neg\, a +$ successor$(n_0){=}$successor(a) cnf(deny_addition_lemma, negated_conjecture)
$\neg$ successor$(a){=}n_0$ cnf(prove_a_contradiction, negated_conjecture)

NUM020^1.p Find N such that N * 3 = 6
include('Axioms/NUM006^0.ax')
$\exists n$: (\$i → \$i) → \$i → \$i: (·@n@three) = six thf(thm, conjecture)

NUM021-1.p If a ≤ b < c, then c cannot divide a
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
include('Axioms/NUM001-2.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
less(b, c) cnf(b_less_than_c, hypothesis)
$\neg$ less(b, a) cnf(b_greater_equal_a, hypothesis)
divides(c, a) cnf(impossible_c_divides_a, negated_conjecture)
$\neg$ successor$(a){=}n_0$ cnf(prove_a_contradiction, negated_conjecture)

NUM021^1.p Find operator o such that 2 o 3 = 5 and 1 o 2 = 3
include('Axioms/NUM006^0.ax')
∃op: ((\$i → \$i) → \$i → \$i) → ((\$i → \$i) → \$i → \$i) → (\$i → \$i) → \$i → \$i: ((op@two@three) = five and (op@1@two) = three) thf(thm, conjecture)

NUM022-1.p Numerator divisble by smaller denominators
If a numerator is divisible by a denominator, then the numerator is divisible by numbers smaller than the denominator.
include('Axioms/NUM001-1.ax')
include('Axioms/NUM001-2.ax')

$(a{=}b$ and $c{=}a) \;\Rightarrow\; c{=}b$ cnf(transitivity, axiom)
$\text{less}(a, b)$ cnf(a_less_than_b, hypothesis)
$\text{divides}(b, d)$ cnf(b_divides_d, hypothesis)
$\neg\,\text{less}(a, d)$ cnf(prove_a_less_than_d, negated_conjecture)

NUM023-1.p Zero is less than all successor numbers
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$\neg\,\text{successor}(a){=}n_0$ cnf(zero_is_first_number, axiom)
$\text{less}(a, \text{successor}(a))$ cnf(numbers_less_than_its_successor, axiom)
$\neg\,\text{less}(n_0, \text{successor}(a))$ cnf(prove_zero_is_less_than_all_successors, negated_conjecture)

NUM024-1.p No number is less than itself
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$a + b{=}c + b \;\Rightarrow\; a{=}c$ cnf(plus_substitution, axiom)
$a + b{=}b + a$ cnf(commutativity_of_plus, axiom)
$\text{less}(a, a)$ cnf(impossible_a_is_less_than_itself, hypothesis)
$\neg\,\text{successor}(a){=}n_0$ cnf(prove_a_contradiction, negated_conjecture)

NUM025-1.p If a<b then not b<a
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$\neg\,\text{successor}(a){=}n_0$ cnf(zero_is_the_first_number, axiom)
$\neg\,\text{less}(a, a)$ cnf(no_number_less_than_itself, axiom)
$\text{less}(a, b)$ cnf(a_less_than_b, hypothesis)
$\text{less}(b, a)$ cnf(prove_b_not_less_than_a, negated_conjecture)

NUM025-2.p If a<b then not b<a
include('Axioms/NUM001-0.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$\text{greater\_or\_equalish}(c, b) \;\Rightarrow\; (\text{greater\_or\_equalish}(a, b)$ or $\text{greater\_or\_equalish}(c, a))$ cnf(transitivity_of_less, axiom)
$\text{successor}(a) + b{=}c \;\Rightarrow\; \neg\,\text{greater\_or\_equalish}(b, c)$ cnf(smaller_number, axiom)
$\text{greater\_or\_equalish}(a, b)$ or $\text{successor}(\text{predecessor\_of\_1st\_minus\_2nd}(b, a)) + a{=}b$ cnf(less_lemma, axiom)
$\neg\,\text{successor}(a){=}n_0$ cnf(zero_is_the_first_number, axiom)
$\text{greater\_or\_equalish}(a, a)$ cnf(no_number_less_than_itself, axiom)
$\neg\,\text{greater\_or\_equalish}(a, b)$ cnf(a_less_than_b, hypothesis)
$\neg\,\text{greater\_or\_equalish}(b, a)$ cnf(prove_b_not_less_than_a, negated_conjecture)

NUM026-1.p Less preserved over multiplication by a number
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \;\Rightarrow\; y{=}x$ cnf(symmetry, axiom)
$(x{=}y$ and $y{=}z) \;\Rightarrow\; x{=}z$ cnf(transitivity, axiom)
$\neg\,\text{successor}(a){=}n_0$ cnf(zero_is_the_first_number, axiom)
$\neg\,\text{less}(a, a)$ cnf(no_number_less_than_itself, axiom)
$\neg\,c{=}n_0$ cnf(c_not$_0$, hypothesis)
$\text{less}(a, b)$ cnf(a_less_than_b, hypothesis)
$\neg\,\text{less}(a \cdot c, b \cdot c)$ cnf(prove_a_times_c_less_b_times_c, negated_conjecture)

NUM027-1.p If a >= b and b*c $\leq$ a*c, then c = 0

include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
$x=x$ cnf(reflexivity, axiom)
$x=y \Rightarrow y=x$ cnf(symmetry, axiom)
$(x=y$ and $y=z) \Rightarrow x=z$ cnf(transitivity, axiom)
$a=b \Rightarrow a \cdot c=b \cdot c$ cnf(equality_preserved_over_times, axiom)
less$(a, b) \Rightarrow \neg a=b$ cnf(not_less_and_equal, axiom)
less(a, b) or $b=a$ or less(b, a) cnf(numbers_either_less_or_equal, axiom)
$\neg$ less(a, a) cnf(number_not_less_than_itself, axiom)
$\neg$ successor$(a)=n_0$ cnf(zero_is_the_first_number, axiom)
less$(a, b) \Rightarrow (c=n_0$ or less$(a \cdot c, b \cdot c))$ cnf(multiply_lemma, axiom)
$\neg$ less(b, a) cnf(b_not_less_than_a, hypothesis)
less$(b \cdot c, a \cdot c)$ cnf(b_times_c_less_than_a_times_c, hypothesis)
$\neg c=n_0$ cnf(prove_c_is$_0$, negated_conjecture)

NUM028-1.p Symmetrization property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
symmetrization_of$(x) = x$ cnf(prove_symmetrization_property1$_1$, negated_conjecture)
symmetrization_of$(x') \neq x'$ cnf(prove_symmetrization_property1$_2$, negated_conjecture)

NUM029-1.p Symmetrization property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
symmetrization_of$(x') = x'$ cnf(prove_symmetrization_property2$_1$, negated_conjecture)
symmetrization_of$(x) \neq x$ cnf(prove_symmetrization_property2$_2$, negated_conjecture)

NUM030-1.p Symmetrization property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
symmetrization_of$(x) = x$ cnf(prove_symmetrization_property3$_1$, negated_conjecture)
$\neg$ subclass(x', x) cnf(prove_symmetrization_property3$_2$, negated_conjecture)

NUM031-1.p Symmetrization property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(x', x) cnf(prove_symmetrization_property4$_1$, negated_conjecture)
symmetrization_of$(x) \neq x$ cnf(prove_symmetrization_property4$_2$, negated_conjecture)

NUM032-1.p Symmetrization property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
symmetrization_of$(x) = x$ cnf(prove_symmetrization_property5$_1$, negated_conjecture)
restrict$(x$, universal_class, universal_class$) \neq x'$ cnf(prove_symmetrization_property5$_2$, negated_conjecture)

NUM033-1.p Symmetrization property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass$(x$, cross_product(universal_class, universal_class)) cnf(prove_symmetrization_property6$_1$, negated_conjecture)
symmetrization_of$(x) = x$ cnf(prove_symmetrization_property6$_2$, negated_conjecture)
$x' \neq x$ cnf(prove_symmetrization_property6$_3$, negated_conjecture)

NUM034-1.p Symmetrization is idempotent
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
symmetrization_of(symmetrization_of$(x)) \neq$ symmetrization_of(x) cnf(prove_idempotency_of_symmetrization$_1$, negated_co

NUM035-1.p Domain equals range of symmetrization
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(symmetrization_of(x)) $\neq$ range_of(symmetrization_of(x))   cnf(prove_domain_equals_range_of_symmetrization$_1$,

NUM036-1.p Symmetrization property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
union(symmetrization_of(x), symmetrization_of(y)) $\neq$ symmetrization_of(union(x, y))   cnf(prove_symmetrization_property

NUM037-1.p Symmetrization property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(restrict(x, universal_class, universal_class), cross_product(domain_of(symmetrization_of(x)), domain_of(symmetrizat

NUM038-1.p Symmetrization property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
restrict(x, universal_class, universal_class) $\neq$ restrict(x, domain_of(symmetrization_of(x)), domain_of(symmetrization_of(x)))

NUM039-1.p Irreflexive class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
irreflexive(x, y)   cnf(prove_irreflexive_class_property1$_1$, negated_conjecture)
ordered_pair(u, u) $\in x$ cnf(prove_irreflexive_class_property1$_2$, negated_conjecture)
$u \in y$ cnf(prove_irreflexive_class_property1$_3$, negated_conjecture)

NUM040-1.p Irreflexive class property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
irreflexive(x, y)   cnf(prove_irreflexive_class_property2$_1$, negated_conjecture)
subclass(z, y)   cnf(prove_irreflexive_class_property2$_2$, negated_conjecture)
$\neg$ irreflexive(x, z)   cnf(prove_irreflexive_class_property2$_3$, negated_conjecture)

NUM041-1.p Irreflexive class property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
irreflexive(x, domain_of(symmetrization_of(x)))   cnf(prove_irreflexive_class_property3$_1$, negated_conjecture)
$\neg$ subclass(x, identity_relation$'$)   cnf(prove_irreflexive_class_property3$_2$, negated_conjecture)

NUM042-1.p Irreflexive class property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(x, identity_relation$'$)   cnf(prove_irreflexive_class_property4$_1$, negated_conjecture)
$\neg$ irreflexive(x, domain_of(symmetrization_of(x)))   cnf(prove_irreflexive_class_property4$_2$, negated_conjecture)

NUM043-1.p Irreflexive class property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
irreflexive(x, domain_of(symmetrization_of(x)))   cnf(prove_irreflexive_class_property5$_1$, negated_conjecture)
ordered_pair(u, u) $\in x$ cnf(prove_irreflexive_class_property5$_2$, negated_conjecture)

NUM044-1.p Irreflexive class property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

irreflexive(x, y) cnf(prove_irreflexive_class_property6_1, negated_conjecture)
restrict(intersection$(x,$ identity_relation$), y, y) \neq$ null_class cnf(prove_irreflexive_class_property6_2, negated_conjecture)

NUM045-1.p Irreflexive class property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
restrict(intersection$(x,$ identity_relation$), y, y) =$ null_class cnf(prove_irreflexive_class_property7_1, negated_conjecture)
$\neg$ irreflexive(x, y) cnf(prove_irreflexive_class_property7_2, negated_conjecture)

NUM046-1.p Connected class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
connected(x, y) cnf(prove_connect_class_property1_1, negated_conjecture)
ordered_pair$(u, v) \in$ cross_product(y, y) cnf(prove_connect_class_property1_2, negated_conjecture)
$\neg$ ordered_pair$(u, v) \in x$ cnf(prove_connect_class_property1_3, negated_conjecture)
$\neg$ ordered_pair$(v, u) \in x$ cnf(prove_connect_class_property1_4, negated_conjecture)
$u \neq v$ cnf(prove_connect_class_property1_5, negated_conjecture)

NUM047-1.p Connected class property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
connected(x, y) cnf(prove_connect_class_property2_1, negated_conjecture)
subclass(z, y) cnf(prove_connect_class_property2_2, negated_conjecture)
$\neg$ connected(x, z) cnf(prove_connect_class_property2_3, negated_conjecture)

NUM048-1.p Connected class property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(cross_product(y, y), identity_relation) cnf(prove_connect_class_property3_1, negated_conjecture)
$\neg$ connected(restrict(x, y, y), domain_of(symmetrization_of(restrict(x, y, y)))) cnf(prove_connect_class_property3_2, negated_

NUM049-1.p Connected class property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(cross_product(y, y), identity_relation) cnf(prove_connect_class_property4_1, negated_conjecture)
$\neg$ connected(x, y) cnf(prove_connect_class_property4_2, negated_conjecture)

NUM050-1.p Connected class property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
connected(x, y) cnf(prove_connect_class_property5_1, negated_conjecture)
$\neg$ connected(restrict(x, y, y), domain_of(symmetrization_of(restrict(x, y, y)))) cnf(prove_connect_class_property5_2, negated_

NUM051-1.p Everything is connected to the null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ connected$(x,$ null_class) cnf(prove_everything_connected_to_null_class$_1$, negated_conjecture)

NUM052-1.p Transitive ordering property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
transitive(xr, y) cnf(prove_transitive_ordering_property1_1, negated_conjecture)
ordered_pair$(u, v) \in$ cross_product(y, y) cnf(prove_transitive_ordering_property1_2, negated_conjecture)
ordered_pair$(u, v) \in$ xr cnf(prove_transitive_ordering_property1_3, negated_conjecture)
ordered_pair$(v, w) \in$ cross_product(y, y) cnf(prove_transitive_ordering_property1_4, negated_conjecture)
ordered_pair$(v, w) \in$ xr cnf(prove_transitive_ordering_property1_5, negated_conjecture)

$\neg$ ordered_pair$(u, w) \in$ xr   cnf(prove_transitive_ordering_property1$_6$, negated_conjecture)

NUM053-1.p Transitive ordering property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
transitive(xr, y)   cnf(prove_transitive_ordering_property2$_1$, negated_conjecture)
subclass(z, y) cnf(prove_transitive_ordering_property2$_2$, negated_conjecture)
$\neg$ transitive(xr, z)   cnf(prove_transitive_ordering_property2$_3$, negated_conjecture)

NUM054-1.p Asymmetric class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
asymmetric(xr, y)   cnf(prove_asymmetric_class_property1$_1$, negated_conjecture)
ordered_pair$(u, v) \in$ cross_product(y, y) cnf(prove_asymmetric_class_property1$_2$, negated_conjecture)
ordered_pair$(u, v) \in$ xr   cnf(prove_asymmetric_class_property1$_3$, negated_conjecture)
ordered_pair$(v, u) \in$ xr   cnf(prove_asymmetric_class_property1$_4$, negated_conjecture)

NUM055-1.p Asymmetric class property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
asymmetric(xr, y)   cnf(prove_asymmetric_class_property2$_1$, negated_conjecture)
subclass(z, y) cnf(prove_asymmetric_class_property2$_2$, negated_conjecture)
$\neg$ asymmetric(xr, z)   cnf(prove_asymmetric_class_property2$_3$, negated_conjecture)

NUM056-1.p Asymmetric class property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
asymmetric(xr, y)   cnf(prove_asymmetric_class_property3$_1$, negated_conjecture)
$\neg$ irreflexive(xr, y)   cnf(prove_asymmetric_class_property3$_2$, negated_conjecture)

NUM057-1.p Segments property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(segment(xr, y, z), y)   cnf(prove_segments_property1$_1$, negated_conjecture)

NUM058-1.p Segments property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg z \in$ universal_class   cnf(prove_segments_property2$_1$, negated_conjecture)
segment(xr, y, z) $\neq$ null_class   cnf(prove_segments_property2$_2$, negated_conjecture)

NUM059-1.p Segments property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$u \in$ segment(xr, y, z)   cnf(prove_segments_property3$_1$, negated_conjecture)
$\neg$ ordered_pair$(u, z) \in$ xr   cnf(prove_segments_property3$_2$, negated_conjecture)

NUM060-1.p Segments property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordered_pair$(u, z) \in$ intersection(xr, cross_product(y, universal_class))   cnf(prove_segments_property4$_1$, negated_conjecture)
$\neg u \in$ segment(xr, y, z)   cnf(prove_segments_property4$_2$, negated_conjecture)

NUM061-1.p Segments property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

$\text{ordered\_pair}(u, z) \in \text{xr}$ cnf(prove_segments_property5$_1$, negated_conjecture)
$u \in y$ cnf(prove_segments_property5$_2$, negated_conjecture)
$z \in \text{universal\_class}$ cnf(prove_segments_property5$_3$, negated_conjecture)
$\neg u \in \text{segment}(\text{xr}, y, z)$ cnf(prove_segments_property5$_4$, negated_conjecture)

NUM062-1.p Segments property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$z \in \text{universal\_class}$ cnf(prove_segments_property6$_1$, negated_conjecture)
$\text{segment}(\text{element\_relation}, y, z) \neq \text{intersection}(y, z)$ cnf(prove_segments_property6$_2$, negated_conjecture)

NUM063-1.p Segments property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$z \in \text{universal\_class}$ cnf(prove_segments_property7$_1$, negated_conjecture)
$\neg \text{segment}(\text{element\_relation}, y, z) \in \text{universal\_class}$ cnf(prove_segments_property7$_2$, negated_conjecture)

NUM064-1.p Least(xr,u) is unique
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_least_is_unique$_1$, negated_conjecture)
$\text{subclass}(u, y)$ cnf(prove_least_is_unique$_2$, negated_conjecture)
$v \in u$ cnf(prove_least_is_unique$_3$, negated_conjecture)
$\text{restrict}(\text{xr}, u, \text{singleton}(v)) = \text{null\_class}$ cnf(prove_least_is_unique$_4$, negated_conjecture)
$\text{least}(\text{xr}, u) \neq v$ cnf(prove_least_is_unique$_5$, negated_conjecture)

NUM065-1.p Well ordering property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_well_ordering_property1$_1$, negated_conjecture)
$\text{subclass}(u, y)$ cnf(prove_well_ordering_property1$_2$, negated_conjecture)
$v \in u$ cnf(prove_well_ordering_property1$_3$, negated_conjecture)
$\neg \text{ordered\_pair}(\text{least}(\text{xr}, u), v) \in \text{xr}$ cnf(prove_well_ordering_property1$_4$, negated_conjecture)
$\text{least}(\text{xr}, u) \neq v$ cnf(prove_well_ordering_property1$_5$, negated_conjecture)

NUM066-1.p Corollary to well ordering property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{element\_relation}, y)$ cnf(prove_corollary_to_well_ordering_property1$_1$, negated_conjecture)
$\text{subclass}(u, y)$ cnf(prove_corollary_to_well_ordering_property1$_2$, negated_conjecture)
$v \in u$ cnf(prove_corollary_to_well_ordering_property1$_3$, negated_conjecture)
$v \in \text{least}(\text{element\_relation}, u)$ cnf(prove_corollary_to_well_ordering_property1$_4$, negated_conjecture)

NUM067-1.p Well ordering property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_well_ordering_property2$_1$, negated_conjecture)
$\text{ordered\_pair}(u, v) \in \text{cross\_product}(y, y)$ cnf(prove_well_ordering_property2$_2$, negated_conjecture)
$\text{ordered\_pair}(u, v) \in \text{xr}$ cnf(prove_well_ordering_property2$_3$, negated_conjecture)
$\text{ordered\_pair}(v, u) \in \text{xr}$ cnf(prove_well_ordering_property2$_4$, negated_conjecture)

NUM068-1.p Well ordering property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_well_ordering_property3$_1$, negated_conjecture)
$u \in \text{cross\_product}(y, y)$ cnf(prove_well_ordering_property3$_2$, negated_conjecture)

$u \in \text{xr}$ cnf(prove_well_ordering_property3$_3$, negated_conjecture)
$u \in \text{xr}'$ cnf(prove_well_ordering_property3$_4$, negated_conjecture)

NUM069-1.p Corollary to well ordering property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(element_relation, y) cnf(prove_corollary_to_well_ordering_property3$_1$, negated_conjecture)
ordered_pair(u, v) $\in$ cross_product(y, y) cnf(prove_corollary_to_well_ordering_property3$_2$, negated_conjecture)
$u \in v$ cnf(prove_corollary_to_well_ordering_property3$_3$, negated_conjecture)
$v \in u$ cnf(prove_corollary_to_well_ordering_property3$_4$, negated_conjecture)

NUM070-1.p A well-order is asymmetric
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y) cnf(prove_well_ordering_is_asymmetric$_1$, negated_conjecture)
$\neg$ asymmetric(xr, y) cnf(prove_well_ordering_is_asymmetric$_2$, negated_conjecture)

NUM071-1.p Well ordering is irreflexive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y) cnf(prove_well_ordering_is_irreflexive$_1$, negated_conjecture)
ordered_pair(u, u) $\in$ xr cnf(prove_well_ordering_is_irreflexive$_2$, negated_conjecture)
$u \in y$ cnf(prove_well_ordering_is_irreflexive$_3$, negated_conjecture)

NUM072-1.p Well ordering property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y) cnf(prove_well_ordering_property4$_1$, negated_conjecture)
ordered_pair(u, v) $\in$ cross_product(y, y) cnf(prove_well_ordering_property4$_2$, negated_conjecture)
ordered_pair(u, v) $\in$ xr cnf(prove_well_ordering_property4$_3$, negated_conjecture)
ordered_pair(v, w) $\in$ cross_product(y, y) cnf(prove_well_ordering_property4$_4$, negated_conjecture)
ordered_pair(v, w) $\in$ xr cnf(prove_well_ordering_property4$_5$, negated_conjecture)
$\neg$ ordered_pair(u, w) $\in$ xr cnf(prove_well_ordering_property4$_6$, negated_conjecture)

NUM073-1.p Corollary to well ordering property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(element_relation, y) cnf(prove_corollary_to_well_ordering_property4$_1$, negated_conjecture)
ordered_pair(u, v) $\in$ cross_product(y, y) cnf(prove_corollary_to_well_ordering_property4$_2$, negated_conjecture)
$u \in v$ cnf(prove_corollary_to_well_ordering_property4$_3$, negated_conjecture)
ordered_pair(v, w) $\in$ cross_product(y, y) cnf(prove_corollary_to_well_ordering_property4$_4$, negated_conjecture)
$v \in w$ cnf(prove_corollary_to_well_ordering_property4$_5$, negated_conjecture)
$\neg u \in w$ cnf(prove_corollary_to_well_ordering_property4$_6$, negated_conjecture)

NUM074-1.p Well ordering property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y) cnf(prove_well_ordering_property5$_1$, negated_conjecture)
$\neg$ transitive(xr, y) cnf(prove_well_ordering_property5$_2$, negated_conjecture)

NUM075-1.p Well ordering property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y) cnf(prove_well_ordering_property6$_1$, negated_conjecture)
subclass(z, y) cnf(prove_well_ordering_property6$_2$, negated_conjecture)
$\neg$ well_ordering(xr, z) cnf(prove_well_ordering_property6$_3$, negated_conjecture)

NUM076-1.p Well ordering property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_well_ordering_property7$_1$, negated_conjecture)
$u \in y$ cnf(prove_well_ordering_property7$_2$, negated_conjecture)
$u \in \text{segment}(\text{xr}, y, u)$ cnf(prove_well_ordering_property7$_3$, negated_conjecture)

NUM077-1.p Corollary 1 to well ordering property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_corollary_1_to_well_ordering_property7$_1$, negated_conjecture)
$u \in y$ cnf(prove_corollary_1_to_well_ordering_property7$_2$, negated_conjecture)
$\text{segment}(\text{xr}, y, u) = y$ cnf(prove_corollary_1_to_well_ordering_property7$_3$, negated_conjecture)

NUM078-1.p Corollary 2 to well ordering property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{element\_relation}, y)$ cnf(prove_corollary_2_to_well_ordering_property7$_1$, negated_conjecture)
$u \in y$ cnf(prove_corollary_2_to_well_ordering_property7$_2$, negated_conjecture)
$u \in \text{intersection}(u, y)$ cnf(prove_corollary_2_to_well_ordering_property7$_3$, negated_conjecture)

NUM079-1.p Well ordering property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{intersection}(y, \text{ordinal\_numbers}) \neq \text{null\_class}$ cnf(prove_well_ordering_property8$_1$, negated_conjecture)
$\neg\, \text{least}(\text{element\_relation}, \text{intersection}(y, \text{ordinal\_numbers})) \in \text{intersection}(y, \text{ordinal\_numbers})$ cnf(prove_well_ordering_prop

NUM080-1.p Well ordering property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{intersection}(y, \text{intersection}(\text{ordinal\_numbers}, \text{least}(\text{element\_relation}, \text{intersection}(y, \text{ordinal\_numbers})))) \neq \text{null\_class}$ cnf(pr

NUM081-1.p Corollary to well ordering property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{intersection}(y, \text{least}(\text{element\_relation}, \text{intersection}(y, \text{ordinal\_numbers}))) \neq \text{null\_class}$ cnf(prove_corollary_to_well_ordering_
$\text{intersection}(y, \text{ordinal\_numbers}) \neq \text{null\_class}$ cnf(prove_corollary_to_well_ordering_property9$_2$, negated_conjecture)

NUM082-1.p Uniqueness of the least element of a non-empty subset
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{well\_ordering}(\text{xr}, y)$ cnf(prove_least_is_unique_in_non_empty_set$_1$, negated_conjecture)
$\text{subclass}(u, y)$ cnf(prove_least_is_unique_in_non_empty_set$_2$, negated_conjecture)
$v \in u$ cnf(prove_least_is_unique_in_non_empty_set$_3$, negated_conjecture)
$\text{segment}(\text{xr}, u, v) = \text{null\_class}$ cnf(prove_least_is_unique_in_non_empty_set$_4$, negated_conjecture)
$\text{least}(\text{xr}, u) \neq v$ cnf(prove_least_is_unique_in_non_empty_set$_5$, negated_conjecture)

NUM083-1.p Transitive class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(\text{sum\_class}(x), x)$ cnf(prove_transitive_class_property1$_1$, negated_conjecture)
$u \in x$ cnf(prove_transitive_class_property1$_2$, negated_conjecture)
$\neg\, \text{subclass}(u, x)$ cnf(prove_transitive_class_property1$_3$, negated_conjecture)

NUM084-1.p Alternate transitive class definition, part 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(sum_class(x), w)   cnf(prove_alternate_trasitive_class_defn1$_1$, negated_conjecture)
$\neg$ subclass(x, power_class(w))   cnf(prove_alternate_trasitive_class_defn1$_2$, negated_conjecture)

NUM085-1.p Alternate transitive class definition, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(x, power_class(w))   cnf(prove_alternate_trasitive_class_defn2$_1$, negated_conjecture)
$\neg$ subclass(sum_class(x), w)   cnf(prove_alternate_trasitive_class_defn2$_2$, negated_conjecture)

NUM086-1.p Transitive class property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(sum_class(x), x)   cnf(prove_transitive_class_property2$_1$, negated_conjecture)
subclass(sum_class(y), y)   cnf(prove_transitive_class_property2$_2$, negated_conjecture)
$\neg$ subclass(sum_class(union(x, y)), union(x, y))   cnf(prove_transitive_class_property2$_3$, negated_conjecture)

NUM087-1.p Transitive class property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(sum_class(x), x)   cnf(prove_transitive_class_property3$_1$, negated_conjecture)
subclass(sum_class(y), y)   cnf(prove_transitive_class_property3$_2$, negated_conjecture)
$\neg$ subclass(sum_class(intersection(x, y)), intersection(x, y))   cnf(prove_transitive_class_property3$_3$, negated_conjecture)

NUM088-1.p Transitive class property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(sum_class(y), y)   cnf(prove_transitive_class_property4$_1$, negated_conjecture)
$z \in y$ cnf(prove_transitive_class_property4$_2$, negated_conjecture)
segment(element_relation, y, z) $\neq z$ cnf(prove_transitive_class_property4$_3$, negated_conjecture)

NUM089-1.p Sections property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
section(xr, y, z)   cnf(prove_sections_property1$_1$, negated_conjecture)
ordered_pair(u, v) $\in$ xr   cnf(prove_sections_property1$_2$, negated_conjecture)
$u \in z$ cnf(prove_sections_property1$_3$, negated_conjecture)
$v \in y$ cnf(prove_sections_property1$_4$, negated_conjecture)
$\neg\, u \in y$ cnf(prove_sections_property1$_5$, negated_conjecture)

NUM090-1.p Corollary to sections property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
section(element_relation, y, z)   cnf(prove_corollary_to_sections_property1$_1$, negated_conjecture)
$u \in$ intersection(v, z)   cnf(prove_corollary_to_sections_property1$_2$, negated_conjecture)
$v \in y$ cnf(prove_corollary_to_sections_property1$_3$, negated_conjecture)
$\neg\, u \in y$ cnf(prove_corollary_to_sections_property1$_4$, negated_conjecture)

NUM091-1.p Sections property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
transitive(xr, y)   cnf(prove_sections_property2$_1$, negated_conjecture)
$u \in y$ cnf(prove_sections_property2$_2$, negated_conjecture)
$\neg$ section(xr, segment(xr, y, u), y)   cnf(prove_sections_property2$_3$, negated_conjecture)

NUM092-1.p Corollary 1 to sections property 2

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y)   cnf(prove_corollary_1_to_sections_property2_1, negated_conjecture)
$u \in y$ cnf(prove_corollary_1_to_sections_property2_2, negated_conjecture)
$\neg$ section(xr, segment(xr, y, u), y)   cnf(prove_corollary_1_to_sections_property2_3, negated_conjecture)

NUM093-1.p Corollary 2 to sections property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(element_relation, y)   cnf(prove_corollary_2_to_sections_property2_1, negated_conjecture)
$u \in y$ cnf(prove_corollary_2_to_sections_property2_2, negated_conjecture)
$\neg$ section(element_relation, intersection(y, u), y)   cnf(prove_corollary_2_to_sections_property2_3, negated_conjecture)

NUM094-1.p Sections property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y)   cnf(prove_sections_property3_1, negated_conjecture)
section(xr, w, y)   cnf(prove_sections_property3_2, negated_conjecture)
$\neg$ least(xr, intersection(w', y)) $\in y$ cnf(prove_sections_property3_3, negated_conjecture)
$y \neq w$ cnf(prove_sections_property3_4, negated_conjecture)

NUM095-1.p Sections property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y)   cnf(prove_sections_property4_1, negated_conjecture)
section(xr, w, y)   cnf(prove_sections_property4_2, negated_conjecture)
$\neg$ least(xr, intersection(w', y)) $\in w'$ cnf(prove_sections_property4_3, negated_conjecture)
$y \neq w$ cnf(prove_sections_property4_4, negated_conjecture)

NUM096-1.p Sections property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(xr, y)   cnf(prove_sections_property5_1, negated_conjecture)
section(xr, w, y)   cnf(prove_sections_property5_2, negated_conjecture)
segment(xr, y, least(xr, intersection(w', y))) $\neq w$ cnf(prove_sections_property5_3, negated_conjecture)
$y \neq w$ cnf(prove_sections_property5_4, negated_conjecture)

NUM097-1.p Corollary to sections property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
well_ordering(element_relation, y)   cnf(prove_corollary_to_sections_property5_1, negated_conjecture)
section(element_relation, w, y)   cnf(prove_corollary_to_sections_property5_2, negated_conjecture)
intersection(y, least(element_relation, intersection(w', y))) $\neq w$ cnf(prove_corollary_to_sections_property5_3, negated_conjec
$y \neq w$ cnf(prove_corollary_to_sections_property5_4, negated_conjecture)

NUM098-1.p Ordinal property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ null_class $\in$ ordinal_numbers   cnf(prove_ordinal_property1_1, negated_conjecture)

NUM099-1.p Corollary to ordinal property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(singleton(null_class), ordinal_numbers)   cnf(prove_corollary_to_ordinal_property1_1, negated_conjecture)

NUM100-1.p Ordinal property 2

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\,\text{singleton}(\text{null\_class}) \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property2$_1$, negated_conjecture)

NUM101-1.p Ordinal property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property3$_1$, negated_conjecture)
$\text{subclass}(y, x)$ cnf(prove_ordinal_property3$_2$, negated_conjecture)
$\text{subclass}(\text{sum\_class}(y), y)$ cnf(prove_ordinal_property3$_3$, negated_conjecture)
$\neg\, y \in x$ cnf(prove_ordinal_property3$_4$, negated_conjecture)
$y \neq x$ cnf(prove_ordinal_property3$_5$, negated_conjecture)

NUM102-1.p Ordinal property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(y, \text{ordinal\_numbers})$ cnf(prove_ordinal_property4$_1$, negated_conjecture)
$\text{subclass}(\text{sum\_class}(y), y)$ cnf(prove_ordinal_property4$_2$, negated_conjecture)
$\neg\, y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property4$_3$, negated_conjecture)
$y \neq \text{ordinal\_numbers}$ cnf(prove_ordinal_property4$_4$, negated_conjecture)

NUM103-1.p Corollary to ordinal property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_corollary_to__ordinal_property4$_1$, negated_conjecture)
$\text{subclass}(y, x)$ cnf(prove_corollary_to__ordinal_property4$_2$, negated_conjecture)
$\text{subclass}(\text{sum\_class}(y), y)$ cnf(prove_corollary_to__ordinal_property4$_3$, negated_conjecture)
$\neg\, y \in \text{successor}(x)$ cnf(prove_corollary_to__ordinal_property4$_4$, negated_conjecture)

NUM104-1.p Ordinal property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property5$_1$, negated_conjecture)
$y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property5$_2$, negated_conjecture)
$\text{subclass}(y, x)$ cnf(prove_ordinal_property5$_3$, negated_conjecture)
$\neg\, y \in x$ cnf(prove_ordinal_property5$_4$, negated_conjecture)
$y \neq x$ cnf(prove_ordinal_property5$_5$, negated_conjecture)

NUM105-1.p Ordinal property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property6$_1$, negated_conjecture)
$\neg\,\text{subclass}(y, \text{ordinal\_numbers})$ cnf(prove_ordinal_property6$_2$, negated_conjecture)

NUM106-1.p Ordinal property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property7$_1$, negated_conjecture)
$y \in x$ cnf(prove_ordinal_property7$_2$, negated_conjecture)
$\neg\,\text{subclass}(y, x)$ cnf(prove_ordinal_property7$_3$, negated_conjecture)

NUM107-1.p Ordinal property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property8$_1$, negated_conjecture)

$\text{intersection}(\text{ordinal\_numbers}, y) \neq y$ cnf(prove_ordinal_property8$_2$, negated_conjecture)

NUM108-1.p Ordinal property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property9$_1$, negated_conjecture)
$y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property9$_2$, negated_conjecture)
$\text{intersection}(x, y) \neq x$ cnf(prove_ordinal_property9$_3$, negated_conjecture)
$x \neq y$ cnf(prove_ordinal_property9$_4$, negated_conjecture)
$\text{intersection}(x, y) \neq y$ cnf(prove_ordinal_property9$_5$, negated_conjecture)

NUM109-1.p Ordinal property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property10$_1$, negated_conjecture)
$y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property10$_2$, negated_conjecture)
$\neg\, x \in y$ cnf(prove_ordinal_property10$_3$, negated_conjecture)
$x \neq y$ cnf(prove_ordinal_property10$_4$, negated_conjecture)
$\neg\, y \in x$ cnf(prove_ordinal_property10$_5$, negated_conjecture)

NUM110-1.p Corollary to ordinal property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{connected}(\text{element\_relation}, \text{ordinal\_numbers})$ cnf(prove_corollary_to_ordinal_property10$_1$, negated_conjecture)

NUM111-1.p Ordinal property 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property11$_1$, negated_conjecture)
$y \in x$ cnf(prove_ordinal_property11$_2$, negated_conjecture)
$\neg\, y \in \text{ordinal\_numbers}$ cnf(prove_ordinal_property11$_3$, negated_conjecture)

NUM112-1.p Ordinal property 12
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{well\_ordering}(\text{element\_relation}, \text{ordinal\_numbers})$ cnf(prove_ordinal_property12$_1$, negated_conjecture)

NUM113-1.p Ordinal property 13
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{ordinal\_class}(x) \Rightarrow \text{well\_ordering}(\text{element\_relation}, x)$ cnf(ordinal_class$_1$, axiom)
$\text{ordinal\_class}(x) \Rightarrow \text{subclass}(\text{sum\_class}(x), x)$ cnf(ordinal_class$_2$, axiom)
$(\text{well\_ordering}(\text{element\_relation}, x) \text{ and } \text{subclass}(\text{sum\_class}(x), x)) \Rightarrow \text{ordinal\_class}(x)$ cnf(ordinal_class$_3$, axiom)
$\neg\, \text{ordinal\_class}(\text{ordinal\_numbers})$ cnf(prove_ordinal_property13$_1$, negated_conjecture)

NUM114-1.p Corollary to ordinal property 13
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_corollary_to_ordinal_property13$_1$, negated_conjecture)
$\neg\, \text{subclass}(x, \text{ordinal\_numbers})$ cnf(prove_corollary_to_ordinal_property13$_2$, negated_conjecture)

NUM115-1.p The class of ordinals is not a set.
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{ordinal\_numbers} \in x$ cnf(prove_class_of_ordinals_is_not_set$_1$, negated_conjecture)

NUM116-1.p Corollary to the class of ordinals is not set

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
successor(ordinal_numbers) $\neq$ ordinal_numbers   cnf(prove_corollary_to_class_of_ordinals_is_not_set$_1$, negated_conjecture)

NUM117-1.p Corollary to ordinal class and numbers
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordinal_class(x) $\Rightarrow$ well_ordering(element_relation, x)   cnf(ordinal_class$_1$, axiom)
ordinal_class(x) $\Rightarrow$ subclass(sum_class(x), x)   cnf(ordinal_class$_2$, axiom)
(well_ordering(element_relation, x) and subclass(sum_class(x), x)) $\Rightarrow$ ordinal_class(x)   cnf(ordinal_class$_3$, axiom)
ordinal_class(x)   cnf(prove_corollary_to_ordinal_class_and_numbers$_1$, negated_conjecture)
$\neg$ subclass(x, ordinal_numbers)   cnf(prove_corollary_to_ordinal_class_and_numbers$_2$, negated_conjecture)

NUM118-1.p Ordinal property 14
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class   cnf(prove_ordinal_property14$_1$, negated_conjecture)
subclass(x, ordinal_numbers)   cnf(prove_ordinal_property14$_2$, negated_conjecture)
subclass(sum_class(x), x)   cnf(prove_ordinal_property14$_3$, negated_conjecture)
$\neg\, x \in$ ordinal_numbers   cnf(prove_ordinal_property14$_4$, negated_conjecture)

NUM119-1.p Corollary to transitive class property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$u \in$ ordinal_numbers   cnf(prove_corollary_to_transitive_class_property4$_1$, negated_conjecture)
segment(element_relation, ordinal_numbers, u) $\neq u$ cnf(prove_corollary_to_transitive_class_property4$_2$, negated_conjecture)

NUM120-1.p Transfinite induction, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ least(element_relation, intersection(y', ordinal_numbers)) $\in$ ordinal_numbers   cnf(prove_transfinite_induction1$_1$, negated_c
$\neg$ subclass(ordinal_numbers, y)   cnf(prove_transfinite_induction1$_2$, negated_conjecture)

NUM121-1.p Transfinite induction, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(least(element_relation, intersection(y', ordinal_numbers)), y)   cnf(prove_transfinite_induction2$_1$, negated_conject
$\neg$ subclass(ordinal_numbers, y)   cnf(prove_transfinite_induction2$_2$, negated_conjecture)

NUM122-1.p Transfinite induction, part 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
least(element_relation, intersection(y', ordinal_numbers)) $\in y$ cnf(prove_transfinite_induction3$_1$, negated_conjecture)
$\neg$ subclass(ordinal_numbers, y)   cnf(prove_transfinite_induction3$_2$, negated_conjecture)

NUM123-1.p Alternate transfinite induction 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ least(element_relation, intersection(y', ordinal_numbers)) $\in y'$ cnf(prove_alternate_transfinite_induction3$_1$, negated_conje
$\neg$ subclass(ordinal_numbers, y)   cnf(prove_alternate_transfinite_induction3$_2$, negated_conjecture)

NUM124-1.p Condensed statement of transfinite induction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(intersection(power_class(x), ordinal_numbers), x)   cnf(prove_condensed_statement_of_transfinite_induction$_1$, negat

$\neg$ subclass(ordinal_numbers, x)   cnf(prove_condensed_statement_of_transfinite_induction$_2$, negated_conjecture)

NUM125-1.p Complete induction upto omega
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(intersection(power_class(x), omega), x)   cnf(prove_complete_induction_upto_omega$_1$, negated_conjecture)
$\neg$ subclass(omega, x)   cnf(prove_complete_induction_upto_omega$_2$, negated_conjecture)

NUM126-1.p Alternate 1 for transfinite induction, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ least(element_relation, intersection(z, ordinal_numbers)) $\in$ ordinal_numbers   cnf(prove_alternate_1_transfinite_induction1
intersection(z, ordinal_numbers) $\neq$ null_class   cnf(prove_alternate_1_transfinite_induction1$_2$, negated_conjecture)

NUM127-1.p Alternate 1 for transfinite induction, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(least(element_relation, intersection(z, ordinal_numbers)), z')   cnf(prove_alternate_1_transfinite_induction2$_1$, nega
intersection(z, ordinal_numbers) $\neq$ null_class   cnf(prove_alternate_1_transfinite_induction2$_2$, negated_conjecture)

NUM128-1.p Alternate 1 for transfinite induction, part 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
least(element_relation, intersection(z, ordinal_numbers)) $\in z'$ cnf(prove_alternate_1_transfinite_induction3$_1$, negated_conjec
intersection(z, ordinal_numbers) $\neq$ null_class   cnf(prove_alternate_1_transfinite_induction3$_2$, negated_conjecture)

NUM129-1.p Alternate 2 for transfinite induction, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(y, ordinal_numbers)   cnf(prove_alternate_2_transfinite_induction1$_1$, negated_conjecture)
$\neg$ least(element_relation, y) $\in$ ordinal_numbers   cnf(prove_alternate_2_transfinite_induction1$_2$, negated_conjecture)
$y \neq$ null_class   cnf(prove_alternate_2_transfinite_induction1$_3$, negated_conjecture)

NUM130-1.p Alternate 2 for transfinite induction, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(y, ordinal_numbers)   cnf(prove_alternate_2_transfinite_induction2$_1$, negated_conjecture)
$\neg$ subclass(least(element_relation, y), y')   cnf(prove_alternate_2_transfinite_induction2$_2$, negated_conjecture)
$y \neq$ null_class   cnf(prove_alternate_2_transfinite_induction2$_3$, negated_conjecture)

NUM131-1.p Alternate 2 for transfinite induction, part 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(y, ordinal_numbers)   cnf(prove_alternate_2_transfinite_induction3$_1$, negated_conjecture)
$\neg$ least(element_relation, y) $\in y$ cnf(prove_alternate_2_transfinite_induction3$_2$, negated_conjecture)
$y \neq$ null_class   cnf(prove_alternate_2_transfinite_induction3$_3$, negated_conjecture)

NUM132-1.p Union of successor relation ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers   cnf(prove_union_of_successor_ordinal$_1$, negated_conjecture)
sum_class(successor(x)) $\neq x$ cnf(prove_union_of_successor_ordinal$_2$, negated_conjecture)

NUM133-1.p Corollary to union of successor ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

$x \in$ ordinal_numbers cnf(prove_corollary_to_union_of_successor_ordinal$_1$, negated_conjecture)
$y \in$ ordinal_numbers cnf(prove_corollary_to_union_of_successor_ordinal$_2$, negated_conjecture)
successor(x) = successor(y) cnf(prove_corollary_to_union_of_successor_ordinal$_3$, negated_conjecture)
$x \neq y$ cnf(prove_corollary_to_union_of_successor_ordinal$_4$, negated_conjecture)

NUM134-1.p Successor relation of an ordinal is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_successor_of_ordinal$_1$, negated_conjecture)
$\neg$ successor(x) $\in$ ordinal_numbers cnf(prove_successor_of_ordinal$_2$, negated_conjecture)

NUM135-1.p The null class is the smallest ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
least(element_relation, ordinal_numbers) $\neq$ null_class cnf(prove_null_class_is_least_ordinal$_1$, negated_conjecture)

NUM136-1.p Transitivity of ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_transitivity_of_ordinals$_1$, negated_conjecture)
$y \in x$ cnf(prove_transitivity_of_ordinals$_2$, negated_conjecture)
$z \in y$ cnf(prove_transitivity_of_ordinals$_3$, negated_conjecture)
$\neg z \in x$ cnf(prove_transitivity_of_ordinals$_4$, negated_conjecture)

NUM137-1.p Condition 1 for complete induction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(intersection(power_class(x), z), x) cnf(prove_complete_induction1$_1$, negated_conjecture)
subclass(y, x) cnf(prove_complete_induction1$_2$, negated_conjecture)
$y \in z$ cnf(prove_complete_induction1$_3$, negated_conjecture)
$\neg y \in x$ cnf(prove_complete_induction1$_4$, negated_conjecture)

NUM138-1.p Condition 2 for complete induction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(not_subclass_element(intersection(power_class(x), z), x), x) cnf(prove_complete_induction2$_1$, negated_conjecture
$\neg$ subclass(intersection(power_class(x), z), x) cnf(prove_complete_induction2$_2$, negated_conjecture)

NUM139-1.p Condition 3 for complete induction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ not_subclass_element(intersection(power_class(x), z), x) $\in z$ cnf(prove_complete_induction3$_1$, negated_conjecture)
$\neg$ subclass(intersection(power_class(x), z), x) cnf(prove_complete_induction3$_2$, negated_conjecture)

NUM140-1.p The successor of a set is a set, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class cnf(prove_successor_of_set_is_set1$_1$, negated_conjecture)
$\neg$ successor(x) $\in$ universal_class cnf(prove_successor_of_set_is_set1$_2$, negated_conjecture)

NUM141-1.p The successor of a set is a set, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class cnf(prove_successor_of_set_is_set2$_1$, negated_conjecture)
$\neg x \in$ successor(x) cnf(prove_successor_of_set_is_set2$_2$, negated_conjecture)

NUM142-1.p The successor of a set is a set, part 3

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(x, successor(x))   cnf(prove_successor_of_set_is_set3$_1$, negated_conjecture)

NUM143-1.p Corollary to the successor of a set being a set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
intersection(successor(x), x) $\neq x$ cnf(prove_corollary_to_successor_of_set_is_set$_1$, negated_conjecture)

NUM144-1.p The successor of a proper class is a class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, x \in$ universal_class   cnf(prove_successor_of_proper_class_is_class$_1$, negated_conjecture)
successor(x) $\neq x$ cnf(prove_successor_of_proper_class_is_class$_2$, negated_conjecture)

NUM145-1.p Corollary to the successor of a proper class being a class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
successor(x) $\in$ universal_class   cnf(prove_corollary$_1$, negated_conjecture)
$\neg\, x \in$ successor(x)   cnf(prove_corollary$_2$, negated_conjecture)

NUM146-1.p The successor of a transitive set is transitive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(sum_class(x), x)   cnf(prove_successor_or_transitive_set_is_set$_1$, negated_conjecture)
$\neg$ subclass(sum_class(successor(x)), successor(x))   cnf(prove_successor_or_transitive_set_is_set$_2$, negated_conjecture)

NUM147-1.p The successor of an ordinal is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers   cnf(prove_successor_of_ordinal_is_ordinal$_1$, negated_conjecture)
$\neg$ successor(x) $\in$ ordinal_numbers   cnf(prove_successor_of_ordinal_is_ordinal$_2$, negated_conjecture)

NUM148-1.p The predecessor of an ordinal is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
successor(x) $\in$ ordinal_numbers   cnf(prove_predcessor_of_ordinal_is_ordinal$_1$, negated_conjecture)
$\neg\, x \in$ ordinal_numbers   cnf(prove_predcessor_of_ordinal_is_ordinal$_2$, negated_conjecture)

NUM149-1.p Successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(image(successor_relation, ordinal_numbers), ordinal_numbers)   cnf(prove_successor_property1$_1$, negated_conjectu

NUM150-1.p Corollary 1 to successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(omega, ordinal_numbers)   cnf(prove_corollary_1_to_successor_property1$_1$, negated_conjecture)

NUM151-1.p Corollary 2 to successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ image(successor_relation, ordinal_numbers)   cnf(prove_corollary_2_to_successor_property1$_1$, negated_conjecture)
successor(dom(successor_relation)) $\neq x$ cnf(prove_corollary_2_to_successor_property1$_2$, negated_conjecture)

NUM152-1.p Corollary 3 to successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{image}(\text{successor\_relation}, \text{ordinal\_numbers})$ cnf(prove_corollary_3_to_successor_property1$_1$, negated_conjecture)
$\neg\, x \in \text{ordinal\_numbers}$ cnf(prove_corollary_3_to_successor_property1$_2$, negated_conjecture)

NUM153-1.p Corollary 4 to successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{image}(\text{successor\_relation}, \text{ordinal\_numbers})$ cnf(prove_corollary_4_to_successor_property1$_1$, negated_conjecture)
$x = \text{null\_class}$ cnf(prove_corollary_4_to_successor_property1$_2$, negated_conjecture)

NUM154-1.p Corollary 5 to successor property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{successor}(x) = \text{null\_class}$ cnf(prove_corollary_5_to_successor_property1$_1$, negated_conjecture)

NUM155-1.p There is no ordinal between x and x + 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_no_ordinal_between$_1$, negated_conjecture)
$y \in \text{ordinal\_numbers}$ cnf(prove_no_ordinal_between$_2$, negated_conjecture)
$x \in y$ cnf(prove_no_ordinal_between$_3$, negated_conjecture)
$y \in \text{successor}(x)$ cnf(prove_no_ordinal_between$_4$, negated_conjecture)

NUM156-1.p Membership condition 1 for kind 1 ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{null\_class} \in \text{kind\_1\_ordinals}$ cnf(prove_null_class_is_kind_1$_1$, negated_conjecture)

NUM157-1.p Membership condition 2 for kind 1 ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_successor_is_kind_1$_1$, negated_conjecture)
$\neg\, \text{successor}(x) \in \text{kind\_1\_ordinals}$ cnf(prove_successor_is_kind_1$_2$, negated_conjecture)

NUM158-1.p Membership condition 3 for kind 1 ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{kind\_1\_ordinals}$ cnf(prove_kind_1_ordinal$_1$, negated_conjecture)
$x \neq \text{null\_class}$ cnf(prove_kind_1_ordinal$_2$, negated_conjecture)
$\text{successor}(\text{dom}(\text{successor\_relation})) \neq x$ cnf(prove_kind_1_ordinal$_3$, negated_conjecture)

NUM159-1.p Membership condition 4 for kind 1 ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{subclass}(\text{image}(\text{successor\_relation}, \text{ordinal\_numbers}), \text{kind\_1\_ordinals})$ cnf(prove_corollary_to_kind_1_ordinal$_1$, negated_co

NUM160-1.p Kind 1 ordinals is a class of ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{subclass}(\text{kind\_1\_ordinals}, \text{ordinal\_numbers})$ cnf(prove_kind_1_ordinals_are_ordinals$_1$, negated_conjecture)

NUM161-1.p Corollary to kind 1 ordinals being a class of ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

include('Axioms/NUM004-0.ax')
$\text{intersection}(\text{ordinal\_numbers}, \text{kind\_1\_ordinals}) \neq \text{kind\_1\_ordinals}$ cnf(prove_corollary_to_kind_1_ordinals_are_ordinals$_1$, neg

NUM162-1.p Successor property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{null\_class} \in \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})$ cnf(prove_successor_property2$_1$, negated_conjectu

NUM163-1.p Inductive is closed under union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, x)), x)$ cnf(prove_inductive_closed_under_union$_1$, negated_co
$\text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, x)), y)$ cnf(prove_inductive_closed_under_union$_2$, negated_co
$\neg\, \text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, \text{union}(x, y))), \text{union}(x, y))$ cnf(prove_inductive_closed_und

NUM164-1.p Inductive is closed under intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, x)), x)$ cnf(prove_inductive_closed_under_intersection$_1$, nega
$\text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, x)), y)$ cnf(prove_inductive_closed_under_intersection$_2$, nega
$\neg\, \text{subclass}(\text{union}(\text{singleton}(\text{null\_class}), \text{image}(\text{successor\_relation}, \text{intersection}(x, y))), \text{intersection}(x, y))$ cnf(prove_inductive

NUM165-1.p Corollary to omega definition, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{omega}$ cnf(prove_corollary_to_omega_defn1$_1$, negated_conjecture)
$\neg\, \text{successor}(x) \in \text{omega}$ cnf(prove_corollary_to_omega_defn1$_2$, negated_conjecture)

NUM166-1.p Corollary to omega definition, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{successor}(\text{null\_class}) \in \text{omega}$ cnf(prove_corollary_to_omega_defn2$_1$, negated_conjecture)

NUM167-1.p Successor property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{subclass}(\text{image}(\text{successor\_relation}, \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})), \text{intersection}(\text{power\_class}(\text{kinc}$

NUM168-1.p Corollary to successor property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \text{subclass}(\text{omega}, \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals}))$ cnf(prove_corollary_to_successor_property3$_1$

NUM169-1.p Successor property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{successor}(x) \in \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})$ cnf(prove_successor_property4$_1$, negated_conject
$\neg\, x \in \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})$ cnf(prove_successor_property4$_2$, negated_conjecture)

NUM170-1.p Successor property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{successor}(x) = y$ cnf(prove_successor_property5$_1$, negated_conjecture)
$y \in \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})$ cnf(prove_successor_property5$_2$, negated_conjecture)
$\neg\, x \in \text{intersection}(\text{power\_class}(\text{kind\_1\_ordinals}), \text{kind\_1\_ordinals})$ cnf(prove_successor_property5$_3$, negated_conjecture)

NUM171-1.p Successor property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(image(successor_relation$'$, intersection(power_class(kind_1_ordinals), kind_1_ordinals)), intersection(power_class(kin

NUM172-1.p The successor relation of a set is different from the set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class   cnf(prove_successor_is_different$_1$, negated_conjecture)
successor(x) $= x$ cnf(prove_successor_is_different$_2$, negated_conjecture)

NUM173-1.p Successor property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in y$ cnf(prove_successor_property7$_1$, negated_conjecture)
$\neg$ successor(x) $\in$ image(successor_relation, y)   cnf(prove_successor_property7$_2$, negated_conjecture)

NUM174-1.p Successor property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(successor_relation)   cnf(prove_successor_property8$_1$, negated_conjecture)

NUM175-1.p Successor property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(successor_relation) $\neq$ universal_class   cnf(prove_successor_property9$_1$, negated_conjecture)

NUM176-1.p Successor property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class   cnf(prove_successor_property10$_1$, negated_conjecture)
apply(successor_relation, x) $\neq$ successor(x)   cnf(prove_successor_property10$_2$, negated_conjecture)

NUM177-1.p Condition 1 for a class to be inductive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(image(successor_relation, x), x)   cnf(prove_inductive_class_condition1$_1$, negated_conjecture)
$u \in x$ cnf(prove_inductive_class_condition1$_2$, negated_conjecture)
$\neg$ successor(u) $\in x$ cnf(prove_inductive_class_condition1$_3$, negated_conjecture)

NUM178-1.p Condition 2 for a class to be inductive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ dom(successor_relation) $\in x$ cnf(prove_inductive_class_condition22$_1$, negated_conjecture)
$\neg$ subclass(image(successor_relation, x), x)   cnf(prove_inductive_class_condition22$_2$, negated_conjecture)

NUM179-1.p Condition 3 for a class to be inductive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
successor(dom(successor_relation)) $\in x$ cnf(prove_inductive_class_condition3$_1$, negated_conjecture)
$\neg$ subclass(image(successor_relation, x), x)   cnf(prove_inductive_class_condition3$_2$, negated_conjecture)

NUM180-1.p Limit ordinals are ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

$\neg$ subclass(limit_ordinals, ordinal_numbers) cnf(prove_limit_ordinals_are_ordinals$_1$, negated_conjecture)

NUM181-1.p The null class is not a limit ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
null_class $\in$ limit_ordinals cnf(prove_null_class_is_not_a_limit_ordinal$_1$, negated_conjecture)

NUM182-1.p Only limit ordinals equal their successors
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ limit_ordinals cnf(prove_only_limit_ordinals_equal_successor$_1$, negated_conjecture)
successor(y) $= x$ cnf(prove_only_limit_ordinals_equal_successor$_2$, negated_conjecture)

NUM183-1.p Ordinals are either kind 1 or limit
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_ordinals_are_kind_1_or_limit$_1$, negated_conjecture)
$\neg x \in$ kind_1_ordinals cnf(prove_ordinals_are_kind_1_or_limit$_2$, negated_conjecture)
$\neg x \in$ limit_ordinals cnf(prove_ordinals_are_kind_1_or_limit$_3$, negated_conjecture)

NUM184-1.p Corollary to ordinals are either kind 1 or limit
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
union(singleton(null_class), union(image(successor_relation, ordinal_numbers), limit_ordinals)) $\neq$ ordinal_numbers cnf(prov

NUM185-1.p Limit ordinals are not members
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
limit_ordinals $\in x$ cnf(prove_limit_ordinals_are_not_members$_1$, negated_conjecture)

NUM186-1.p Omega property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
intersection(power_class(kind_1_ordinals), kind_1_ordinals) $\neq$ omega cnf(prove_omega_property1$_1$, negated_conjecture)

NUM187-1.p Lemma for successor property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in y$ cnf(prove_lemma_for_successor_property8$_1$, negated_conjecture)
$y \in$ intersection(power_class(kind_1_ordinals), kind_1_ordinals) cnf(prove_lemma_for_successor_property8$_2$, negated_conject
$\neg x \in$ intersection(power_class(kind_1_ordinals), kind_1_ordinals) cnf(prove_lemma_for_successor_property8$_3$, negated_conje

NUM188-1.p Omega is transitive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(sum_class(omega), omega) cnf(prove_transitivity_of_omega$_1$, negated_conjecture)

NUM189-1.p Omega is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ omega $\in$ ordinal_numbers cnf(prove_omega_is_an_ordinal$_1$, negated_conjecture)

NUM190-1.p Omega is not the null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

$\mathrm{omega} = \mathrm{null\_class}$ cnf(prove_omega_is_not_null$_1$, negated_conjecture)

NUM191-1.p Omega is a limit ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg\, \mathrm{omega} \in \mathrm{limit\_ordinals}$ cnf(prove_omega_is_a_limit_ordinal$_1$, negated_conjecture)

NUM192-1.p Omega is the smallest limit ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \mathrm{limit\_ordinals}$ cnf(prove_omega_is_the_smallest_limit_ordinal$_1$, negated_conjecture)
$\neg\, \mathrm{omega} \in \mathrm{successor}(x)$ cnf(prove_omega_is_the_smallest_limit_ordinal$_2$, negated_conjecture)

NUM193-1.p The sum of ordinals is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{sum\_class}(\mathrm{ordinal\_numbers}) \neq \mathrm{ordinal\_numbers}$ cnf(prove_sum_of_ordinals_is_ordinal$_1$, negated_conjecture)

NUM194-1.p The union of a class of ordinals is a class of ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{subclass}(x, \mathrm{ordinal\_numbers})$ cnf(prove_union_of_ordinal_class_is_ordinal_class$_1$, negated_conjecture)
$\neg\, \mathrm{subclass}(\mathrm{sum\_class}(x), \mathrm{ordinal\_numbers})$ cnf(prove_union_of_ordinal_class_is_ordinal_class$_2$, negated_conjecture)

NUM195-1.p The union of a class of ordinals is transitive
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{subclass}(x, \mathrm{ordinal\_numbers})$ cnf(prove_transitivity_of_union_of_ordinal_class$_1$, negated_conjecture)
$\neg\, \mathrm{subclass}(\mathrm{sum\_class}(\mathrm{sum\_class}(x)), \mathrm{sum\_class}(x))$ cnf(prove_transitivity_of_union_of_ordinal_class$_2$, negated_conjecture)

NUM196-1.p The union of a set of ordinals is an ordinal
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{subclass}(x, \mathrm{ordinal\_numbers})$ cnf(prove_union_of_ordinal_set_is_ordinal$_1$, negated_conjecture)
$x \in \mathrm{universal\_class}$ cnf(prove_union_of_ordinal_set_is_ordinal$_2$, negated_conjecture)
$\neg\, \mathrm{sum\_class}(x) \in \mathrm{ordinal\_numbers}$ cnf(prove_union_of_ordinal_set_is_ordinal$_3$, negated_conjecture)

NUM197-1.p The union of a proper class of ordinals is the class of ordinals
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{subclass}(x, \mathrm{ordinal\_numbers})$ cnf(prove_union_of_proper_ordinal_class_is_ordinal$_1$, negated_conjecture)
$\mathrm{sum\_class}(x) \neq \mathrm{ordinal\_numbers}$ cnf(prove_union_of_proper_ordinal_class_is_ordinal$_2$, negated_conjecture)
$\neg\, x \in \mathrm{universal\_class}$ cnf(prove_union_of_proper_ordinal_class_is_ordinal$_3$, negated_conjecture)

NUM198-1.p The union of a set of ordinals is >= each ordinal in the set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\mathrm{subclass}(x, \mathrm{ordinal\_numbers})$ cnf(prove_union_of_ordinal_set_exceeds_members$_1$, negated_conjecture)
$x \in \mathrm{universal\_class}$ cnf(prove_union_of_ordinal_set_exceeds_members$_2$, negated_conjecture)
$y \in x$ cnf(prove_union_of_ordinal_set_exceeds_members$_3$, negated_conjecture)
$\neg\, y \in \mathrm{sum\_class}(x)$ cnf(prove_union_of_ordinal_set_exceeds_members$_4$, negated_conjecture)
$\mathrm{sum\_class}(x) \neq y$ cnf(prove_union_of_ordinal_set_exceeds_members$_5$, negated_conjecture)

NUM199-1.p Least upper bound property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

$\text{subclass}(x, \text{ordinal\_numbers})$ cnf(prove_least_upper_bound_property1$_1$, negated_conjecture)
$x \in \text{universal\_class}$ cnf(prove_least_upper_bound_property1$_2$, negated_conjecture)
$\neg\,\text{subclass}(x, \text{successor}(\text{sum\_class}(x)))$ cnf(prove_least_upper_bound_property1$_3$, negated_conjecture)

NUM200-1.p If every element of x is ≤ y, then sum class(x) ≤ y
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(x, \text{successor}(y))$ cnf(prove_least_upper_bound_property2$_1$, negated_conjecture)
$y \in \text{ordinal\_numbers}$ cnf(prove_least_upper_bound_property2$_2$, negated_conjecture)
$\neg\,\text{sum\_class}(x) \in \text{successor}(y)$ cnf(prove_least_upper_bound_property2$_3$, negated_conjecture)

NUM201-1.p Least upper bound property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(x, \text{ordinal\_numbers})$ cnf(prove_least_upper_bound_property3$_1$, negated_conjecture)
$x \in \text{universal\_class}$ cnf(prove_least_upper_bound_property3$_2$, negated_conjecture)
$\neg\,\text{sum\_class}(x) \in x$ cnf(prove_least_upper_bound_property3$_3$, negated_conjecture)
$\neg\,\text{subclass}(x, \text{sum\_class}(x))$ cnf(prove_least_upper_bound_property3$_4$, negated_conjecture)

NUM202-1.p If the lub of a set of ordinals is a successor, it's in the set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(x, \text{ordinal\_numbers})$ cnf(prove_least_upper_bound_property4$_1$, negated_conjecture)
$x \in \text{universal\_class}$ cnf(prove_least_upper_bound_property4$_2$, negated_conjecture)
$\text{sum\_class}(x) = \text{successor}(y)$ cnf(prove_least_upper_bound_property4$_3$, negated_conjecture)
$\neg\,\text{sum\_class}(x) \in x$ cnf(prove_least_upper_bound_property4$_4$, negated_conjecture)

NUM203-1.p Corollary to least upper bound being a successor relation
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\text{subclass}(x, \text{ordinal\_numbers})$ cnf(prove_corollary_to_least_upper_bound_property4$_1$, negated_conjecture)
$x \in \text{universal\_class}$ cnf(prove_corollary_to_least_upper_bound_property4$_2$, negated_conjecture)
$\neg\,\text{sum\_class}(x) \in x$ cnf(prove_corollary_to_least_upper_bound_property4$_3$, negated_conjecture)
$\neg\,\text{sum\_class}(x) \in \text{limit\_ordinals}$ cnf(prove_corollary_to_least_upper_bound_property4$_4$, negated_conjecture)
$x \neq \text{null\_class}$ cnf(prove_corollary_to_least_upper_bound_property4$_5$, negated_conjecture)

NUM204-1.p Least upper bound property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{image}(\text{successor\_relation}, \text{ordinal\_numbers})$ cnf(prove_least_upper_bound_property5$_1$, negated_conjecture)
$\text{successor}(\text{sum\_class}(x)) \neq x$ cnf(prove_least_upper_bound_property5$_2$, negated_conjecture)

NUM205-1.p Corollary 1 to least upper bound property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{image}(\text{successor\_relation}, \text{ordinal\_numbers})$ cnf(prove_corollary_1_to_least_upper_bound_property5$_1$, negated_conjectur
$\text{dom}(\text{successor\_relation}) \neq \text{sum\_class}(x)$ cnf(prove_corollary_1_to_least_upper_bound_property5$_2$, negated_conjecture)

NUM206-1.p Corollary 2 to least upper bound property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in \text{ordinal\_numbers}$ cnf(prove_corollary_2_to_least_upper_bound_property1$_1$, negated_conjecture)
$\text{dom}(\text{successor\_relation}) \neq x$ cnf(prove_corollary_2_to_least_upper_bound_property1$_2$, negated_conjecture)

NUM207-1.p Least upper bound property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_least_upper_bound_property6$_1$, negated_conjecture)
sum_class(x) $\in x$ cnf(prove_least_upper_bound_property6$_2$, negated_conjecture)
successor(sum_class(x)) $\neq x$ cnf(prove_least_upper_bound_property6$_3$, negated_conjecture)

NUM208-1.p Least upper bound property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ limit_ordinals cnf(prove_least_upper_bound_property7$_1$, negated_conjecture)
sum_class(x) $\neq x$ cnf(prove_least_upper_bound_property7$_2$, negated_conjecture)

NUM209-1.p Corollary to least upper bound property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_corollary_to_least_upper_bound_property7$_1$, negated_conjecture)
$\neg\, x \in$ image(successor_relation, ordinal_numbers) cnf(prove_corollary_to_least_upper_bound_property7$_2$, negated_conjectur
sum_class(x) $\neq x$ cnf(prove_corollary_to_least_upper_bound_property7$_3$, negated_conjecture)

NUM210-1.p Lemma 1 for least upper bound property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
null_class $\in x$ cnf(prove_lemma_1_for_least_upper_bound_property8$_1$, negated_conjecture)
least(element_relation, intersection(intersection(power_class(x), x)$'$, ordinal_numbers)) = null_class cnf(prove_lemma_1_for
$\neg$ subclass(ordinal_numbers, intersection(power_class(x), x)) cnf(prove_lemma_1_for_least_upper_bound_property8$_3$, negate

NUM211-1.p Lemma 2 for least upper bound property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(image(successor_relation, x), x) cnf(prove_lemma_2_for_least_upper_bound_property8$_1$, negated_conjecture)
least(element_relation, intersection(intersection(power_class(x), x)$'$, ordinal_numbers)) $\in$ image(successor_relation, ordinal_nun
$\neg$ subclass(ordinal_numbers, intersection(power_class(x), x)) cnf(prove_lemma_2_for_least_upper_bound_property8$_3$, negate

NUM212-1.p Lemma 3 for least upper bound property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(intersection(power_class(x), limit_ordinals), x) cnf(prove_lemma_3_for_least_upper_bound_property8$_1$, negated_co
least(element_relation, intersection(intersection(power_class(x), x)$'$, ordinal_numbers)) $\in$ limit_ordinals cnf(prove_lemma_3
$\neg$ subclass(ordinal_numbers, intersection(power_class(x), x)) cnf(prove_lemma_3_for_least_upper_bound_property8$_3$, negate

NUM213-1.p Alternate 3 for transfinite induction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
null_class $\in x$ cnf(prove_alternate_3_transfinite_induction$_1$, negated_conjecture)
subclass(image(successor_relation, x), x) cnf(prove_alternate_3_transfinite_induction$_2$, negated_conjecture)
subclass(intersection(power_class(x), limit_ordinals), x) cnf(prove_alternate_3_transfinite_induction$_3$, negated_conjecture)
$\neg$ subclass(ordinal_numbers, x) cnf(prove_alternate_3_transfinite_induction$_4$, negated_conjecture)

NUM214-1.p Induction up to y
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ ordinal_numbers cnf(prove_induction_upto_y$_1$, negated_conjecture)
null_class $\in x$ cnf(prove_induction_upto_y$_2$, negated_conjecture)
subclass(image(successor_relation, intersection(x, y)), x) cnf(prove_induction_upto_y$_3$, negated_conjecture)
subclass(intersection(power_class(x), intersection(limit_ordinals, y)), x) cnf(prove_induction_upto_y$_4$, negated_conjecture)
$\neg$ subclass(y, x) cnf(prove_induction_upto_y$_5$, negated_conjecture)

NUM215-1.p Corollary to induction upto y

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
null_class $\in x$ cnf(prove_corollary_to_induction_upto_y_1, negated_conjecture)
subclass(image(successor_relation, intersection(x, omega)), x) cnf(prove_corollary_to_induction_upto_y_2, negated_conjectur
$\neg$ subclass(omega, x) cnf(prove_corollary_to_induction_upto_y_3, negated_conjecture)

NUM216-1.p Corollary 1 to rest definition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$u \in$ domain_of(x) cnf(prove_corollary_1_to_rest_defn_1, negated_conjecture)
apply(rest_of(x), u) $\neq$ restrict(x, u, universal_class) cnf(prove_corollary_1_to_rest_defn_2, negated_conjecture)

NUM217-1.p Corollary 2 to rest definition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
rest_of(null_class) $\neq$ null_class cnf(prove_corollary_2_to_rest_defn_1, negated_conjecture)

NUM218-1.p Rest of is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(rest_of(u)) cnf(prove_rest_of_is_a_function_1, negated_conjecture)

NUM219-1.p The domain of rest_of(X) is the domain of X
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(rest_of(u)) $\neq$ domain_of(u) cnf(prove_domain_of_rest_of_1, negated_conjecture)

NUM220-1.p Corollary to the domain of rest_of(X) being the domain of X
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(rest_of(union(u, singleton(ordered_pair(x, y))))) $\neq$ union(domain_of(u), singleton(x)) cnf(prove_corollary_to_do

NUM221-1.p Rest_of property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(x) $\in$ ordinal_numbers cnf(prove_rest_of_property1_1, negated_conjecture)
rest_of(union(x, singleton(ordered_pair(domain_of(x), y)))) $\neq$ union(rest_of(x), singleton(ordered_pair(domain_of(x), x))) c

NUM222-1.p Rest_of is monotonic.
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(x, y) cnf(prove_monotonicity_of_rest_of_1, negated_conjecture)
$\neg$ subclass(rest_of(x), rest_of(y)) cnf(prove_monotonicity_of_rest_of_2, negated_conjecture)

NUM223-1.p Rest relation is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(rest_relation) cnf(prove_rest_relation_is_a_function_1, negated_conjecture)

NUM224-1.p Rest relation property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(compose_class(x) $\circ$ rest_relation) cnf(prove_rest_relation_property1_1, negated_conjecture)

NUM225-1.p Rest relation property 2

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(rest_relation) $\neq$ universal_class cnf(prove_rest_relation_property2_1, negated_conjecture)

NUM226-1.p Rest relation property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ universal_class cnf(prove_rest_relation_property3_1, negated_conjecture)
apply(rest_relation, x) $\neq$ rest_of(x) cnf(prove_rest_relation_property3_2, negated_conjecture)

NUM227-1.p Rest relation property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
sum_class(image(rest_relation, x)) $\neq$ rest_of(sum_class(x)) cnf(prove_rest_relation_property4_1, negated_conjecture)

NUM228-1.p Corollary to recursion equation functions definition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(z) cnf(prove_corollary$_1$, negated_conjecture)
recursion_equation_functions(z) $\neq$ null_class cnf(prove_corollary$_2$, negated_conjecture)

NUM229-1.p Transfinite recursion lemma 0
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma0_1, negated_conjecture)
$y \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma0_2, negated_conjecture)
$\neg$ subclass(domain_of(intersection(y', x)), ordinal_numbers) cnf(prove_transfinite_recursion_lemma0_3, negated_conjecture)

NUM230-1.p Transfinite recursion lemma 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma1_1, negated_conjecture)
$y \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma1_2, negated_conjecture)
ordered_pair(u, v) $\in x$ cnf(prove_transfinite_recursion_lemma1_3, negated_conjecture)
$u \in$ least(element_relation, domain_of(intersection(y', x))) cnf(prove_transfinite_recursion_lemma1_4, negated_conjecture)
$\neg$ ordered_pair(u, v) $\in y$ cnf(prove_transfinite_recursion_lemma1_5, negated_conjecture)

NUM231-1.p Transfinite recursion lemma 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma2_1, negated_conjecture)
$y \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma2_2, negated_conjecture)
ordered_pair(u, v) $\in y$ cnf(prove_transfinite_recursion_lemma2_3, negated_conjecture)
$u \in$ least(element_relation, domain_of(intersection(y', x))) cnf(prove_transfinite_recursion_lemma2_4, negated_conjecture)
$\neg$ subclass(x, y) cnf(prove_transfinite_recursion_lemma2_5, negated_conjecture)
$\neg$ ordered_pair(u, v) $\in x$ cnf(prove_transfinite_recursion_lemma2_6, negated_conjecture)

NUM232-1.p Transfinite recursion lemma 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma3_1, negated_conjecture)
$y \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_lemma3_2, negated_conjecture)
$\neg$ subclass(x, y) cnf(prove_transfinite_recursion_lemma3_3, negated_conjecture)
restrict(x, least(element_relation, domain_of(intersection(y', x))), universal_class) $\neq$ restrict(y, least(element_relation, domain_o

NUM233-1.p Transfinite recursion lemma 4

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma4$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma4$_2$, negated_conjecture)
domain_of(x) $\in$ domain_of(y)   cnf(prove_transfinite_recursion_lemma4$_3$, negated_conjecture)
$\neg$ subclass(x, y)   cnf(prove_transfinite_recursion_lemma4$_4$, negated_conjecture)
apply(y, least(element_relation, domain_of(intersection(y', x)))) $\neq$ apply(x, least(element_relation, domain_of(intersection(y', x

NUM234-1.p Transfinite recursion lemma 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma5$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma5$_2$, negated_conjecture)
domain_of(x) $\in$ domain_of(y)   cnf(prove_transfinite_recursion_lemma5$_3$, negated_conjecture)
$\neg$ subclass(x, y)   cnf(prove_transfinite_recursion_lemma5$_4$, negated_conjecture)
$\neg$ ordered_pair(least(element_relation, domain_of(intersection(y', x))), apply(y, least(element_relation, domain_of(intersection(
y cnf(prove_transfinite_recursion_lemma5$_5$, negated_conjecture)

NUM235-1.p Transfinite recursion lemma 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma6$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma6$_2$, negated_conjecture)
domain_of(x) $\in$ domain_of(y)   cnf(prove_transfinite_recursion_lemma6$_3$, negated_conjecture)
$\neg$ subclass(x, y)   cnf(prove_transfinite_recursion_lemma6$_4$, negated_conjecture)

NUM236-1.p Corollary 1 to transfinite recursion lemma 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_corollary_1_to_transfinite_recursion_lemma6$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_corollary_1_to_transfinite_recursion_lemma6$_2$, negated_conjecture)
$\neg$ union(x, y) $\in$ recursion_equation_functions(z)   cnf(prove_corollary_1_to_transfinite_recursion_lemma6$_3$, negated_conjectur

NUM237-1.p Corollary 2 to transfinite recursion lemma 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_corollary_2_to_transfinite_recursion_lemma6$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_corollary_2_to_transfinite_recursion_lemma6$_2$, negated_conjecture)
$\neg$ function(union(x, y))   cnf(prove_corollary_2_to_transfinite_recursion_lemma6$_3$, negated_conjecture)

NUM238-1.p Transfinite recursion lemma 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma7$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma7$_2$, negated_conjecture)
domain_of(x) $\in$ domain_of(y)   cnf(prove_transfinite_recursion_lemma7$_3$, negated_conjecture)
$u \in$ domain_of(x)   cnf(prove_transfinite_recursion_lemma7$_4$, negated_conjecture)
restrict(x, u, universal_class) $\neq$ restrict(y, u, universal_class)   cnf(prove_transfinite_recursion_lemma7$_5$, negated_conjecture)

NUM239-1.p Transfinite recursion lemma 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma8$_1$, negated_conjecture)
$y \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_lemma8$_2$, negated_conjecture)
domain_of(x) $\in$ domain_of(y)   cnf(prove_transfinite_recursion_lemma8$_3$, negated_conjecture)
$\neg$ subclass(rest_of(x), rest_of(y))   cnf(prove_transfinite_recursion_lemma8$_4$, negated_conjecture)

NUM240-1.p Transfinite recursion lemma 9.1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$z \in$ universal_class cnf(prove_transfinite_recursion_lemma9_1$_1$, negated_conjecture)
image(image(composition_function, singleton(z)), image(rest, recursion_equation_functions(z))) $\neq$ recursion_equation_function

NUM241-1.p Transfinite recursion lemma 9.2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
image(comp(z), image(rest, recursion_equation_functions(z))) $\neq$ recursion_equation_functions(z) cnf(prove_transfinite_recu

NUM242-1.p Transfinite recursion lemma 9.3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(x) cnf(prove_transfinite_recursion_lemma9_3$_1$, negated_conjecture)
function(y) cnf(prove_transfinite_recursion_lemma9_3$_2$, negated_conjecture)
domain_of(x) = ordinal_numbers cnf(prove_transfinite_recursion_lemma9_3$_3$, negated_conjecture)
domain_of(y) = ordinal_numbers cnf(prove_transfinite_recursion_lemma9_3$_4$, negated_conjecture)
$x \neq y$ cnf(prove_transfinite_recursion_lemma9_3$_5$, negated_conjecture)
restrict(x, least(element_relation, domain_of(intersection(x', y))), universal_class) $\neq$ restrict(y, least(element_relation, domain_o

NUM243-1.p Transfinite recursion lemma 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(x) cnf(prove_transfinite_recursion_lemma10$_1$, negated_conjecture)
$z \circ$ rest(x) = x cnf(prove_transfinite_recursion_lemma10$_2$, negated_conjecture)
domain_of(x) = ordinal_numbers cnf(prove_transfinite_recursion_lemma10$_3$, negated_conjecture)
$\neg$ subclass(sum_class(recursion_equation_functions(z)), x) cnf(prove_transfinite_recursion_lemma10$_4$, negated_conjecture)
apply(sum_class(recursion_equation_functions(z)), least(element_relation, domain_of(intersection(x', sum_class(recursion_equa
apply(x, least(element_relation, domain_of(intersection(x', sum_class(recursion_equation_functions(z)))))) cnf(prove_transf

NUM244-1.p Transfinite recursion lemma 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(x) cnf(prove_transfinite_recursion_lemma11$_1$, negated_conjecture)
$z \circ$ rest(x) = x cnf(prove_transfinite_recursion_lemma11$_2$, negated_conjecture)
domain_of(x) = ordinal_numbers cnf(prove_transfinite_recursion_lemma11$_3$, negated_conjecture)
ordered_pair(least(element_relation, domain_of(intersection(x', sum_class(recursion_equation_functions(z))))), apply(sum_class
intersection(x', sum_class(recursion_equation_functions(z))) cnf(prove_transfinite_recursion_lemma11$_4$, negated_conjecture
$\neg$ subclass(sum_class(recursion_equation_functions(z)), x) cnf(prove_transfinite_recursion_lemma11$_5$, negated_conjecture)

NUM245-1.p Transfinite recursion property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_property1$_1$, negated_conjecture)
$u \in$ domain_of(x) cnf(prove_transfinite_recursion_property1$_2$, negated_conjecture)
apply(z, restrict(x, u, universal_class)) $\neq$ apply(x, u) cnf(prove_transfinite_recursion_property1$_3$, negated_conjecture)

NUM246-1.p Transfinite recursion property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(x) cnf(prove_transfinite_recursion_property2$_1$, negated_conjecture)
function(z) cnf(prove_transfinite_recursion_property2$_2$, negated_conjecture)
domain_of(x) = ordinal_numbers cnf(prove_transfinite_recursion_property2$_3$, negated_conjecture)
$z \circ$ rest_of(x) = x cnf(prove_transfinite_recursion_property2$_4$, negated_conjecture)
$u \in$ ordinal_numbers cnf(prove_transfinite_recursion_property2$_5$, negated_conjecture)

apply(z, restrict(x, u, universal_class)) $\neq$ apply(x, u)   cnf(prove_transfinite_recursion_property2$_6$, negated_conjecture)

NUM247-1.p Transfinite recursion property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(sum_class(recursion_equation_functions(z)), cross_product(universal_class, universal_class))   cnf(prove_transfinit

NUM248-1.p Transfinite recursion property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(sum_class(recursion_equation_functions(z)))   cnf(prove_transfinite_recursion_property4$_1$, negated_conjecture)

NUM249-1.p Transfinite recursion property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$z\circ$rest_of(sum_class(recursion_equation_functions(z))) $\neq$ sum_class(recursion_equation_functions(z))   cnf(prove_transfinite_

NUM250-1.p Transfinite recursion property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(image(domain_relation, recursion_equation_functions(z)), ordinal_numbers)   cnf(prove_transfinite_recursion_prop

NUM251-1.p Transfinite recursion property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ subclass(domain_of(sum_class(recursion_equation_functions(z))), ordinal_numbers)   cnf(prove_transfinite_recursion_prope

NUM252-1.p Transfinite recursion property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ domain_of(sum_class(recursion_equation_functions(z))) $\in$ ordinal_numbers   cnf(prove_transfinite_recursion_property8$_1$, n
domain_of(sum_class(recursion_equation_functions(z))) $\neq$ ordinal_numbers   cnf(prove_transfinite_recursion_property8$_2$, neg

NUM253-1.p Transfinite recursion property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_property9$_1$, negated_conjecture)
$\neg$ function(union(singleton(ordered_pair(domain_of(x), apply(z, x))), x))   cnf(prove_transfinite_recursion_property9$_2$, negat

NUM254-1.p Transfinite recursion property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_property10$_1$, negated_conjecture)
$\neg$ domain_of(union(singleton(ordered_pair(domain_of(x), apply(z, x))), x)) $\in$ ordinal_numbers   cnf(prove_transfinite_recursi

NUM255-1.p Transfinite recursion property 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_property11$_1$, negated_conjecture)
domain_of(union(singleton(ordered_pair(domain_of(x), apply(z, x))), x)) $\neq$ successor(domain_of(x))   cnf(prove_transfinite_

NUM256-1.p Transfinite recursion property 12
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z)   cnf(prove_transfinite_recursion_property12$_1$, negated_conjecture)

$x \in$ domain_of(z) cnf(prove_transfinite_recursion_property12$_2$, negated_conjecture)
$z\circ$rest_of(union(singleton(ordered_pair(domain_of(x), apply(z,x))), x)) $\neq$ union(singleton(ordered_pair(domain_of(x), apply(z

NUM257-1.p Transfinite recursion property 13
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ recursion_equation_functions(z) cnf(prove_transfinite_recursion_property13$_1$, negated_conjecture)
$\neg$ union(singleton(ordered_pair(domain_of(x), apply(z,x))), x) $\in$ recursion_equation_functions(z) cnf(prove_transfinite_rec

NUM258-1.p Transfinite recursion property 14
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(z) cnf(prove_transfinite_recursion_property14$_1$, negated_conjecture)
domain_of(sum_class(recursion_equation_functions(z))) $\neq$ ordinal_numbers cnf(prove_transfinite_recursion_property14$_2$, ne

NUM259-1.p The uniqueness of the function defined by transfinite recursion
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
function(x) cnf(prove_transfinite_recursion_function_unique$_1$, negated_conjecture)
domain_of(x) = ordinal_numbers cnf(prove_transfinite_recursion_function_unique$_2$, negated_conjecture)
$z \circ$ rest_of(x) = x cnf(prove_transfinite_recursion_function_unique$_3$, negated_conjecture)
sum_class(recursion_equation_functions(z)) $\neq x$ cnf(prove_transfinite_recursion_function_unique$_4$, negated_conjecture)

NUM260-1.p Alternate 4 for transfinite induction, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$\neg$ function(recursion(x,y,z)) cnf(prove_alternate_4_transfinite_induction1$_1$, negated_conjecture)

NUM261-1.p Alternate 4 for transfinite induction, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
domain_of(recursion(x,y,z)) $\neq$ ordinal_numbers cnf(prove_alternate_4_transfinite_induction2$_1$, negated_conjecture)

NUM262-1.p Alternate 4 for transfinite induction, part 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
apply(recursion(x,y,z), null_class) $\neq x$ cnf(prove_alternate_4_transfinite_induction3$_1$, negated_conjecture)

NUM263-1.p Alternate 4 for transfinite induction, part 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$u \in$ ordinal_numbers cnf(prove_alternate_4_transfinite_induction4$_1$, negated_conjecture)
apply(recursion(x,y,z), successor(u)) $\neq$ apply(y, apply(recursion(x,y,z), u)) cnf(prove_alternate_4_transfinite_induction4

NUM264-1.p Alternate 4 for transfinite induction, part 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$u \in$ limit_ordinals cnf(prove_alternate_4_transfinite_induction5$_1$, negated_conjecture)
apply(z, restrict(recursion(x,y,z), u, universal_class)) $\neq$ apply(recursion(x,y,z), u) cnf(prove_alternate_4_transfinite_induc

NUM265-1.p Ordinal addition property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordinal_add(x, null_class) $\neq x$ cnf(prove_ordinal_addition_property1$_1$, negated_conjecture)

NUM266-1.p Ordinal addition property 2

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ ordinal_numbers cnf(prove_ordinal_addition_property2$_1$, negated_conjecture)
ordinal_add(x, successor(y)) $\neq$ successor(ordinal_add(x, y)) cnf(prove_ordinal_addition_property2$_2$, negated_conjecture)

NUM267-1.p Ordinal addition property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ limit_ordinals cnf(prove_ordinal_addition_property3$_1$, negated_conjecture)
sum_class(image(recursion(x, successor_relation, union_of_range_map), y)) $\neq$ ordinal_add(x, y) cnf(prove_ordinal_addition_

NUM268-1.p Ordinal addition property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ limit_ordinals cnf(prove_ordinal_addition_property4$_1$, negated_conjecture)
sum_class(image(image(add_relation, singleton(x)), y)) $\neq$ ordinal_add(x, y) cnf(prove_ordinal_addition_property4$_2$, negated

NUM269-1.p Ordinal addition property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_ordinal_addition_property5$_1$, negated_conjecture)
$y \in$ ordinal_numbers cnf(prove_ordinal_addition_property5$_2$, negated_conjecture)
$\neg$ ordinal_add(x, y) $\in$ ordinal_numbers cnf(prove_ordinal_addition_property5$_3$, negated_conjecture)

NUM270-1.p Ordinal addition property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordinal_add(x, successor(null_class)) $\neq$ successor(x) cnf(prove_ordinal_addition_property6$_1$, negated_conjecture)

NUM271-1.p Lemma 1 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ null_class $\in$ ordinals_with_null_class_as_identity cnf(prove_lemma_1_for_ordinal_addition_property7$_1$, negated_conjecture)

NUM272-1.p Lemma 2 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ subclass(image(successor_relation, ordinals_with_null_class_as_identity), image(successor_relation, ordinal_numbers)) cnf(

NUM273-1.p Lemma 3 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ subclass(image(successor_relation, ordinals_with_null_class_as_identity), ordinals_with_null_class_as_identity) cnf(prove_le

NUM274-1.p Lemma 4 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ subclass(ordinals_with_null_class_as_identity, domain_of(recursion(null_class, successor_relation, union_of_range_map))) c

NUM275-1.p Lemma 5 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ subclass(ordinals_with_null_class_as_identity, domain_of(intersection(recursion(null_class, successor_relation, union_of_range_

NUM276-1.p Lemma 6 for ordinal addition property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity_def$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity_def$_2$, axi
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
$\neg$ subclass(intersection(power_class(ordinals_with_null_class_as_identity), limit_ordinals), ordinals_with_null_class_as_identity)

NUM277-1.p Ordinal addition property 7_1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity$_2$, axiom)
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
ordinals_with_null_class_as_identity $\neq$ ordinal_numbers cnf(prove_ordinal_addition_property7_1$_1$, negated_conjecture)

NUM277-2.p Ordinal addition property 7.1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
subclass(ordinals_with_null_class_as_identity, ordinal_numbers) cnf(ordinals_with_null_class_as_identity$_1$, axiom)
$x \in$ ordinals_with_null_class_as_identity $\Rightarrow$ ordinal_add(null_class, x) $= x$ cnf(ordinals_with_null_class_as_identity$_2$, axiom)
($x \in$ ordinal_numbers and ordinal_add(null_class, x) $= x$) $\Rightarrow$ $x \in$ ordinals_with_null_class_as_identity cnf(ordinals_with_nu
null_class $\in$ ordinals_with_null_class_as_identity cnf(lemma_1_for_ordinal_addition_property$_7$, axiom)
subclass(image(successor_relation, ordinals_with_null_class_as_identity), image(successor_relation, ordinal_numbers)) cnf(le
subclass(image(successor_relation, ordinals_with_null_class_as_identity), ordinals_with_null_class_as_identity) cnf(lemma_3_
subclass(ordinals_with_null_class_as_identity, domain_of(recursion(null_class, successor_relation, union_of_range_map))) cnf
subclass(ordinals_with_null_class_as_identity, domain_of(intersection(recursion(null_class, successor_relation, union_of_range_ma
subclass(intersection(power_class(ordinals_with_null_class_as_identity), limit_ordinals), ordinals_with_null_class_as_identity)
ordinals_with_null_class_as_identity $\neq$ ordinal_numbers cnf(prove_ordinal_addition_property7_1$_1$, negated_conjecture)

NUM278-1.p Ordinal addition property 7_2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$x \in$ ordinal_numbers cnf(prove_ordinal_addition_property7_2$_1$, negated_conjecture)
ordinal_add(null_class, x) $\neq x$ cnf(prove_ordinal_addition_property7_2$_2$, negated_conjecture)

NUM279-1.p Ordinal addition property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordinal_add(x, successor(null_class)) $\neq$ successor(x) cnf(prove_ordinal_adition_property8$_1$, negated_conjecture)

NUM280-1.p Ordinal multiplication property 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
ordinal_multiply(x, null_class) $\neq$ null_class   cnf(prove_ordinal_multiplication_property1$_1$, negated_conjecture)

NUM281-1.p Ordinal multiplication property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ ordinal_numbers   cnf(prove_ordinal_multiplication_property2$_1$, negated_conjecture)
ordinal_multiply(x, successor(y)) $\neq$ ordinal_add(ordinal_multiply(x, y), y)   cnf(prove_ordinal_multiplication_property2$_2$, neg

NUM282-1.p Ordinal multiplication property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')
$y \in$ limit_ordinals   cnf(prove_ordinal_multiplication_property3$_1$, negated_conjecture)
sum_class(image(recursion(null_class, apply(add_relation, x), union_of_range_map), y)) $\neq$ ordinal_multiply(x, y)   cnf(prove_

NUM283-1.005.p Calculation of factorial
Compute 5 factorial.
$x + n_0 = x$ cnf(add$_0$, axiom)
$x + y = z \;\Rightarrow\; x + s(y) = s(z)$ cnf(add, axiom)
$s(n_0) \cdot x = x$ cnf(times$_1$, axiom)
$(r + y = z$ and $x \cdot y = r) \;\Rightarrow\; s(x) \cdot y = z$ cnf(times, axiom)
factorial($n_0, s(n_0)$)   cnf(factorial$_0$, axiom)
(factorial(x, z) and $s(x) \cdot z = y$) $\Rightarrow$ factorial($s(x), y$)   cnf(factorial, axiom)
$\neg$ factorial($s(s(s(s(s(n_0)))))$, result)   cnf(prove_factorial, negated_conjecture)

NUM284-1.014.p Calculation of fibonacci numbers
Compute the 14th Fibonacci number.
fibonacci($n_0, s(n_0)$)   cnf(fibonacci$_0$, axiom)
fibonacci($s(n_0), s(n_0)$)   cnf(fibonacci$_1$, axiom)
$(n_1 + s(n_0) = n$ and $n_2 + s(s(n_0)) = n$ and fibonacci(n_1, f_1) and fibonacci(n_2, f_2) and $f_1 + f_2 = \mathrm{fN}) \;\Rightarrow\;$ fibonacci(n, fN)   cnf(fib
$x + n_0 = x$ cnf(add$_0$, axiom)
$x + y = z \;\Rightarrow\; x + s(y) = s(z)$ cnf(add, axiom)
$\neg$ fibonacci($s(s(s(s(s(s(s(s(s(s(s(s(s(s(n_0))))))))))))))$, result)   cnf(prove_fibonacci, negated_conjecture)

NUM286-1.p Number theory axioms
include('Axioms/NUM001-0.ax')

NUM286-2.p Number theory (equality) axioms
include('Axioms/NUM002-0.ax')

NUM286-3.p Number theory axioms, based on Godel set theory
include('Axioms/SET003-0.ax')
include('Axioms/ALG001-0.ax')
include('Axioms/NUM003-0.ax')

NUM287-1.p Number theory less axioms
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')

NUM288-1.p Number theory div axioms
include('Axioms/NUM001-0.ax')
include('Axioms/NUM001-1.ax')
include('Axioms/NUM001-2.ax')

NUM289-1.p Number theory (ordinals) axioms, based on NBG set theory
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
include('Axioms/NUM004-0.ax')

NUM290+1.p $2 < 3$
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\mathrm{less}(n_2, n_3)$ fof(n2_less_n$_3$, conjecture)

NUM291+1.p 3 !< 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\neg\,\mathrm{less}(n_3, n_2)$ fof(n3_not_less_n$_2$, conjecture)

NUM292+1.p 2 < 13
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\mathrm{less}(n_2, n_{13})$ fof(n2_less_n$_{13}$, conjecture)

NUM293+1.p ? < 13
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x\colon \mathrm{less}(x, n_{13})$ fof(something_less_n$_{13}$, conjecture)

NUM294+1.p 12 < ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x\colon \mathrm{less}(n_{12}, x)$ fof(n12_less_something, conjecture)

NUM295+1.p ? < ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y\colon \mathrm{less}(x, y)$ fof(something_less_something, conjecture)

NUM296+1.p -2 < 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\mathrm{less}(\mathrm{nn}_2, n_2)$ fof(nn2_less_n$_2$, conjecture)

NUM297+1.p -4 < -2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\mathrm{less}(\mathrm{nn}_4, \mathrm{nn}_2)$ fof(nn4_less_nn$_2$, conjecture)

NUM298+1.p 2 !< -2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\neg\,\mathrm{less}(n_2, \mathrm{nn}_2)$ fof(n2_not_less_nn$_2$, conjecture)

NUM299+1.p -2 !< -4
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\neg\,\mathrm{less}(\mathrm{nn}_2, \mathrm{nn}_4)$ fof(nn2_not_less_nn$_4$, conjecture)

NUM300+1.p ? < 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x\colon \mathrm{less}(x, n_0)$ fof(something_less_n$_0$, conjecture)

NUM301+1.p ? < -2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\exists x$: less(x, nn$_2$) fof(something_less_nn$_2$, conjecture)

NUM302+1.p -2 < ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: less(nn$_2$, x) fof(nn2_less_something, conjecture)

NUM303+1.p 31 != 21
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_{31} \neq n_{21}$ fof(n31_not_n$_{12}$, conjecture)

NUM304+1.p ? != 12
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x \neq n_{12}$ fof(something_not_n$_{12}$, conjecture)

NUM305+1.p 2 + 3 = 5
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_2 + n_3 = n_5$ fof(sum_n2_n3_n$_5$, conjecture)

NUM306+1.p 23 + 34 = 57
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_{23} + n_{34} = n_{57}$ fof(sum_n23_n34_n$_{57}$, conjecture)

NUM307+1.p 23 + 34 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_{23} + n_{34} = x$ fof(summ_n23_n34_something, conjecture)

NUM308+1.p ? + 23 = 34
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + n_{23} = n_{34}$ fof(sum_something_n23_n$_{34}$, conjecture)

NUM309+1.p 23 + ? = 34
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_{23} + x = n_{34}$ fof(sum_n23_something_n$_{34}$, conjecture)

NUM310+1.p 2 + 3 != 6
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\neg\, n_2 + n_3 = n_6$ fof(sum_n2_n3_not_n$_6$, conjecture)

NUM311+1.p 2 + 3 = only 5
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_2 + n_3 = x \Rightarrow x = n_5)$ fof(sum_n2_n3_only_n$_5$, conjecture)

NUM312+1.p only 2 + 3 = 5
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\forall x$: $(x + n_3{=}n_5 \;\Rightarrow\; x = n_2)$ fof(sum_only_n2_n3_n$_5$, conjecture)

NUM313+1.p 2 + only 3 = 5
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_2 + x{=}n_5 \;\Rightarrow\; x = n_3)$ fof(sumn2_only_n3_n$_5$, conjecture)

NUM314+1.p Show upper boundary
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_{126} + n_1{=}x$ fof(show_upper_boundary, conjecture)

NUM315+1.p -2 + -5 = -7
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\mathrm{nn}_2 + \mathrm{nn}_5{=}\mathrm{nn}_7$ fof(sum_nn2_nn5_nn$_7$, conjecture)

NUM316+1.p 2 + -5 = -3
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_2 + \mathrm{nn}_5{=}\mathrm{nn}_3$ fof(sum_n2_nn5_nn$_3$, conjecture)

NUM317+1.p 5 + -2 = 3
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_5 + \mathrm{nn}_2{=}n_3$ fof(sum_n5_nn2_n$_3$, conjecture)

NUM318+1.p 5 + -5 = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_5 + \mathrm{nn}_5{=}n_0$ fof(sum_n5_nn5_n$_0$, conjecture)

NUM319+1.p -2 + -5 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $\mathrm{nn}_2 + \mathrm{nn}_5{=}x$ fof(sum_nn2_nn5_what, conjecture)

NUM320+1.p 2 + -5 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists y$: $n_2 + \mathrm{nn}_5{=}y$ fof(sum_n2_nn5_what, conjecture)

NUM321+1.p 5 + -2 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_5 + \mathrm{nn}_2{=}x$ fof(sum_n5_nn2_what, conjecture)

NUM322+1.p 5 + -5 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_5 + \mathrm{nn}_5{=}x$ fof(sum_n5_nn5_what, conjecture)

NUM323+1.p ? + -5 = -7
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\exists x$: $x + \mathrm{nn}_5 = \mathrm{nn}_7$ fof(sum_what_nn5_nn$_7$, conjecture)

NUM324+1.p ? + -5 = -3
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + \mathrm{nn}_5 = \mathrm{nn}_3$ fof(sum_what_nn5_nn$_3$, conjecture)

NUM325+1.p ? + -2 = 3
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + \mathrm{nn}_2 = n_3$ fof(sum_what_nn2_n$_3$, conjecture)

NUM326+1.p ? + -5 = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + \mathrm{nn}_5 = n_0$ fof(sum_what_nn5_n$_0$, conjecture)

NUM327+1.p ? + 0 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + n_0 = x$ fof(sum_zero_identity, conjecture)

NUM328+1.p ?1 + ? = ?1
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $x + y = x$ fof(sum_something_anotherthing_firstthing, conjecture)

NUM329+1.p ? + ? = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x + x = x$ fof(idempotent_element, conjecture)

NUM330+1.p XY (X+Y = 8) & X = 4 & Y = 4
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $(x + y = n_8 \text{ and } x = n_4 \text{ and } y = n_4)$ fof(sum_n4_n4_n$_8$, conjecture)

NUM331+1.p 6 + 7 = 7 + 6
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall z_1, z_2$: $((n_6 + n_7 = z_1 \text{ and } n_7 + n_6 = z_2) \Rightarrow z_1 = z_2)$ fof(communative_sum_n6_n$_7$, conjecture)

NUM332+1.p (2 + 3) + 6 = 2 + (3 + 6)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall z_1, z_2, z_3, z_4$: $((n_2 + n_3 = z_1 \text{ and } z_1 + n_6 = z_2 \text{ and } n_3 + n_6 = z_3 \text{ and } n_2 + z_3 = z_4) \Rightarrow z_2 = z_4)$ fof(associative_sum, conjecture)

NUM333+1.p ! XYZ, ((X+Y)+Z) = (X+(Y+Z))
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z, z_1, z_2, z_3, z_4$: $((x + y = z_1 \text{ and } z_1 + z = z_2 \text{ and } y + z = z_3 \text{ and } x + z_3 = z_4) \Rightarrow z_2 = z_4)$ fof(associative, conjecture)

NUM334+1.p 7 - 5 = 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$n_7 \setminus n_5{=}n_2$ fof(diff_n7_n5_n$_2$, conjecture)

NUM335+1.p 5 - 3 = only 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_5 \setminus n_3{=}x \Rightarrow x = n_2)$ fof(diff_n5_n3_only_n$_2$, conjecture)

NUM336+1.p only 5 - 2 = 3
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(x \setminus n_2{=}n_3 \Rightarrow x = n_5)$ fof(diff_only_n5_n2_n$_3$, conjecture)

NUM337+1.p 5 - only 3 = 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_5 \setminus x{=}n_2 \Rightarrow x = n_3)$ fof(diff_n5_only_n3_n$_2$, conjecture)

NUM338+1.p 5 - 3 = only 2
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_5 \setminus n_3{=}x \Rightarrow x = n_2)$ fof(diff_n5_n3_only_n$_2$, conjecture)

NUM339+1.p Show lower boundary
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $\mathrm{nn}_{127} \setminus n_1{=}x$ fof(show_lower_boundary, conjecture)

NUM340+1.p ? - 0 = ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $x \setminus n_0{=}x$ fof(diff_zero_identity, conjecture)

NUM341+1.p x + y = z $\leq$> z - y = x & z - x = y
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y, z$: $(x + y{=}z \iff (z \setminus y{=}x \text{ and } z \setminus x{=}y))$ fof(add_same_as_subtract, conjecture)

NUM342+1.p XY (X + Y = 8) => X - Y = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $(x + y{=}n_8 \text{ and } x \setminus y{=}n_0)$ fof(sum_and_difference, conjecture)

NUM343+1.p -1 < ? & ? < 1 => 21 + ? = 21
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $((\mathrm{less}(\mathrm{nn}_1, x) \text{ and } \mathrm{less}(x, n_1)) \Rightarrow n_{21} + x{=}n_{21})$ fof(sum_something_n0_samething, conjecture)

NUM344+1.p x+1 = z => z > x
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $(x + n_1{=}y \text{ and } \mathrm{less}(x, y))$ fof(exist_bigger_plus_one, conjecture)

NUM345+1.p 2 + 3 < 6
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\forall x$: $(n_2 + n_3 = x \Rightarrow \text{less}(x, n_6))$ fof(sum_n2_n3_less_n_6, conjecture)

NUM346+1.p 2 + 3 > 4
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_2 + n_3 = x \Rightarrow \text{less}(n_4, x))$ fof(sum_n2_n3_greater_n_4, conjecture)

NUM347+1.p 2 + 2 = 5
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$n_2 + n_2 = n_5$ fof(anti_sum_n2_n2_n_5, conjecture)

NUM348+1.p X (127 + 1 = X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $n_{127} + n_1 = x$ fof(anti_upper_boundary, conjecture)

NUM349+1.p X (-128 - 1 = X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $\text{nn}_{128} \setminus n_1 = x$ fof(anti_lower_boundary, conjecture)

NUM350+1.p !XY, (X + X) = Y
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $x + x = y$ fof(anti_sum_x_x_y, conjecture)

NUM351+1.p XY (X + Y) = X
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $x + y = x$ fof(anti_sum_x_y_x, conjecture)

NUM352+1.p ?XY (X+Y) != (X+Y)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y, z_1, z_2$: $(x + y = z_1 \text{ and } x + y = z_2 \text{ and } z_1 \neq z_2)$ fof(anti_unique, conjecture)

NUM353+1.p XYZ ((X+Y)+Z) != (Z+X)+Y)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y, z, z_1, z_2, z_3, z_4$: $(x+y=z_1 \text{ and } z_1+z=z_2 \text{ and } z+x=z_3 \text{ and } z_3+y=z_4 \text{ and } z_2 \neq z_4)$ fof(anti_associativity, conjecture)

NUM354+1.p ? != 0 such that ? + ? = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x$: $(x \neq n_0 \text{ and } x + x = n_0)$ fof(what_what_n_0, conjecture)

NUM355+1.p XY (X+Y = 8) => X = 4, Y = 4
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $(x + y = n_8 \Rightarrow (x = n_4 \text{ and } y = n_4))$ fof(anti_sum_only_n4_only_n4_n_8, conjecture)

NUM356+1.p ? + 0 != ?
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\exists x$: $\neg\, x + n_0 = x$ fof(anti_sum_identity, conjecture)

NUM357+1.p ?X (X + 0 != X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $(x + n_0 = y$ and $y \neq x)$ fof(anti_sum_identity, conjecture)

NUM358+1.p !X (X + X = X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $x + x = x$ fof(anti_sum_idempotence, conjecture)

NUM359+1.p !X (X + X != X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $\neg\, x + x = x$ fof(anti_not_sum_idempotence, conjecture)

NUM360+1.p ?XY (X + Y = 8) => X - Y = 1
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\exists x, y$: $(x + y = n_8$ and $x \setminus y = n_1)$ fof(exists_sum_consecutive_n$_8$, conjecture)

NUM361+1.p !XY (X+Y = 8) => X - Y = 1,
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $(x + y = n_8 \;\Rightarrow\; x \setminus y = n_1)$ fof(all_sum_consecutive_n$_8$, conjecture)

NUM362+1.p !XY (X+Y = 8) => X - Y = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $(x + y = n_8 \;\Rightarrow\; x \setminus y = n_0)$ fof(all_sum_same_n$_8$, conjecture)

NUM363+1.p if (X+Y) = Z then Z > X & Z > Y
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z$: $(x + y = z \;\Rightarrow\; (\mathrm{less}(x, z)$ and $\mathrm{less}(y, z)))$ fof(sum_larger, conjecture)

NUM364+1.p !XY (X + Y > X - Y)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z_1, z_2$: $(x + y = z_1$ and $x \setminus y = z_2$ and $\mathrm{less}(z_1, z_2))$ fof(anti_sum_diff_less$_1$, conjecture)

NUM365+1.p !XY (X - Y > X + Y)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z_1, z_2$: $(x + y = z_1$ and $x \setminus y = z_2$ and $\mathrm{less}(z_2, z_1))$ fof(anti_sum_diff_less$_2$, conjecture)

NUM366+1.p 2 + 3 > 7
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $(n_2 + n_3 = x \;\Rightarrow\; \mathrm{less}(n_7, x))$ fof(anti_sum_n2_n3_greater_n$_7$, conjecture)

NUM367+1.p ?XY (X + Y != Y + X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

$\forall x, y, z_1, z_2$: $((x + y{=}z_1 \text{ and } x \setminus y{=}z_2) \Rightarrow \text{less}(z_2, z_1))$ fof(x_plus_y_greater_x_minus_y, conjecture)

NUM368+1.p ! - ! = 0
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $x \setminus x{=}n_0$ fof(x_minus_x_equals$_0$, conjecture)

NUM369+1.p ! + 0 = !
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $x + n_0{=}x$ fof(n0_identity, conjecture)

NUM370+1.p 0 + ! = !
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x$: $n_0 + x{=}x$ fof(n0_identy_rev, conjecture)

NUM371+1.p if (X - Y) = Z and Z > 0, then X > Y
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z$: $((x \setminus y{=}z \text{ and less}(n_0, z)) \Rightarrow \text{less}(y, x))$ fof(difference_greater, conjecture)

NUM372+1.p if (X - Y) = 0, then X = Y
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y$: $(x \setminus y{=}n_0 \Rightarrow x = y)$ fof(identity, conjecture)

NUM373+1.p ?XYZ, (X+Y) = (Y+X)
include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')
$\forall x, y, z_1, z_2$: $((x + y{=}z_1 \text{ and } y + x{=}z_2) \Rightarrow z_1 = z_2)$ fof(communative, conjecture)

NUM374+1.p Disprove Wilkie identity from Tarski's identities
$\forall x, y$: $x + y = y + x$ fof(sum_symmetry, axiom)
$\forall x, y, z$: $x + (y + z) = (x + y) + z$ fof(sum_associativity, axiom)
$\forall x$: $x \cdot n_1 = x$ fof(product_identity, axiom)
$\forall x, y$: $x \cdot y = y \cdot x$ fof(product_symmetry, axiom)
$\forall x, y, z$: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ fof(product_associativity, axiom)
$\forall x, y, z$: $x \cdot (y + z) = x \cdot y + x \cdot z$ fof(sum_product_distribution, axiom)
$\forall x$: $\text{exponent}(n_1, x) = n_1$ fof(exponent_n$_1$, axiom)
$\forall x$: $\text{exponent}(x, n_1) = x$ fof(exponent_identity, axiom)
$\forall x, y, z$: $\text{exponent}(x, y + z) = \text{exponent}(x, y) \cdot \text{exponent}(x, z)$ fof(exponent_sum_product, axiom)
$\forall x, y, z$: $\text{exponent}(x \cdot y, z) = \text{exponent}(x, z) \cdot \text{exponent}(y, z)$ fof(exponent_product_distribution, axiom)
$\forall x, y, z$: $\text{exponent}(\text{exponent}(x, y), z) = \text{exponent}(x, y \cdot z)$ fof(exponent_exponent, axiom)
$\forall c, p, q, r, s, a, b$: $((c = a \cdot a \text{ and } p = n_1 + a \text{ and } q = p + c \text{ and } r = n_1 + a \cdot c \text{ and } s = (n_1 + c) + c \cdot c) \Rightarrow \text{exponent}(\text{exponent}(p, a) + \text{exponent}(q, a), b) \cdot \text{exponent}(\text{exponent}(r, b) + \text{exponent}(s, b), a) = \text{exponent}(\text{exponent}(p, b) + \text{exponent}(q, b), a) \cdot \text{exponent}(\text{exponent}(r, a) + \text{exponent}(s, a), b))$ fof(wilkie, conjecture)

NUM374+2.p Disprove Wilkie identity from Tarski's identities
$\forall x, y$: $x + y = y + x$ fof(sum_symmetry, axiom)
$\forall x, y, z$: $x + (y + z) = (x + y) + z$ fof(sum_associativity, axiom)
$\forall x$: $x \cdot n_1 = x$ fof(product_identity, axiom)
$\forall x, y$: $x \cdot y = y \cdot x$ fof(product_symmetry, axiom)
$\forall x, y, z$: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ fof(product_associativity, axiom)
$\forall x, y, z$: $x \cdot (y + z) = x \cdot y + x \cdot z$ fof(sum_product_distribution, axiom)
$\forall x$: $\text{exponent}(n_1, x) = n_1$ fof(exponent_n$_1$, axiom)
$\forall x$: $\text{exponent}(x, n_1) = x$ fof(exponent_identity, axiom)
$\forall x, y, z$: $\text{exponent}(x, y + z) = \text{exponent}(x, y) \cdot \text{exponent}(x, z)$ fof(exponent_sum_product, axiom)

$\forall x, y, z$: exponent$(x \cdot y, z) =$ exponent$(x, z) \cdot$ exponent(y, z) fof(exponent_product_distribution, axiom)
$\forall x, y, z$: exponent(exponent$(x, y), z) =$ exponent$(x, y \cdot z)$ fof(exponent_exponent, axiom)
$\forall c, p, q, r, s, b$: (lemmas$(c, p, q, r, s, b) \iff (n_2 = n_1 + n_1$ and $b \neq n_0$ and $b \neq n_1$ and $b \neq n_2$ and $\forall x$: $b \neq n_0 \cdot x$ and $\forall x$: $p \neq q \cdot x$ and $\forall x$: $q \neq p \cdot x$ and $\forall x$: $r \neq s \cdot x$ and $\forall x$: $s \neq r \cdot x$ and $n_1 + n_0 \neq n_1$ and $n_2 + n_0 \neq n_1$ and $n_0 + n_0 \neq n_1$ and $c \neq n_1$ and $n_1 + c \neq n_1$ and $c \cdot n_0 \neq n_1$ and $n_1 + n_0 \neq n_0$ and $n_2 + n_0 \neq n_0$ and $n_0 + n_0 \neq n_0$ and $c \neq n_0$ and $n_1 + c \neq n_0$ and $n_2 + n_0 \neq n_1 + n_0$ and $c \neq n_1 + n_0$ and $c \cdot n_0 \neq n_1 + n_0$ and $c \neq n_2 + n_0$ and $c \neq n_0 + n_0$ and $n_1 + c \neq c))$ fof(lemmas, axiom)
$n_0 \neq n_1$ fof(n0_n$_1$, axiom)
$n_0 \neq n_2$ fof(n0_n$_2$, axiom)
$n_1 \neq n_2$ fof(n1_n$_2$, axiom)
$\forall c, p, q, r, s, a, b$: $((c = a \cdot a$ and $p = n_1 + a$ and $q = p + c$ and $r = n_1 + a \cdot c$ and $s = (n_1 + c) + c \cdot c$ and lemmas$(c, p, q, r, s, b)) \Rightarrow$ exponent(exponent$(p, a)+$exponent$(q, a), b)\cdot$exponent(exponent$(r, b)+$exponent$(s, b), a) =$ exponent(exponent$(p, b)+$ exponent$(q, b), a) \cdot$ exponent(exponent$(r, a) +$ exponent$(s, a), b))$ fof(wilkie, conjecture)

NUM374+3.p Disprove Wilkie identity from Tarski's identities
$\forall x, y$: $x + y = y + x$ fof(sum_symmetry, axiom)
$\forall x, y, z$: $x + (y + z) = (x + y) + z$ fof(sum_associativity, axiom)
$\forall x$: $x \cdot n_1 = x$ fof(product_identity, axiom)
$\forall x, y$: $x \cdot y = y \cdot x$ fof(product_symmetry, axiom)
$\forall x, y, z$: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ fof(product_associativity, axiom)
$\forall x, y, z$: $x \cdot (y + z) = x \cdot y + x \cdot z$ fof(sum_product_distribution, axiom)
$\forall x$: exponent$(n_1, x) = n_1$ fof(exponent_n$_1$, axiom)
$\forall x$: exponent$(x, n_1) = x$ fof(exponent_identity, axiom)
$\forall x, y, z$: exponent$(x, y + z) =$ exponent$(x, y) \cdot$ exponent(x, z) fof(exponent_sum_product, axiom)
$\forall x, y, z$: exponent$(x \cdot y, z) =$ exponent$(x, z) \cdot$ exponent(y, z) fof(exponent_product_distribution, axiom)
$\forall x, y, z$: exponent(exponent$(x, y), z) =$ exponent$(x, y \cdot z)$ fof(exponent_exponent, axiom)
$n_0 \neq n_1$ fof(n0_n$_1$, axiom)
$\forall c, p, q, r, s, b$: $((c = n_0 \cdot n_0$ and $p = n_1 + n_0$ and $q = p + c$ and $r = n_1 + n_0 \cdot c$ and $s = (n_1 + c) + c \cdot c) \Rightarrow$ exponent(exponent$(p, n_0)+$exponent$(q, n_0), b)\cdot$exponent(exponent$(r, b)+$exponent$(s, b), n_0) =$ exponent(exponent$(p, b)+$ exponent$(q, b), n_0) \cdot$ exponent(exponent$(r, n_0) +$ exponent$(s, n_0), b))$ fof(wilkie, conjecture)

NUM374+4.p Disprove Wilkie identity from Tarski's identities
$\forall x, y$: $x + y = y + x$ fof(sum_symmetry, axiom)
$\forall x, y, z$: $x + (y + z) = (x + y) + z$ fof(sum_associativity, axiom)
$\forall x$: $x \cdot n_1 = x$ fof(product_identity, axiom)
$\forall x, y$: $x \cdot y = y \cdot x$ fof(product_symmetry, axiom)
$\forall x, y, z$: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ fof(product_associativity, axiom)
$\forall x, y, z$: $x \cdot (y + z) = x \cdot y + x \cdot z$ fof(sum_product_distribution, axiom)
$\forall x$: exponent$(n_1, x) = n_1$ fof(exponent_n$_1$, axiom)
$\forall x$: exponent$(x, n_1) = x$ fof(exponent_identity, axiom)
$\forall x, y, z$: exponent$(x, y + z) =$ exponent$(x, y) \cdot$ exponent(x, z) fof(exponent_sum_product, axiom)
$\forall x, y, z$: exponent$(x \cdot y, z) =$ exponent$(x, z) \cdot$ exponent(y, z) fof(exponent_product_distribution, axiom)
$\forall x, y, z$: exponent(exponent$(x, y), z) =$ exponent$(x, y \cdot z)$ fof(exponent_exponent, axiom)
$\forall c, p, q, r, s, b$: (lemmas$(c, p, q, r, s, b) \iff (n_2 = n_1 + n_1$ and $b \neq n_0$ and $b \neq n_1$ and $b \neq n_2$ and $\forall x$: $b \neq n_0 \cdot x$ and $\forall x$: $p \neq q \cdot x$ and $\forall x$: $q \neq p \cdot x$ and $\forall x$: $r \neq s \cdot x$ and $\forall x$: $s \neq r \cdot x$ and $n_1 + n_0 \neq n_1$ and $n_2 + n_0 \neq n_1$ and $n_0 + n_0 \neq n_1$ and $c \neq n_1$ and $n_1 + c \neq n_1$ and $c \cdot n_0 \neq n_1$ and $n_1 + n_0 \neq n_0$ and $n_2 + n_0 \neq n_0$ and $n_0 + n_0 \neq n_0$ and $c \neq n_0$ and $n_1 + c \neq n_0$ and $n_2 + n_0 \neq n_1 + n_0$ and $c \neq n_1 + n_0$ and $c \cdot n_0 \neq n_1 + n_0$ and $c \neq n_2 + n_0$ and $c \neq n_0 + n_0$ and $n_1 + c \neq c))$ fof(lemmas, axiom)
$n_0 \neq n_1$ fof(n0_n$_1$, axiom)
$n_0 \neq n_2$ fof(n0_n$_2$, axiom)
$n_1 \neq n_2$ fof(n1_n$_2$, axiom)
$\forall c, p, q, r, s, b$: $((c = n_0 \cdot n_0$ and $p = n_1 + n_0$ and $q = p + c$ and $r = n_1 + n_0 \cdot c$ and $s = (n_1 + c) + c \cdot c$ and lemmas$(c, p, q, r, s, b)) \Rightarrow$ exponent(exponent$(p, n_0)+$exponent$(q, n_0), b)\cdot$exponent(exponent$(r, b)+$exponent$(s, b), n_0) =$ exponent(exponent$(p, b)+$ exponent$(q, b), n_0) \cdot$ exponent(exponent$(r, n_0) +$ exponent$(s, n_0), b))$ fof(wilkie, conjecture)

NUM380+1.p Ordinal numbers, theorem 4
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty$(a) \Rightarrow$ function$(a))$ fof(cc1_funct$_1$, axiom)
$\forall a$: (empty$(a) \Rightarrow$ relation$(a))$ fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function$(a)) \Rightarrow$ (relation(a) and function(a) and one_to_one$(a)))$ fof(cc2_funct$_1$, axio

$\forall a, b, c, d, e$: ($e =$ unordered_quadruple(a, b, c, d) $\iff$ $\forall f$: (in(f, e) $\iff$ $\neg f \neq a$ and $f \neq b$ and $f \neq c$ and $f \neq d$)) fof(d2_enumset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b, c, d$: $\neg$ in(a, b) and in(b, c) and in(c, d) and in(d, a) fof(t4_ordinal$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $\forall c$: $\neg$ in(c, b) and $\forall d$: $\neg$ in(d, b) and in(d, c) fof(t7_tarski, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM381+1.p Ordinal numbers, theorem 5
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b, c, d, e, f$: ($f =$ unordered_quintuple(a, b, c, d, e) $\iff$ $\forall g$: (in(g, f) $\iff$ $\neg g \neq a$ and $g \neq b$ and $g \neq c$ and $g \neq d$ and $g \neq e$)) fof(d3_enumset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b, c, d, e$: $\neg$ in(a, b) and in(b, c) and in(c, d) and in(d, e) and in(e, a) fof(t5_ordinal$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $\forall c$: $\neg$ in(c, b) and $\forall d$: $\neg$ in(d, b) and in(d, c) fof(t7_tarski, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM382+1.p Ordinal numbers, theorem 6
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b, c, d, e, f, g$: ($g =$ unordered_sextuple(a, b, c, d, e, f) $\iff$ $\forall h$: (in(h, g) $\iff$ $\neg h \neq a$ and $h \neq b$ and $h \neq c$ and $h \neq d$ and $h \neq e$ and $h \neq f$)) fof(d4_enumset$_1$, axiom)

$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b, c, d, e, f$: $\neg$ in(a, b) and in(b, c) and in(c, d) and in(d, e) and in(e, f) and in(f, a) fof(t6_ordinal$_1$, conjecture)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $\forall c$: $\neg$ in(c, b) and $\forall d$: $\neg$ in(d, b) and in(d, c) fof(t7_tarski, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM383+1.p Ordinal numbers, theorem 7
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element$(a$, powerset$(b))$ $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element$(b$, powerset$(c))$) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element$(b$, powerset$(c))$ and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $b \subseteq a$ fof(t7_ordinal$_1$, conjecture)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM385+1.p Ordinal numbers, theorem 12
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: succ$(a) =$ set_union$_2(a$, singleton$(a))$ fof(d1_ordinal$_1$, axiom)
$\forall a, b$: ($b =$ singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)

$\forall a, b, c$: (c = set_union$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_union$_2$(a, b))) fof(fc2_relat$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a$: in(a, succ(a)) fof(t10_ordinal$_1$, axiom)
$\forall a, b$: (succ(a) = succ(b) $\Rightarrow$ $a = b$) fof(t12_ordinal$_1$, conjecture)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM386+1.p Ordinal numbers, theorem 13
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2$(a, b) = set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: succ(a) = set_union$_2$(a, singleton(a)) fof(d1_ordinal$_1$, axiom)
$\forall a, b$: (b = singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: (c = set_union$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_union$_2$(a, b))) fof(fc2_relat$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: (in(a, succ(b)) $\iff$ (in(a, b) or $a = b$)) fof(t13_ordinal$_1$, conjecture)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)

$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM387+1.p Ordinal numbers, theorem 14
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty$(a) \Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty$(a) \Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: succ$(a) =$ set_union$_2(a,$ singleton$(a))$ fof(d1_ordinal$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_union$_2(a, b)$)) fof(fc2_relat$_1$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a$: in$(a,$ succ$(a))$ fof(t10_ordinal$_1$, axiom)
$\forall a$: $a \neq$ succ(a) fof(t14_ordinal$_1$, conjecture)
$\forall a$: set_union$_2(a,$ empty_set$) = a$ fof(t1_boole, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM388+1.p Ordinal numbers, theorem 19
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty$(a) \Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (ordinal$(a) \Rightarrow$ (epsilon_transitive(a) and epsilon_connected(a))) fof(cc1_ordinal$_1$, axiom)
$\forall a$: (empty$(a) \Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a$: ((epsilon_transitive(a) and epsilon_connected(a)) $\Rightarrow$ ordinal(a)) fof(cc2_ordinal$_1$, axiom)
$\forall a$: (epsilon_transitive$(a) \iff \forall b$: (in$(b, a) \Rightarrow b \subseteq a$)) fof(d2_ordinal$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)) fof(rc1_ordinal$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)

$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: (ordinal(a) $\Rightarrow$ $\forall b$: (ordinal(b) $\Rightarrow$ $\forall c$: (epsilon_transitive(c) $\Rightarrow$ ((in(c, a) and in(a, b)) $\Rightarrow$ in(c, b))))) fof(t19_ordinal$_1$,
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

NUM395+1.p Ordinal numbers, theorem 27
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a$: (ordinal(a) $\Rightarrow$ (epsilon_transitive(a) and epsilon_connected(a))) fof(cc1_ordinal$_1$, axiom)
$\forall a$: ((epsilon_transitive(a) and epsilon_connected(a)) $\Rightarrow$ ordinal(a)) fof(cc2_ordinal$_1$, axiom)
$\exists a$: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)) fof(rc1_ordinal$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
ordinal(empty_set) fof(t27_ordinal$_1$, conjecture)
$\forall a$: (ordinal(a) $\iff$ (epsilon_transitive(a) and epsilon_connected(a))) fof(d4_ordinal$_1$, axiom)
epsilon_transitive(empty_set) and epsilon_connected(empty_set) fof(l18_ordinal$_1$, axiom)

NUM415^1.p 2 * (3 + 7) = (2 * 5) * (1 + 1)
include('Axioms/NUM006^0.ax')
($\cdot$@two@(+@three@seven)) = ($\cdot$@($\cdot$@two@five)@(+@1@1)) thf(thm, conjecture)

NUM416^1.p 10 * (10 * 10) = (10 + 10) * (5 * 10)
include('Axioms/NUM006^0.ax')
($\cdot$@ten@($\cdot$@ten@ten)) = ($\cdot$@(+@ten@ten)@($\cdot$@five@ten)) thf(thm, conjecture)

NUM417^1.p (10 * 10) * (10 * 10) = ((10 * 10) * 10) * 10
include('Axioms/NUM006^0.ax')
($\cdot$@($\cdot$@ten@ten)@($\cdot$@ten@ten)) = ($\cdot$@($\cdot$@($\cdot$@ten@ten)@ten)@ten) thf(thm, conjecture)

NUM418^1.p Find N such that N + 3 = 3
include('Axioms/NUM006^0.ax')
$\exists n$: (\$i $\to$ \$i) $\to$ \$i $\to$ \$i: (+@n@three) = three thf(thm, conjecture)

NUM419^1.p Find N such that N + 3 = 4
include('Axioms/NUM006^0.ax')

$\exists n\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{i} \to \$\mathrm{i}\colon (+@n@\mathrm{three}) = \mathrm{four}$ thf(thm, conjecture)

NUM421+1.p Fuerstenberg's infinitude of primes 01, 00 expansion
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\mathrm{aInteger}_0(\mathrm{xa})$ fof(m_419, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{xa}, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, \mathrm{xa})$ fof(m__, conjecture)

NUM422+1.p Fuerstenberg's infinitude of primes 02, 00 expansion
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)
$\mathrm{aInteger}_0(\mathrm{xa})$ fof(m_446, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), \mathrm{xa}) = \mathrm{smndt}_0(\mathrm{xa})$ fof(m__, conjecture)

NUM423+1.p Fuerstenberg's infinitude of primes 03, 00 expansion
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, a

$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aDivisorOf$_0(w_1, w_0)$ $\iff$ (aInteger$_0(w_1)$ and $w_1 \neq$ sz$_{00}$ and $\exists w_2$: (aInteger$_0(w_2)$ and sdtasdt$_0$
w_0)))) fof(mDivisor, definition)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$ and $w_2 \neq$ sz$_{00}$) $\Rightarrow$ (sdteqdtlpzmzozddtrp$_0(w_0, w_1, w_2)$ $\Leftarrow$
aDivisorOf$_0(w_2$, sdtpldt$_0(w_0$, smndt$_0(w_1)))))$ fof(mEquMod, definition)
aInteger$_0$(xa) and aInteger$_0$(xq) and xq $\neq$ sz$_{00}$ fof(m$_{_671}$, hypothesis)
sdteqdtlpzmzozddtrp$_0$(xa, xa, xq) fof(m$_{__}$, conjecture)

NUM423+3.p Fuerstenberg's infinitude of primes 03, 02 expansion
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ \$true) fof(mIntegers, axiom)
aInteger$_0$(sz$_{00}$) fof(mIntZero, axiom)
aInteger$_0$(sz$_{10}$) fof(mIntOne, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ aInteger$_0$(smndt$_0(w_0)$)) fof(mIntNeg, axiom)
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ aInteger$_0$(sdtpldt$_0(w_0, w_1)$)) fof(mIntPlus, axiom)
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ aInteger$_0$(sdtasdt$_0(w_0, w_1)$)) fof(mIntMult, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ sdtpldt$_0(w_0$, sdtpldt$_0(w_1, w_2))$ = sdtpldt$_0$(sdtpldt$_0(w_0$
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ sdtpldt$_0(w_0, w_1)$ = sdtpldt$_0(w_1, w_0)$) fof(mAddComm, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtpldt$_0(w_0$, sz$_{00})$ = w_0 and w_0 = sdtpldt$_0$(sz$_{00}, w_0$))) fof(mAddZero, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtpldt$_0(w_0$, smndt$_0(w_0))$ = sz$_{00}$ and sz$_{00}$ = sdtpldt$_0$(smndt$_0(w_0), w_0$))) fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ sdtasdt$_0(w_0$, sdtasdt$_0(w_1, w_2))$ = sdtasdt$_0$(sdtasdt$_0(w$
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ sdtasdt$_0(w_0, w_1)$ = sdtasdt$_0(w_1, w_0)$) fof(mMulComm, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0(w_0$, sz$_{10})$ = w_0 and w_0 = sdtasdt$_0$(sz$_{10}, w_0$))) fof(mMulOne, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ (sdtasdt$_0(w_0$, sdtpldt$_0(w_1, w_2))$ = sdtpldt$_0$(sdtasdt$_0(w$
sdtpldt$_0$(sdtasdt$_0(w_0, w_2)$, sdtasdt$_0(w_1, w_2)$)))) fof(mDistrib, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0(w_0$, sz$_{00})$ = sz$_{00}$ and sz$_{00}$ = sdtasdt$_0$(sz$_{00}, w_0$))) fof(mMulZero, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0$(smndt$_0$(sz$_{10}$), w_0) = smndt$_0(w_0)$ and smndt$_0(w_0)$ = sdtasdt$_0(w_0$, smndt$_0$(sz$_{10}$)))) fof(m
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ (sdtasdt$_0(w_0, w_1)$ = sz$_{00}$ $\Rightarrow$ (w_0 = sz$_{00}$ or w_1 = sz$_{00}$))) fof(mZeroDiv, a
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aDivisorOf$_0(w_1, w_0)$ $\iff$ (aInteger$_0(w_1)$ and $w_1 \neq$ sz$_{00}$ and $\exists w_2$: (aInteger$_0(w_2)$ and sdtasdt$_0$
w_0)))) fof(mDivisor, definition)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$ and $w_2 \neq$ sz$_{00}$) $\Rightarrow$ (sdteqdtlpzmzozddtrp$_0(w_0, w_1, w_2)$ $\Leftarrow$
aDivisorOf$_0(w_2$, sdtpldt$_0(w_0$, smndt$_0(w_1)))))$ fof(mEquMod, definition)
aInteger$_0$(xa) and aInteger$_0$(xq) and xq $\neq$ sz$_{00}$ fof(m$_{_671}$, hypothesis)
$\exists w_0$: (aInteger$_0(w_0)$ and sdtasdt$_0$(xq, w_0) = sdtpldt$_0$(xa, smndt$_0$(xa))) or aDivisorOf$_0$(xq, sdtpldt$_0$(xa, smndt$_0$(xa))) or sdteq

NUM424+1.p Fuerstenberg's infinitude of primes 04, 00 expansion
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ \$true) fof(mIntegers, axiom)
aInteger$_0$(sz$_{00}$) fof(mIntZero, axiom)
aInteger$_0$(sz$_{10}$) fof(mIntOne, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ aInteger$_0$(smndt$_0(w_0)$)) fof(mIntNeg, axiom)
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ aInteger$_0$(sdtpldt$_0(w_0, w_1)$)) fof(mIntPlus, axiom)
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ aInteger$_0$(sdtasdt$_0(w_0, w_1)$)) fof(mIntMult, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ sdtpldt$_0(w_0$, sdtpldt$_0(w_1, w_2))$ = sdtpldt$_0$(sdtpldt$_0(w_0$
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ sdtpldt$_0(w_0, w_1)$ = sdtpldt$_0(w_1, w_0)$) fof(mAddComm, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtpldt$_0(w_0$, sz$_{00})$ = w_0 and w_0 = sdtpldt$_0$(sz$_{00}, w_0$))) fof(mAddZero, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtpldt$_0(w_0$, smndt$_0(w_0))$ = sz$_{00}$ and sz$_{00}$ = sdtpldt$_0$(smndt$_0(w_0), w_0$))) fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ sdtasdt$_0(w_0$, sdtasdt$_0(w_1, w_2))$ = sdtasdt$_0$(sdtasdt$_0(w$
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ sdtasdt$_0(w_0, w_1)$ = sdtasdt$_0(w_1, w_0)$) fof(mMulComm, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0(w_0$, sz$_{10})$ = w_0 and w_0 = sdtasdt$_0$(sz$_{10}, w_0$))) fof(mMulOne, axiom)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$) $\Rightarrow$ (sdtasdt$_0(w_0$, sdtpldt$_0(w_1, w_2))$ = sdtpldt$_0$(sdtasdt$_0(w$
sdtpldt$_0$(sdtasdt$_0(w_0, w_2)$, sdtasdt$_0(w_1, w_2)$)))) fof(mDistrib, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0(w_0$, sz$_{00})$ = sz$_{00}$ and sz$_{00}$ = sdtasdt$_0$(sz$_{00}, w_0$))) fof(mMulZero, axiom)
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ (sdtasdt$_0$(smndt$_0$(sz$_{10}$), w_0) = smndt$_0(w_0)$ and smndt$_0(w_0)$ = sdtasdt$_0(w_0$, smndt$_0$(sz$_{10}$)))) fof(m
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$) $\Rightarrow$ (sdtasdt$_0(w_0, w_1)$ = sz$_{00}$ $\Rightarrow$ (w_0 = sz$_{00}$ or w_1 = sz$_{00}$))) fof(mZeroDiv, a
$\forall w_0$: (aInteger$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aDivisorOf$_0(w_1, w_0)$ $\iff$ (aInteger$_0(w_1)$ and $w_1 \neq$ sz$_{00}$ and $\exists w_2$: (aInteger$_0(w_2)$ and sdtasdt$_0$
w_0)))) fof(mDivisor, definition)
$\forall w_0, w_1, w_2$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and aInteger$_0(w_2)$ and $w_2 \neq$ sz$_{00}$) $\Rightarrow$ (sdteqdtlpzmzozddtrp$_0(w_0, w_1, w_2)$ $\Leftarrow$
aDivisorOf$_0(w_2$, sdtpldt$_0(w_0$, smndt$_0(w_1)))))$ fof(mEquMod, definition)
$\forall w_0, w_1$: ((aInteger$_0(w_0)$ and aInteger$_0(w_1)$ and $w_1 \neq$ sz$_{00}$) $\Rightarrow$ sdteqdtlpzmzozddtrp$_0(w_0, w_0, w_1)$) fof(mEquModRef, axi
aInteger$_0$(xa) and aInteger$_0$(xb) and aInteger$_0$(xq) and xq $\neq$ sz$_{00}$ fof(m$_{_704}$, hypothesis)
sdteqdtlpzmzozddtrp$_0$(xa, xb, xq) $\Rightarrow$ sdteqdtlpzmzozddtrp$_0$(xb, xa, xq) fof(m$_{__}$, conjecture)

NUM425+1.p Fuerstenberg's infinitude of primes 04_01, 00 expansion

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0$

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, a

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aDivisorOf}_0(w_1, w_0) \iff (\mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00} \text{ and } \exists w_2\colon (\mathrm{aInteger}_0(w_2) \text{ and } \mathrm{sdtasdt}_0$
$w_0))))$ fof(mDivisor, definition)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Longleftarrow$
$\mathrm{aDivisorOf}_0(w_2, \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_1)))))$ fof(mEquMod, definition)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00}) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_0, w_1))$ fof(mEquModRef, axi

$\mathrm{aInteger}_0(\mathrm{xa}) \text{ and } \mathrm{aInteger}_0(\mathrm{xb}) \text{ and } \mathrm{aInteger}_0(\mathrm{xq}) \text{ and } \mathrm{xq} \neq \mathrm{sz}_{00}$ fof(m$_{_704}$, hypothesis)

$\mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xa}, \mathrm{xb}, \mathrm{xq})$ fof(m$_{_724}$, hypothesis)

$\exists w_0\colon (\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{xq}, w_0) = \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb})))$ fof(m$_{__}$, conjecture)

NUM425+3.p Fuerstenberg's infinitude of primes 04_01, 02 expansion

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0$

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, a

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aDivisorOf}_0(w_1, w_0) \iff (\mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00} \text{ and } \exists w_2\colon (\mathrm{aInteger}_0(w_2) \text{ and } \mathrm{sdtasdt}_0$
$w_0))))$ fof(mDivisor, definition)

$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Longleftarrow$
$\mathrm{aDivisorOf}_0(w_2, \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_1)))))$ fof(mEquMod, definition)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00}) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_0, w_1))$ fof(mEquModRef, axi

$\mathrm{aInteger}_0(\mathrm{xa}) \text{ and } \mathrm{aInteger}_0(\mathrm{xb}) \text{ and } \mathrm{aInteger}_0(\mathrm{xq}) \text{ and } \mathrm{xq} \neq \mathrm{sz}_{00}$ fof(m$_{_704}$, hypothesis)

$\exists w_0\colon (\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{xq}, w_0) = \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb}))) \text{ and } \mathrm{aDivisorOf}_0(\mathrm{xq}, \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb}))) \text{ and sd}$

$\exists w_0\colon (\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{xq}, w_0) = \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb})))$ fof(m$_{__}$, conjecture)

NUM426+1.p Fuerstenberg's infinitude of primes 04_02, 00 expansion

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)

$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)

$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, ax
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aDivisorOf}_0(w_1, w_0) \iff (\mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00} \text{ and } \exists w_2\colon (\mathrm{aInteger}_0(w_2) \text{ and } \mathrm{sdtasdt}_0$
$w_0))))$ fof(mDivisor, definition)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Leftarrow$
$\mathrm{aDivisorOf}_0(w_2, \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_1)))))$ fof(mEquMod, definition)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00}) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_0, w_1))$ fof(mEquModRef, axi
$\mathrm{aInteger}_0(\mathrm{xa}) \text{ and } \mathrm{aInteger}_0(\mathrm{xb}) \text{ and } \mathrm{aInteger}_0(\mathrm{xq}) \text{ and } \mathrm{xq} \neq \mathrm{sz}_{00}$ fof(m_704, hypothesis)
$\mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xa}, \mathrm{xb}, \mathrm{xq})$ fof(m_724, hypothesis)
$\mathrm{aInteger}_0(\mathrm{xn}) \text{ and } \mathrm{sdtasdt}_0(\mathrm{xq}, \mathrm{xn}) = \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb}))$ fof(m_747, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{xq}, \mathrm{smndt}_0(\mathrm{xn})) = \mathrm{sdtpldt}_0(\mathrm{xb}, \mathrm{smndt}_0(\mathrm{xa}))$ fof(m__, conjecture)

NUM427+1.p Fuerstenberg's infinitude of primes 04_03, 00 expansion
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, ax
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aDivisorOf}_0(w_1, w_0) \iff (\mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00} \text{ and } \exists w_2\colon (\mathrm{aInteger}_0(w_2) \text{ and } \mathrm{sdtasdt}_0$
$w_0))))$ fof(mDivisor, definition)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Leftarrow$
$\mathrm{aDivisorOf}_0(w_2, \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_1)))))$ fof(mEquMod, definition)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00}) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_0, w_1))$ fof(mEquModRef, axi
$\mathrm{aInteger}_0(\mathrm{xa}) \text{ and } \mathrm{aInteger}_0(\mathrm{xb}) \text{ and } \mathrm{aInteger}_0(\mathrm{xq}) \text{ and } \mathrm{xq} \neq \mathrm{sz}_{00}$ fof(m_704, hypothesis)
$\mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xa}, \mathrm{xb}, \mathrm{xq})$ fof(m_724, hypothesis)
$\mathrm{aInteger}_0(\mathrm{xn}) \text{ and } \mathrm{sdtasdt}_0(\mathrm{xq}, \mathrm{xn}) = \mathrm{sdtpldt}_0(\mathrm{xa}, \mathrm{smndt}_0(\mathrm{xb}))$ fof(m_747, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{xq}, \mathrm{smndt}_0(\mathrm{xn})) = \mathrm{sdtpldt}_0(\mathrm{xb}, \mathrm{smndt}_0(\mathrm{xa}))$ fof(m_767, hypothesis)
$\mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xb}, \mathrm{xa}, \mathrm{xq})$ fof(m__, conjecture)

NUM428+1.p Fuerstenberg's infinitude of primes 05, 00 expansion
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mIntegers, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{00})$ fof(mIntZero, axiom)
$\mathrm{aInteger}_0(\mathrm{sz}_{10})$ fof(mIntOne, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \mathrm{aInteger}_0(\mathrm{smndt}_0(w_0)))$ fof(mIntNeg, axiom)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mIntPlus, axiom)

$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{aInteger}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mIntMult, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddNeg, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) = \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulOne, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistrib, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mMulZero, axiom)
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(m
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ or } w_1 = \mathrm{sz}_{00})))$ fof(mZeroDiv, a
$\forall w_0\colon (\mathrm{aInteger}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aDivisorOf}_0(w_1, w_0) \iff (\mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00} \text{ and } \exists w_2\colon (\mathrm{aInteger}_0(w_2) \text{ and } \mathrm{sdtasdt}$
$w_0))))$ fof(mDivisor, definition)
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Leftarrow$
$\mathrm{aDivisorOf}_0(w_2, \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_1)))))$ fof(mEquMod, definition)
$\forall w_0, w_1\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } w_1 \neq \mathrm{sz}_{00}) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_0, w_1))$ fof(mEquModRef, axi
$\forall w_0, w_1, w_2\colon ((\mathrm{aInteger}_0(w_0) \text{ and } \mathrm{aInteger}_0(w_1) \text{ and } \mathrm{aInteger}_0(w_2) \text{ and } w_2 \neq \mathrm{sz}_{00}) \Rightarrow (\mathrm{sdteqdtlpzmzozddtrp}_0(w_0, w_1, w_2) \Rightarrow$
$\mathrm{sdteqdtlpzmzozddtrp}_0(w_1, w_0, w_2)))$ fof(mEquModSym, axiom)
$\mathrm{aInteger}_0(\mathrm{xa}) \text{ and } \mathrm{aInteger}_0(\mathrm{xb}) \text{ and } \mathrm{aInteger}_0(\mathrm{xq}) \text{ and } \mathrm{xq} \neq \mathrm{sz}_{00} \text{ and } \mathrm{aInteger}_0(\mathrm{xc})$ fof(m$_{818}$, hypothesis)
$(\mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xa}, \mathrm{xb}, \mathrm{xq}) \text{ and } \mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xb}, \mathrm{xc}, \mathrm{xq})) \Rightarrow \mathrm{sdteqdtlpzmzozddtrp}_0(\mathrm{xa}, \mathrm{xc}, \mathrm{xq})$ fof(m$_{--}$, c

NUM457+1.p Square root of a prime is irrational 01, 00 expansion
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mNatSort, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{sz}_{00})$ fof(mSortsC, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{sz}_{10}) \text{ and } \mathrm{sz}_{10} \neq \mathrm{sz}_{00}$ fof(mSortsC$_{01}$, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow \mathrm{aNaturalNumber}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSortsB, axi
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow \mathrm{aNaturalNumber}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mSortsB$_{02}$, a
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm,
$\forall w_0, w_1, w_2\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1) \text{ and } \mathrm{aNaturalNumber}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1),$
$\mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)))$ fof(mAddAsso, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(m_AddZero, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm,
$\forall w_0, w_1, w_2\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1) \text{ and } \mathrm{aNaturalNumber}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1),$
$\mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)))$ fof(mMulAsso, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(m_MulUnit, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, w_0)))$ fof(m_MulZero, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1) \text{ and } \mathrm{aNaturalNumber}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1,$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_1), \mathrm{sdtasdt}_0(w_0, w_2)) \text{ and } \mathrm{sdtasdt}_0(\mathrm{sdtpldt}_0(w_1, w_2), w_0) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_1, w_0), \mathrm{sdtasdt}_0(w_2, w_0$
$\forall w_0, w_1, w_2\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1) \text{ and } \mathrm{aNaturalNumber}_0(w_2)) \Rightarrow ((\mathrm{sdtpldt}_0(w_0, w_1) =$
$\mathrm{sdtpldt}_0(w_0, w_2) \text{ or } \mathrm{sdtpldt}_0(w_1, w_0) = \mathrm{sdtpldt}_0(w_2, w_0)) \Rightarrow w_1 = w_2))$ fof(mAddCanc, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (w_0 \neq \mathrm{sz}_{00} \Rightarrow \forall w_1, w_2\colon ((\mathrm{aNaturalNumber}_0(w_1) \text{ and } \mathrm{aNaturalNumber}_0(w_2)) \Rightarrow$
$((\mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_0, w_2) \text{ or } \mathrm{sdtasdt}_0(w_1, w_0) = \mathrm{sdtasdt}_0(w_2, w_0)) \Rightarrow w_1 = w_2))))$ fof(mMulCanc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sz}_{00} \Rightarrow (w_0 = \mathrm{sz}_{00} \text{ and } w_1 =$
$\mathrm{sz}_{00})))$ fof(mZeroAdd, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{xm}) \text{ and } \mathrm{aNaturalNumber}_0(\mathrm{xn})$ fof(m$_{624}$, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{xm}, \mathrm{xn}) = \mathrm{sz}_{00} \Rightarrow (\mathrm{xm} = \mathrm{sz}_{00} \text{ or } \mathrm{xn} = \mathrm{sz}_{00})$ fof(m$_{--}$, conjecture)

NUM531+1.p Ramsey's Infinite Theorem 01, 00 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof(m$_{--}$, conjecture)

NUM531+2.p Ramsey's Infinite Theorem 01, 01 expansion

$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isFinite}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isCountable}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; \neg\,\neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0) \text{ and } w_0 = \mathrm{slcrc}_0)$ fof(m__, conjecture)

NUM532+1.p Ramsey's Infinite Theorem 02, 00 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isFinite}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isCountable}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; w_0 \neq \mathrm{slcrc}_0)$ fof($\mathrm{mCountNFin}_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2, w_1) \;\Rightarrow\; \mathrm{aElementOf}_0(w_2, w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\mathrm{aSet}_0(\mathrm{xA})$ fof(m_{-467}, hypothesis)
$\mathrm{aSubsetOf}_0(\mathrm{xA}, \mathrm{xA})$ fof(m__, conjecture)

NUM533+1.p Ramsey's Infinite Theorem 03, 00 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isFinite}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isCountable}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; w_0 \neq \mathrm{slcrc}_0)$ fof($\mathrm{mCountNFin}_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2, w_1) \;\Rightarrow\; \mathrm{aElementOf}_0(w_2, w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \mathrm{aSubsetOf}_0(w_0, w_0))$ fof(mSubRefl, axiom)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \;\Rightarrow\; ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_0)) \;\Rightarrow\; w_0 = w_1))$ fof(mSubASymm
$\mathrm{aSet}_0(\mathrm{xA}) \text{ and } \mathrm{aSet}_0(\mathrm{xB}) \text{ and } \mathrm{aSet}_0(\mathrm{xC})$ fof(m_{-522}, hypothesis)
$(\mathrm{aSubsetOf}_0(\mathrm{xA}, \mathrm{xB}) \text{ and } \mathrm{aSubsetOf}_0(\mathrm{xB}, \mathrm{xC})) \;\Rightarrow\; \mathrm{aSubsetOf}_0(\mathrm{xA}, \mathrm{xC})$ fof(m__, conjecture)

NUM533+2.p Ramsey's Infinite Theorem 03, 01 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \;\Rightarrow\; \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isFinite}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\,\exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; (\mathrm{isCountable}_0(w_0) \;\Rightarrow\; \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \;\Rightarrow\; w_0 \neq \mathrm{slcrc}_0)$ fof($\mathrm{mCountNFin}_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2, w_1) \;\Rightarrow\; \mathrm{aElementOf}_0(w_2, w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \;\Rightarrow\; \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \;\Rightarrow\; \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \;\Rightarrow\; \mathrm{aSubsetOf}_0(w_0, w_0))$ fof(mSubRefl, axiom)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \;\Rightarrow\; ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_0)) \;\Rightarrow\; w_0 = w_1))$ fof(mSubASymm
$\mathrm{aSet}_0(\mathrm{xA}) \text{ and } \mathrm{aSet}_0(\mathrm{xB}) \text{ and } \mathrm{aSet}_0(\mathrm{xC})$ fof(m_{-522}, hypothesis)
$(\forall w_0\colon (\mathrm{aElementOf}_0(w_0, \mathrm{xA}) \;\Rightarrow\; \mathrm{aElementOf}_0(w_0, \mathrm{xB})) \text{ and } \mathrm{aSubsetOf}_0(\mathrm{xA}, \mathrm{xB}) \text{ and } \forall w_0\colon (\mathrm{aElementOf}_0(w_0, \mathrm{xB}) \;\Rightarrow$
$\mathrm{aElementOf}_0(w_0, \mathrm{xC})) \text{ and } \mathrm{aSubsetOf}_0(\mathrm{xB}, \mathrm{xC})) \;\Rightarrow\; (\forall w_0\colon (\mathrm{aElementOf}_0(w_0, \mathrm{xA}) \;\Rightarrow\; \mathrm{aElementOf}_0(w_0, \mathrm{xC})) \text{ or } \mathrm{aSubsetOf}_0(\mathrm{x}$

NUM534+1.p Ramsey's Infinite Theorem 04, 00 expansion

$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1,w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\exists w_1\colon \mathrm{aElementOf}_0(w_1,w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof(mCountNFin$_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2,w_1) \Rightarrow \mathrm{aElementOf}_0(w_2,w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \Rightarrow \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \mathrm{aSubsetOf}_0(w_0,w_0))$ fof(mSubRefl, axiom)
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0,w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1,w_0)) \Rightarrow w_0 = w_1))$ fof(mSubASymm
$\forall w_0,w_1,w_2\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1) \text{ and } \mathrm{aSet}_0(w_2)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0,w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1,w_2)) \Rightarrow \mathrm{aSubsetOf}_0(w$
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtpldt}_0(w_0,w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3,w_2)$
$(\mathrm{aElement}_0(w_3) \text{ and } (\mathrm{aElementOf}_0(w_3,w_0) \text{ or } w_3 = w_1))))))$ fof(mDefCons, definition)
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtmndt}_0(w_0,w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3,w_2$
$(\mathrm{aElement}_0(w_3) \text{ and } \mathrm{aElementOf}_0(w_3,w_0) \text{ and } w_3 \neq w_1)))))$ fof(mDefDiff, definition)
$\mathrm{aSet}_0(\mathrm{xS})$ fof(m_617, hypothesis)
$\mathrm{aElementOf}_0(\mathrm{xx},\mathrm{xS})$ fof(m__617$_{02}$, hypothesis)
$\mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx}),\mathrm{xx}) = \mathrm{xS}$ fof(m__, conjecture)

NUM534+2.p Ramsey's Infinite Theorem 04, 01 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1,w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\exists w_1\colon \mathrm{aElementOf}_0(w_1,w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof(mCountNFin$_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2,w_1) \Rightarrow \mathrm{aElementOf}_0(w_2,w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \Rightarrow \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \mathrm{aSubsetOf}_0(w_0,w_0))$ fof(mSubRefl, axiom)
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0,w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1,w_0)) \Rightarrow w_0 = w_1))$ fof(mSubASymm
$\forall w_0,w_1,w_2\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1) \text{ and } \mathrm{aSet}_0(w_2)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0,w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1,w_2)) \Rightarrow \mathrm{aSubsetOf}_0(w$
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtpldt}_0(w_0,w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3,w_2)$
$(\mathrm{aElement}_0(w_3) \text{ and } (\mathrm{aElementOf}_0(w_3,w_0) \text{ or } w_3 = w_1))))))$ fof(mDefCons, definition)
$\forall w_0,w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtmndt}_0(w_0,w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3,w_2$
$(\mathrm{aElement}_0(w_3) \text{ and } \mathrm{aElementOf}_0(w_3,w_0) \text{ and } w_3 \neq w_1)))))$ fof(mDefDiff, definition)
$\mathrm{aSet}_0(\mathrm{xS})$ fof(m_617, hypothesis)
$\mathrm{aElementOf}_0(\mathrm{xx},\mathrm{xS})$ fof(m__617$_{02}$, hypothesis)
$(\mathrm{aSet}_0(\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx})) \text{ and } \forall w_0\colon (\mathrm{aElementOf}_0(w_0,\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx})) \iff (\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElementOf}_0(w_0,\mathrm{xS}) \text{ and}$
$\mathrm{xx}))) \Rightarrow ((\mathrm{aSet}_0(\mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx}),\mathrm{xx})) \text{ and } \forall w_0\colon (\mathrm{aElementOf}_0(w_0,\mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx}),\mathrm{xx})) \iff$
$(\mathrm{aElement}_0(w_0) \text{ and } (\mathrm{aElementOf}_0(w_0,\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx})) \text{ or } w_0 = \mathrm{xx})))) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS},\mathrm{xx}),\mathrm{xx}) =$
$\mathrm{xS})$ fof(m__, conjecture)

NUM535+1.p Ramsey's Infinite Theorem 04_01, 00 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1,w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg\exists w_1\colon \mathrm{aElementOf}_0(w_1,w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg\,\mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof(mCountNFin$_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2,w_1) \Rightarrow \mathrm{aElementOf}_0(w_2,w_0)))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1,w_0) \Rightarrow \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)

$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \mathrm{aSubsetOf}_0(w_0, w_0))$ fof(mSubRefl, axiom)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_0)) \Rightarrow w_0 = w_1))$ fof(mSubASymm
$\forall w_0, w_1, w_2\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1) \text{ and } \mathrm{aSet}_0(w_2)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_2)) \Rightarrow \mathrm{aSubsetOf}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtpldt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2)$
$(\mathrm{aElement}_0(w_3) \text{ and } (\mathrm{aElementOf}_0(w_3, w_0) \text{ or } w_3 = w_1))))))$ fof(mDefCons, definition)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtmndt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2$
$(\mathrm{aElement}_0(w_3) \text{ and } \mathrm{aElementOf}_0(w_3, w_0) \text{ and } w_3 \neq w_1)))))$ fof(mDefDiff, definition)
$\mathrm{aSet}_0(\mathrm{xS})$ fof($\mathrm{m}_{\_617}$, hypothesis)
$\mathrm{aElementOf}_0(\mathrm{xx}, \mathrm{xS})$ fof($\mathrm{m}\_\_617_{02}$, hypothesis)
$\mathrm{aSubsetOf}_0(\mathrm{xS}, \mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx})) \text{ and } \mathrm{aSubsetOf}_0(\mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx}), \mathrm{xS})$ fof(m__, conjecture)

NUM536+1.p Ramsey's Infinite Theorem 05, 00 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg \exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg \mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof($\mathrm{mCountNFin}_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2, w_1) \Rightarrow \mathrm{aElementOf}_0(w_2, w_0))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \Rightarrow \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \mathrm{aSubsetOf}_0(w_0, w_0))$ fof(mSubRefl, axiom)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_0)) \Rightarrow w_0 = w_1))$ fof(mSubASymm
$\forall w_0, w_1, w_2\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1) \text{ and } \mathrm{aSet}_0(w_2)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_2)) \Rightarrow \mathrm{aSubsetOf}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtpldt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2)$
$(\mathrm{aElement}_0(w_3) \text{ and } (\mathrm{aElementOf}_0(w_3, w_0) \text{ or } w_3 = w_1))))))$ fof(mDefCons, definition)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtmndt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2$
$(\mathrm{aElement}_0(w_3) \text{ and } \mathrm{aElementOf}_0(w_3, w_0) \text{ and } w_3 \neq w_1)))))$ fof(mDefDiff, definition)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(w_0, w_1), w_1) = w_0))$ fof(mConsDiff, axiom)
$\mathrm{aElement}_0(\mathrm{xx}) \text{ and } \mathrm{aSet}_0(\mathrm{xS})$ fof($\mathrm{m}_{\_679}$, hypothesis)
$\neg \mathrm{aElementOf}_0(\mathrm{xx}, \mathrm{xS})$ fof($\mathrm{m}\_\_679_{02}$, hypothesis)
$\mathrm{sdtmndt}_0(\mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx}) = \mathrm{xS}$ fof(m__, conjecture)

NUM536+2.p Ramsey's Infinite Theorem 05, 01 expansion
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mSetSort, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \Rightarrow \mathrm{aElement}_0(w_1)))$ fof(mEOfElem, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isFinite}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mFinRel, axiom)
$\forall w_0\colon (w_0 = \mathrm{slcrc}_0 \iff (\mathrm{aSet}_0(w_0) \text{ and } \neg \exists w_1\colon \mathrm{aElementOf}_0(w_1, w_0)))$ fof(mDefEmp, definition)
$\mathrm{isFinite}_0(\mathrm{slcrc}_0)$ fof(mEmpFin, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow (\mathrm{isCountable}_0(w_0) \Rightarrow \$\mathrm{true}))$ fof(mCntRel, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow \neg \mathrm{isFinite}_0(w_0))$ fof(mCountNFin, axiom)
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isCountable}_0(w_0)) \Rightarrow w_0 \neq \mathrm{slcrc}_0)$ fof($\mathrm{mCountNFin}_{01}$, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \iff (\mathrm{aSet}_0(w_1) \text{ and } \forall w_2\colon (\mathrm{aElementOf}_0(w_2, w_1) \Rightarrow \mathrm{aElementOf}_0(w_2, w_0))))$
$\forall w_0\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{isFinite}_0(w_0)) \Rightarrow \forall w_1\colon (\mathrm{aSubsetOf}_0(w_1, w_0) \Rightarrow \mathrm{isFinite}_0(w_1)))$ fof(mSubFSet, axiom)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \mathrm{aSubsetOf}_0(w_0, w_0))$ fof(mSubRefl, axiom)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_0)) \Rightarrow w_0 = w_1))$ fof(mSubASymm
$\forall w_0, w_1, w_2\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aSet}_0(w_1) \text{ and } \mathrm{aSet}_0(w_2)) \Rightarrow ((\mathrm{aSubsetOf}_0(w_0, w_1) \text{ and } \mathrm{aSubsetOf}_0(w_1, w_2)) \Rightarrow \mathrm{aSubsetOf}_0(w$
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtpldt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2)$
$(\mathrm{aElement}_0(w_3) \text{ and } (\mathrm{aElementOf}_0(w_3, w_0) \text{ or } w_3 = w_1))))))$ fof(mDefCons, definition)
$\forall w_0, w_1\colon ((\mathrm{aSet}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \forall w_2\colon (w_2 = \mathrm{sdtmndt}_0(w_0, w_1) \iff (\mathrm{aSet}_0(w_2) \text{ and } \forall w_3\colon (\mathrm{aElementOf}_0(w_3, w_2$
$(\mathrm{aElement}_0(w_3) \text{ and } \mathrm{aElementOf}_0(w_3, w_0) \text{ and } w_3 \neq w_1)))))$ fof(mDefDiff, definition)
$\forall w_0\colon (\mathrm{aSet}_0(w_0) \Rightarrow \forall w_1\colon (\mathrm{aElementOf}_0(w_1, w_0) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtmndt}_0(w_0, w_1), w_1) = w_0))$ fof(mConsDiff, axiom)
$\mathrm{aElement}_0(\mathrm{xx}) \text{ and } \mathrm{aSet}_0(\mathrm{xS})$ fof($\mathrm{m}_{\_679}$, hypothesis)
$\neg \mathrm{aElementOf}_0(\mathrm{xx}, \mathrm{xS})$ fof($\mathrm{m}\_\_679_{02}$, hypothesis)
$(\mathrm{aSet}_0(\mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx})) \text{ and } \forall w_0\colon (\mathrm{aElementOf}_0(w_0, \mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx})) \iff (\mathrm{aElement}_0(w_0) \text{ and } (\mathrm{aElementOf}_0(w_0, \mathrm{xS}) \text{ or } w$
$\mathrm{xx})))) \Rightarrow ((\mathrm{aSet}_0(\mathrm{sdtmndt}_0(\mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx})) \text{ and } \forall w_0\colon (\mathrm{aElementOf}_0(w_0, \mathrm{sdtmndt}_0(\mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx})) \iff$
$(\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElementOf}_0(w_0, \mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx})) \text{ and } w_0 \neq \mathrm{xx}))) \Rightarrow \mathrm{sdtmndt}_0(\mathrm{sdtpldt}_0(\mathrm{xS}, \mathrm{xx}), \mathrm{xx}) = \mathrm{xS})$ fof(m__

NUM537+1.p Ramsey's Infinite Theorem 05_01, 00 expansion
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ \$true) fof(mSetSort, axiom)
$\forall w_0$: (aElement$_0(w_0)$ $\Rightarrow$ \$true) fof(mElmSort, axiom)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aElementOf$_0(w_1, w_0)$ $\Rightarrow$ aElement$_0(w_1)$)) fof(mEOfElem, axiom)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ (isFinite$_0(w_0)$ $\Rightarrow$ \$true)) fof(mFinRel, axiom)
$\forall w_0$: (w_0 = slcrc$_0$ $\iff$ (aSet$_0(w_0)$ and $\neg\exists w_1$: aElementOf$_0(w_1, w_0)$)) fof(mDefEmp, definition)
isFinite$_0$(slcrc$_0$) fof(mEmpFin, axiom)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ (isCountable$_0(w_0)$ $\Rightarrow$ \$true)) fof(mCntRel, axiom)
$\forall w_0$: ((aSet$_0(w_0)$ and isCountable$_0(w_0)$) $\Rightarrow$ $\neg$ isFinite$_0(w_0)$) fof(mCountNFin, axiom)
$\forall w_0$: ((aSet$_0(w_0)$ and isCountable$_0(w_0)$) $\Rightarrow$ $w_0 \neq$ slcrc$_0$) fof(mCountNFin$_{01}$, axiom)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aSubsetOf$_0(w_1, w_0)$ $\iff$ (aSet$_0(w_1)$ and $\forall w_2$: (aElementOf$_0(w_2, w_1)$ $\Rightarrow$ aElementOf$_0(w_2, w_0)$))))
$\forall w_0$: ((aSet$_0(w_0)$ and isFinite$_0(w_0)$) $\Rightarrow$ $\forall w_1$: (aSubsetOf$_0(w_1, w_0)$ $\Rightarrow$ isFinite$_0(w_1)$)) fof(mSubFSet, axiom)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ aSubsetOf$_0(w_0, w_0)$) fof(mSubRefl, axiom)
$\forall w_0, w_1$: ((aSet$_0(w_0)$ and aSet$_0(w_1)$) $\Rightarrow$ ((aSubsetOf$_0(w_0, w_1)$ and aSubsetOf$_0(w_1, w_0)$) $\Rightarrow$ $w_0 = w_1$)) fof(mSubASymm
$\forall w_0, w_1, w_2$: ((aSet$_0(w_0)$ and aSet$_0(w_1)$ and aSet$_0(w_2)$) $\Rightarrow$ ((aSubsetOf$_0(w_0, w_1)$ and aSubsetOf$_0(w_1, w_2)$) $\Rightarrow$ aSubsetOf$_0$(
$\forall w_0, w_1$: ((aSet$_0(w_0)$ and aElement$_0(w_1)$) $\Rightarrow$ $\forall w_2$: (w_2 = sdtpldt$_0(w_0, w_1)$ $\iff$ (aSet$_0(w_2)$ and $\forall w_3$: (aElementOf$_0(w_3, w_2)$
(aElement$_0(w_3)$ and (aElementOf$_0(w_3, w_0)$ or $w_3 = w_1$)))))) fof(mDefCons, definition)
$\forall w_0, w_1$: ((aSet$_0(w_0)$ and aElement$_0(w_1)$) $\Rightarrow$ $\forall w_2$: (w_2 = sdtmndt$_0(w_0, w_1)$ $\iff$ (aSet$_0(w_2)$ and $\forall w_3$: (aElementOf$_0(w_3, w_2$
(aElement$_0(w_3)$ and aElementOf$_0(w_3, w_0)$ and $w_3 \neq w_1$))))) fof(mDefDiff, definition)
$\forall w_0$: (aSet$_0(w_0)$ $\Rightarrow$ $\forall w_1$: (aElementOf$_0(w_1, w_0)$ $\Rightarrow$ sdtpldt$_0$(sdtmndt$_0(w_0, w_1), w_1) = w_0$)) fof(mConsDiff, axiom)
aElement$_0$(xx) and aSet$_0$(xS) fof(m$_{-679}$, hypothesis)
$\neg$ aElementOf$_0$(xx, xS) fof(m__679$_{02}$, hypothesis)
aSubsetOf$_0$(xS, sdtmndt$_0$(sdtpldt$_0$(xS, xx), xx)) and aSubsetOf$_0$(sdtmndt$_0$(sdtpldt$_0$(xS, xx), xx), xS) fof(m__, conjecture)

NUM635^1.p Landau theorem 1
(suc x = suc y)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
$x \neq y$ thf(n, axiom)
suc: nat $\rightarrow$ nat thf(suc, type)
$\forall$xx: nat, xy: nat: ((suc@xx) = (suc@xy) $\Rightarrow$ xx = xy) thf(ax$_4$, axiom)
(suc@x) $\neq$ (suc@y) thf(satz$_1$, conjecture)

NUM635^2.p Landau theorem 1
1: \$i thf(one_type, type)
succ: \$i $\rightarrow$ \$i thf(succ_type, type)
$\forall x$: \$i: (succ@x) $\neq$ 1 thf(one_is_first, axiom)
$\forall x$: \$i, y: \$i: ((succ@x) = (succ@y) $\Rightarrow$ $x = y$) thf(succ_injective, axiom)
$\forall m$: \$i $\rightarrow$ \$o: ((m@1 and $\forall x$: \$i: ((m@x) $\Rightarrow$ (m@(succ@x)))) $\Rightarrow$ $\forall y$: \$i: (m@y)) thf(induction, axiom)
$\forall x$: \$i, y: \$i: ($x \neq y$ $\Rightarrow$ (succ@x) $\neq$ (succ@y)) thf(satz$_1$, conjecture)

NUM636^1.p Landau theorem 2
(suc x = x)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
suc: nat $\rightarrow$ nat thf(suc, type)
set: \$tType thf(set_type, type)
esti: nat $\rightarrow$ set $\rightarrow$ \$o thf(esti, type)
setof: (nat $\rightarrow$ \$o) $\rightarrow$ set thf(setof, type)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax$_5$, axio
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xx: nat: (suc@xx) $\neq n_1$ thf(ax$_3$, axiom)
$\forall$xx: nat, xy: nat: (xx $\neq$ xy $\Rightarrow$ (suc@xx) $\neq$ (suc@xy)) thf(satz$_1$, axiom)
(suc@x) $\neq x$ thf(satz$_2$, conjecture)

NUM636^2.p Landau theorem 2
1: \$i thf(one_type, type)
succ: \$i $\rightarrow$ \$i thf(succ_type, type)

$\forall x\colon \$\mathrm{i}\colon (\mathrm{succ}@x) \neq 1$ thf(one_is_first, axiom)
$\forall x\colon \$\mathrm{i}, y\colon \$\mathrm{i}\colon ((\mathrm{succ}@x) = (\mathrm{succ}@y) \Rightarrow x = y)$ thf(succ_injective, axiom)
$\forall m\colon \$\mathrm{i} \to \$\mathrm{o}\colon ((m@1 \text{ and } \forall x\colon \$\mathrm{i}\colon ((m@x) \Rightarrow (m@(\mathrm{succ}@x)))) \Rightarrow \forall y\colon \$\mathrm{i}\colon (m@y))$ thf(induction, axiom)
$\forall x\colon \$\mathrm{i}\colon (\mathrm{succ}@x) \neq x$ thf(satz$_2$, conjecture)

NUM636^3.p Landau theorem 2
$1\colon \$\mathrm{i}$ thf(one_type, type)
$\mathrm{succ}\colon \$\mathrm{i} \to \$\mathrm{i}$ thf(succ_type, type)
$\forall x\colon \$\mathrm{i}\colon (\mathrm{succ}@x) \neq 1$ thf(one_is_first, axiom)
$\forall x\colon \$\mathrm{i}, y\colon \$\mathrm{i}\colon ((\mathrm{succ}@x) = (\mathrm{succ}@y) \Rightarrow x = y)$ thf(succ_injective, axiom)
$\forall m\colon \$\mathrm{i} \to \$\mathrm{o}\colon ((m@1 \text{ and } \forall x\colon \$\mathrm{i}\colon ((m@x) \Rightarrow (m@(\mathrm{succ}@x)))) \Rightarrow \forall y\colon \$\mathrm{i}\colon (m@y))$ thf(induction, axiom)
$m\colon \$\mathrm{i} \to \$\mathrm{o}$ thf(m_type, type)
$m = (\lambda e\colon \$\mathrm{i}\colon (\mathrm{succ}@e) \neq e)$ thf(m_defn, definition)
$m@1$ thf(m_is_one, lemma)
$\forall x\colon \$\mathrm{i}\colon ((m@x) \Rightarrow (m@(\mathrm{succ}@x)))$ thf(m_is_next, lemma)
$\forall x\colon \$\mathrm{i}\colon (m@x)$ thf(m_is_all, conjecture)

NUM637^1.p Landau theorem 3
(forall x_0:nat. (x = suc x_0))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
n_1: nat thf(n_1, type)
$x \neq n_1$ thf(n, axiom)
suc: nat $\to$ nat thf(suc, type)
set: \$tType thf(set_type, type)
esti: nat $\to$ set $\to$ \$o thf(esti, type)
setof: (nat $\to$ \$o) $\to$ set thf(setof, type)
$\forall$xp: nat $\to$ \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax$_5$, axio
$\forall$xp: nat $\to$ \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xa: \$o: ($\neg\neg$xa $\Rightarrow$ xa) thf(et, axiom)
$\neg\forall$xx$_0$: nat: $x \neq$ (suc@xx$_0$) thf(satz$_3$, conjecture)

NUM637^2.p Landau theorem 3
$1\colon \$\mathrm{i}$ thf(one_type, type)
$\mathrm{succ}\colon \$\mathrm{i} \to \$\mathrm{i}$ thf(succ_type, type)
$\forall x\colon \$\mathrm{i}\colon (\mathrm{succ}@x) \neq 1$ thf(one_is_first, axiom)
$\forall x\colon \$\mathrm{i}, y\colon \$\mathrm{i}\colon ((\mathrm{succ}@x) = (\mathrm{succ}@y) \Rightarrow x = y)$ thf(succ_injective, axiom)
$\forall m\colon \$\mathrm{i} \to \$\mathrm{o}\colon ((m@1 \text{ and } \forall x\colon \$\mathrm{i}\colon ((m@x) \Rightarrow (m@(\mathrm{succ}@x)))) \Rightarrow \forall y\colon \$\mathrm{i}\colon (m@y))$ thf(induction, axiom)
$\forall x\colon \$\mathrm{i}\colon (x \neq 1 \Rightarrow \exists u\colon \$\mathrm{i}\colon x = (\mathrm{succ}@u))$ thf(satz$_3$, conjecture)

NUM638^1.p Landau theorem 3a
((forall x_0:nat.forall y:nat.x = suc x_0 $\to$ x = suc y $\to$ x_0 = y) $\to$ (some (lambda u.x = suc u)))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
n_1: nat thf(n_1, type)
$x \neq n_1$ thf(n, axiom)
suc: nat $\to$ nat thf(suc, type)
some: (nat $\to$ \$o) $\to$ \$o thf(some, type)
$\forall$xx: nat, xy: nat: ((suc@xx) = (suc@xy) $\Rightarrow$ xx = xy) thf(ax$_4$, axiom)
$\forall$xx: nat: (xx $\neq n_1$ $\Rightarrow$ (some@λxu: nat: xx = (suc@xu))) thf(satz$_3$, axiom)
$\neg\forall$xx$_0$: nat, xy: nat: (x = (suc@xx$_0$) $\Rightarrow$ (x = (suc@xy) $\Rightarrow$ xx$_0$ = xy)) $\Rightarrow$ $\neg$some@λxu: nat: x = (suc@xu) thf(satz3a, conjecture)

NUM639^1.p Landau theorem 4e
suc x = pl x n_1
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
suc: nat $\to$ nat thf(suc, type)
pl: nat $\to$ nat $\to$ nat thf(pl, type)
n_1: nat thf(n_1, type)
$\forall$xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)

(suc@x) = (pl@x@n_1) thf(satz4e, conjecture)

NUM640∧1.p Landau theorem 4f
suc (pl x y) = pl x (suc y)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat: (pl@xx@(suc@xy)) = (suc@(pl@xx@xy)) thf(satz4b, axiom)
(suc@(pl@x@y)) = (pl@x@(suc@y)) thf(satz4f, conjecture)

NUM641∧1.p Landau theorem 4g
suc x = pl n_1 x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
∀xx: nat: (pl@n_1@xx) = (suc@xx) thf(satz4c, axiom)
(suc@x) = (pl@n_1@x) thf(satz4g, conjecture)

NUM642∧1.p Landau theorem 4h
suc (pl x y) = pl (suc x) y
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat: (pl@(suc@xx)@xy) = (suc@(pl@xx@xy)) thf(satz4d, axiom)
(suc@(pl@x@y)) = (pl@(suc@x)@y) thf(satz4h, conjecture)

NUM643∧1.p Landau theorem 5
pl (pl x y) z = pl x (pl y z)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
pl: nat → nat → nat thf(pl, type)
set: $tType thf(set_type, type)
esti: nat → set → $o thf(esti, type)
setof: (nat → $o) → set thf(setof, type)
∀xp: nat → $o, xs: nat: ((esti@xs@(setof@xp)) ⇒ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat → nat thf(suc, type)
∀xs: set: ((esti@n_1@xs) ⇒ (∀xx: nat: ((esti@xx@xs) ⇒ (esti@(suc@xx)@xs)) ⇒ ∀xx: nat: (esti@xx@xs))) thf(ax_5, axior
∀xp: nat → $o, xs: nat: ((xp@xs) ⇒ (esti@xs@(setof@xp))) thf(estii, axiom)
∀xx: nat: (suc@xx) = (pl@xx@n_1) thf(satz4e, axiom)
∀xx: nat, xy: nat: (suc@(pl@xx@xy)) = (pl@xx@(suc@xy)) thf(satz4f, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
∀xx: nat, xy: nat: (pl@xx@(suc@xy)) = (suc@(pl@xx@xy)) thf(satz4b, axiom)
(pl@(pl@x@y)@z) = (pl@x@(pl@y@z)) thf($satz_5$, conjecture)

NUM644∧1.p Landau theorem 6
pl x y = pl y x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
set: $tType thf(set_type, type)
esti: nat → set → $o thf(esti, type)
setof: (nat → $o) → set thf(setof, type)

$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat $\rightarrow$ nat thf(suc, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax$_5$, axiom)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
$\forall$xx: nat: (pl@n_1@xx) = (suc@xx) thf(satz4c, axiom)
$\forall$xx: nat, xy: nat: (suc@(pl@xx@xy)) = (pl@xx@(suc@xy)) thf(satz4f, axiom)
$\forall$xx: nat, xy: nat: (pl@(suc@xx)@xy) = (suc@(pl@xx@xy)) thf(satz4d, axiom)
(pl@x@y) = (pl@y@x) thf(satz$_6$, conjecture)

NUM645^1.p Landau theorem 7
(y = pl x y)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat $\rightarrow$ nat $\rightarrow$ nat thf(pl, type)
set: \$tType thf(set_type, type)
esti: nat $\rightarrow$ set $\rightarrow$ \$o thf(esti, type)
setof: (nat $\rightarrow$ \$o) $\rightarrow$ set thf(setof, type)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat $\rightarrow$ nat thf(suc, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax$_5$, axiom)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xx: nat: (suc@xx) $\neq$ n_1 thf(ax$_3$, axiom)
$\forall$xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
$\forall$xx: nat, xy: nat: (xx $\neq$ xy $\Rightarrow$ (suc@xx) $\neq$ (suc@xy)) thf(satz$_1$, axiom)
$\forall$xx: nat, xy: nat: (pl@xx@(suc@xy)) = (suc@(pl@xx@xy)) thf(satz4b, axiom)
y $\neq$ (pl@x@y) thf(satz$_7$, conjecture)

NUM646^1.p Landau theorem 8
(pl x y = pl x z)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
$y \neq z$ thf(n, axiom)
pl: nat $\rightarrow$ nat $\rightarrow$ nat thf(pl, type)
set: \$tType thf(set_type, type)
esti: nat $\rightarrow$ set $\rightarrow$ \$o thf(esti, type)
setof: (nat $\rightarrow$ \$o) $\rightarrow$ set thf(setof, type)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat $\rightarrow$ nat thf(suc, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax$_5$, axiom)
$\forall$xp: nat $\rightarrow$ \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xx: nat, xy: nat: (xx $\neq$ xy $\Rightarrow$ (suc@xx) $\neq$ (suc@xy)) thf(satz$_1$, axiom)
$\forall$xx: nat: (suc@xx) = (pl@n_1@xx) thf(satz4g, axiom)
$\forall$xx: nat, xy: nat: (suc@(pl@xx@xy)) = (pl@(suc@xx)@xy) thf(satz4h, axiom)
(pl@x@y) $\neq$ (pl@x@z) thf(satz$_8$, conjecture)

NUM647^1.p Landau theorem 8a
y = z
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
pl: nat $\rightarrow$ nat $\rightarrow$ nat thf(pl, type)
(pl@x@y) = (pl@x@z) thf(i, axiom)
$\forall$xa: \$o: ($\neg\neg$xa $\Rightarrow$ xa) thf(et, axiom)

$\forall$xx: nat, xy: nat, xz: nat: (xy $\neq$ xz $\Rightarrow$ (pl@xx@xy) $\neq$ (pl@xx@xz)) thf($satz_8$, axiom)
$y = z$ thf(satz8a, conjecture)

NUM648^1.p Landau theorem 8b
(forall x_0:nat.forall y_0:nat.x = pl y x_0 → x = pl y y_0 → x_0 = y_0)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
$\forall$xx: nat, xy: nat, xz: nat: ((pl@xx@xy) = (pl@xx@xz) $\Rightarrow$ xy = xz) thf(satz8a, axiom)
$\forall xx_0$: nat, xy_0: nat: (x = (pl@y@xx_0) $\Rightarrow$ (x = (pl@y@xy_0) $\Rightarrow$ $xx_0 = xy_0$)) thf(satz8b, conjecture)

NUM649^1.p Landau theorem 9
(((x = y) → ((forall x_0:nat. (x = pl y x_0))) → (forall x_0:nat. (y = pl x x_0))) → (((x = y → ((forall x_0:nat. (x = pl y x_0)))) → ((((forall x_0:nat. (x = pl y x_0)) → ((forall x_0:nat. (y = pl x x_0)))) → ((forall x_0:nat. (y = pl x x_0)) → (x = y)))))))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
set: \$tType thf(set_type, type)
esti: nat → set → \$o thf(esti, type)
setof: (nat → \$o) → set thf(setof, type)
$\forall$xp: nat → \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat → nat thf(suc, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax_5, axiom
$\forall$xp: nat → \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xa: \$o: ($\neg\neg$xa $\Rightarrow$ xa) thf(et, axiom)
$\forall$xx: nat: (xx $\neq n_1$ $\Rightarrow$ $\neg\forall xx_0$: nat: xx $\neq$ (suc@xx_0)) thf($satz_3$, axiom)
$\forall$xx: nat: (suc@xx) = (pl@n_1@xx) thf(satz4g, axiom)
$\forall$xx: nat: (suc@xx) = (pl@xx@n_1) thf(satz4e, axiom)
$\forall$xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
$\forall$xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz)) thf($satz_5$, axiom)
$\forall$xx: nat, xy: nat: (suc@(pl@xx@xy)) = (pl@xx@(suc@xy)) thf(satz4f, axiom)
$\forall$xx: nat, xy: nat: xy $\neq$ (pl@xx@xy) thf($satz_7$, axiom)
$\forall$xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf($satz_6$, axiom)
$\neg$($x \neq y$ $\Rightarrow$ ($\neg\neg\forall xx_0$: nat: $x \neq$ (pl@y@xx_0) $\Rightarrow$ $\neg\forall xx_0$: nat: $y \neq$ (pl@x@xx_0))) $\Rightarrow$ $\neg\neg$($x = y$ $\Rightarrow$ $\neg\neg\forall xx_0$: nat: $x \neq$ (pl@y@xx_0)) $\Rightarrow$ $\neg\neg$($\neg\forall xx_0$: nat: $x \neq$ (pl@y@xx_0) $\Rightarrow$ $\neg\neg\forall xx_0$: nat: $y \neq$ (pl@x@xx_0)) $\Rightarrow$ $\neg\neg\forall xx_0$: nat: $y \neq$ (pl@x@xx_0) $\Rightarrow$ $x \neq y$ thf($satz_9$, conjecture)

NUM650^1.p Landau theorem 9a
(x = y) → ((forall x_0:nat. (x = pl y x_0))) → (forall x_0:nat. (y = pl x x_0))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
set: \$tType thf(set_type, type)
esti: nat → set → \$o thf(esti, type)
setof: (nat → \$o) → set thf(setof, type)
$\forall$xp: nat → \$o, xs: nat: ((esti@xs@(setof@xp)) $\Rightarrow$ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat → nat thf(suc, type)
$\forall$xs: set: ((esti@n_1@xs) $\Rightarrow$ ($\forall$xx: nat: ((esti@xx@xs) $\Rightarrow$ (esti@(suc@xx)@xs)) $\Rightarrow$ $\forall$xx: nat: (esti@xx@xs))) thf(ax_5, axiom
$\forall$xp: nat → \$o, xs: nat: ((xp@xs) $\Rightarrow$ (esti@xs@(setof@xp))) thf(estii, axiom)
$\forall$xa: \$o: ($\neg\neg$xa $\Rightarrow$ xa) thf(et, axiom)
$\forall$xx: nat: (xx $\neq n_1$ $\Rightarrow$ $\neg\forall xx_0$: nat: xx $\neq$ (suc@xx_0)) thf($satz_3$, axiom)
$\forall$xx: nat: (suc@xx) = (pl@n_1@xx) thf(satz4g, axiom)
$\forall$xx: nat: (suc@xx) = (pl@xx@n_1) thf(satz4e, axiom)
$\forall$xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
$\forall$xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz)) thf($satz_5$, axiom)

$\forall$xx: nat, xy: nat: (suc@(pl@xx@xy)) = (pl@xx@(suc@xy)) thf(satz4f, axiom)
$x \neq y \Rightarrow (\neg\neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon x \neq (\mathrm{pl}@y@\mathrm{xx}_0) \Rightarrow \neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon y \neq (\mathrm{pl}@x@\mathrm{xx}_0))$ thf(satz9a, conjecture)

NUM651∧1.p Landau theorem 9b
((x = y → ((forall x_0:nat. (x = pl y x_0)))) → ((((forall x_0:nat. (x = pl y x_0)) → ((forall x_0:nat. (y = pl x x_0)))) → ((forall x_0:nat. (y = pl x x_0)) → (x = y)))))
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
$\forall$xx: nat, xy: nat: xy ≠ (pl@xx@xy) thf(satz_7, axiom)
$\forall$xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz_6, axiom)
$\forall$xa: $o: (¬¬xa ⇒ xa) thf(et, axiom)
$\forall$xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz)) thf(satz_5, axiom)
$\neg(x = y \Rightarrow \neg\neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon x \neq (\mathrm{pl}@y@\mathrm{xx}_0)) \Rightarrow \neg\neg(\neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon x \neq (\mathrm{pl}@y@\mathrm{xx}_0) \Rightarrow \neg\neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon y \neq (\mathrm{pl}@x@\mathrm{xx}_0)) \Rightarrow \neg\neg\forall \mathrm{xx}_0\colon \mathrm{nat}\colon y \neq (\mathrm{pl}@x@\mathrm{xx}_0) \Rightarrow x \neq y$ thf(satz9b, conjecture)

NUM652∧1.p Landau theorem 10c
(less x y)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
more: nat → nat → $o thf(more, type)
¬more@x@y ⇒ $x = y$ thf(m, axiom)
less: nat → nat → $o thf(less, type)
$\forall$xa: $o: (¬¬xa ⇒ xa) thf(et, axiom)
$\forall$xx: nat, xy: nat: ¬(xx = xy ⇒ ¬more@xx@xy) ⇒ ¬¬((more@xx@xy) ⇒ ¬less@xx@xy) ⇒ ¬(less@xx@xy) ⇒ xx ≠ xy thf(satz10b, axiom)
¬less@x@y thf(satz10c, conjecture)

NUM653∧1.p Landau theorem 10d
(more x y)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
less: nat → nat → $o thf(less, type)
¬less@x@y ⇒ $x = y$ thf(l, axiom)
more: nat → nat → $o thf(more, type)
$\forall$xa: $o: (¬¬xa ⇒ xa) thf(et, axiom)
$\forall$xx: nat, xy: nat: ¬(xx = xy ⇒ ¬more@xx@xy) ⇒ ¬¬((more@xx@xy) ⇒ ¬less@xx@xy) ⇒ ¬(less@xx@xy) ⇒ xx ≠ xy thf(satz10b, axiom)
¬more@x@y thf(satz10d, conjecture)

NUM654∧1.p Landau theorem 10e
(less x y) → x = y
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
more: nat → nat → $o thf(more, type)
¬more@x@y thf(n, axiom)
less: nat → nat → $o thf(less, type)
$\forall$xa: $o: (¬¬xa ⇒ xa) thf(et, axiom)
$\forall$xx: nat, xy: nat: (xx ≠ xy ⇒ (¬more@xx@xy ⇒ (less@xx@xy))) thf(satz10a, axiom)
¬less@x@y ⇒ $x = y$ thf(satz10e, conjecture)

NUM655∧1.p Landau theorem 10f
(more x y) → x = y
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
less: nat → nat → $o thf(less, type)
¬less@x@y thf(n, axiom)

more: nat → nat → $o   thf(more, type)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: (xx ≠ xy ⇒ (¬more@xx@xy ⇒ (less@xx@xy)))   thf(satz10a, axiom)
¬more@x@y ⇒ x = y   thf(satz10f, conjecture)

NUM656^1.p Landau theorem 10g
((less x y) → x = y)
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
more: nat → nat → $o   thf(more, type)
more@x@y   thf(m, axiom)
less: nat → nat → $o   thf(less, type)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: ¬(xx = xy ⇒ ¬more@xx@xy) ⇒ ¬¬((more@xx@xy) ⇒ ¬less@xx@xy) ⇒ ¬(less@xx@xy) ⇒ xx ≠ xy   thf(satz10b, axiom)
¬¬less@x@y ⇒ x = y   thf(satz10g, conjecture)

NUM657^1.p Landau theorem 10h
((more x y) → x = y)
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
less: nat → nat → $o   thf(less, type)
less@x@y   thf(l, axiom)
more: nat → nat → $o   thf(more, type)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: ¬(xx = xy ⇒ ¬more@xx@xy) ⇒ ¬¬((more@xx@xy) ⇒ ¬less@xx@xy) ⇒ ¬(less@xx@xy) ⇒ xx ≠ xy   thf(satz10b, axiom)
¬¬more@x@y ⇒ x = y   thf(satz10h, conjecture)

NUM658^1.p Landau theorem 10j
less x y
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
more: nat → nat → $o   thf(more, type)
¬¬more@x@y ⇒ x = y   thf(n, axiom)
less: nat → nat → $o   thf(less, type)
∀xx: nat, xy: nat: (xx ≠ xy ⇒ (¬more@xx@xy ⇒ (less@xx@xy)))   thf(satz10a, axiom)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
less@x@y   thf(satz10j, conjecture)

NUM659^1.p Landau theorem 10k
more x y
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
less: nat → nat → $o   thf(less, type)
¬¬less@x@y ⇒ x = y   thf(n, axiom)
more: nat → nat → $o   thf(more, type)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: (xx ≠ xy ⇒ (¬more@xx@xy ⇒ (less@xx@xy)))   thf(satz10a, axiom)
more@x@y   thf(satz10k, conjecture)

NUM660^1.p Landau theorem 13
(more y x) → y = x
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
more: nat → nat → $o   thf(more, type)
¬more@x@y ⇒ x = y   thf(m, axiom)

less: nat → nat → \$o thf(less, type)
∀xx: nat, xy: nat: ((more@xx@xy) ⇒ (less@xy@xx)) thf(satz$_{11}$, axiom)
¬ less@y@x ⇒ $y = x$ thf(satz$_{13}$, conjecture)

NUM662∧1.p Landau theorem 15
(forall x_0:nat. (z = pl x x_0))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
pl: nat → nat → nat thf(pl, type)
¬ ∀xx$_0$: nat: $y \neq$ (pl@x@xx$_0$) thf(l, axiom)
¬ ∀xx: nat: $z \neq$ (pl@y@xx) thf(k, axiom)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz)) thf(satz$_5$, axiom)
¬ ∀xx$_0$: nat: $z \neq$ (pl@x@xx$_0$) thf(satz$_{15}$, conjecture)

NUM663∧1.p Landau theorem 16a
less x z
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
less: nat → nat → \$o thf(less, type)
¬ less@x@y ⇒ $x = y$ thf(l, axiom)
less@y@z thf(k, axiom)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat, xz: nat: ((less@xx@xy) ⇒ ((less@xy@xz) ⇒ (less@xx@xz))) thf(satz$_{15}$, axiom)
less@x@z thf(satz16a, conjecture)

NUM664∧1.p Landau theorem 16b
less x z
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
less: nat → nat → \$o thf(less, type)
less@x@y thf(l, axiom)
¬ less@y@z ⇒ $y = z$ thf(k, axiom)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat, xz: nat: ((less@xx@xy) ⇒ ((less@xy@xz) ⇒ (less@xx@xz))) thf(satz$_{15}$, axiom)
less@x@z thf(satz16b, conjecture)

NUM665∧1.p Landau theorem 16c
some (lambda u.diffprop x z u)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
moreis: nat → nat → \$o thf(moreis, type)
moreis@x@y thf(m, axiom)
some: (nat → \$o) → \$o thf(some, type)
diffprop: nat → nat → nat → \$o thf(diffprop, type)
some@λxu: nat: (diffprop@y@z@xu) thf(n, axiom)
lessis: nat → nat → \$o thf(lessis, type)
∀xx: nat, xy: nat, xz: nat: ((some@λxv: nat: (diffprop@xy@xx@xv)) ⇒ ((lessis@xy@xz) ⇒ (some@λxv: nat: (diffprop@xz@x
∀xx: nat, xy: nat: ((moreis@xx@xy) ⇒ (lessis@xy@xx)) thf(satz$_{13}$, axiom)
some@λxu: nat: (diffprop@x@z@xu) thf(satz16c, conjecture)

NUM666∧1.p Landau theorem 16d
some (lambda u.diffprop x z u)
nat: \$tType thf(nat_type, type)

x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
some: (nat → \$o) → \$o thf(some, type)
diffprop: nat → nat → nat → \$o thf(diffprop, type)
some@λxu: nat: (diffprop@x@y@xu) thf(m, axiom)
moreis: nat → nat → \$o thf(moreis, type)
moreis@y@z thf(n, axiom)
lessis: nat → nat → \$o thf(lessis, type)
∀xx: nat, xy: nat, xz: nat: ((lessis@xx@xy) ⇒ ((some@λxv: nat: (diffprop@xz@xy@xv)) ⇒ (some@λxv: nat: (diffprop@xz@x
∀xx: nat, xy: nat: ((moreis@xx@xy) ⇒ (lessis@xy@xx)) thf(satz$_{13}$, axiom)
some@λxu: nat: (diffprop@x@z@xu) thf(satz16d, conjecture)

NUM667^1.p Landau theorem 17
(less x z) → x = z
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
less: nat → nat → \$o thf(less, type)
¬ less@x@y ⇒ $x = y$ thf(l, axiom)
¬ less@y@z ⇒ $y = z$ thf(k, axiom)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat, xz: nat: ((¬ less@xx@xy ⇒ xx = xy) ⇒ ((less@xy@xz) ⇒ (less@xx@xz))) thf(satz16a, axiom)
∀xx: nat, xy: nat, xz: nat: ((less@xx@xy) ⇒ ((¬ less@xy@xz ⇒ xy = xz) ⇒ (less@xx@xz))) thf(satz16b, axiom)
¬ less@x@z ⇒ $x = z$ thf(satz$_{17}$, conjecture)

NUM668^1.p Landau theorem 18
(forall x_0:nat. (pl x y = pl x x_0))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
¬ ∀xx$_0$: nat: (pl@x@y) ≠ (pl@x@xx$_0$) thf(satz$_{18}$, conjecture)

NUM669^1.p Landau theorem 18b
more (suc x) x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
more: nat → nat → \$o thf(more, type)
suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
∀xx: nat, xy: nat: (more@(pl@xx@xy)@xx) thf(satz$_{18}$, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
more@(suc@x)@x thf(satz18b, conjecture)

NUM670^1.p Landau theorem 19a
(forall x_0:nat. (pl x z = pl (pl y z) x_0))
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
pl: nat → nat → nat thf(pl, type)
¬ ∀xx$_0$: nat: x ≠ (pl@y@xx$_0$) thf(m, axiom)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz$_6$, axiom)
∀xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz)) thf(satz$_5$, axiom)
¬ ∀xx$_0$: nat: (pl@x@z) ≠ (pl@(pl@y@z)@xx$_0$) thf(satz19a, conjecture)

NUM671^1.p Landau theorem 19b
pl x z = pl y z

nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
$x = y$ thf(i, axiom)
pl: nat → nat → nat   thf(pl, type)
(pl@x@z) = (pl@y@z)   thf(satz19b, conjecture)

NUM672∧1.p Landau theorem 19c
less (pl x z) (pl y z)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
less: nat → nat → \$o   thf(less, type)
less@x@y thf(l, axiom)
pl: nat → nat → nat   thf(pl, type)
more: nat → nat → \$o   thf(more, type)
∀xx: nat, xy: nat: ((more@xx@xy) ⇒ (less@xy@xx))   thf(satz_{11}, axiom)
∀xx: nat, xy: nat, xz: nat: ((more@xx@xy) ⇒ (more@(pl@xx@xz)@(pl@xy@xz)))   thf(satz19a, axiom)
∀xx: nat, xy: nat: ((less@xx@xy) ⇒ (more@xy@xx))   thf(satz_{12}, axiom)
less@(pl@x@z)@(pl@y@z)   thf(satz19c, conjecture)

NUM673∧1.p Landau theorem 19d
more (pl z x) (pl z y)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
more: nat → nat → \$o   thf(more, type)
more@x@y thf(m, axiom)
pl: nat → nat → nat   thf(pl, type)
∀xx: nat, xy: nat, xz: nat: ((more@xx@xy) ⇒ (more@(pl@xx@xz)@(pl@xy@xz)))   thf(satz19a, axiom)
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx)   thf(satz_{6}, axiom)
more@(pl@z@x)@(pl@z@y)   thf(satz19d, conjecture)

NUM674∧1.p Landau theorem 19e
pl z x = pl z y
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
$x = y$ thf(i, axiom)
pl: nat → nat → nat   thf(pl, type)
(pl@z@x) = (pl@z@y)   thf(satz19e, conjecture)

NUM676∧1.p Landau theorem 19g
more (pl x z) (pl y u)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
u: nat   thf(u, type)
$x = y$ thf(i, axiom)
more: nat → nat → \$o   thf(more, type)
more@z@u thf(m, axiom)
pl: nat → nat → nat   thf(pl, type)
∀xx: nat, xy: nat, xz: nat: ((more@xx@xy) ⇒ (more@(pl@xz@xx)@(pl@xz@xy)))   thf(satz19d, axiom)
more@(pl@x@z)@(pl@y@u)   thf(satz19g, conjecture)

NUM677∧1.p Landau theorem 19h
more (pl z x) (pl u y)

$\mathrm{nat}\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{nat\_type}, \mathrm{type})$
$x\colon \mathrm{nat} \quad \mathrm{thf}(x, \mathrm{type})$
$y\colon \mathrm{nat} \quad \mathrm{thf}(y, \mathrm{type})$
$z\colon \mathrm{nat} \quad \mathrm{thf}(z, \mathrm{type})$
$u\colon \mathrm{nat} \quad \mathrm{thf}(u, \mathrm{type})$
$x = y \quad \mathrm{thf}(i, \mathrm{axiom})$
$\mathrm{more}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{more}, \mathrm{type})$
$\mathrm{more}@z@u \quad \mathrm{thf}(m, \mathrm{axiom})$
$\mathrm{pl}\colon \mathrm{nat} \to \mathrm{nat} \to \mathrm{nat} \quad \mathrm{thf}(\mathrm{pl}, \mathrm{type})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}, \mathrm{xz}\colon \mathrm{nat}, \mathrm{xu}\colon \mathrm{nat}\colon (\mathrm{xx} = \mathrm{xy} \Rightarrow ((\mathrm{more}@\mathrm{xz}@\mathrm{xu}) \Rightarrow (\mathrm{more}@(\mathrm{pl}@\mathrm{xx}@\mathrm{xz})@(\mathrm{pl}@\mathrm{xy}@\mathrm{xu})))) \quad \mathrm{thf}(\mathrm{satz19g}, \mathrm{axio}$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}\colon (\mathrm{pl}@\mathrm{xx}@\mathrm{xy}) = (\mathrm{pl}@\mathrm{xy}@\mathrm{xx}) \quad \mathrm{thf}(\mathrm{satz}_6, \mathrm{axiom})$
$\mathrm{more}@(\mathrm{pl}@z@x)@(\mathrm{pl}@u@y) \quad \mathrm{thf}(\mathrm{satz19h}, \mathrm{conjecture})$

NUM680∧1.p Landau theorem 20a
more x y
$\mathrm{nat}\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{nat\_type}, \mathrm{type})$
$x\colon \mathrm{nat} \quad \mathrm{thf}(x, \mathrm{type})$
$y\colon \mathrm{nat} \quad \mathrm{thf}(y, \mathrm{type})$
$z\colon \mathrm{nat} \quad \mathrm{thf}(z, \mathrm{type})$
$\mathrm{more}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{more}, \mathrm{type})$
$\mathrm{pl}\colon \mathrm{nat} \to \mathrm{nat} \to \mathrm{nat} \quad \mathrm{thf}(\mathrm{pl}, \mathrm{type})$
$\mathrm{more}@(\mathrm{pl}@x@z)@(\mathrm{pl}@y@z) \quad \mathrm{thf}(m, \mathrm{axiom})$
$\forall \mathrm{xa}\colon \$\mathrm{o}\colon (\neg\neg \mathrm{xa} \Rightarrow \mathrm{xa}) \quad \mathrm{thf}(\mathrm{et}, \mathrm{axiom})$
$\mathrm{less}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{less}, \mathrm{type})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}\colon \neg(\mathrm{xx} = \mathrm{xy} \Rightarrow \neg\,\mathrm{more}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg\neg((\mathrm{more}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg\,\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg(\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow$
$\mathrm{xx} \neq \mathrm{xy} \quad \mathrm{thf}(\mathrm{satz10b}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}, \mathrm{xz}\colon \mathrm{nat}\colon (\mathrm{xx} = \mathrm{xy} \Rightarrow (\mathrm{pl}@\mathrm{xx}@\mathrm{xz}) = (\mathrm{pl}@\mathrm{xy}@\mathrm{xz})) \quad \mathrm{thf}(\mathrm{satz19b}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}, \mathrm{xz}\colon \mathrm{nat}\colon ((\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow (\mathrm{less}@(\mathrm{pl}@\mathrm{xx}@\mathrm{xz})@(\mathrm{pl}@\mathrm{xy}@\mathrm{xz}))) \quad \mathrm{thf}(\mathrm{satz19c}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}\colon (\mathrm{xx} \neq \mathrm{xy} \Rightarrow (\neg\,\mathrm{more}@\mathrm{xx}@\mathrm{xy} \Rightarrow (\mathrm{less}@\mathrm{xx}@\mathrm{xy}))) \quad \mathrm{thf}(\mathrm{satz10a}, \mathrm{axiom})$
$\mathrm{more}@x@y \quad \mathrm{thf}(\mathrm{satz20a}, \mathrm{conjecture})$

NUM681∧1.p Landau theorem 20b
x = y
$\mathrm{nat}\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{nat\_type}, \mathrm{type})$
$x\colon \mathrm{nat} \quad \mathrm{thf}(x, \mathrm{type})$
$y\colon \mathrm{nat} \quad \mathrm{thf}(y, \mathrm{type})$
$z\colon \mathrm{nat} \quad \mathrm{thf}(z, \mathrm{type})$
$\mathrm{pl}\colon \mathrm{nat} \to \mathrm{nat} \to \mathrm{nat} \quad \mathrm{thf}(\mathrm{pl}, \mathrm{type})$
$(\mathrm{pl}@x@z) = (\mathrm{pl}@y@z) \quad \mathrm{thf}(i, \mathrm{axiom})$
$\forall \mathrm{xa}\colon \$\mathrm{o}\colon (\neg\neg \mathrm{xa} \Rightarrow \mathrm{xa}) \quad \mathrm{thf}(\mathrm{et}, \mathrm{axiom})$
$\mathrm{less}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{less}, \mathrm{type})$
$\mathrm{more}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{more}, \mathrm{type})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}\colon \neg(\mathrm{xx} = \mathrm{xy} \Rightarrow \neg\,\mathrm{more}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg\neg((\mathrm{more}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg\,\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow \neg(\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow$
$\mathrm{xx} \neq \mathrm{xy} \quad \mathrm{thf}(\mathrm{satz10b}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}, \mathrm{xz}\colon \mathrm{nat}\colon ((\mathrm{less}@\mathrm{xx}@\mathrm{xy}) \Rightarrow (\mathrm{less}@(\mathrm{pl}@\mathrm{xx}@\mathrm{xz})@(\mathrm{pl}@\mathrm{xy}@\mathrm{xz}))) \quad \mathrm{thf}(\mathrm{satz19c}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}\colon (\mathrm{xx} \neq \mathrm{xy} \Rightarrow (\neg\,\mathrm{more}@\mathrm{xx}@\mathrm{xy} \Rightarrow (\mathrm{less}@\mathrm{xx}@\mathrm{xy}))) \quad \mathrm{thf}(\mathrm{satz10a}, \mathrm{axiom})$
$\forall \mathrm{xx}\colon \mathrm{nat}, \mathrm{xy}\colon \mathrm{nat}, \mathrm{xz}\colon \mathrm{nat}\colon ((\mathrm{more}@\mathrm{xx}@\mathrm{xy}) \Rightarrow (\mathrm{more}@(\mathrm{pl}@\mathrm{xx}@\mathrm{xz})@(\mathrm{pl}@\mathrm{xy}@\mathrm{xz}))) \quad \mathrm{thf}(\mathrm{satz19a}, \mathrm{axiom})$
$x = y \quad \mathrm{thf}(\mathrm{satz20b}, \mathrm{conjecture})$

NUM682∧1.p Landau theorem 20c
less x y
$\mathrm{nat}\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{nat\_type}, \mathrm{type})$
$x\colon \mathrm{nat} \quad \mathrm{thf}(x, \mathrm{type})$
$y\colon \mathrm{nat} \quad \mathrm{thf}(y, \mathrm{type})$
$z\colon \mathrm{nat} \quad \mathrm{thf}(z, \mathrm{type})$
$\mathrm{less}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{less}, \mathrm{type})$
$\mathrm{pl}\colon \mathrm{nat} \to \mathrm{nat} \to \mathrm{nat} \quad \mathrm{thf}(\mathrm{pl}, \mathrm{type})$
$\mathrm{less}@(\mathrm{pl}@x@z)@(\mathrm{pl}@y@z) \quad \mathrm{thf}(l, \mathrm{axiom})$
$\forall \mathrm{xa}\colon \$\mathrm{o}\colon (\neg\neg \mathrm{xa} \Rightarrow \mathrm{xa}) \quad \mathrm{thf}(\mathrm{et}, \mathrm{axiom})$
$\mathrm{more}\colon \mathrm{nat} \to \mathrm{nat} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{more}, \mathrm{type})$

∀xx: nat, xy: nat: ¬ (xx = xy ⇒ ¬ more@xx@xy) ⇒ ¬ ¬ ((more@xx@xy) ⇒ ¬ less@xx@xy) ⇒ ¬ (less@xx@xy) ⇒ xx ≠ xy thf(satz10b, axiom)
∀xx: nat, xy: nat, xz: nat: ((more@xx@xy) ⇒ (more@(pl@xx@xz)@(pl@xy@xz))) thf(satz19a, axiom)
∀xx: nat, xy: nat, xz: nat: (xx = xy ⇒ (pl@xx@xz) = (pl@xy@xz)) thf(satz19b, axiom)
∀xx: nat, xy: nat: (xx ≠ xy ⇒ (¬ more@xx@xy ⇒ (less@xx@xy))) thf(satz10a, axiom)
less@x@y thf(satz20c, conjecture)

NUM683∧1.p Landau theorem 20d
more x y
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
more: nat → nat → $o thf(more, type)
pl: nat → nat → nat thf(pl, type)
more@(pl@z@x)@(pl@z@y) thf(m, axiom)
∀xx: nat, xy: nat, xz: nat: ((more@(pl@xx@xz)@(pl@xy@xz)) ⇒ (more@xx@xy)) thf(satz20a, axiom)
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz_6, axiom)
more@x@y thf(satz20d, conjecture)

NUM684∧1.p Landau theorem 20e
x = y
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
pl: nat → nat → nat thf(pl, type)
(pl@z@x) = (pl@z@y) thf(i, axiom)
∀xx: nat, xy: nat, xz: nat: ((pl@xx@xz) = (pl@xy@xz) ⇒ xx = xy) thf(satz20b, axiom)
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz_6, axiom)
$x = y$ thf(satz20e, conjecture)

NUM686∧1.p Landau theorem 21
some (lambda u_0.diffprop (pl x z) (pl y u) u_0)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
u: nat thf(u, type)
some: (nat → $o) → $o thf(some, type)
diffprop: nat → nat → nat → $o thf(diffprop, type)
some@λxu: nat: (diffprop@x@y@xu) thf(m, axiom)
some@λxu_0: nat: (diffprop@z@u@xu_0) thf(n, axiom)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat, xz: nat: ((some@λxv: nat: (diffprop@xy@xx@xv)) ⇒ ((some@λxv: nat: (diffprop@xz@xy@xv)) ⇒ (some@λxv: nat: (diffprop@xz@xx@xv)))) thf(satz_{15}, axiom)
∀xx: nat, xy: nat, xz: nat: ((some@λxu: nat: (diffprop@xx@xy@xu)) ⇒ (some@λxu: nat: (diffprop@(pl@xx@xz)@(pl@xy@xz
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz_6, axiom)
some@λxu_0: nat: (diffprop@(pl@x@z)@(pl@y@u)@xu_0) thf(satz_{21}, conjecture)

NUM687∧1.p Landau theorem 22a
more (pl x z) (pl y u)
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
u: nat thf(u, type)
more: nat → nat → $o thf(more, type)
¬ more@x@y ⇒ $x = y$ thf(m, axiom)
more@z@u thf(n, axiom)
pl: nat → nat → nat thf(pl, type)

∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat, xz: nat, xu: nat: (xx = xy ⇒ ((more@xz@xu) ⇒ (more@(pl@xx@xz)@(pl@xy@xu))))   thf(satz19g, axio
∀xx: nat, xy: nat, xz: nat, xu: nat: ((more@xx@xy) ⇒ ((more@xz@xu) ⇒ (more@(pl@xx@xz)@(pl@xy@xu))))   thf(satz2
more@(pl@x@z)@(pl@y@u)   thf(satz22a, conjecture)

NUM688^1.p Landau theorem 22b
more (pl x z) (pl y u)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
u: nat   thf(u, type)
more: nat → nat → \$o   thf(more, type)
more@x@y thf(m, axiom)
¬more@z@u ⇒ $z = u$ thf(n, axiom)
pl: nat → nat → nat   thf(pl, type)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat, xz: nat, xu: nat: (xx = xy ⇒ ((more@xz@xu) ⇒ (more@(pl@xz@xx)@(pl@xu@xy))))   thf(satz19h, axio
∀xx: nat, xy: nat, xz: nat, xu: nat: ((more@xx@xy) ⇒ ((more@xz@xu) ⇒ (more@(pl@xx@xz)@(pl@xy@xu))))   thf(satz2
more@(pl@x@z)@(pl@y@u)   thf(satz22b, conjecture)

NUM689^1.p Landau theorem 22c
some (lambda v.diffprop (pl y u) (pl x z) v)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
u: nat   thf(u, type)
lessis: nat → nat → \$o   thf(lessis, type)
lessis@x@y thf(l, axiom)
some: (nat → \$o) → \$o   thf(some, type)
diffprop: nat → nat → nat → \$o   thf(diffprop, type)
some@λxv: nat: (diffprop@u@z@xv)   thf(k, axiom)
pl: nat → nat → nat   thf(pl, type)
moreis: nat → nat → \$o   thf(moreis, type)
∀xx: nat, xy: nat, xz: nat, xu: nat: ((moreis@xx@xy) ⇒ ((some@λxu_0: nat: (diffprop@xz@xu@xu_0)) ⇒ (some@λxu_0: nat: (di
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ (moreis@xy@xx))   thf($satz_{14}$, axiom)
some@λxv: nat: (diffprop@(pl@y@u)@(pl@x@z)@xv)   thf(satz22c, conjecture)

NUM690^1.p Landau theorem 22d
some (lambda v.diffprop (pl y u) (pl x z) v)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
u: nat   thf(u, type)
some: (nat → \$o) → \$o   thf(some, type)
diffprop: nat → nat → nat → \$o   thf(diffprop, type)
some@λxv: nat: (diffprop@y@x@xv)   thf(l, axiom)
lessis: nat → nat → \$o   thf(lessis, type)
lessis@z@u thf(k, axiom)
pl: nat → nat → nat   thf(pl, type)
moreis: nat → nat → \$o   thf(moreis, type)
∀xx: nat, xy: nat, xz: nat, xu: nat: ((some@λxu: nat: (diffprop@xx@xy@xu)) ⇒ ((moreis@xz@xu) ⇒ (some@λxu_0: nat: (diff
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ (moreis@xy@xx))   thf($satz_{14}$, axiom)
some@λxv: nat: (diffprop@(pl@y@u)@(pl@x@z)@xv)   thf(satz22d, conjecture)

NUM691^1.p Landau theorem 23
(more (pl x z) (pl y u)) → pl x z = pl y u
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)

y: nat thf(y, type)
z: nat thf(z, type)
u: nat thf(u, type)
more: nat → nat → \$o thf(more, type)
$\neg$ more@x@y $\Rightarrow$ $x = y$ thf(m, axiom)
$\neg$ more@z@u $\Rightarrow$ $z = u$ thf(n, axiom)
pl: nat → nat → nat thf(pl, type)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat, xy: nat, xz: nat, xu: nat: ((¬ more@xx@xy ⇒ xx = xy) ⇒ ((more@xz@xu) ⇒ (more@(pl@xx@xz)@(pl@xy@xu)))
∀xx: nat, xy: nat, xz: nat, xu: nat: ((more@xx@xy) ⇒ ((¬ more@xz@xu ⇒ xz = xu) ⇒ (more@(pl@xx@xz)@(pl@xy@xu)))
¬ more@(pl@x@z)@(pl@y@u) ⇒ (pl@x@z) = (pl@y@u) thf(satz_{23}, conjecture)

NUM692^1.p Landau theorem 23a
lessis (pl x z) (pl y u)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
u: nat thf(u, type)
lessis: nat → nat → \$o thf(lessis, type)
lessis@x@y thf(l, axiom)
lessis@z@u thf(k, axiom)
ts: nat → nat → nat thf(ts, type)
moreis: nat → nat → \$o thf(moreis, type)
∀xx: nat, xy: nat: ((moreis@xx@xy) ⇒ (lessis@xy@xx)) thf(satz_{13}, axiom)
∀xx: nat, xy: nat, xz: nat, xu: nat: ((moreis@xx@xy) ⇒ ((moreis@xz@xu) ⇒ (moreis@(ts@xx@xz)@(ts@xy@xu)))) thf(s
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ (moreis@xy@xx)) thf(satz_{14}, axiom)
lessis@(ts@x@z)@(ts@y@u) thf(satz23a, conjecture)

NUM693^1.p Landau theorem 24
(more x n_1) → x = n_1
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
more: nat → nat → \$o thf(more, type)
n_1: nat thf(n_1, type)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
suc: nat → nat thf(suc, type)
∀xx: nat: (xx ≠ n_1 ⇒ ¬ ∀xx_0: nat: xx ≠ (suc@xx_0)) thf(satz_3, axiom)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat: (more@(pl@xx@xy)@xx) thf(satz_{18}, axiom)
∀xx: nat: (suc@xx) = (pl@n_1@xx) thf(satz4g, axiom)
¬ more@x@n_1 ⇒ $x = n_1$ thf(satz_{24}, conjecture)

NUM694^1.p Landau theorem 24a
lessis n_1 x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
lessis: nat → nat → \$o thf(lessis, type)
n_1: nat thf(n_1, type)
moreis: nat → nat → \$o thf(moreis, type)
∀xx: nat, xy: nat: ((moreis@xx@xy) ⇒ (lessis@xy@xx)) thf(satz_{13}, axiom)
∀xx: nat: (moreis@xx@n_1) thf(satz_{24}, axiom)
lessis@n_1@x thf(satz24a, conjecture)

NUM695^1.p Landau theorem 24b
more (suc x) n_1
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
more: nat → nat → \$o thf(more, type)
suc: nat → nat thf(suc, type)
n_1: nat thf(n_1, type)

∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat: (xx ≠ n_1 ⇒ ¬ ∀xx_0: nat: xx ≠ (suc@xx_0)) thf(satz_3, axiom)
∀xx: nat: (suc@xx) ≠ n_1 thf(ax_3, axiom)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat: (more@(pl@xx@xy)@xx) thf(satz_{18}, axiom)
∀xx: nat: (suc@xx) = (pl@n_1@xx) thf(satz4g, axiom)
more@(suc@x)@n_1 thf(satz24b, conjecture)

NUM696∧1.p Landau theorem 25
((forall x_0:nat. (y = pl (pl x n_1) x_0))) → y = pl x n_1
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
¬ ∀xx_0: nat: y ≠ (pl@x@xx_0) thf(m, axiom)
n_1: nat thf(n_1, type)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: nat: (¬ ¬ ∀xx_0: nat: xx ≠ (pl@n_1@xx_0) ⇒ xx = n_1) thf(satz_{24}, axiom)
∀xx: nat, xy: nat, xz: nat: (¬ ∀xx_0: nat: xx ≠ (pl@xy@xx_0) ⇒ ¬ ∀xx_0: nat: (pl@xx@xz) ≠ (pl@(pl@xy@xz)@xx_0)) thf(sat
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx) thf(satz_6, axiom)
¬ ¬ ∀xx_0: nat: y ≠ (pl@(pl@x@n_1)@xx_0) ⇒ y = (pl@x@n_1) thf(satz_{25}, conjecture)

NUM697∧1.p Landau theorem 25a
moreis y (suc x)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
more: nat → nat → \$o thf(more, type)
more@y@x thf(m, axiom)
moreis: nat → nat → \$o thf(moreis, type)
suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
∀xx: nat, xy: nat: ((more@xy@xx) ⇒ (moreis@xy@(pl@xx@n_1))) thf(satz_{25}, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
moreis@y@(suc@x) thf(satz25a, conjecture)

NUM698∧1.p Landau theorem 25b
lessis (pl y n_1) x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
some: (nat → \$o) → \$o thf(some, type)
diffprop: nat → nat → nat → \$o thf(diffprop, type)
some@λxv: nat: (diffprop@x@y@xv) thf(l, axiom)
lessis: nat → nat → \$o thf(lessis, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
moreis: nat → nat → \$o thf(moreis, type)
∀xx: nat, xy: nat: ((moreis@xx@xy) ⇒ (lessis@xy@xx)) thf(satz_{13}, axiom)
∀xx: nat, xy: nat: ((some@λxu: nat: (diffprop@xy@xx@xu)) ⇒ (moreis@xy@(pl@xx@n_1))) thf(satz_{25}, axiom)
lessis@(pl@y@n_1)@x thf(satz25b, conjecture)

NUM699∧1.p Landau theorem 25c
lessis (suc y) x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
less: nat → nat → \$o thf(less, type)
less@y@x thf(l, axiom)
lessis: nat → nat → \$o thf(lessis, type)

suc: nat → nat thf(suc, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
∀xx: nat, xy: nat: ((less@xy@xx) ⇒ (lessis@(pl@xy@n_1)@xx)) thf(satz25b, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx) thf(satz4a, axiom)
lessis@(suc@y)@x thf(satz25c, conjecture)

NUM700∧1.p Landau theorem 26
lessis y x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
less: nat → nat → $o thf(less, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
less@y@(pl@x@n_1) thf(l, axiom)
lessis: nat → nat → $o thf(lessis, type)
more: nat → nat → $o thf(more, type)
∀xx: nat, xy: nat: (¬ more@xx@xy ⇒ (lessis@xx@xy)) thf(satz10e, axiom)
moreis: nat → nat → $o thf(moreis, type)
∀xx: nat, xy: nat: ((less@xx@xy) ⇒ ¬ moreis@xx@xy) thf(satz10h, axiom)
∀xx: nat, xy: nat: ((more@xy@xx) ⇒ (moreis@xy@(pl@xx@n_1))) thf($satz_{25}$, axiom)
lessis@y@x thf($satz_{26}$, conjecture)

NUM701∧1.p Landau theorem 26a
lessis y x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
less: nat → nat → $o thf(less, type)
suc: nat → nat thf(suc, type)
less@y@(suc@x) thf(l, axiom)
lessis: nat → nat → $o thf(lessis, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
∀xx: nat, xy: nat: ((less@xy@(pl@xx@n_1)) ⇒ (lessis@xy@xx)) thf($satz_{26}$, axiom)
∀xx: nat: (suc@xx) = (pl@xx@n_1) thf(satz4e, axiom)
lessis@y@x thf(satz26a, conjecture)

NUM702∧1.p Landau theorem 26b
moreis y x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
some: (nat → $o) → $o thf(some, type)
diffprop: nat → nat → nat → $o thf(diffprop, type)
pl: nat → nat → nat thf(pl, type)
n_1: nat thf(n_1, type)
some@λxu: nat: (diffprop@(pl@y@n_1)@x@xu) thf(m, axiom)
moreis: nat → nat → $o thf(moreis, type)
lessis: nat → nat → $o thf(lessis, type)
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ (moreis@xy@xx)) thf($satz_{14}$, axiom)
∀xx: nat, xy: nat: ((some@λxv: nat: (diffprop@(pl@xx@n_1)@xy@xv)) ⇒ (lessis@xy@xx)) thf($satz_{26}$, axiom)
moreis@y@x thf(satz26b, conjecture)

NUM703∧1.p Landau theorem 26c
moreis y x
nat: $tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
more: nat → nat → $o thf(more, type)

suc: nat → nat   thf(suc, type)
more@(suc@y)@x thf(m, axiom)
moreis: nat → nat → $o   thf(moreis, type)
pl: nat → nat → nat   thf(pl, type)
n_1: nat   thf(n_1, type)
∀xx: nat, xy: nat: ((more@(pl@xy@n_1)@xx) ⇒ (moreis@xy@xx))   thf(satz26b, axiom)
∀xx: nat: (suc@xx) = (pl@xx@n_1)   thf(satz4e, axiom)
moreis@y@x thf(satz26c, conjecture)

NUM704^1.p Landau theorem 27
(forall x:nat. (((forall x_0_0:nat.p x_0_0 → (less x x_0_0) → x = x_0_0) → (p x))))
nat: $tType   thf(nat_type, type)
p: nat → $o   thf($p$, type)
¬∀xx: nat: ¬ p@xx   thf(s, axiom)
less: nat → nat → $o   thf(less, type)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
more: nat → nat → $o   thf(more, type)
∀xx: nat, xy: nat: ((more@xx@xy) ⇒ ¬¬less@xx@xy ⇒ xx = xy)   thf(satz10g, axiom)
pl: nat → nat → nat   thf(pl, type)
n_1: nat   thf(n_1, type)
∀xx: nat, xy: nat: (more@(pl@xx@xy)@xx)   thf(satz_{18}, axiom)
set: $tType   thf(set_type, type)
esti: nat → set → $o   thf(esti, type)
setof: (nat → $o) → set   thf(setof, type)
∀xp: nat → $o, xs: nat: ((esti@xs@(setof@xp)) ⇒ (xp@xs))   thf(estie, axiom)
suc: nat → nat   thf(suc, type)
∀xs: set: ((esti@n_1@xs) ⇒ (∀xx: nat: ((esti@xx@xs) ⇒ (esti@(suc@xx)@xs)) ⇒ ∀xx: nat: (esti@xx@xs)))   thf(ax_5, axio
∀xp: nat → $o, xs: nat: ((xp@xs) ⇒ (esti@xs@(setof@xp)))   thf(estii, axiom)
∀xx: nat: (¬less@n_1@xx ⇒ n_1 = xx)   thf(satz24a, axiom)
∀xx: nat, xy: nat: ((less@xy@xx) ⇒ (¬less@(pl@xy@n_1)@xx ⇒ (pl@xy@n_1) = xx))   thf(satz25b, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx)   thf(satz4a, axiom)
¬∀xx: nat: ¬¬∀xx_0_0: nat: ((p@xx_0_0) ⇒ (¬less@xx@xx_0_0 ⇒ xx = xx_0_0)) ⇒ ¬ p@xx   thf(satz_{27}, conjecture)

NUM705^1.p Landau theorem 27a
((forall x:nat.forall y:nat. ((forall x_0:nat.p x_0 → lessis x x_0) → (p x)) → ((forall x_0:nat.p x_0 → lessis y x_0) → (p y)) → x = y) → (some (lambda x. ((forall x_0:nat.p x_0 → lessis x x_0) → (p x)))))
nat: $tType   thf(nat_type, type)
p: nat → $o   thf($p$, type)
some: (nat → $o) → $o   thf(some, type)
some@p thf(s, axiom)
lessis: nat → nat → $o   thf(lessis, type)
more: nat → nat → $o   thf(more, type)
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ (¬more@xy@xx ⇒ xy = xx))   thf(satz_{14}, axiom)
∀xa: $o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: ((lessis@xx@xy) ⇒ ¬more@xx@xy)   thf(satz10d, axiom)
∀xp: nat → $o: ((some@xp) ⇒ (some@λxx: nat: ¬∀xx$_0$: nat: ((xp@xx$_0$) ⇒ (lessis@xx@xx$_0$)) ⇒ ¬xp@xx))   thf(satz_{27}, a
¬∀xx: nat, xy: nat: (¬∀xx$_0$: nat: ((p@xx$_0$) ⇒ (lessis@xx@xx$_0$)) ⇒ ¬ p@xx ⇒ (¬∀xx$_0$: nat: ((p@xx$_0$) ⇒ (lessis@xy@xx$_0$)) ⇒ ¬ p@xy ⇒ xx = xy)) ⇒ ¬some@λxx: nat: ¬∀xx$_0$: nat: ((p@xx$_0$) ⇒ (lessis@xx@xx$_0$)) ⇒ ¬ p@xx   thf(satz27a, conjecture)

NUM706^1.p Landau theorem 28e
x = ts x n_1
nat: $tType   thf(nat_type, type)
x: nat   thf(x, type)
ts: nat → nat → nat   thf(ts, type)
n_1: nat   thf(n_1, type)
∀xx: nat: (ts@xx@n_1) = xx   thf(satz28a, axiom)
x = (ts@x@n_1)   thf(satz28e, conjecture)

NUM707^1.p Landau theorem 28f
pl (ts x y) x = ts x (suc y)

nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
ts: nat → nat → nat thf(ts, type)
suc: nat → nat thf(suc, type)
∀xx: nat, xy: nat: (ts@xx@(suc@xy)) = (pl@(ts@xx@xy)@xx) thf(satz28b, axiom)
(pl@(ts@x@y)@x) = (ts@x@(suc@y)) thf(satz28f, conjecture)

NUM708^1.p Landau theorem 28g
x = ts n_1 x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
ts: nat → nat → nat thf(ts, type)
n_1: nat thf(n_1, type)
∀xx: nat: (ts@n_1@xx) = xx thf(satz28c, axiom)
x = (ts@n_1@x) thf(satz28g, conjecture)

NUM709^1.p Landau theorem 28h
pl (ts x y) y = ts (suc x) y
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
pl: nat → nat → nat thf(pl, type)
ts: nat → nat → nat thf(ts, type)
suc: nat → nat thf(suc, type)
∀xx: nat, xy: nat: (ts@(suc@xx)@xy) = (pl@(ts@xx@xy)@xy) thf(satz28d, axiom)
(pl@(ts@x@y)@y) = (ts@(suc@x)@y) thf(satz28h, conjecture)

NUM710^1.p Landau theorem 29
ts x y = ts y x
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
ts: nat → nat → nat thf(ts, type)
set: \$tType thf(set_type, type)
esti: nat → set → \$o thf(esti, type)
setof: (nat → \$o) → set thf(setof, type)
∀xp: nat → \$o, xs: nat: ((esti@xs@(setof@xp)) ⇒ (xp@xs)) thf(estie, axiom)
n_1: nat thf(n_1, type)
suc: nat → nat thf(suc, type)
∀xs: set: ((esti@n_1@xs) ⇒ (∀xx: nat: ((esti@xx@xs) ⇒ (esti@(suc@xx)@xs)) ⇒ ∀xx: nat: (esti@xx@xs))) thf(ax$_5$, axiom)
∀xp: nat → \$o, xs: nat: ((xp@xs) ⇒ (esti@xs@(setof@xp))) thf(estii, axiom)
∀xx: nat: (ts@xx@n_1) = xx thf(satz28a, axiom)
∀xx: nat: (ts@n_1@xx) = xx thf(satz28c, axiom)
pl: nat → nat → nat thf(pl, type)
∀xx: nat, xy: nat: (pl@(ts@xx@xy)@xx) = (ts@xx@(suc@xy)) thf(satz28f, axiom)
∀xx: nat, xy: nat: (ts@(suc@xx)@xy) = (pl@(ts@xx@xy)@xy) thf(satz28d, axiom)
(ts@x@y) = (ts@y@x) thf(satz$_{29}$, conjecture)

NUM711^1.p Landau theorem 30
ts x (pl y z) = pl (ts x y) (ts x z)
nat: \$tType thf(nat_type, type)
x: nat thf(x, type)
y: nat thf(y, type)
z: nat thf(z, type)
ts: nat → nat → nat thf(ts, type)
pl: nat → nat → nat thf(pl, type)
set: \$tType thf(set_type, type)
esti: nat → set → \$o thf(esti, type)
setof: (nat → \$o) → set thf(setof, type)

∀xp: nat → \$o, xs: nat: ((esti@xs@(setof@xp)) ⇒ (xp@xs))   thf(estie, axiom)
n_1: nat   thf(n_1, type)
suc: nat → nat   thf(suc, type)
∀xs: set: ((esti@n_1@xs) ⇒ (∀xx: nat: ((esti@xx@xs) ⇒ (esti@(suc@xx)@xs)) ⇒ ∀xx: nat: (esti@xx@xs)))   thf(ax_5, axio
∀xp: nat → \$o, xs: nat: ((xp@xs) ⇒ (esti@xs@(setof@xp)))   thf(estii, axiom)
∀xx: nat: xx = (ts@xx@n_1)   thf(satz28e, axiom)
∀xx: nat, xy: nat: (ts@xx@(suc@xy)) = (pl@(ts@xx@xy)@xx)   thf(satz28b, axiom)
∀xx: nat: (pl@xx@n_1) = (suc@xx)   thf(satz4a, axiom)
∀xx: nat, xy: nat: (pl@(ts@xx@xy)@xx) = (ts@xx@(suc@xy))   thf(satz28f, axiom)
∀xx: nat, xy: nat, xz: nat: (pl@(pl@xx@xy)@xz) = (pl@xx@(pl@xy@xz))   thf($satz_5$, axiom)
∀xx: nat, xy: nat: (pl@xx@(suc@xy)) = (suc@(pl@xx@xy))   thf(satz4b, axiom)
(ts@x@(pl@y@z)) = (pl@(ts@x@y)@(ts@x@z))   thf($satz_{30}$, conjecture)

NUM712∧1.p Landau theorem 31
ts (ts x y) z = ts x (ts y z)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
ts: nat → nat → nat   thf(ts, type)
set: \$tType   thf(set_type, type)
esti: nat → set → \$o   thf(esti, type)
setof: (nat → \$o) → set   thf(setof, type)
∀xp: nat → \$o, xs: nat: ((esti@xs@(setof@xp)) ⇒ (xp@xs))   thf(estie, axiom)
n_1: nat   thf(n_1, type)
suc: nat → nat   thf(suc, type)
∀xs: set: ((esti@n_1@xs) ⇒ (∀xx: nat: ((esti@xx@xs) ⇒ (esti@(suc@xx)@xs)) ⇒ ∀xx: nat: (esti@xx@xs)))   thf(ax_5, axio
∀xp: nat → \$o, xs: nat: ((xp@xs) ⇒ (esti@xs@(setof@xp)))   thf(estii, axiom)
∀xx: nat: xx = (ts@xx@n_1)   thf(satz28e, axiom)
∀xx: nat: (ts@xx@n_1) = xx   thf(satz28a, axiom)
pl: nat → nat → nat   thf(pl, type)
∀xx: nat, xy: nat: (pl@(ts@xx@xy)@xx) = (ts@xx@(suc@xy))   thf(satz28f, axiom)
∀xx: nat, xy: nat, xz: nat: (ts@xx@(pl@xy@xz)) = (pl@(ts@xx@xy)@(ts@xx@xz))   thf($satz_{30}$, axiom)
∀xx: nat, xy: nat: (ts@xx@(suc@xy)) = (pl@(ts@xx@xy)@xx)   thf(satz28b, axiom)
(ts@(ts@x@y)@z) = (ts@x@(ts@y@z))   thf($satz_{31}$, conjecture)

NUM713∧1.p Landau theorem 32a
(forall x_0:nat. (ts x z = pl (ts y z) x_0))
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
pl: nat → nat → nat   thf(pl, type)
¬∀xx_0: nat: x ≠ (pl@y@xx_0)   thf(m, axiom)
ts: nat → nat → nat   thf(ts, type)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx)   thf($satz_{29}$, axiom)
∀xx: nat, xy: nat, xz: nat: (ts@xx@(pl@xy@xz)) = (pl@(ts@xx@xy)@(ts@xx@xz))   thf($satz_{30}$, axiom)
¬∀xx_0: nat: (ts@x@z) ≠ (pl@(ts@y@z)@xx_0)   thf(satz32a, conjecture)

NUM721∧1.p Landau theorem 34a
some (lambda v.diffprop (ts y u) (ts x z) v)
nat: \$tType   thf(nat_type, type)
x: nat   thf(x, type)
y: nat   thf(y, type)
z: nat   thf(z, type)
u: nat   thf(u, type)
some: (nat → \$o) → \$o   thf(some, type)
diffprop: nat → nat → nat → \$o   thf(diffprop, type)
some@λxv: nat: (diffprop@y@x@xv)   thf(l, axiom)
some@λxv: nat: (diffprop@u@z@xv)   thf(k, axiom)

ts: nat $\rightarrow$ nat $\rightarrow$ nat thf(ts, type)
$\forall$xx: nat, xy: nat, xz: nat, xu: nat: ((some@λxu: nat: (diffprop@xx@xy@xu)) $\Rightarrow$ ((some@λxu$_0$: nat: (diffprop@xz@xu@xu$_0$)) $\Rightarrow$
(some@λxu$_0$: nat: (diffprop@(ts@xx@xz)@(ts@xy@xu)@xu$_0$)))) thf(satz$_{34}$, axiom)
some@λxv: nat: (diffprop@(ts@y@u)@(ts@x@z)@xv) thf(satz34a, conjecture)

NUM726^1.p Landau theorem 38
ts (num y) (den x) = ts (num x) (den y)
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
nat: \$tType thf(nat_type, type)
ts: nat $\rightarrow$ nat $\rightarrow$ nat thf(ts, type)
num: frac $\rightarrow$ nat thf(num, type)
den: frac $\rightarrow$ nat thf(den, type)
(ts@(num@x)@(den@y)) = (ts@(num@y)@(den@x)) thf(e, axiom)
(ts@(num@y)@(den@x)) = (ts@(num@x)@(den@y)) thf(satz$_{38}$, conjecture)

NUM727^1.p Landau theorem 39
ts (num x) (den z) = ts (num z) (den x)
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
nat: \$tType thf(nat_type, type)
ts: nat $\rightarrow$ nat $\rightarrow$ nat thf(ts, type)
num: frac $\rightarrow$ nat thf(num, type)
den: frac $\rightarrow$ nat thf(den, type)
(ts@(num@x)@(den@y)) = (ts@(num@y)@(den@x)) thf(e, axiom)
(ts@(num@y)@(den@z)) = (ts@(num@z)@(den@y)) thf(f, axiom)
$\forall$xx: nat, xy: nat, xz: nat: ((ts@xx@xz) = (ts@xy@xz) $\Rightarrow$ xx = xy) thf(satz33b, axiom)
$\forall$xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx) thf(satz$_{29}$, axiom)
$\forall$xx: nat, xy: nat, xz: nat: (ts@(ts@xx@xy)@xz) = (ts@xx@(ts@xy@xz)) thf(satz$_{31}$, axiom)
(ts@(num@x)@(den@z)) = (ts@(num@z)@(den@x)) thf(satz$_{39}$, conjecture)

NUM728^1.p Landau theorem 40a
eq (fr (ts (1x x) n) (ts (2x x) n)) x
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
nat: \$tType thf(nat_type, type)
n: nat thf(n, type)
eq: frac $\rightarrow$ frac $\rightarrow$ \$o thf(eq, type)
fr: nat $\rightarrow$ nat $\rightarrow$ frac thf(fr, type)
ts: nat $\rightarrow$ nat $\rightarrow$ nat thf(ts, type)
c1x: frac $\rightarrow$ nat thf(c1x, type)
c2x: frac $\rightarrow$ nat thf(c2x, type)
$\forall$xx: frac, xy: frac: ((eq@xx@xy) $\Rightarrow$ (eq@xy@xx)) thf(satz$_{38}$, axiom)
$\forall$xx: frac, xn: nat: (eq@xx@(fr@(ts@(c1x@xx)@xn)@(ts@(c2x@xx)@xn))) thf(satz$_{40}$, axiom)
eq@(fr@(ts@(c1x@x)@n)@(ts@(c2x@x)@n))@x thf(satz40a, conjecture)

NUM729^1.p Landau theorem 40c
eq (fr (ts x1 n) (ts x2 n)) (fr x1 x2)
nat: \$tType thf(nat_type, type)
x_1: nat thf(x_1, type)
x_2: nat thf(x_2, type)
n: nat thf(n, type)
frac: \$tType thf(frac_type, type)
eq: frac $\rightarrow$ frac $\rightarrow$ \$o thf(eq, type)
fr: nat $\rightarrow$ nat $\rightarrow$ frac thf(fr, type)
ts: nat $\rightarrow$ nat $\rightarrow$ nat thf(ts, type)
$\forall$xx: frac, xy: frac: ((eq@xx@xy) $\Rightarrow$ (eq@xy@xx)) thf(satz$_{38}$, axiom)
$\forall$xx$_1$: nat, xx$_2$: nat, xn: nat: (eq@(fr@xx$_1$@xx$_2$)@(fr@(ts@xx$_1$@xn)@(ts@xx$_2$@xn))) thf(satz40b, axiom)

eq@(fr@(ts@x_1@n)@(ts@x_2@n))@(fr@x_1@x_2) thf(satz40c, conjecture)

NUM730^1.p Landau theorem 41
orec3 (ts (1x x) (2y y) = ts (1y y) (2x x)) (more (ts (1x x) (2y y)) (ts (1y y) (2x x))) (less (ts (1x x) (2y y)) (ts (1y y) (2x x)))
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
orec$_3$: \$o → \$o → \$o → \$o thf(orec$_3$, type)
nat: \$tType thf(nat_type, type)
ts: nat → nat → nat thf(ts, type)
c1x: frac → nat thf(c1x, type)
c2y: frac → nat thf(c2y, type)
c1y: frac → nat thf(c1y, type)
c2x: frac → nat thf(c2x, type)
more: nat → nat → \$o thf(more, type)
less: nat → nat → \$o thf(less, type)
∀xx: nat, xy: nat: (orec$_3$@xx = xy@(more@xx@xy)@(less@xx@xy)) thf(satz$_{10}$, axiom)
orec$_3$@(ts@(c1x@x)@(c2y@y)) = (ts@(c1y@y)@(c2x@x))@(more@(ts@(c1x@x)@(c2y@y))@(ts@(c1y@y)@(c2x@x)))@(less@

NUM736^1.p Landau theorem 42
less (ts (num y) (den x)) (ts (num x) (den y))
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
nat: \$tType thf(nat_type, type)
more: nat → nat → \$o thf(more, type)
ts: nat → nat → nat thf(ts, type)
num: frac → nat thf(num, type)
den: frac → nat thf(den, type)
more@(ts@(num@x)@(den@y))@(ts@(num@y)@(den@x)) thf(m, axiom)
less: nat → nat → \$o thf(less, type)
∀xx: nat, xy: nat: ((more@xx@xy) ⇒ (less@xy@xx)) thf(satz$_{11}$, axiom)
less@(ts@(num@y)@(den@x))@(ts@(num@x)@(den@y)) thf(satz$_{42}$, conjecture)

NUM737^1.p Landau theorem 44
more (ts (num z) (den u)) (ts (num u) (den z))
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
nat: \$tType thf(nat_type, type)
more: nat → nat → \$o thf(more, type)
ts: nat → nat → nat thf(ts, type)
num: frac → nat thf(num, type)
den: frac → nat thf(den, type)
more@(ts@(num@x)@(den@y))@(ts@(num@y)@(den@x)) thf(m, axiom)
(ts@(num@x)@(den@z)) = (ts@(num@z)@(den@x)) thf(e, axiom)
(ts@(num@y)@(den@u)) = (ts@(num@u)@(den@y)) thf(f, axiom)
∀xx: nat, xy: nat, xz: nat: ((more@(ts@xx@xz)@(ts@xy@xz)) ⇒ (more@xx@xy)) thf(satz33a, axiom)
∀xx: nat, xy: nat, xz: nat: ((more@xx@xy) ⇒ (more@(ts@xz@xx)@(ts@xz@xy))) thf(satz32d, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx) thf(satz$_{29}$, axiom)
∀xx: nat, xy: nat, xz: nat: (ts@(ts@xx@xy)@xz) = (ts@xx@(ts@xy@xz)) thf(satz$_{31}$, axiom)
∀xx: frac, xy: frac: ((ts@(num@xx)@(den@xy)) = (ts@(num@xy)@(den@xx)) ⇒ (ts@(num@xy)@(den@xx)) = (ts@(num@xx)@(den@xy))) thf(satz$_{38}$, axiom)
more@(ts@(num@z)@(den@u))@(ts@(num@u)@(den@z)) thf(satz$_{44}$, conjecture)

NUM738^1.p Landau theorem 45
lessf z u
frac: \$tType thf(frac_type, type)

x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
lessf: frac → frac → $o thf(lessf, type)
lessf@x@y thf(l, axiom)
eq: frac → frac → $o thf(eq, type)
eq@x@z thf(e, axiom)
eq@y@u thf(f, axiom)
moref: frac → frac → $o thf(moref, type)
∀xx: frac, xy: frac: ((moref@xx@xy) ⇒ (lessf@xy@xx)) thf($satz_{42}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz4
∀xx: frac, xy: frac: ((lessf@xx@xy) ⇒ (moref@xy@xx)) thf($satz_{43}$, axiom)
lessf@z@u thf($satz_{45}$, conjecture)

NUM739^1.p Landau theorem 46
(moref z u) → eq z u
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
moref: frac → frac → $o thf(moref, type)
eq: frac → frac → $o thf(eq, type)
¬ moref@x@y ⇒ (eq@x@y) thf(m, axiom)
eq@x@z thf(e, axiom)
eq@y@u thf(f, axiom)
∀xa: $o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz))) thf($satz_{39}$, axiom)
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx)) thf($satz_{38}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz4
¬ moref@z@u ⇒ (eq@z@u) thf($satz_{46}$, conjecture)

NUM740^1.p Landau theorem 48
(lessf y x) → eq y x
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
moref: frac → frac → $o thf(moref, type)
eq: frac → frac → $o thf(eq, type)
¬ moref@x@y ⇒ (eq@x@y) thf(m, axiom)
lessf: frac → frac → $o thf(lessf, type)
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx)) thf($satz_{38}$, axiom)
∀xx: frac, xy: frac: ((moref@xx@xy) ⇒ (lessf@xy@xx)) thf($satz_{42}$, axiom)
¬ lessf@y@x ⇒ (eq@y@x) thf($satz_{48}$, conjecture)

NUM741^1.p Landau theorem 50
less (ts (num x) (den z)) (ts (num z) (den x))
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
nat: $tType thf(nat_type, type)
less: nat → nat → $o thf(less, type)
ts: nat → nat → nat thf(ts, type)
num: frac → nat thf(num, type)
den: frac → nat thf(den, type)
less@(ts@(num@x)@(den@y))@(ts@(num@y)@(den@x)) thf(l, axiom)
less@(ts@(num@y)@(den@z))@(ts@(num@z)@(den@y)) thf(k, axiom)
∀xx: nat, xy: nat, xz: nat: ((less@(ts@xx@xz)@(ts@xy@xz)) ⇒ (less@xx@xy)) thf(satz33c, axiom)
∀xx: nat, xy: nat, xz: nat, xu: nat: ((less@xx@xy) ⇒ ((less@xz@xu) ⇒ (less@(ts@xx@xz)@(ts@xy@xu)))) thf(satz34a, ax

∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx)   thf(satz$_{29}$, axiom)
∀xx: nat, xy: nat, xz: nat: (ts@(ts@xx@xy)@xz) = (ts@xx@(ts@xy@xz))   thf(satz$_{31}$, axiom)
less@(ts@(num@x)@(den@z))@(ts@(num@z)@(den@x))   thf(satz$_{50}$, conjecture)

NUM742∧1.p Landau theorem 51a
lessf x z
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
z: frac   thf(z, type)
lessf: frac → frac → $o   thf(lessf, type)
eq: frac → frac → $o   thf(eq, type)
¬ lessf@x@y ⇒ (eq@x@y)   thf(l, axiom)
lessf@y@z thf(k, axiom)
∀xa: $o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((lessf@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (lessf@xz@xu))))   thf(satz$_{45}$, a
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx))   thf(satz$_{38}$, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ ((lessf@xy@xz) ⇒ (lessf@xx@xz)))   thf(satz$_{50}$, axiom)
∀xx: frac: (eq@xx@xx)   thf(satz$_{37}$, axiom)
lessf@x@z thf(satz51a, conjecture)

NUM743∧1.p Landau theorem 51b
lessf x z
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
z: frac   thf(z, type)
lessf: frac → frac → $o   thf(lessf, type)
lessf@x@y thf(l, axiom)
eq: frac → frac → $o   thf(eq, type)
¬ lessf@y@z ⇒ (eq@y@z)   thf(k, axiom)
∀xa: $o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((lessf@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (lessf@xz@xu))))   thf(satz$_{45}$, a
∀xx: frac: (eq@xx@xx)   thf(satz$_{37}$, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ ((lessf@xy@xz) ⇒ (lessf@xx@xz)))   thf(satz$_{50}$, axiom)
lessf@x@z thf(satz51b, conjecture)

NUM746∧1.p Landau theorem 52
(lessf x z) → eq x z
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
z: frac   thf(z, type)
lessf: frac → frac → $o   thf(lessf, type)
eq: frac → frac → $o   thf(eq, type)
¬ lessf@x@y ⇒ (eq@x@y)   thf(l, axiom)
¬ lessf@y@z ⇒ (eq@y@z)   thf(k, axiom)
∀xa: $o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz)))   thf(satz$_{39}$, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ ((¬ lessf@xy@xz ⇒ (eq@xy@xz)) ⇒ (lessf@xx@xz)))   thf(satz51b, axiom)
∀xx: frac, xy: frac, xz: frac: ((¬ lessf@xx@xy ⇒ (eq@xx@xy)) ⇒ ((lessf@xy@xz) ⇒ (lessf@xx@xz)))   thf(satz51a, axiom)
¬ lessf@x@z ⇒ (eq@x@z)   thf(satz$_{52}$, conjecture)

NUM747∧1.p Landau theorem 57a
eq (fr (pl x1 x2) n) (pf (fr x1 n) (fr x2 n))
nat: $tType   thf(nat_type, type)
x_1: nat   thf(x_1, type)
x_2: nat   thf(x_2, type)
n: nat   thf(n, type)
frac: $tType   thf(frac_type, type)
eq: frac → frac → $o   thf(eq, type)

fr: nat → nat → frac   thf(fr, type)
pl: nat → nat → nat   thf(pl, type)
pf: frac → frac → frac   thf(pf, type)
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx))   thf($satz_{38}$, axiom)
∀xx_1: nat, xx_2: nat, xn: nat: (eq@(pf@(fr@xx_1@xn)@(fr@xx_2@xn))@(fr@(pl@xx_1@xx_2)@xn))   thf($satz_{57}$, axiom)
eq@(fr@(pl@x_1@x_2)@n)@(pf@(fr@x_1@n)@(fr@x_2@n))   thf(satz57a, conjecture)

NUM748∧1.p Landau theorem 58
eq (fr (pl (ts (num x) (den y)) (ts (num y) (den x))) (ts (den x) (den y))) (fr (pl (ts (num y) (den x)) (ts (num x) (den y))) (ts (den y) (den x)))
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
eq: frac → frac → $o   thf(eq, type)
nat: $tType   thf(nat_type, type)
fr: nat → nat → frac   thf(fr, type)
pl: nat → nat → nat   thf(pl, type)
ts: nat → nat → nat   thf(ts, type)
num: frac → nat   thf(num, type)
den: frac → nat   thf(den, type)
∀xx: frac: (eq@xx@xx)   thf($satz_{37}$, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx)   thf($satz_{29}$, axiom)
∀xx: nat, xy: nat: (pl@xx@xy) = (pl@xy@xx)   thf($satz_{6}$, axiom)
eq@(fr@(pl@(ts@(num@x)@(den@y))@(ts@(num@y)@(den@x)))@(ts@(den@x)@(den@y)))@(fr@(pl@(ts@(num@y)@(den@

NUM749∧1.p Landau theorem 60a
less x (pf x y)
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
less: frac → frac → $o   thf(less, type)
pf: frac → frac → frac   thf(pf, type)
more: frac → frac → $o   thf(more, type)
∀xx: frac, xy: frac: ((more@xx@xy) ⇒ (less@xy@xx))   thf($satz_{42}$, axiom)
∀xx: frac, xy: frac: (more@(pf@xx@xy)@xx)   thf($satz_{60}$, axiom)
less@x@(pf@x@y)   thf(satz60a, conjecture)

NUM750∧1.p Landau theorem 62b
eq (pf x z) (pf y z)
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
z: frac   thf(z, type)
eq: frac → frac → $o   thf(eq, type)
eq@x@y thf(e, axiom)
pf: frac → frac → frac   thf(pf, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu))))   thf($satz_{56}$, axio
∀xx: frac: (eq@xx@xx)   thf($satz_{37}$, axiom)
eq@(pf@x@z)@(pf@y@z)   thf(satz62b, conjecture)

NUM751∧1.p Landau theorem 62d
moref (pf z x) (pf z y)
frac: $tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
z: frac   thf(z, type)
moref: frac → frac → $o   thf(moref, type)
moref@x@y thf(m, axiom)
pf: frac → frac → frac   thf(pf, type)
eq: frac → frac → $o   thf(eq, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu))))   thf(satz$_{4}$

∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xx@xz)@(pf@xy@xz))) thf(satz62a, axiom)
∀xx: frac, xy: frac: (eq@(pf@xx@xy)@(pf@xy@xx)) thf(satz$_{58}$, axiom)
moref@(pf@z@x)@(pf@z@y) thf(satz62d, conjecture)

NUM752^1.p Landau theorem 62e
eq (pf z x) (pf z y)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
eq: frac → frac → $o thf(eq, type)
eq@x@y thf(e, axiom)
pf: frac → frac → frac thf(pf, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu)))) thf(satz$_{56}$, axion
∀xx: frac: (eq@xx@xx) thf(satz$_{37}$, axiom)
eq@(pf@z@x)@(pf@z@y) thf(satz62e, conjecture)

NUM753^1.p Landau theorem 62g
moref (pf x z) (pf y u)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
eq: frac → frac → $o thf(eq, type)
eq@x@y thf(e, axiom)
moref: frac → frac → $o thf(moref, type)
moref@z@u thf(m, axiom)
pf: frac → frac → frac thf(pf, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz$_{4}$
∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xz@xx)@(pf@xz@xy))) thf(satz62d, axiom)
∀xx: frac: (eq@xx@xx) thf(satz$_{37}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu)))) thf(satz$_{56}$, axion
moref@(pf@x@z)@(pf@y@u) thf(satz62g, conjecture)

NUM754^1.p Landau theorem 62h
moref (pf z x) (pf u y)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
eq: frac → frac → $o thf(eq, type)
eq@x@y thf(e, axiom)
moref: frac → frac → $o thf(moref, type)
moref@z@u thf(m, axiom)
pf: frac → frac → frac thf(pf, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz$_{4}$
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((moref@xz@xu) ⇒ (moref@(pf@xx@xz)@(pf@xy@xu)))) thf(satz6
∀xx: frac, xy: frac: (eq@(pf@xx@xy)@(pf@xy@xx)) thf(satz$_{58}$, axiom)
moref@(pf@z@x)@(pf@u@y) thf(satz62h, conjecture)

NUM755^1.p Landau theorem 63a
moref x y
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
moref: frac → frac → $o thf(moref, type)
pf: frac → frac → frac thf(pf, type)
moref@(pf@x@z)@(pf@y@z) thf(m, axiom)

∀xa: \$o: (¬¬xa ⇒ xa) thf(et, axiom)
eq: frac → frac → \$o thf(eq, type)
lessf: frac → frac → \$o thf(lessf, type)
∀xx: frac, xy: frac: ¬((eq@xx@xy) ⇒ ¬moref@xx@xy) ⇒ ¬¬((moref@xx@xy) ⇒ ¬lessf@xx@xy) ⇒ ¬(lessf@xx@xy) ⇒ ¬eq@xx@xy thf(satz41b, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ (eq@(pf@xx@xz)@(pf@xy@xz))) thf(satz62b, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ (lessf@(pf@xx@xz)@(pf@xy@xz))) thf(satz62c, axiom)
∀xx: frac, xy: frac: (¬eq@xx@xy ⇒ (¬moref@xx@xy ⇒ (lessf@xx@xy))) thf(satz41a, axiom)
moref@x@y thf(satz63a, conjecture)

NUM756^1.p Landau theorem 63b
eq x y
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
eq: frac → frac → \$o thf(eq, type)
pf: frac → frac → frac thf(pf, type)
eq@(pf@x@z)@(pf@y@z) thf(e, axiom)
∀xa: \$o: (¬¬xa ⇒ xa) thf(et, axiom)
lessf: frac → frac → \$o thf(lessf, type)
moref: frac → frac → \$o thf(moref, type)
∀xx: frac, xy: frac: ¬((eq@xx@xy) ⇒ ¬moref@xx@xy) ⇒ ¬¬((moref@xx@xy) ⇒ ¬lessf@xx@xy) ⇒ ¬(lessf@xx@xy) ⇒ ¬eq@xx@xy thf(satz41b, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ (lessf@(pf@xx@xz)@(pf@xy@xz))) thf(satz62c, axiom)
∀xx: frac, xy: frac: (¬eq@xx@xy ⇒ (¬moref@xx@xy ⇒ (lessf@xx@xy))) thf(satz41a, axiom)
∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xx@xz)@(pf@xy@xz))) thf(satz62a, axiom)
eq@x@y thf(satz63b, conjecture)

NUM757^1.p Landau theorem 63c
lessf x y
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
lessf: frac → frac → \$o thf(lessf, type)
pf: frac → frac → frac thf(pf, type)
lessf@(pf@x@z)@(pf@y@z) thf(l, axiom)
∀xa: \$o: (¬¬xa ⇒ xa) thf(et, axiom)
moref: frac → frac → \$o thf(moref, type)
eq: frac → frac → \$o thf(eq, type)
∀xx: frac, xy: frac: ¬((eq@xx@xy) ⇒ ¬moref@xx@xy) ⇒ ¬¬((moref@xx@xy) ⇒ ¬lessf@xx@xy) ⇒ ¬(lessf@xx@xy) ⇒ ¬eq@xx@xy thf(satz41b, axiom)
∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xx@xz)@(pf@xy@xz))) thf(satz62a, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ (eq@(pf@xx@xz)@(pf@xy@xz))) thf(satz62b, axiom)
∀xx: frac, xy: frac: (¬eq@xx@xy ⇒ (¬moref@xx@xy ⇒ (lessf@xx@xy))) thf(satz41a, axiom)
lessf@x@y thf(satz63c, conjecture)

NUM758^1.p Landau theorem 63d
moref x y
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
moref: frac → frac → \$o thf(moref, type)
pf: frac → frac → frac thf(pf, type)
moref@(pf@z@x)@(pf@z@y) thf(m, axiom)
∀xx: frac, xy: frac, xz: frac: ((moref@(pf@xx@xz)@(pf@xy@xz)) ⇒ (moref@xx@xy)) thf(satz63a, axiom)
eq: frac → frac → \$o thf(eq, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz$_{4}$
∀xx: frac, xy: frac: (eq@(pf@xx@xy)@(pf@xy@xx)) thf(satz$_{58}$, axiom)

moref@x@y thf(satz63d, conjecture)

NUM759∧1.p Landau theorem 63e
eq x y
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
eq: frac → frac → $o thf(eq, type)
pf: frac → frac → frac thf(pf, type)
eq@(pf@z@x)@(pf@z@y) thf(e, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@(pf@xx@xz)@(pf@xy@xz)) ⇒ (eq@xx@xy)) thf(satz63b, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz))) thf(satz$_{39}$, axiom)
∀xx: frac, xy: frac: (eq@(pf@xx@xy)@(pf@xy@xx)) thf(satz$_{58}$, axiom)
eq@x@y thf(satz63e, conjecture)

NUM760∧1.p Landau theorem 64
moref (pf x z) (pf y u)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
moref: frac → frac → $o thf(moref, type)
moref@x@y thf(m, axiom)
moref@z@u thf(n, axiom)
pf: frac → frac → frac thf(pf, type)
lessf: frac → frac → $o thf(lessf, type)
∀xx: frac, xy: frac: ((lessf@xx@xy) ⇒ (moref@xy@xx)) thf(satz$_{43}$, axiom)
∀xx: frac, xy: frac, xz: frac: ((lessf@xx@xy) ⇒ ((lessf@xy@xz) ⇒ (lessf@xx@xz))) thf(satz$_{50}$, axiom)
∀xx: frac, xy: frac: ((moref@xx@xy) ⇒ (lessf@xy@xx)) thf(satz$_{42}$, axiom)
eq: frac → frac → $o thf(eq, type)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz$_{4}$
∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xx@xz)@(pf@xy@xz))) thf(satz$_{61}$, axiom)
∀xx: frac, xy: frac: (eq@(pf@xx@xy)@(pf@xy@xx)) thf(satz$_{58}$, axiom)
moref@(pf@x@z)@(pf@y@u) thf(satz$_{64}$, conjecture)

NUM761∧1.p Landau theorem 65a
moref (pf x z) (pf y u)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
moref: frac → frac → $o thf(moref, type)
eq: frac → frac → $o thf(eq, type)
¬ moref@x@y ⇒ (eq@x@y) thf(m, axiom)
moref@z@u thf(n, axiom)
pf: frac → frac → frac thf(pf, type)
∀xa: $o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((moref@xz@xu) ⇒ (moref@(pf@xx@xz)@(pf@xy@xu)))) thf(satz6
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((moref@xz@xu) ⇒ (moref@(pf@xx@xz)@(pf@xy@xu)))) thf(sa
moref@(pf@x@z)@(pf@y@u) thf(satz65a, conjecture)

NUM762∧1.p Landau theorem 65b
moref (pf x z) (pf y u)
frac: $tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)

moref: frac → frac → \$o thf(moref, type)
moref@x@y thf(m, axiom)
eq: frac → frac → \$o thf(eq, type)
¬ moref@z@u ⇒ (eq@z@u) thf(n, axiom)
pf: frac → frac → frac thf(pf, type)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((eq@xx@xz) ⇒ ((eq@xy@xu) ⇒ (moref@xz@xu)))) thf(satz$_{44}$
∀xx: frac, xy: frac, xz: frac: ((moref@xx@xy) ⇒ (moref@(pf@xx@xz)@(pf@xy@xz))) thf(satz$_{61}$, axiom)
∀xx: frac: (eq@xx@xx) thf(satz$_{37}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu)))) thf(satz$_{56}$, axior
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((moref@xz@xu) ⇒ (moref@(pf@xx@xz)@(pf@xy@xu)))) thf(sa
moref@(pf@x@z)@(pf@y@u) thf(satz65b, conjecture)

NUM765∧1.p Landau theorem 66
(moref (pf x z) (pf y u)) → eq (pf x z) (pf y u)
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
z: frac thf(z, type)
u: frac thf(u, type)
moref: frac → frac → \$o thf(moref, type)
eq: frac → frac → \$o thf(eq, type)
¬ moref@x@y ⇒ (eq@x@y) thf(m, axiom)
¬ moref@z@u ⇒ (eq@z@u) thf(n, axiom)
pf: frac → frac → frac thf(pf, type)
∀xa: \$o: (¬ ¬ xa ⇒ xa) thf(et, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu)))) thf(satz$_{56}$, axior
∀xx: frac, xy: frac, xz: frac, xu: frac: ((moref@xx@xy) ⇒ ((¬ moref@xz@xu ⇒ (eq@xz@xu)) ⇒ (moref@(pf@xx@xz)@(pf@x
∀xx: frac, xy: frac, xz: frac, xu: frac: ((¬ moref@xx@xy ⇒ (eq@xx@xy)) ⇒ ((moref@xz@xu) ⇒ (moref@(pf@xx@xz)@(pf@x
¬ moref@(pf@x@z)@(pf@y@u) ⇒ (eq@(pf@x@z)@(pf@y@u)) thf(satz$_{66}$, conjecture)

NUM766∧1.p Landau theorem 67a
(forall x_0:frac. (eq (pf y x_0) x))
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
nat: \$tType thf(nat_type, type)
ts: nat → nat → nat thf(ts, type)
num: frac → nat thf(num, type)
den: frac → nat thf(den, type)
pl: nat → nat → nat thf(pl, type)
¬ ∀xx$_0$: nat: (ts@(num@x)@(den@y)) ≠ (pl@(ts@(num@y)@(den@x))@xx$_0$) thf(m, axiom)
eq: frac → frac → \$o thf(eq, type)
pf: frac → frac → frac thf(pf, type)
fr: nat → nat → frac thf(fr, type)
ind: (nat → \$o) → nat thf(ind, type)
amone: (nat → \$o) → \$o thf(amone, type)
∀xx: nat, xy: nat: (amone@λxz: nat: xx = (pl@xy@xz)) thf(satz8b, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz))) thf(satz$_{39}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu)))) thf(satz$_{56}$, axior
∀xx: frac, xn: nat: (eq@xx@(fr@(ts@(num@xx)@xn)@(ts@(den@xx)@xn))) thf(satz$_{40}$, axiom)
∀xx: frac: (eq@xx@xx) thf(satz$_{37}$, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx) thf(satz$_{29}$, axiom)
∀xx$_1$: nat, xx$_2$: nat, xn: nat: (eq@(pf@(fr@xx$_1$@xn)@(fr@xx$_2$@xn))@(fr@(pl@xx$_1$@xx$_2$)@xn)) thf(satz$_{57}$, axiom)
∀xp: nat → \$o: (¬ (amone@xp) ⇒ ¬ ¬ ∀xx: nat: ¬ xp@xx ⇒ (xp@(ind@xp))) thf(oneax, axiom)
∀xx: frac, xn: nat: (eq@(fr@(ts@(num@xx)@xn)@(ts@(den@xx)@xn))@xx) thf(satz40a, axiom)
¬ ∀xx$_0$: frac: ¬ eq@(pf@y@xx$_0$)@x thf(satz67a, conjecture)

NUM767∧1.p Landau theorem 67b
eq ∨ w
frac: \$tType thf(frac_type, type)

x: frac   thf(x, type)
y: frac   thf(y, type)
v: frac   thf(v, type)
w: frac   thf(w, type)
eq: frac → frac → \$o   thf(eq, type)
pf: frac → frac → frac   thf(pf, type)
eq@(pf@y@v)@x thf(e, axiom)
eq@(pf@y@w)@x thf(f, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@(pf@xz@xx)@(pf@xz@xy)) ⇒ (eq@xx@xy))   thf(satz63e, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz)))   thf(satz$_{39}$, axiom)
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx))   thf(satz$_{38}$, axiom)
eq@v@w thf(satz67b, conjecture)

NUM768^1.p Landau theorem 67c
eq (pf y (fr (ind (lambda t.ts (num x) (den y) = pl (ts (num y) (den x)) t)) (ts (den x) (den y)))) x
frac: \$tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
nat: \$tType   thf(nat_type, type)
some: (nat → \$o) → \$o   thf(some, type)
ts: nat → nat → nat   thf(ts, type)
num: frac → nat   thf(num, type)
den: frac → nat   thf(den, type)
pl: nat → nat → nat   thf(pl, type)
some@λxu: nat: (ts@(num@x)@(den@y)) = (pl@(ts@(num@y)@(den@x))@xu)   thf(m, axiom)
eq: frac → frac → \$o   thf(eq, type)
pf: frac → frac → frac   thf(pf, type)
fr: nat → nat → frac   thf(fr, type)
ind: (nat → \$o) → nat   thf(ind, type)
amone: (nat → \$o) → \$o   thf(amone, type)
∀xx: nat, xy: nat: (amone@λxz: nat: xx = (pl@xy@xz))   thf(satz8b, axiom)
∀xx: frac, xy: frac, xz: frac: ((eq@xx@xy) ⇒ ((eq@xy@xz) ⇒ (eq@xx@xz)))   thf(satz$_{39}$, axiom)
∀xx: frac, xy: frac, xz: frac, xu: frac: ((eq@xx@xy) ⇒ ((eq@xz@xu) ⇒ (eq@(pf@xx@xz)@(pf@xy@xu))))   thf(satz$_{56}$, axio
∀xx: frac, xn: nat: (eq@xx@(fr@(ts@(num@xx)@xn)@(ts@(den@xx)@xn)))   thf(satz$_{40}$, axiom)
∀xx: frac: (eq@xx@xx)   thf(satz$_{37}$, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx)   thf(satz$_{29}$, axiom)
∀xx$_1$: nat, xx$_2$: nat, xn: nat: (eq@(pf@(fr@xx$_1$@xn)@(fr@xx$_2$@xn))@(fr@(pl@xx$_1$@xx$_2$)@xn))   thf(satz$_{57}$, axiom)
∀xp: nat → \$o: (¬(amone@xp) ⇒ ¬some@xp ⇒ (xp@(ind@xp)))   thf(oneax, axiom)
∀xx: frac, xn: nat: (eq@(fr@(ts@(num@xx)@xn)@(ts@(den@xx)@xn))@xx)   thf(satz40a, axiom)
eq@(pf@y@(fr@(ind@λxt: nat: (ts@(num@x)@(den@y)) = (pl@(ts@(num@y)@(den@x))@xt))@(ts@(den@x)@(den@y))))@x

NUM769^1.p Landau theorem 67d
eq x (pf y (mf x y))
frac: \$tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
moref: frac → frac → \$o   thf(moref, type)
moref@x@y thf(m, axiom)
eq: frac → frac → \$o   thf(eq, type)
pf: frac → frac → frac   thf(pf, type)
mf: frac → frac → frac   thf(mf, type)
∀xx: frac, xy: frac: ((eq@xx@xy) ⇒ (eq@xy@xx))   thf(satz$_{38}$, axiom)
∀xx: frac, xy: frac: ((moref@xx@xy) ⇒ (eq@(pf@xy@(mf@xx@xy))@xx))   thf(satz67c, axiom)
eq@x@(pf@y@(mf@x@y))   thf(satz67d, conjecture)

NUM770^1.p Landau theorem 67e
eq ∨ (mf x y)
frac: \$tType   thf(frac_type, type)
x: frac   thf(x, type)
y: frac   thf(y, type)
v: frac   thf(v, type)

moref: frac → frac → \$o thf(moref, type)
moref@x@y thf(m, axiom)
eq: frac → frac → \$o thf(eq, type)
pf: frac → frac → frac thf(pf, type)
eq@(pf@y@v)@x thf(e, axiom)
mf: frac → frac → frac thf(mf, type)
∀xx: frac, xy: frac, xv: frac, xw: frac: ((eq@(pf@xy@xv)@xx) ⇒ ((eq@(pf@xy@xw)@xx) ⇒ (eq@xv@xw))) thf(satz67b, a
∀xx: frac, xy: frac: ((moref@xx@xy) ⇒ (eq@(pf@xy@(mf@xx@xy))@xx)) thf(satz67c, axiom)
eq@v@(mf@x@y) thf(satz67e, conjecture)

NUM771∧1.p Landau theorem 69
eq (fr (ts (num x) (num y)) (ts (den x) (den y))) (fr (ts (num y) (num x)) (ts (den y) (den x)))
frac: \$tType thf(frac_type, type)
x: frac thf(x, type)
y: frac thf(y, type)
eq: frac → frac → \$o thf(eq, type)
nat: \$tType thf(nat_type, type)
fr: nat → nat → frac thf(fr, type)
ts: nat → nat → nat thf(ts, type)
num: frac → nat thf(num, type)
den: frac → nat thf(den, type)
∀xx: frac: (eq@xx@xx) thf($satz_{37}$, axiom)
∀xx: nat, xy: nat: (ts@xx@xy) = (ts@xy@xx) thf($satz_{29}$, axiom)
eq@(fr@(ts@(num@x)@(num@y))@(ts@(den@x)@(den@y)))@(fr@(ts@(num@y)@(num@x))@(ts@(den@y)@(den@x))) t

NUM781∧1.p Landau theorem 79
y0 = x0
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
$x_0 = y_0$ thf(i, axiom)
$y_0 = x_0$ thf($satz_{79}$, conjecture)

NUM781∧2.p Landau theorem 79
a: \$tType thf(a_type, type)
∀a: a, b: a: ($a = b$ ⇒ $b = a$) thf(cES_eq_, conjecture)

NUM781∧3.p Landau theorem 79
Symmetry of equality.
cY: \$i thf(cY, type)
cX: \$i thf(cX, type)
cY = cX ⇒ cX = cY thf($cTHM_{76}$, conjecture)

NUM782∧1.p Landau theorem 80
x0 = z0
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
z_0: rat thf(z_0, type)
$x_0 = y_0$ thf(i, axiom)
$y_0 = z_0$ thf(j, axiom)
$x_0 = z_0$ thf($satz_{80}$, conjecture)

NUM783∧1.p Landau theorem 81a
or3 (is x0 y0) (more x0 y0) (less x0 y0)
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
or_3: \$o → \$o → \$o → \$o thf(or_3, type)
is: rat → rat → \$o thf(is, type)
more: rat → rat → \$o thf(more, type)
less: rat → rat → \$o thf(less, type)
∀xa: \$o: (¬¬xa ⇒ xa) thf(et, axiom)

ec$_3$: \$o → \$o → \$o → \$o   thf(ec$_3$, type)
$\forall$xx$_0$: rat, xy$_0$: rat: ¬ (or$_3$@(is@xx$_0$@xy$_0$)@(more@xx$_0$@xy$_0$)@(less@xx$_0$@xy$_0$)) ⇒ ¬ ec$_3$@(is@xx$_0$@xy$_0$)@(more@xx$_0$@xy$_0$)@(
or$_3$@(is@x_0@y_0)@(more@x_0@y_0)@(less@x_0@y_0)   thf(satz81a, conjecture)

NUM784^1.p Landau theorem 81c
(less x0 y0)
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
more: rat → rat → \$o   thf(more, type)
is: rat → rat → \$o   thf(is, type)
¬ more@x_0@y_0 ⇒ (is@x_0@y_0)   thf(m, axiom)
less: rat → rat → \$o   thf(less, type)
$\forall$xa: \$o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
$\forall$xx$_0$: rat, xy$_0$: rat: ¬ ((is@xx$_0$@xy$_0$) ⇒ ¬ more@xx$_0$@xy$_0$) ⇒ ¬ ¬ ((more@xx$_0$@xy$_0$) ⇒ ¬ less@xx$_0$@xy$_0$) ⇒
¬ (less@xx$_0$@xy$_0$) ⇒ ¬ is@xx$_0$@xy$_0$ thf(satz81b, axiom)
¬ less@x_0@y_0 thf(satz81c, conjecture)

NUM785^1.p Landau theorem 81d
(more x0 y0)
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
less: rat → rat → \$o   thf(less, type)
is: rat → rat → \$o   thf(is, type)
¬ less@x_0@y_0 ⇒ (is@x_0@y_0)   thf(l, axiom)
more: rat → rat → \$o   thf(more, type)
$\forall$xa: \$o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
$\forall$xx$_0$: rat, xy$_0$: rat: ¬ ((is@xx$_0$@xy$_0$) ⇒ ¬ more@xx$_0$@xy$_0$) ⇒ ¬ ¬ ((more@xx$_0$@xy$_0$) ⇒ ¬ less@xx$_0$@xy$_0$) ⇒
¬ (less@xx$_0$@xy$_0$) ⇒ ¬ is@xx$_0$@xy$_0$ thf(satz81b, axiom)
¬ more@x_0@y_0 thf(satz81d, conjecture)

NUM786^1.p Landau theorem 81e
(less x0 y0) → is x0 y0
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
more: rat → rat → \$o   thf(more, type)
¬ more@x_0@y_0 thf(n, axiom)
less: rat → rat → \$o   thf(less, type)
is: rat → rat → \$o   thf(is, type)
$\forall$xa: \$o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
$\forall$xx$_0$: rat, xy$_0$: rat: (¬ is@xx$_0$@xy$_0$ ⇒ (¬ more@xx$_0$@xy$_0$ ⇒ (less@xx$_0$@xy$_0$)))   thf(satz81a, axiom)
¬ less@x_0@y_0 ⇒ (is@x_0@y_0)   thf(satz81e, conjecture)

NUM787^1.p Landau theorem 81f
(more x0 y0) → is x0 y0
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
less: rat → rat → \$o   thf(less, type)
¬ less@x_0@y_0 thf(n, axiom)
more: rat → rat → \$o   thf(more, type)
is: rat → rat → \$o   thf(is, type)
$\forall$xa: \$o: (¬ ¬ xa ⇒ xa)   thf(et, axiom)
$\forall$xx$_0$: rat, xy$_0$: rat: (¬ is@xx$_0$@xy$_0$ ⇒ (¬ more@xx$_0$@xy$_0$ ⇒ (less@xx$_0$@xy$_0$)))   thf(satz81a, axiom)
¬ more@x_0@y_0 ⇒ (is@x_0@y_0)   thf(satz81f, conjecture)

NUM788^1.p Landau theorem 81g
((less x0 y0) → is x0 y0)
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)

y_0: rat   thf(y_0, type)
more: rat → rat → \$o   thf(more, type)
more@x_0@y_0 thf(m, axiom)
less: rat → rat → \$o   thf(less, type)
is: rat → rat → \$o   thf(is, type)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx_0: rat, xy_0: rat: ¬((is@xx_0@xy_0) ⇒ ¬more@xx_0@xy_0) ⇒ ¬¬((more@xx_0@xy_0) ⇒ ¬less@xx_0@xy_0) ⇒ ¬(less@xx_0@xy_0) ⇒ ¬is@xx_0@xy_0 thf(satz81b, axiom)
¬¬less@x_0@y_0 ⇒ (is@x_0@y_0)   thf(satz81g, conjecture)

NUM789∧1.p Landau theorem 81h
((more x0 y0) → is x0 y0)
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
less: rat → rat → \$o   thf(less, type)
less@x_0@y_0 thf(l, axiom)
more: rat → rat → \$o   thf(more, type)
is: rat → rat → \$o   thf(is, type)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx_0: rat, xy_0: rat: ¬((is@xx_0@xy_0) ⇒ ¬more@xx_0@xy_0) ⇒ ¬¬((more@xx_0@xy_0) ⇒ ¬less@xx_0@xy_0) ⇒ ¬(less@xx_0@xy_0) ⇒ ¬is@xx_0@xy_0 thf(satz81b, axiom)
¬¬more@x_0@y_0 ⇒ (is@x_0@y_0)   thf(satz81h, conjecture)

NUM790∧1.p Landau theorem 81j
less x0 y0
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
more: rat → rat → \$o   thf(more, type)
is: rat → rat → \$o   thf(is, type)
¬¬more@x_0@y_0 ⇒ (is@x_0@y_0)   thf(n, axiom)
less: rat → rat → \$o   thf(less, type)
∀xx_0: rat, xy_0: rat: (¬is@xx_0@xy_0 ⇒ (¬more@xx_0@xy_0 ⇒ (less@xx_0@xy_0)))   thf(satz81a, axiom)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
less@x_0@y_0 thf(satz81j, conjecture)

NUM791∧1.p Landau theorem 81k
more x0 y0
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
less: rat → rat → \$o   thf(less, type)
is: rat → rat → \$o   thf(is, type)
¬¬less@x_0@y_0 ⇒ (is@x_0@y_0)   thf(n, axiom)
more: rat → rat → \$o   thf(more, type)
∀xa: \$o: (¬¬xa ⇒ xa)   thf(et, axiom)
∀xx_0: rat, xy_0: rat: (¬is@xx_0@xy_0 ⇒ (¬more@xx_0@xy_0 ⇒ (less@xx_0@xy_0)))   thf(satz81a, axiom)
more@x_0@y_0 thf(satz81k, conjecture)

NUM792∧1.p Landau theorem 87c
more x0 z0
rat: \$tType   thf(rat_type, type)
x_0: rat   thf(x_0, type)
y_0: rat   thf(y_0, type)
z_0: rat   thf(z_0, type)
moreis: rat → rat → \$o   thf(moreis, type)
moreis@x_0@y_0 thf(m, axiom)
more: rat → rat → \$o   thf(more, type)
more@y_0@z_0 thf(n, axiom)
less: rat → rat → \$o   thf(less, type)

$\forall xx_0\colon rat, xy_0\colon rat\colon ((less@xx_0@xy_0) \Rightarrow (more@xy_0@xx_0))$ thf($satz_{83}$, axiom)
lessis: rat → rat → \$o thf(lessis, type)
$\forall xx_0\colon rat, xy_0\colon rat, xz_0\colon rat\colon ((less@xx_0@xy_0) \Rightarrow ((lessis@xy_0@xz_0) \Rightarrow (less@xx_0@xz_0)))$ thf(satz87b, axiom)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((more@xx_0@xy_0) \Rightarrow (less@xy_0@xx_0))$ thf($satz_{82}$, axiom)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((moreis@xx_0@xy_0) \Rightarrow (lessis@xy_0@xx_0))$ thf($satz_{84}$, axiom)
more@x_0@z_0 thf(satz87c, conjecture)

NUM793^1.p Landau theorem 87d
more x0 z0
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
z_0: rat thf(z_0, type)
more: rat → rat → \$o thf(more, type)
more@x_0@y_0 thf(m, axiom)
moreis: rat → rat → \$o thf(moreis, type)
moreis@y_0@z_0 thf(n, axiom)
less: rat → rat → \$o thf(less, type)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((less@xx_0@xy_0) \Rightarrow (more@xy_0@xx_0))$ thf($satz_{83}$, axiom)
lessis: rat → rat → \$o thf(lessis, type)
$\forall xx_0\colon rat, xy_0\colon rat, xz_0\colon rat\colon ((lessis@xx_0@xy_0) \Rightarrow ((less@xy_0@xz_0) \Rightarrow (less@xx_0@xz_0)))$ thf(satz87a, axiom)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((moreis@xx_0@xy_0) \Rightarrow (lessis@xy_0@xx_0))$ thf($satz_{84}$, axiom)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((more@xx_0@xy_0) \Rightarrow (less@xy_0@xx_0))$ thf($satz_{82}$, axiom)
more@x_0@z_0 thf(satz87d, conjecture)

NUM795^1.p Landau theorem 99c
less (pl x0 z0) (pl y0 u0)
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
z_0: rat thf(z_0, type)
u_0: rat thf(u_0, type)
lessis: rat → rat → \$o thf(lessis, type)
lessis@x_0@y_0 thf(l, axiom)
less: rat → rat → \$o thf(less, type)
less@z_0@u_0 thf(k, axiom)
pl: rat → rat → rat thf(pl, type)
more: rat → rat → \$o thf(more, type)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((more@xx_0@xy_0) \Rightarrow (less@xy_0@xx_0))$ thf($satz_{82}$, axiom)
moreis: rat → rat → \$o thf(moreis, type)
$\forall xx_0\colon rat, xy_0\colon rat, xz_0\colon rat, xu_0\colon rat\colon ((moreis@xx_0@xy_0) \Rightarrow ((more@xz_0@xu_0) \Rightarrow (more@(pl@xx_0@xz_0)@(pl@xy_0@xu_0))))$
$\forall xx_0\colon rat, xy_0\colon rat\colon ((lessis@xx_0@xy_0) \Rightarrow (moreis@xy_0@xx_0))$ thf($satz_{85}$, axiom)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((less@xx_0@xy_0) \Rightarrow (more@xy_0@xx_0))$ thf($satz_{83}$, axiom)
less@(pl@x_0@z_0)@(pl@y_0@u_0) thf(satz99c, conjecture)

NUM796^1.p Landau theorem 99d
less (pl x0 z0) (pl y0 u0)
rat: \$tType thf(rat_type, type)
x_0: rat thf(x_0, type)
y_0: rat thf(y_0, type)
z_0: rat thf(z_0, type)
u_0: rat thf(u_0, type)
less: rat → rat → \$o thf(less, type)
less@x_0@y_0 thf(l, axiom)
lessis: rat → rat → \$o thf(lessis, type)
lessis@z_0@u_0 thf(k, axiom)
pl: rat → rat → rat thf(pl, type)
more: rat → rat → \$o thf(more, type)
$\forall xx_0\colon rat, xy_0\colon rat\colon ((more@xx_0@xy_0) \Rightarrow (less@xy_0@xx_0))$ thf($satz_{82}$, axiom)
moreis: rat → rat → \$o thf(moreis, type)
$\forall xx_0\colon rat, xy_0\colon rat, xz_0\colon rat, xu_0\colon rat\colon ((more@xx_0@xy_0) \Rightarrow ((moreis@xz_0@xu_0) \Rightarrow (more@(pl@xx_0@xz_0)@(pl@xy_0@xu_0))))$

$\forall \mathrm{xx}_0$: rat, xy_0: rat: ((less@xx_0@xy_0) $\Rightarrow$ (more@xy_0@xx_0)) thf(satz$_{83}$, axiom)
$\forall \mathrm{xx}_0$: rat, xy_0: rat: ((lessis@xx_0@xy_0) $\Rightarrow$ (moreis@xy_0@xx_0)) thf(satz$_{85}$, axiom)
less@(pl@x_0@z_0)@(pl@y_0@u_0) thf(satz99d, conjecture)

NUM797^1.p Landau theorem 4
1: \$i thf(one_type, type)
succ: \$i → \$i thf(succ_type, type)
$\forall x$: \$i: (succ@x) ≠ 1 thf(one_is_first, axiom)
$\forall x$: \$i, y: \$i: ((succ@x) = (succ@y) $\Rightarrow$ $x = y$) thf(succ_injective, axiom)
$\forall m$: \$i → \$o: ((m@1 and $\forall x$: \$i: ((m@x) $\Rightarrow$ (m@(succ@x)))) $\Rightarrow$ $\forall y$: \$i: (m@y)) thf(induction, axiom)
$\exists p$: \$i → \$i → \$i: ($\forall x$: \$i: (p@x@1) = (succ@x) and $\forall x$: \$i, y: \$i: (p@x@(succ@y)) = (succ@(p@x@y))) thf(satz$_4$, conjectu

NUM798^1.p Something times one is one
include('Axioms/NUM006^0.ax')
$\exists n$: (\$i → \$i) → \$i → \$i: (·@n@1) = 1 thf(thm, conjecture)

NUM799^1.p Something times four equal five plus seven
include('Axioms/NUM006^0.ax')
$\exists n$: (\$i → \$i) → \$i → \$i: (·@n@four) = (+@five@seven) thf(thm, conjecture)

NUM800^1.p Some function of two and three is six, and of one and two is two
include('Axioms/NUM006^0.ax')
$\exists h$: ((\$i → \$i) → \$i → \$i) → ((\$i → \$i) → \$i → \$i) → (\$i → \$i) → \$i → \$i: ((h@two@three) = six and (h@1@two) = two) thf(thm, conjecture)

NUM801^1.p Something times four equals five plus something
include('Axioms/NUM006^0.ax')
$\exists n$: (\$i → \$i) → \$i → \$i, m: (\$i → \$i) → \$i → \$i: (·@n@four) = (+@five@m) thf(thm, conjecture)

NUM802^5.p TPS problem BLEDSOE-FENG-8
There is a set containing no nonnegative numbers and containing -2.
c_2: \$i thf(c_2, type)
absval: \$i → \$i thf(absval, type)
c_0: \$i thf(c_0, type)
c_less_: \$i → \$i → \$o thf(c_less_, type)
(c_less_@c_2@c_0) $\Rightarrow$ ($\forall$xu: \$i, xv: \$i: ((c_less_@xu@c_0) $\Rightarrow$ xu ≠ (absval@xv)) $\Rightarrow$ $\exists a$: \$i → \$o: ($\forall$xy: \$i: ¬ a@(absval@xy) and

NUM803^5.p TPS problem from NATS
n: \$tType thf(n_type, type)
c_0: n thf(c_0, type)
c_star: $n \to n \to n$ thf(c_star, type)
$\forall$xx: n: (c_star@xx@c_0) = c_0 thf(cPA$_3$, conjecture)

NUM804^5.p TPS problem from NATS
n: \$tType thf(n_type, type)
c_0: n thf(c_0, type)
c_plus: $n \to n \to n$ thf(c_plus, type)
$\forall$xx: n: (c_plus@xx@c_0) = xx thf(cPA$_1$, conjecture)

NUM805^5.p TPS problem from NATS
n: \$tType thf(n_type, type)
c_plus: $n \to n \to n$ thf(c_plus, type)
cS: $n \to n$ thf(cS, type)
$\forall$xx: n, xy: n: (c_plus@xx@(cS@xy)) = (cS@(c_plus@xx@xy)) thf(cPA$_2$, conjecture)

NUM806^5.p TPS problem from NATS
n: \$tType thf(n_type, type)
c_star: $n \to n \to n$ thf(c_star, type)
c_plus: $n \to n \to n$ thf(c_plus, type)
cS: $n \to n$ thf(cS, type)
$\forall$xx: n, xy: n: (c_star@xx@(cS@xy)) = (c_plus@(c_star@xx@xy)@xx) thf(cPA$_4$, conjecture)

NUM807^5.p TPS problem from NATS
n: \$tType thf(n_type, type)
cS: $n \to n$ thf(cS, type)
c_0: n thf(c_0, type)

∀xp: $n \to n$, xq: $n \to n$: (((xp@c_0) = (xq@c_0) and ∀xx: n: ((xp@xx) = (xq@xx) ⇒ (xp@(cS@xx)) = (xq@(cS@xx)))) ⇒ ∀xx: n: (xp@xx) = (xq@xx))   thf(cPA_IND_EQ, conjecture)

NUM808^5.p TPS problem THM130A
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
r: $i → $i → $o   thf(r_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cIND and r@c_0@c_0 and ∀xx: $i: (($r$@xx@xx) ⇒ ($r$@(cS@xx)@(cS@xx)))) ⇒ ∀xx: $i: ∃xy: $i: ($r$@xx@xy)   thf(cTHM13

NUM809^5.p TPS problem THM130
Induction theorem in which the conclusion is weaker than the statement which must be proved by induction.
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
r: $i → $i → $o   thf(r_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cIND and r@c_0@c_0 and ∀xx: $i, xy: $i: (($r$@xx@xy) ⇒ ($r$@(cS@xx)@(cS@xy)))) ⇒ ∀xx: $i: ∃xy: $i: ($r$@xx@xy)   thf(cT

NUM810^5.p TPS problem THM140
Existence of doubles of naturals.
c_0: $i   thf(c0_type, type)
cDOUBLE: $i → $i → $o   thf(cDOUBLE_type, type)
cS: $i → $i   thf(cS_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cIND and cDOUBLE@c_0@c_0 and ∀xx: $i, xy: $i: ((cDOUBLE@xx@xy) ⇒ (cDOUBLE@(cS@xx)@(cS@(cS@xy))))) ⇒ ∀xx: $i: ∃xy: $i: (cDOUBLE@xx@xy)   thf(cTHM$_{140}$, conjecture)

NUM811^5.p TPS problem THM129
Induction theorem for addition.
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
c_plus: $i → $i → $i → $o   thf(c_plus_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cIND and ∀xx: $i: (c_plus@$c_0$@xx@xx) and ∀xx: $i, xy: $i, xz: $i: ((c_plus@xy@xx@xz) ⇒ (c_plus@(cS@xy)@xx@(cS@xz))))
∀xy: $i, xx: $i: ∃xz: $i: (c_plus@xy@xx@xz)   thf(cTHM$_{129}$, conjecture)

NUM812^5.p TPS problem THM578
Variant of THM6104, including induction in the hypothesis.
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
cIND ⇒ ∀xn: $i: (xn = c_0 or ∃xm: $i: xn = (cS@xm))   thf(cTHM$_{578}$, conjecture)

NUM813^5.p TPS problem THM303
c_0: $i   thf(c0_type, type)
cEVEN: $i → $o   thf(cEVEN_type, type)
cNUMBER: $i → $o   thf(cNUMBER_type, type)
cODD: $i → $o   thf(cODD_type, type)
cS: $i → $i   thf(cS_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cEVEN@c_0 and ∀xn: $i: ((cEVEN@xn) ⇒ (cEVEN@(cS@(cS@xn)))) and cODD@(cS@c_0) and ∀xn: $i: ((cODD@xn) ⇒ (cODD@(cS@(cS@xn)))) and cIND and ∀xn: $i: ((cNUMBER@xn) ⟺ (cEVEN@xn or cODD@xn))) ⇒ ∀xn: $i: (cNUMI

NUM814^5.p TPS problem from IND-THMS
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
cEVEN$_1$: $i → $o   thf(cEVEN1_type, type)

cIND: \$o thf(cIND_type, type)
cODD$_1$: \$i → \$o thf(cODD1_type, type)
cPEANO$_1$: \$o thf(cPEANO1_type, type)
cEVEN$_1$ = (λxn: \$i: ∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@(cS@xx))))) ⇒ (xp@xn))) thf(cEVEN1_
cIND = (∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: \$i: (xp@xx))) thf(cIND_def, definitio
cODD$_1$ = (λxn: \$i: ¬ cEVEN$_1$@xn) thf(cODD1_def, definition)
cPEANO$_1$ = (∀xu: \$i: (cS@xu) ≠ c_0 and ∀xv: \$i, xw: \$i: ((cS@xv) = (cS@xw) ⇒ xv = xw) and cIND) thf(cPEANO1_de
cPEANO$_1$ ⇒ ∃xn: \$i: (cODD$_1$@xn) thf(cTHM$_{404}$, conjecture)

NUM815^5.p TPS problem from IND-THMS
c_0: \$i thf(c0_type, type)
cPSI: \$i → \$i → \$i thf(cPSI_type, type)
cS: \$i → \$i thf(cS_type, type)
cIND: \$o thf(cIND_type, type)
cPETER_INDEQS: \$i → (\$i → \$i) → (\$i → \$i → \$i) → \$o thf(cPETER_INDEQS_type, type)
cIND = (∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: \$i: (xp@xx))) thf(cIND_def, definitio
cPETER_INDEQS = (λx_0: \$i, s: \$i → \$i, pSI: \$i → \$i → \$i: (∀xn: \$i: (pSI@x_0@xn) = (s@xn) and ∀xm: \$i: (pSI@(s@xm)@x_0)
(pSI@xm@(s@x_0)) and ∀xm: \$i, xn: \$i: (pSI@(s@xm)@(s@xn)) = (pSI@xm@(pSI@(s@xm)@xn)))) thf(cPETER_INDEQ
(cIND and cPETER_INDEQS@c_0@cS@cPSI) ⇒ ∀xm: \$i, xn: \$i: ∃xk: \$i: (cPSI@xm@xn) = (cS@xk) thf(cTHM$_{585}$, conje

NUM816^5.p TPS problem from IND-THMS
c_0: \$i thf(c0_type, type)
cS: \$i → \$i thf(cS_type, type)
cEVEN$_1$: \$i → \$o thf(cEVEN1_type, type)
cODD$_1$: \$i → \$o thf(cODD1_type, type)
cEVEN$_1$ = (λxn: \$i: ∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@(cS@xx))))) ⇒ (xp@xn))) thf(cEVEN1_
cODD$_1$ = (λxn: \$i: ¬ cEVEN$_1$@xn) thf(cODD1_def, definition)
(∀xu: \$i: (cS@xu) ≠ c_0 and ∀xv: \$i, xw: \$i: ((cS@xv) = (cS@xw) ⇒ xv = xw)) ⇒ (cODD$_1$@(cS@c_0)) thf(cTHM$_{406}$, conj

NUM817^5.p TPS problem from IND-THMS
c_0: \$i thf(c0_type, type)
cS: \$i → \$i thf(cS_type, type)
cEVEN$_1$: \$i → \$o thf(cEVEN1_type, type)
cODD$_1$: \$i → \$o thf(cODD1_type, type)
cEVEN$_1$ = (λxn: \$i: ∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@(cS@xx))))) ⇒ (xp@xn))) thf(cEVEN1_
cODD$_1$ = (λxn: \$i: ¬ cEVEN$_1$@xn) thf(cODD1_def, definition)
(∀xu: \$i: (cS@xu) ≠ c_0 and ∀xv: \$i, xw: \$i: ((cS@xv) = (cS@xw) ⇒ xv = xw)) ⇒ ∃xn: \$i: (cODD$_1$@xn) thf(cTHM$_{405}$, c

NUM818^5.p TPS problem from IND-THMS
a: \$tType thf(a_type, type)
h: ((\$i → \$o) → \$o) → a → a thf(h, type)
cS: ((\$i → \$o) → \$o) → (\$i → \$o) → \$o thf(cS, type)
cO: (\$i → \$o) → \$o thf(cO, type)
g: a thf(g, type)
∃f: ((\$i → \$o) → \$o) → a: ((f@cO) = g and ∀xn: (\$i → \$o) → \$o: (∀xp: ((\$i → \$o) → \$o) → \$o: ((xp@cO and ∀xx: (\$i →
\$o) → \$o: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ (xp@xn)) ⇒ (f@(cS@xn)) = (h@xn@(f@xn)))) thf(cT6400A_pme, conjectu

NUM819^5.p TPS problem from IND-THMS
c_0: \$i thf(c0_type, type)
cEVEN: \$i → \$o thf(cEVEN_type, type)
cODD: \$i → \$o thf(cODD_type, type)
cS: \$i → \$i thf(cS_type, type)
cIND: \$o thf(cIND_type, type)
cIND = (∀xp: \$i → \$o: ((xp@c_0 and ∀xx: \$i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: \$i: (xp@xx))) thf(cIND_def, definitio
(cEVEN@c_0 and ∀xn: \$i: ((cEVEN@xn) ⇒ (cEVEN@(cS@(cS@xn)))) and cODD@(cS@c_0) and ∀xn: \$i: ((cODD@xn) ⇒
(cODD@(cS@(cS@xn)))) and cIND) ⇒ ∀xn: \$i: (cEVEN@xn or cODD@xn) thf(cEVEN_ODD$_1$, conjecture)

NUM821^5.p TPS problem from IND-THMS
c_0: \$i thf(c0_type, type)
cEVEN: \$i → \$o thf(cEVEN_type, type)
cNUMBER: \$i → \$o thf(cNUMBER_type, type)
cODD: \$i → \$o thf(cODD_type, type)
cS: \$i → \$i thf(cS_type, type)

cIND: $o thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx))) thf(cIND_def, definition
(cIND and cEVEN@c_0 and ∀xn: $i: ((cEVEN@xn) ⇒ (cEVEN@(cS@(cS@xn)))) and cODD@(cS@c_0) and ∀xn: $i: ((cODD
(cODD@(cS@(cS@xn)))) and ∀xn: $i: ((cNUMBER@xn) ⟺ (cEVEN@xn or cODD@xn))) ⇒ ∀xn: $i: (cNUMBER@xn)

NUM822^5.p TPS problem from IND-THMS
cODD: $i → $o thf(cODD, type)
cEVEN: $i → $o thf(cEVEN, type)
cS: $i → $i thf(cS, type)
c_0: $i thf($c_0$, type)
(cEVEN@c_0 and ∀xn: $i: ((cEVEN@xn) ⇒ (cEVEN@(cS@(cS@xn)))) and cODD@(cS@c_0) and ∀xn: $i: ((cODD@xn) ⇒ (cODD@(cS@(cS@xn)))) and ∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx))) ⇒ ∀xn: $i: (cEVEN@xn or cODD@xn) thf(cEVEN_ODD$_2$, conjecture)

NUM823^5.p TPS problem from IND-THMS
cODD: $i → $o thf(cODD, type)
cEVEN: $i → $o thf(cEVEN, type)
cS: $i → $i thf(cS, type)
c_0: $i thf($c_0$, type)
(∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)) and cEVEN@$c_0$ and ∀xn: $i: ((cEV
(cEVEN@(cS@(cS@xn)))) and cODD@(cS@c_0) and ∀xn: $i: ((cODD@xn) ⇒ (cODD@(cS@(cS@xn))))) ⇒ ∀xn: $i: (cEVEN

NUM824^5.p TPS problem from IND-THMS
cN: $i → $o thf(cN, type)
cODD: $i → $o thf(cODD, type)
cEVEN: $i → $o thf(cEVEN, type)
cS: $i → $i thf(cS, type)
c_0: $i thf($c_0$, type)
(∀xp: $i → $o, xq: $i → $o: ((xp@$c_0$ and ∀xu: $i: ((xp@xu) ⇒ (xq@(cS@xu))) and ∀xv: $i: ((xq@xv) ⇒ (xp@(cS@xv)))) ⇒ (∀xx: $i: ((cEVEN@xx) ⇒ (xp@xx)) and ∀xx: $i: ((cODD@xx) ⇒ (xq@xx)))) and cN@c_0 and ∀xn: $i:
(cN@(cS@xn)))) ⇒ ∀xm: $i: ((cEVEN@xm or cODD@xm) ⇒ (cN@xm)) thf(cTHM623_pme, conjecture)

NUM825^5.p TPS problem from IND-THMS
cS: $i → $i thf(cS, type)
cEVEN: $i → $o thf(cEVEN, type)
cODD: $i → $o thf(cODD, type)
c_0: $i thf($c_0$, type)
(cEVEN@c_0 and ∀xx: $i: ((cEVEN@xx) ⇒ (cODD@(cS@xx))) and ∀xy: $i: ((cODD@xy) ⇒ (cEVEN@(cS@xy))) and ∀xp:
$o, xq: $i → $o: ((xp@$c_0$ and ∀xu: $i: ((xp@xu) ⇒ (xq@(cS@xu))) and ∀xv: $i: ((xq@xv) ⇒ (xp@(cS@xv)))) ⇒ (∀xx: $i: ((cEVEN@xx) ⇒ (xp@xx)) and ∀xx: $i: ((cODD@xx) ⇒ (xq@xx))))) ⇒ ∀xn: $i: ((cODD@xn) ⇒ ∃xm: $i: (cEVEN@xm and (cS@xm) = xn)) thf(cTHM624_pme, conjecture)

NUM826^5.p TPS problem from IND-THMS
cG: $i → $i thf(cG, type)
cQ: $i → $o thf(cQ, type)
cP: $i → $o thf(cP, type)
cF: $i → $i → $i thf(cF, type)
cA: $i thf(cA, type)
cB: $i thf(cB, type)
(cP@cA and cQ@cB and ∀xx: $i, xy: $i: ((cP@xx and cQ@xy) ⇒ (cQ@(cF@xx@xy))) and ∀xx: $i, xy: $i: ((cQ@xx and cP@
(cP@(cF@xx@xy))) and ∀xp: $i → $o, xq: $i → $o: ((xp@cA and xq@cB and ∀xx: $i, xy: $i: ((xp@xx and xq@xy) ⇒ (xq@(cF@xx@xy))) and ∀xx: $i, xy: $i: ((xq@xx and xp@xy) ⇒ (xp@(cF@xx@xy)))) ⇒ (∀xx: $i: ((cP@xx) ⇒ (xp@xx)) and ∀xx: $i: ((cQ@xx) ⇒ (xq@xx)))) and (cG@cA) = cB and (cG@cB) = cA and ∀xx: $i, xy: $i: (cG@(cF@xx@xy
(cF@(cG@xx)@(cG@xy))) ⇒ ∀xx: $i: ((cP@xx) ⇒ (cQ@(cG@xx))) thf(cTHM622_pme, conjecture)

NUM827^5.p TPS problem PA-THM2
n: $tType thf(n_type, type)
c_0: n thf(c0_type, type)
cS: $n \to n$ thf(cS_type, type)
c_plus: $n \to n \to n$ thf(c_plus_type, type)
cPA$_1$: $o thf(cPA_1_type, type)
cPA$_2$: $o thf(cPA_2_type, type)
cPA_IND_EQ: $o thf(cPA_IND_EQ_type, type)

cPA$_1$ = (∀xx: n: (c_plus@xx@c_0) = xx)   thf(cPA_1_def, definition)
cPA$_2$ = (∀xx: n, xy: n: (c_plus@xx@(cS@xy)) = (cS@(c_plus@xx@xy)))   thf(cPA_2_def, definition)
cPA_IND_EQ = (∀xp: $n \to n$, xq: $n \to n$: (((xp@c_0) = (xq@c_0) and ∀xx: n: ((xp@xx) = (xq@xx) ⇒ (xp@(cS@xx)) = (xq@(cS@xx)))) ⇒ ∀xx: n: (xp@xx) = (xq@xx)))   thf(cPA_IND_EQ_def, definition)
(cPA$_1$ and cPA$_2$ and cPA_IND_EQ) ⇒ ∀xx: n: (c_plus@xx@c_0) = (c_plus@c_0@xx)   thf(cPA_THM$_2$, conjecture)

NUM828^5.p TPS problem from PA-THMS
n: $tType   thf(n_type, type)
c_0: n thf(c0_type, type)
cS: $n \to n$ thf(cS_type, type)
c_plus: $n \to n \to n$ thf(c_plus_type, type)
cPA$_1$: $o   thf(cPA_1_type, type)
cPA$_2$: $o   thf(cPA_2_type, type)
cPA_IND_EQ: $o   thf(cPA_IND_EQ_type, type)
cPA$_1$ = (∀xx: n: (c_plus@xx@c_0) = xx)   thf(cPA_1_def, definition)
cPA$_2$ = (∀xx: n, xy: n: (c_plus@xx@(cS@xy)) = (cS@(c_plus@xx@xy)))   thf(cPA_2_def, definition)
cPA_IND_EQ = (∀xp: $n \to n$, xq: $n \to n$: (((xp@c_0) = (xq@c_0) and ∀xx: n: ((xp@xx) = (xq@xx) ⇒ (xp@(cS@xx)) = (xq@(cS@xx)))) ⇒ ∀xx: n: (xp@xx) = (xq@xx)))   thf(cPA_IND_EQ_def, definition)
(cPA$_1$ and cPA$_2$ and cPA_IND_EQ) ⇒ ∀xx: n, xy: n: (c_plus@xx@xy) = (c_plus@xy@xx)   thf(cPA_THM$_4$, conjecture)

NUM829^5.p TPS problem from PA-THMS
n: $tType   thf(n_type, type)
c_0: n thf(c0_type, type)
cS: $n \to n$ thf(cS_type, type)
c_plus: $n \to n \to n$ thf(c_plus_type, type)
cPA$_1$: $o   thf(cPA_1_type, type)
cPA$_2$: $o   thf(cPA_2_type, type)
cPA_IND_EQ: $o   thf(cPA_IND_EQ_type, type)
cPA$_1$ = (∀xx: n: (c_plus@xx@c_0) = xx)   thf(cPA_1_def, definition)
cPA$_2$ = (∀xx: n, xy: n: (c_plus@xx@(cS@xy)) = (cS@(c_plus@xx@xy)))   thf(cPA_2_def, definition)
cPA_IND_EQ = (∀xp: $n \to n$, xq: $n \to n$: (((xp@c_0) = (xq@c_0) and ∀xx: n: ((xp@xx) = (xq@xx) ⇒ (xp@(cS@xx)) = (xq@(cS@xx)))) ⇒ ∀xx: n: (xp@xx) = (xq@xx)))   thf(cPA_IND_EQ_def, definition)
(cPA$_1$ and cPA$_2$ and cPA_IND_EQ) ⇒ ∀xx: n, xy: n: (c_plus@(cS@xx)@xy) = (c_plus@xy@(cS@xx))   thf(cPA_THM$_3$, co

NUM830^5.p TPS problem from PA-THMS
n: $tType   thf(n_type, type)
c_0: n thf(c0_type, type)
cS: $n \to n$ thf(cS_type, type)
c_plus: $n \to n \to n$ thf(c_plus_type, type)
c_star: $n \to n \to n$ thf(c_star_type, type)
cPA$_1$: $o   thf(cPA_1_type, type)
cPA$_2$: $o   thf(cPA_2_type, type)
cPA$_3$: $o   thf(cPA_3_type, type)
cPA$_4$: $o   thf(cPA_4_type, type)
cPA$_1$ = (∀xx: n: (c_plus@xx@c_0) = xx)   thf(cPA_1_def, definition)
cPA$_2$ = (∀xx: n, xy: n: (c_plus@xx@(cS@xy)) = (cS@(c_plus@xx@xy)))   thf(cPA_2_def, definition)
cPA$_3$ = (∀xx: n: (c_star@xx@c_0) = c_0)   thf(cPA_3_def, definition)
cPA$_4$ = (∀xx: n, xy: n: (c_star@xx@(cS@xy)) = (c_plus@(c_star@xx@xy)@xx))   thf(cPA_4_def, definition)
(cPA$_1$ and cPA$_2$ and cPA$_3$ and cPA$_4$) ⇒ (c_star@(cS@(cS@c_0))@(cS@(cS@c_0))) = (c_plus@(cS@(cS@c_0))@(cS@(cS@c_0)))

NUM831^5.p TPS problem from PETER-THMS
c_0: $i   thf(c0_type, type)
cS: $i → $i   thf(cS_type, type)
cIND: $o   thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx)))   thf(cIND_def, definitio
(cIND and ∀xx: $i, xy: $i: ((cS@xx) = (cS@xy) ⇒ xx = xy) and ∀xn: $i: (cS@xn) ≠ c_0) ⇒ ∃xd: $i → $i → $o: (xd@$c_0$@$c_0$ and ∀xx: $i, xy: $i: ((xd@xx@xy) ⇒ (xd@(cS@xx)@(cS@(cS@xy)))) and ∀xx: $i: ∃$x$: $i: (xd@xx@$x$ and ∀y: $
$x = y$)))   thf(cTHM606_pme, conjecture)

NUM832^5.p TPS problem from PETER-THMS
c_0: $i   thf(c0_type, type)
cR: $i → $i → $i → $o   thf(cR_type, type)

cS: $i → $i thf(cS_type, type)
cIND: $o thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx))) thf(cIND_def, definitio
(cIND and ∀xn: $i: (cR@$c_0$@xn@(cS@xn)) and ∀xm: $i, xk: $i: ((cR@xm@(cS@$c_0$)@xk) ⇒ (cR@(cS@xm)@$c_0$@xk)) and ∀xn
((cR@xm@xl@xk) ⇒ (cR@(cS@xm)@(cS@xn)@xk)))) ⇒ ∀xx: $i, xy: $i: ∃xz: $i: (cR@xx@xy@xz) thf(cTHM$_{604}$, conject

NUM833^5.p TPS problem from PETER-THMS
c_0: $i thf(c0_type, type)
cS: $i → $i thf(cS_type, type)
cIND: $o thf(cIND_type, type)
cIND = (∀xp: $i → $o: ((xp@$c_0$ and ∀xx: $i: ((xp@xx) ⇒ (xp@(cS@xx)))) ⇒ ∀xx: $i: (xp@xx))) thf(cIND_def, definitio
(cIND and ∀xx: $i, xy: $i: ((cS@xx) = (cS@xy) ⇒ xx = xy) and ∀xn: $i: (cS@xn) ≠ c_0) ⇒ ∃xr: $i → $i → $i →
$o: (∀xn: $i: (xr@$c_0$@xn@(cS@xn)) and ∀xm: $i, xk: $i: ((xr@xm@(cS@$c_0$)@xk) ⇒ (xr@(cS@xm)@$c_0$@xk)) and ∀xm: $i, xn: $
((xr@xm@xl@xk) ⇒ (xr@(cS@xm)@(cS@xn)@xk))) and ∀xx: $i, xy: $i: ∃$x$: $i: (xr@xx@xy@$x$ and ∀y: $i: ((xr@xx@xy@$y$) =
$x = y$))) thf(cTHM605_pme, conjecture)

NUM834^5.p TPS problem from PETER-THMS
cS: $i → $i thf(cS, type)
c_0: $i thf($c_0$, type)
∃xr: $i → $i → $i → $o: (∀xn: $i: (xr@$c_0$@xn@(cS@xn)) and ∀xm: $i, xk: $i: ((xr@xm@(cS@$c_0$)@xk) ⇒ (xr@(cS@xm)@$c_0$@x
((xr@xm@xl@xk) ⇒ (xr@(cS@xm)@(cS@xn)@xk))) and ∀t: $i → $i → $i → $o: ((∀xn: $i: ($t$@$c_0$@xn@(cS@xn)) and ∀xm: $i
(t@(cS@xm)@c_0@xk)) and ∀xm: $i, xn: $i, xk: $i, xl: $i: (($t$@(cS@xm)@xn@xl) ⇒ (($t$@xm@xl@xk) ⇒ ($t$@(cS@xm)@(cS@xn
∀xx: $i, xy: $i, xz: $i: ((xr@xx@xy@xz) ⇒ ($t$@xx@xy@xz)))) thf(cTHM$_{603}$, conjecture)

NUM836+1.p dis(ex(cond(conseq(131),0),1))
¬ greater(vd$_{203}$, vd$_{204}$) or ¬ less(vd$_{203}$, vd$_{204}$) fof('dis(ex(cond(conseq(131), 0), 1))', conjecture)
vd$_{203}$ ≠ vd$_{204}$ or ¬ greater(vd$_{203}$, vd$_{204}$) fof('dis(ex(cond(conseq(131), 0), 0))', axiom)
∀vd$_{198}$, vd$_{199}$: (less(vd$_{199}$, vd$_{198}$) ⟺ ∃vd$_{201}$: vd$_{198}$ = vplus(vd$_{199}$, vd$_{201}$)) fof('def(cond(conseq(axiom(3)), 12), 1)', axio
∀vd$_{193}$, vd$_{194}$: (greater(vd$_{194}$, vd$_{193}$) ⟺ ∃vd$_{196}$: vd$_{194}$ = vplus(vd$_{193}$, vd$_{196}$)) fof('def(cond(conseq(axiom(3)), 11), 1)', a
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ = vd$_{121}$ or ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$) or ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ ≠ vd$_{121}$ or ¬ ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond(goal(88), 0), 1)', axiom)
∀vd$_{120}$, vd$_{121}$: (¬ ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$) or ¬ ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond(goal(88), 0),
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ ≠ vd$_{121}$ or ¬ ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$)) fof('ass(cond(goal(88), 0), 3)', axiom)
∀vd$_{104}$, vd$_{105}$: (vd$_{104}$ ≠ vd$_{105}$ ⇒ ∀vd$_{107}$: vplus(vd$_{107}$, vd$_{104}$) ≠ vplus(vd$_{107}$, vd$_{105}$)) fof('ass(cond(81, 0), 0)', axiom)
∀vd$_{92}$, vd$_{93}$: vd$_{93}$ ≠ vplus(vd$_{92}$, vd$_{93}$) fof('ass(cond(73, 0), 0)', axiom)
∀vd$_{78}$, vd$_{79}$: vplus(vd$_{79}$, vd$_{78}$) = vplus(vd$_{78}$, vd$_{79}$) fof('ass(cond(61, 0), 0)', axiom)
∀vd$_{68}$, vd$_{69}$: vplus(vsucc(vd$_{68}$), vd$_{69}$) = vsucc(vplus(vd$_{68}$, vd$_{69}$)) fof('ass(cond(52, 0), 0)', axiom)
∀vd$_{59}$: vplus(v_1, vd$_{59}$) = vsucc(vd$_{59}$) fof('ass(cond(43, 0), 0)', axiom)
∀vd$_{46}$, vd$_{47}$, vd$_{48}$: vplus(vplus(vd$_{46}$, vd$_{47}$), vd$_{48}$) = vplus(vd$_{46}$, vplus(vd$_{47}$, vd$_{48}$)) fof('ass(cond(33, 0), 0)', axiom)
∀vd$_{42}$, vd$_{43}$: (vplus(vd$_{42}$, vsucc(vd$_{43}$)) = vsucc(vplus(vd$_{42}$, vd$_{43}$)) and vplus(vd$_{42}$, v_1) = vsucc(vd$_{42}$)) fof('qu(cond(conseq
∀vd$_{24}$: (vd$_{24}$ ≠ v_1 ⇒ vd$_{24}$ = vsucc(vskolem$_2$(vd$_{24}$))) fof('ass(cond(20, 0), 0)', axiom)
∀vd$_{16}$: vsucc(vd$_{16}$) ≠ vd$_{16}$ fof('ass(cond(12, 0), 0)', axiom)
∀vd$_7$, vd$_8$: (vd$_7$ ≠ vd$_8$ ⇒ vsucc(vd$_7$) ≠ vsucc(vd$_8$)) fof('ass(cond(6, 0), 0)', axiom)
∀vd$_3$, vd$_4$: (vsucc(vd$_3$) = vsucc(vd$_4$) ⇒ vd$_3$ = vd$_4$) fof('qu(antec(axiom(3)), imp(antec(axiom(3))))', axiom)
∀vd$_1$: vsucc(vd$_1$) ≠ v_1 fof('qu(restrictor(axiom(1)), holds(scope(axiom(1)), 2, 0))', axiom)

NUM836+2.p dis(ex(cond(conseq(131),0),1))
¬ greater(vd$_{203}$, vd$_{204}$) or ¬ less(vd$_{203}$, vd$_{204}$) fof('dis(ex(cond(conseq(131), 0), 1))', conjecture)
vd$_{203}$ ≠ vd$_{204}$ or ¬ greater(vd$_{203}$, vd$_{204}$) fof('dis(ex(cond(conseq(131), 0), 0))', axiom)
∀vd$_{198}$, vd$_{199}$: (less(vd$_{199}$, vd$_{198}$) ⟺ ∃vd$_{201}$: vd$_{198}$ = vplus(vd$_{199}$, vd$_{201}$)) fof('def(cond(conseq(axiom(3)), 12), 1)', axio
∀vd$_{193}$, vd$_{194}$: (greater(vd$_{194}$, vd$_{193}$) ⟺ ∃vd$_{196}$: vd$_{194}$ = vplus(vd$_{193}$, vd$_{196}$)) fof('def(cond(conseq(axiom(3)), 11), 1)', a
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ = vd$_{121}$ or ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$) or ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ ≠ vd$_{121}$ or ¬ ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond(goal(88), 0), 1)', axiom)
∀vd$_{120}$, vd$_{121}$: (¬ ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$) or ¬ ∃vd$_{125}$: vd$_{121}$ = vplus(vd$_{120}$, vd$_{125}$)) fof('ass(cond(goal(88), 0),
∀vd$_{120}$, vd$_{121}$: (vd$_{120}$ ≠ vd$_{121}$ or ¬ ∃vd$_{123}$: vd$_{120}$ = vplus(vd$_{121}$, vd$_{123}$)) fof('ass(cond(goal(88), 0), 3)', axiom)
∀vd$_{104}$, vd$_{105}$: (vd$_{104}$ ≠ vd$_{105}$ ⇒ ∀vd$_{107}$: vplus(vd$_{107}$, vd$_{104}$) ≠ vplus(vd$_{107}$, vd$_{105}$)) fof('ass(cond(81, 0), 0)', axiom)
∀vd$_{46}$, vd$_{47}$, vd$_{48}$: vplus(vplus(vd$_{46}$, vd$_{47}$), vd$_{48}$) = vplus(vd$_{46}$, vplus(vd$_{47}$, vd$_{48}$)) fof('ass(cond(33, 0), 0)', axiom)
∀vd$_{42}$, vd$_{43}$: (vplus(vd$_{42}$, vsucc(vd$_{43}$)) = vsucc(vplus(vd$_{42}$, vd$_{43}$)) and vplus(vd$_{42}$, v_1) = vsucc(vd$_{42}$)) fof('qu(cond(conseq

NUM837+2.p qe(171)
∃vd$_{273}$: vd$_{269}$ = vplus(vd$_{268}$, vd$_{273}$) fof('qe(171)', conjecture)
less(vd$_{269}$, vd$_{271}$) fof('(conjunct2(170),272,0)', axiom)

less(vd_{268}, vd_{269}) fof('holds(conjunct1(170), 270, 0)', axiom)
$\forall vd_{258}, vd_{259}$: ($vd_{258} \leq vd_{259} \Rightarrow$ geq(vd_{259}, vd_{258})) fof('ass(cond(163, 0), 0)', axiom)
$\forall vd_{254}, vd_{255}$: (geq($vd_{254}, vd_{255}$) $\Rightarrow vd_{255} \leq vd_{254}$) fof('ass(cond(158, 0), 0)', axiom)
$\forall vd_{249}, vd_{250}$: ($vd_{250} \leq vd_{249} \iff$ (less(vd_{250}, vd_{249}) or $vd_{250} = vd_{249}$)) fof('def(cond(conseq(axiom(3)), 17), 1)', axiom)
$\forall vd_{244}, vd_{245}$: (geq($vd_{245}, vd_{244}$) $\iff$ (greater(vd_{245}, vd_{244}) or $vd_{245} = vd_{244}$)) fof('def(cond(conseq(axiom(3)), 16), 1)',
$\forall vd_{226}, vd_{227}$: (less($vd_{226}, vd_{227}$) $\Rightarrow$ greater(vd_{227}, vd_{226})) fof('ass(cond(147, 0), 0)', axiom)
$\forall vd_{208}, vd_{209}$: (greater($vd_{208}, vd_{209}$) $\Rightarrow$ less(vd_{209}, vd_{208})) fof('ass(cond(140, 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} = vd_{204}$ or greater(vd_{203}, vd_{204}) or less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} \neq vd_{204}$ or $\neg$ less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 1)', axiom)
$\forall vd_{203}, vd_{204}$: ($\neg$ greater(vd_{203}, vd_{204}) or $\neg$ less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 2)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} \neq vd_{204}$ or $\neg$ greater(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 3)', axiom)
$\forall vd_{198}, vd_{199}$: (less($vd_{199}, vd_{198}$) $\iff \exists vd_{201}$: vd_{198} = vplus(vd_{199}, vd_{201})) fof('def(cond(conseq(axiom(3)), 12), 1)', axio
$\forall vd_{193}, vd_{194}$: (greater($vd_{194}, vd_{193}$) $\iff \exists vd_{196}$: vd_{194} = vplus(vd_{193}, vd_{196})) fof('def(cond(conseq(axiom(3)), 11), 1)', a
$\forall vd_{120}, vd_{121}$: ($vd_{120} = vd_{121}$ or $\exists vd_{123}$: vd_{120} = vplus(vd_{121}, vd_{123}) or $\exists vd_{125}$: vd_{121} = vplus(vd_{120}, vd_{125})) fof('ass(cond
$\forall vd_{120}, vd_{121}$: ($vd_{120} \neq vd_{121}$ or $\neg \exists vd_{125}$: vd_{121} = vplus(vd_{120}, vd_{125})) fof('ass(cond(goal(88), 0), 1)', axiom)
$\forall vd_{120}, vd_{121}$: ($\neg \exists vd_{123}$: vd_{120} = vplus(vd_{121}, vd_{123}) or $\neg \exists vd_{125}$: vd_{121} = vplus(vd_{120}, vd_{125})) fof('ass(cond(goal(88), 0),
$\forall vd_{120}, vd_{121}$: ($vd_{120} \neq vd_{121}$ or $\neg \exists vd_{123}$: vd_{120} = vplus(vd_{121}, vd_{123})) fof('ass(cond(goal(88), 0), 3)', axiom)
$\forall vd_{104}, vd_{105}$: ($vd_{104} \neq vd_{105} \Rightarrow \forall vd_{107}$: vplus($vd_{107}, vd_{104}$) $\neq$ vplus(vd_{107}, vd_{105})) fof('ass(cond(81, 0), 0)', axiom)
$\forall vd_{46}, vd_{47}, vd_{48}$: vplus(vplus($vd_{46}, vd_{47}$), vd_{48}) = vplus(vd_{46}, vplus(vd_{47}, vd_{48})) fof('ass(cond(33, 0), 0)', axiom)
$\forall vd_{42}, vd_{43}$: (vplus(vd_{42}, vsucc(vd_{43})) = vsucc(vplus(vd_{42}, vd_{43})) and vplus(vd_{42}, v_1) = vsucc(vd_{42})) fof('qu(cond(conseq

NUM841+2.p holds(214,352,0)
greater(vplus(vd_{344}, vd_{347}), vplus(vd_{345}, vd_{348})) fof('holds(214, 352, 0)', conjecture)
vplus(vd_{348}, vd_{345}) = vplus(vd_{345}, vd_{348}) fof('holds(213, 351, 2)', axiom)
greater(vplus(vd_{347}, vd_{345}), vplus(vd_{348}, vd_{345})) fof('holds(213, 351, 1)', axiom)
vplus(vd_{345}, vd_{347}) = vplus(vd_{347}, vd_{345}) fof('holds(213, 351, 0)', axiom)
greater(vplus(vd_{344}, vd_{347}), vplus(vd_{345}, vd_{347})) fof('holds(212, 350, 0)', axiom)
greater(vd_{347}, vd_{348}) fof('holds(conjunct2(211), 349, 0)', axiom)
greater(vd_{344}, vd_{345}) fof('holds(conjunct1(211), 346, 0)', axiom)
$\forall vd_{328}, vd_{329}, vd_{330}$: (less(vplus($vd_{328}, vd_{330}$), vplus($vd_{329}, vd_{330}$)) $\Rightarrow$ less(vd_{328}, vd_{329})) fof('ass(cond(goal(202), 0), 0)', a
$\forall vd_{328}, vd_{329}, vd_{330}$: (vplus($vd_{328}, vd_{330}$) = vplus($vd_{329}, vd_{330}$) $\Rightarrow vd_{328} = vd_{329}$) fof('ass(cond(goal(202), 0), 1)', axiom)
$\forall vd_{328}, vd_{329}, vd_{330}$: (greater(vplus($vd_{328}, vd_{330}$), vplus($vd_{329}, vd_{330}$)) $\Rightarrow$ greater(vd_{328}, vd_{329})) fof('ass(cond(goal(202), 0
$\forall vd_{301}, vd_{302}, vd_{303}$: (less($vd_{301}, vd_{302}$) $\Rightarrow$ less(vplus(vd_{301}, vd_{303}), vplus(vd_{302}, vd_{303}))) fof('ass(cond(goal(193), 0), 0)', a
$\forall vd_{301}, vd_{302}, vd_{303}$: ($vd_{301} = vd_{302} \Rightarrow$ vplus(vd_{301}, vd_{303}) = vplus(vd_{302}, vd_{303})) fof('ass(cond(goal(193), 0), 1)', axiom)
$\forall vd_{301}, vd_{302}, vd_{303}$: (greater($vd_{301}, vd_{302}$) $\Rightarrow$ greater(vplus(vd_{301}, vd_{303}), vplus(vd_{302}, vd_{303}))) fof('ass(cond(goal(193), 0
$\forall vd_{295}, vd_{296}$: greater(vplus($vd_{295}, vd_{296}$), vd_{295}) fof('ass(cond(189, 0), 0)', axiom)
$\forall vd_{226}, vd_{227}$: (less($vd_{226}, vd_{227}$) $\Rightarrow$ greater(vd_{227}, vd_{226})) fof('ass(cond(147, 0), 0)', axiom)
$\forall vd_{208}, vd_{209}$: (greater($vd_{208}, vd_{209}$) $\Rightarrow$ less(vd_{209}, vd_{208})) fof('ass(cond(140, 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} = vd_{204}$ or greater(vd_{203}, vd_{204}) or less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($\neg$ greater(vd_{203}, vd_{204}) or $\neg$ less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 2)', axiom)
$\forall vd_{193}, vd_{194}$: (greater($vd_{194}, vd_{193}$) $\iff \exists vd_{196}$: vd_{194} = vplus(vd_{193}, vd_{196})) fof('def(cond(conseq(axiom(3)), 11), 1)', a
$\forall vd_{120}, vd_{121}$: ($vd_{120} \neq vd_{121}$ or $\neg \exists vd_{123}$: vd_{120} = vplus(vd_{121}, vd_{123})) fof('ass(cond(goal(88), 0), 3)', axiom)
$\forall vd_{78}, vd_{79}$: vplus(vd_{79}, vd_{78}) = vplus(vd_{78}, vd_{79}) fof('ass(cond(61, 0), 0)', axiom)
$\forall vd_{68}, vd_{69}$: vplus(vsucc(vd_{68}), vd_{69}) = vsucc(vplus(vd_{68}, vd_{69})) fof('ass(cond(52, 0), 0)', axiom)
$\forall vd_{59}$: vplus(v_1, vd_{59}) = vsucc(vd_{59}) fof('ass(cond(43, 0), 0)', axiom)
$\forall vd_{46}, vd_{47}, vd_{48}$: vplus(vplus($vd_{46}, vd_{47}$), vd_{48}) = vplus(vd_{46}, vplus(vd_{47}, vd_{48})) fof('ass(cond(33, 0), 0)', axiom)
$\forall vd_{42}, vd_{43}$: (vplus(vd_{42}, vsucc(vd_{43})) = vsucc(vplus(vd_{42}, vd_{43})) and vplus(vd_{42}, v_1) = vsucc(vd_{42})) fof('qu(cond(conseq
$\forall vd_{24}$: ($vd_{24} \neq v_1 \Rightarrow vd_{24}$ = vsucc(vskolem$_2$(vd_{24}))) fof('ass(cond(20, 0), 0)', axiom)

NUM844+2.p holds(266,415,3)
vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{413}, vsucc(vd_{411}))) = vplus(vmul(vd_{411}, vd_{413}), vplus(vsucc(vd_{411}), vd_{413})) fof('holds(2
vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{413}), vsucc(vd_{411})) = vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{413}, vsucc(vd_{411}))) fof('holds(2
vplus(vmul(vsucc(vd_{411}), vd_{413}), vsucc(vd_{411})) = vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{413}), vsucc(vd_{411})) fof('holds(266, 41
vmul(vsucc(vd_{411}), vsucc(vd_{413})) = vplus(vmul(vsucc(vd_{411}), vd_{413}), vsucc(vd_{411})) fof('holds(266, 415, 0)', axiom)
vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) fof('holds(265, 414, 0)', axiom)
vsucc(vmul(vd_{411}, v_1)) = vplus(vmul(vd_{411}, v_1), v_1) fof('holds(264, 412, 2)', axiom)
vsucc(vd_{411}) = vsucc(vmul(vd_{411}, v_1)) fof('holds(264, 412, 1)', axiom)
vmul(vsucc(vd_{411}), v_1) = vsucc(vd_{411}) fof('holds(264, 412, 0)', axiom)
$\forall vd_{400}$: vmul(v_1, vd_{400}) = vd_{400} fof('ass(cond(253, 0), 0)', axiom)
$\forall vd_{396}, vd_{397}$: (vmul($vd_{396}$, vsucc($vd_{397}$)) = vplus(vmul($vd_{396}, vd_{397}$), vd_{396}) and vmul(vd_{396}, v_1) = vd_{396}) fof('qu(cond(c

$\forall vd_{386}, vd_{387}$: $(less(vd_{386}, vplus(vd_{387}, v_1)) \Rightarrow vd_{386} \leq vd_{387})$ fof('ass(cond(241, 0), 0)', axiom)
$\forall vd_{375}, vd_{376}$: $(greater(vd_{375}, vd_{376}) \Rightarrow geq(vd_{375}, vplus(vd_{376}, v_1)))$ fof('ass(cond(234, 0), 0)', axiom)
$\forall vd_{78}, vd_{79}$: $vplus(vd_{79}, vd_{78}) = vplus(vd_{78}, vd_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall vd_{68}, vd_{69}$: $vplus(vsucc(vd_{68}), vd_{69}) = vsucc(vplus(vd_{68}, vd_{69}))$ fof('ass(cond(52, 0), 0)', axiom)
$\forall vd_{59}$: $vplus(v_1, vd_{59}) = vsucc(vd_{59})$ fof('ass(cond(43, 0), 0)', axiom)
$\forall vd_{46}, vd_{47}, vd_{48}$: $vplus(vplus(vd_{46}, vd_{47}), vd_{48}) = vplus(vd_{46}, vplus(vd_{47}, vd_{48}))$ fof('ass(cond(33, 0), 0)', axiom)
$\forall vd_{42}, vd_{43}$: $(vplus(vd_{42}, vsucc(vd_{43})) = vsucc(vplus(vd_{42}, vd_{43}))$ and $vplus(vd_{42}, v_1) = vsucc(vd_{42}))$ fof('qu(cond(conseq
$\forall vd_{24}$: $(vd_{24} \neq v_1 \Rightarrow vd_{24} = vsucc(vskolem_2(vd_{24})))$ fof('ass(cond(20, 0), 0)', axiom)
$\forall vd_{16}$: $vsucc(vd_{16}) \neq vd_{16}$ fof('ass(cond(12, 0), 0)', axiom)
$\forall vd_7, vd_8$: $(vd_7 \neq vd_8 \Rightarrow vsucc(vd_7) \neq vsucc(vd_8))$ fof('ass(cond(6, 0), 0)', axiom)

NUM845+2.p qu(ind(267),imp(267))
$\forall vd_{416}$: $(vmul(vsucc(vd_{411}), vd_{416}) = vplus(vmul(vd_{411}, vd_{416}), vd_{416}) \Rightarrow vmul(vsucc(vd_{411}), vsucc(vd_{416})) = vplus(vmul(vd$
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{411}), vsucc(vd_{413}$
$vplus(vmul(vd_{411}, vsucc(vd_{413})), vsucc(vd_{413})))$ fof('ass(cond(conseq(263), 1), 0)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{411}, vsucc(vd_{413})$
$vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{411}), vsucc(vd_{413})))$ fof('ass(cond(conseq(263), 1), 1)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vmul(vd_{411}, vd_{413}), vsucc(vplus(vd_{411}, vd_{413})$
$vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{411}, vsucc(vd_{413}))))$ fof('ass(cond(conseq(263), 1), 2)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vmul(vd_{411}, vd_{413}), vplus(vsucc(vd_{411}), vd_{413}$
$vplus(vmul(vd_{411}, vd_{413}), vsucc(vplus(vd_{411}, vd_{413}))))$ fof('ass(cond(conseq(263), 1), 3)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{413}, vsucc(vd_{411})$
$vplus(vmul(vd_{411}, vd_{413}), vplus(vsucc(vd_{411}), vd_{413})))$ fof('ass(cond(conseq(263), 1), 4)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{413}), vsucc(vd_{411}$
$vplus(vmul(vd_{411}, vd_{413}), vplus(vd_{413}, vsucc(vd_{411}))))$ fof('ass(cond(conseq(263), 1), 5)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vplus(vmul(vsucc(vd_{411}), vd_{413}), vsucc(vd_{411})) =$
$vplus(vplus(vmul(vd_{411}, vd_{413}), vd_{413}), vsucc(vd_{411})))$ fof('ass(cond(conseq(263), 1), 6)', axiom)
$\forall vd_{413}$: $(vmul(vsucc(vd_{411}), vd_{413}) = vplus(vmul(vd_{411}, vd_{413}), vd_{413}) \Rightarrow vmul(vsucc(vd_{411}), vsucc(vd_{413})) = vplus(vmul(vsu$
$vsucc(vmul(vd_{411}, v_1)) = vplus(vmul(vd_{411}, v_1), v_1)$ fof('holds(264, 412, 2)', axiom)
$\forall vd_{396}, vd_{397}$: $(vmul(vd_{396}, vsucc(vd_{397})) = vplus(vmul(vd_{396}, vd_{397}), vd_{396})$ and $vmul(vd_{396}, v_1) = vd_{396})$ fof('qu(cond(c
$\forall vd_{78}, vd_{79}$: $vplus(vd_{79}, vd_{78}) = vplus(vd_{78}, vd_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall vd_{59}$: $vplus(v_1, vd_{59}) = vsucc(vd_{59})$ fof('ass(cond(43, 0), 0)', axiom)
$\forall vd_{46}, vd_{47}, vd_{48}$: $vplus(vplus(vd_{46}, vd_{47}), vd_{48}) = vplus(vd_{46}, vplus(vd_{47}, vd_{48}))$ fof('ass(cond(33, 0), 0)', axiom)
$\forall vd_{42}, vd_{43}$: $(vplus(vd_{42}, vsucc(vd_{43})) = vsucc(vplus(vd_{42}, vd_{43}))$ and $vplus(vd_{42}, v_1) = vsucc(vd_{42}))$ fof('qu(cond(conseq

NUM846+2.p holds(286,441,2)
$vplus(vmul(vd_{436}, vplus(vd_{437}, vd_{439})), vd_{436}) = vplus(vplus(vmul(vd_{436}, vd_{437}), vmul(vd_{436}, vd_{439})), vd_{436})$ fof('holds(28
$vmul(vd_{436}, vsucc(vplus(vd_{437}, vd_{439}))) = vplus(vmul(vd_{436}, vplus(vd_{437}, vd_{439})), vd_{436})$ fof('holds(286, 441, 1)', axiom)
$vmul(vd_{436}, vplus(vd_{437}, vsucc(vd_{439}))) = vmul(vd_{436}, vsucc(vplus(vd_{437}, vd_{439})))$ fof('holds(286, 441, 0)', axiom)
$vmul(vd_{436}, vplus(vd_{437}, vd_{439})) = vplus(vmul(vd_{436}, vd_{437}), vmul(vd_{436}, vd_{439}))$ fof('holds(285, 440, 0)', axiom)
$vplus(vmul(vd_{436}, vd_{437}), vd_{436}) = vplus(vmul(vd_{436}, vd_{437}), vmul(vd_{436}, v_1))$ fof('holds(284, 438, 2)', axiom)
$vmul(vd_{436}, vsucc(vd_{437})) = vplus(vmul(vd_{436}, vd_{437}), vd_{436})$ fof('holds(284, 438, 1)', axiom)
$vmul(vd_{436}, vplus(vd_{437}, v_1)) = vmul(vd_{436}, vsucc(vd_{437}))$ fof('holds(284, 438, 0)', axiom)
$\forall vd_{418}, vd_{419}$: $vmul(vd_{418}, vd_{419}) = vmul(vd_{419}, vd_{418})$ fof('ass(cond(270, 0), 0)', axiom)
$\forall vd_{400}$: $vmul(v_1, vd_{400}) = vd_{400}$ fof('ass(cond(253, 0), 0)', axiom)
$\forall vd_{396}, vd_{397}$: $(vmul(vd_{396}, vsucc(vd_{397})) = vplus(vmul(vd_{396}, vd_{397}), vd_{396})$ and $vmul(vd_{396}, v_1) = vd_{396})$ fof('qu(cond(c
$\forall vd_{386}, vd_{387}$: $(less(vd_{386}, vplus(vd_{387}, v_1)) \Rightarrow vd_{386} \leq vd_{387})$ fof('ass(cond(241, 0), 0)', axiom)
$\forall vd_{375}, vd_{376}$: $(greater(vd_{375}, vd_{376}) \Rightarrow geq(vd_{375}, vplus(vd_{376}, v_1)))$ fof('ass(cond(234, 0), 0)', axiom)
$\forall vd_{369}$: $geq(vd_{369}, v_1)$ fof('ass(cond(228, 0), 0)', axiom)
$\forall vd_{78}, vd_{79}$: $vplus(vd_{79}, vd_{78}) = vplus(vd_{78}, vd_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall vd_{68}, vd_{69}$: $vplus(vsucc(vd_{68}), vd_{69}) = vsucc(vplus(vd_{68}, vd_{69}))$ fof('ass(cond(52, 0), 0)', axiom)
$\forall vd_{59}$: $vplus(v_1, vd_{59}) = vsucc(vd_{59})$ fof('ass(cond(43, 0), 0)', axiom)
$\forall vd_{46}, vd_{47}, vd_{48}$: $vplus(vplus(vd_{46}, vd_{47}), vd_{48}) = vplus(vd_{46}, vplus(vd_{47}, vd_{48}))$ fof('ass(cond(33, 0), 0)', axiom)
$\forall vd_{42}, vd_{43}$: $(vplus(vd_{42}, vsucc(vd_{43})) = vsucc(vplus(vd_{42}, vd_{43}))$ and $vplus(vd_{42}, v_1) = vsucc(vd_{42}))$ fof('qu(cond(conseq
$\forall vd_{24}$: $(vd_{24} \neq v_1 \Rightarrow vd_{24} = vsucc(vskolem_2(vd_{24})))$ fof('ass(cond(20, 0), 0)', axiom)
$\forall vd_{16}$: $vsucc(vd_{16}) \neq vd_{16}$ fof('ass(cond(12, 0), 0)', axiom)
$\forall vd_7, vd_8$: $(vd_7 \neq vd_8 \Rightarrow vsucc(vd_7) \neq vsucc(vd_8))$ fof('ass(cond(6, 0), 0)', axiom)

NUM847+2.p holds(286,441,3)
$vplus(vplus(vmul(vd_{436}, vd_{437}), vmul(vd_{436}, vd_{439})), vd_{436}) = vplus(vmul(vd_{436}, vd_{437}), vplus(vmul(vd_{436}, vd_{439}), vd_{436}))$
$vplus(vmul(vd_{436}, vplus(vd_{437}, vd_{439})), vd_{436}) = vplus(vplus(vmul(vd_{436}, vd_{437}), vmul(vd_{436}, vd_{439})), vd_{436})$ fof('holds(28

$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439}))) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})), \mathrm{vd}_{436})$ fof('holds(286, 441, 1)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vsucc}(\mathrm{vd}_{439}))) = \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})))$ fof('holds(286, 441, 0)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439}))$ fof('holds(285, 440, 0)', axiom)
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vd}_{436}) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, v_1))$ fof('holds(284, 438, 2)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vd}_{437})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vd}_{436})$ fof('holds(284, 438, 1)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, v_1)) = \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vd}_{437}))$ fof('holds(284, 438, 0)', axiom)
$\forall \mathrm{vd}_{400}$: $\mathrm{vmul}(v_1, \mathrm{vd}_{400}) = \mathrm{vd}_{400}$ fof('ass(cond(253, 0), 0)', axiom)
$\forall \mathrm{vd}_{396}, \mathrm{vd}_{397}$: $(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vsucc}(\mathrm{vd}_{397})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vd}_{397}), \mathrm{vd}_{396})$ and $\mathrm{vmul}(\mathrm{vd}_{396}, v_1) = \mathrm{vd}_{396})$ fof('qu(cond(c
$\forall \mathrm{vd}_{386}, \mathrm{vd}_{387}$: $(\mathrm{less}(\mathrm{vd}_{386}, \mathrm{vplus}(\mathrm{vd}_{387}, v_1)) \Rightarrow \mathrm{vd}_{386} \leq \mathrm{vd}_{387})$ fof('ass(cond(241, 0), 0)', axiom)
$\forall \mathrm{vd}_{375}, \mathrm{vd}_{376}$: $(\mathrm{greater}(\mathrm{vd}_{375}, \mathrm{vd}_{376}) \Rightarrow \mathrm{geq}(\mathrm{vd}_{375}, \mathrm{vplus}(\mathrm{vd}_{376}, v_1)))$ fof('ass(cond(234, 0), 0)', axiom)
$\forall \mathrm{vd}_{78}, \mathrm{vd}_{79}$: $\mathrm{vplus}(\mathrm{vd}_{79}, \mathrm{vd}_{78}) = \mathrm{vplus}(\mathrm{vd}_{78}, \mathrm{vd}_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall \mathrm{vd}_{68}, \mathrm{vd}_{69}$: $\mathrm{vplus}(\mathrm{vsucc}(\mathrm{vd}_{68}), \mathrm{vd}_{69}) = \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{68}, \mathrm{vd}_{69}))$ fof('ass(cond(52, 0), 0)', axiom)
$\forall \mathrm{vd}_{59}$: $\mathrm{vplus}(v_1, \mathrm{vd}_{59}) = \mathrm{vsucc}(\mathrm{vd}_{59})$ fof('ass(cond(43, 0), 0)', axiom)
$\forall \mathrm{vd}_{46}, \mathrm{vd}_{47}, \mathrm{vd}_{48}$: $\mathrm{vplus}(\mathrm{vplus}(\mathrm{vd}_{46}, \mathrm{vd}_{47}), \mathrm{vd}_{48}) = \mathrm{vplus}(\mathrm{vd}_{46}, \mathrm{vplus}(\mathrm{vd}_{47}, \mathrm{vd}_{48}))$ fof('ass(cond(33, 0), 0)', axiom)
$\forall \mathrm{vd}_{42}, \mathrm{vd}_{43}$: $(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vsucc}(\mathrm{vd}_{43})) = \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vd}_{43}))$ and $\mathrm{vplus}(\mathrm{vd}_{42}, v_1) = \mathrm{vsucc}(\mathrm{vd}_{42}))$ fof('qu(cond(conseq
$\forall \mathrm{vd}_{24}$: $(\mathrm{vd}_{24} \neq v_1 \Rightarrow \mathrm{vd}_{24} = \mathrm{vsucc}(\mathrm{vskolem}_2(\mathrm{vd}_{24})))$ fof('ass(cond(20, 0), 0)', axiom)
$\forall \mathrm{vd}_{16}$: $\mathrm{vsucc}(\mathrm{vd}_{16}) \neq \mathrm{vd}_{16}$ fof('ass(cond(12, 0), 0)', axiom)

NUM848+2.p holds(286,441,4)
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439}), \mathrm{vd}_{436})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vd}_{439})))$ fof('ho
$\mathrm{vplus}(\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439})), \mathrm{vd}_{436}) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439}), \mathrm{vd}_{436}))$
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})), \mathrm{vd}_{436}) = \mathrm{vplus}(\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439})), \mathrm{vd}_{436})$ fof('holds(28
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439}))) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})), \mathrm{vd}_{436})$ fof('holds(286, 441, 1)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vsucc}(\mathrm{vd}_{439}))) = \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})))$ fof('holds(286, 441, 0)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vplus}(\mathrm{vd}_{437}, \mathrm{vd}_{439})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{439}))$ fof('holds(285, 440, 0)', axiom)
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vd}_{436}) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vmul}(\mathrm{vd}_{436}, v_1))$ fof('holds(284, 438, 2)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vsucc}(\mathrm{vd}_{437})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{436}, \mathrm{vd}_{437}), \mathrm{vd}_{436})$ fof('holds(284, 438, 1)', axiom)
$\forall \mathrm{vd}_{396}, \mathrm{vd}_{397}$: $(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vsucc}(\mathrm{vd}_{397})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vd}_{397}), \mathrm{vd}_{396})$ and $\mathrm{vmul}(\mathrm{vd}_{396}, v_1) = \mathrm{vd}_{396})$ fof('qu(cond(c
$\forall \mathrm{vd}_{386}, \mathrm{vd}_{387}$: $(\mathrm{less}(\mathrm{vd}_{386}, \mathrm{vplus}(\mathrm{vd}_{387}, v_1)) \Rightarrow \mathrm{vd}_{386} \leq \mathrm{vd}_{387})$ fof('ass(cond(241, 0), 0)', axiom)
$\forall \mathrm{vd}_{78}, \mathrm{vd}_{79}$: $\mathrm{vplus}(\mathrm{vd}_{79}, \mathrm{vd}_{78}) = \mathrm{vplus}(\mathrm{vd}_{78}, \mathrm{vd}_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall \mathrm{vd}_{68}, \mathrm{vd}_{69}$: $\mathrm{vplus}(\mathrm{vsucc}(\mathrm{vd}_{68}), \mathrm{vd}_{69}) = \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{68}, \mathrm{vd}_{69}))$ fof('ass(cond(52, 0), 0)', axiom)
$\forall \mathrm{vd}_{59}$: $\mathrm{vplus}(v_1, \mathrm{vd}_{59}) = \mathrm{vsucc}(\mathrm{vd}_{59})$ fof('ass(cond(43, 0), 0)', axiom)
$\forall \mathrm{vd}_{46}, \mathrm{vd}_{47}, \mathrm{vd}_{48}$: $\mathrm{vplus}(\mathrm{vplus}(\mathrm{vd}_{46}, \mathrm{vd}_{47}), \mathrm{vd}_{48}) = \mathrm{vplus}(\mathrm{vd}_{46}, \mathrm{vplus}(\mathrm{vd}_{47}, \mathrm{vd}_{48}))$ fof('ass(cond(33, 0), 0)', axiom)
$\forall \mathrm{vd}_{42}, \mathrm{vd}_{43}$: $(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vsucc}(\mathrm{vd}_{43})) = \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vd}_{43}))$ and $\mathrm{vplus}(\mathrm{vd}_{42}, v_1) = \mathrm{vsucc}(\mathrm{vd}_{42}))$ fof('qu(cond(conseq
$\forall \mathrm{vd}_{24}$: $(\mathrm{vd}_{24} \neq v_1 \Rightarrow \mathrm{vd}_{24} = \mathrm{vsucc}(\mathrm{vskolem}_2(\mathrm{vd}_{24})))$ fof('ass(cond(20, 0), 0)', axiom)

NUM849+2.p qu(ind(296),imp(296))
$\forall \mathrm{vd}_{454}$: $(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{454}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{454})) \Rightarrow \mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vsucc}(\mathrm{vd}_{454})) =$
$\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vsucc}(\mathrm{vd}_{454}))))$ fof('qu(ind(296), imp(296))', conjecture)
$\forall \mathrm{vd}_{451}$: $(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})) \Rightarrow \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451}), \mathrm{vd}_{449}))$
$\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vsucc}(\mathrm{vd}_{451}))))$ fof('ass(cond(conseq(292), 1), 0)', axiom)
$\forall \mathrm{vd}_{451}$: $(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})) \Rightarrow \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})), \mathrm{vmul}($
$\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451}), \mathrm{vd}_{449})))$ fof('ass(cond(conseq(292), 1), 1)', axiom)
$\forall \mathrm{vd}_{451}$: $(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})) \Rightarrow \mathrm{vplus}(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}), \mathrm{vmul}($
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})), \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449})))$ fof('ass(cond(conseq(292), 1), 2)', axiom)
$\forall \mathrm{vd}_{451}$: $(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, \mathrm{vd}_{451})) \Rightarrow \mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vsucc}(\mathrm{vd}_{451})) =$
$\mathrm{vplus}(\mathrm{vmul}(\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}), \mathrm{vd}_{451}), \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449})))$ fof('ass(cond(conseq(292), 1), 3)', axiom)
$\mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vd}_{449}) = \mathrm{vmul}(\mathrm{vd}_{448}, \mathrm{vmul}(\mathrm{vd}_{449}, v_1))$ fof('holds(293, 450, 1)', axiom)
$\forall \mathrm{vd}_{432}, \mathrm{vd}_{433}, \mathrm{vd}_{434}$: $\mathrm{vmul}(\mathrm{vd}_{432}, \mathrm{vplus}(\mathrm{vd}_{433}, \mathrm{vd}_{434})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{432}, \mathrm{vd}_{433}), \mathrm{vmul}(\mathrm{vd}_{432}, \mathrm{vd}_{434}))$ fof('ass(cond(281, 0
$\forall \mathrm{vd}_{418}, \mathrm{vd}_{419}$: $\mathrm{vmul}(\mathrm{vd}_{418}, \mathrm{vd}_{419}) = \mathrm{vmul}(\mathrm{vd}_{419}, \mathrm{vd}_{418})$ fof('ass(cond(270, 0), 0)', axiom)
$\forall \mathrm{vd}_{408}, \mathrm{vd}_{409}$: $\mathrm{vmul}(\mathrm{vsucc}(\mathrm{vd}_{408}), \mathrm{vd}_{409}) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{408}, \mathrm{vd}_{409}), \mathrm{vd}_{409})$ fof('ass(cond(261, 0), 0)', axiom)
$\forall \mathrm{vd}_{396}, \mathrm{vd}_{397}$: $(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vsucc}(\mathrm{vd}_{397})) = \mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{396}, \mathrm{vd}_{397}), \mathrm{vd}_{396})$ and $\mathrm{vmul}(\mathrm{vd}_{396}, v_1) = \mathrm{vd}_{396})$ fof('qu(cond(c
$\forall \mathrm{vd}_{78}, \mathrm{vd}_{79}$: $\mathrm{vplus}(\mathrm{vd}_{79}, \mathrm{vd}_{78}) = \mathrm{vplus}(\mathrm{vd}_{78}, \mathrm{vd}_{79})$ fof('ass(cond(61, 0), 0)', axiom)
$\forall \mathrm{vd}_{42}, \mathrm{vd}_{43}$: $(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vsucc}(\mathrm{vd}_{43})) = \mathrm{vsucc}(\mathrm{vplus}(\mathrm{vd}_{42}, \mathrm{vd}_{43}))$ and $\mathrm{vplus}(\mathrm{vd}_{42}, v_1) = \mathrm{vsucc}(\mathrm{vd}_{42}))$ fof('qu(cond(conseq

NUM852+2.p holds(conseq_conjunct1(conseq_conjunct2(conseq(304))),484,0)
$\mathrm{greater}(\mathrm{vmul}(\mathrm{vd}_{481}, \mathrm{vd}_{469}), \mathrm{vmul}(\mathrm{vd}_{480}, \mathrm{vd}_{469}))$ fof('holds(conseq_conjunct1(conseq_conjunct2(conseq(304))), 484, 0)', con
$\mathrm{greater}(\mathrm{vd}_{481}, \mathrm{vd}_{480})$ fof('holds(conseq_conjunct1(conseq(304)), 483, 0)', axiom)
$\mathrm{less}(\mathrm{vd}_{480}, \mathrm{vd}_{481})$ fof('holds(antec(304), 482, 0)', axiom)
$\forall \mathrm{vd}_{476}, \mathrm{vd}_{477}$: $(\mathrm{vd}_{476} = \mathrm{vd}_{477} \Rightarrow \mathrm{vmul}(\mathrm{vd}_{476}, \mathrm{vd}_{469}) = \mathrm{vmul}(\mathrm{vd}_{477}, \mathrm{vd}_{469}))$ fof('ass(cond(303, 0), 0)', axiom)
$\forall \mathrm{vd}_{470}, \mathrm{vd}_{471}$: $(\mathrm{greater}(\mathrm{vd}_{470}, \mathrm{vd}_{471}) \Rightarrow \mathrm{greater}(\mathrm{vplus}(\mathrm{vmul}(\mathrm{vd}_{471}, \mathrm{vd}_{469}), \mathrm{vmul}(\mathrm{vskolem}_9(\mathrm{vd}_{470}, \mathrm{vd}_{471}), \mathrm{vd}_{469})), \mathrm{vmul}(\mathrm{vd}_{471}, \mathrm{v}$

$\forall vd_{470}, vd_{471}$: (greater($vd_{470}, vd_{471}$) $\Rightarrow$ vmul(vplus(vd_{471}, vskolem$_9$(vd_{470}, vd_{471})), vd_{469}) = vplus(vmul(vd_{471}, vd_{469}), vmul(v
$\forall vd_{470}, vd_{471}$: (greater($vd_{470}, vd_{471}$) $\Rightarrow$ vmul(vd_{470}, vd_{469}) = vmul(vplus(vd_{471}, vskolem$_9$(vd_{470}, vd_{471})), vd_{469})) fof('ass(c
$\forall vd_{470}, vd_{471}$: (greater($vd_{470}, vd_{471}$) $\Rightarrow$ vd_{470} = vplus(vd_{471}, vskolem$_9$(vd_{470}, vd_{471}))) fof('ass(cond(302, 0), 3)', axiom)
$\forall vd_{444}, vd_{445}, vd_{446}$: vmul(vmul($vd_{444}, vd_{445}$), vd_{446}) = vmul(vd_{444}, vmul(vd_{445}, vd_{446})) fof('ass(cond(290, 0), 0)', axiom)
$\forall vd_{432}, vd_{433}, vd_{434}$: vmul($vd_{432}$, vplus($vd_{433}, vd_{434}$)) = vplus(vmul($vd_{432}, vd_{433}$), vmul($vd_{432}, vd_{434}$)) fof('ass(cond(281, 0
$\forall vd_{418}, vd_{419}$: vmul($vd_{418}, vd_{419}$) = vmul($vd_{419}, vd_{418}$) fof('ass(cond(270, 0), 0)', axiom)
$\forall vd_{408}, vd_{409}$: vmul(vsucc($vd_{408}$), vd_{409}) = vplus(vmul(vd_{408}, vd_{409}), vd_{409}) fof('ass(cond(261, 0), 0)', axiom)
$\forall vd_{226}, vd_{227}$: (less($vd_{226}, vd_{227}$) $\Rightarrow$ greater(vd_{227}, vd_{226})) fof('ass(cond(147, 0), 0)', axiom)
$\forall vd_{208}, vd_{209}$: (greater($vd_{208}, vd_{209}$) $\Rightarrow$ less(vd_{209}, vd_{208})) fof('ass(cond(140, 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} = vd_{204}$ or greater(vd_{203}, vd_{204}) or less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 0)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} \neq vd_{204}$ or $\neg$ less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 1)', axiom)
$\forall vd_{203}, vd_{204}$: ($\neg$ greater(vd_{203}, vd_{204}) or $\neg$ less(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 2)', axiom)
$\forall vd_{203}, vd_{204}$: ($vd_{203} \neq vd_{204}$ or $\neg$ greater(vd_{203}, vd_{204})) fof('ass(cond(goal(130), 0), 3)', axiom)
$\forall vd_{198}, vd_{199}$: (less($vd_{199}, vd_{198}$) $\iff$ $\exists vd_{201}$: vd_{198} = vplus(vd_{199}, vd_{201})) fof('def(cond(conseq(axiom(3)), 12), 1)', axio
$\forall vd_{193}, vd_{194}$: (greater($vd_{194}, vd_{193}$) $\iff$ $\exists vd_{196}$: vd_{194} = vplus(vd_{193}, vd_{196})) fof('def(cond(conseq(axiom(3)), 11), 1)', a

NUM858+1.p Basic upper bound replace maximum
This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.
$\forall x$: lesseq(x, x) fof(lesseq_ref, axiom)
$\forall x, y, z$: ((lesseq(x, y) and lesseq(y, z)) $\Rightarrow$ lesseq(x, z)) fof(lesseq_trans, axiom)
$\forall x, y$: ((lesseq(x, y) and lesseq(y, x)) $\Rightarrow$ $x = y$) fof(lesseq_antisymmetric, axiom)
$\forall x, y$: (lesseq(x, y) or lesseq(y, x)) fof(lesseq_total, axiom)
$\forall x, y, z$: (lesseq(x, y) $\iff$ lesseq($z + x, z + y$)) fof(sum_monotone$_1$, axiom)
$\forall x, y$: (lesseq(x, y) $\iff$ lesseq(summation(x), summation(y))) fof(summation_monotone, axiom)
$\forall x, y$: (max(x, y) = x or $\neg$ lesseq(y, x)) fof(max$_1$, axiom)
$\forall x, y$: (max(x, y) = y or $\neg$ lesseq(x, y)) fof(max$_2$, axiom)
$\forall x, y, z$: (ub(x, y, z) $\iff$ (lesseq(x, z) and lesseq(y, z))) fof(ub, axiom)
$\forall x, y, n$: (model_max(x, y, n) $\iff$ n = max(x, y)) fof(model_max$_1$, axiom)
$\forall x, y, n$: (model_ub(x, y, n) $\iff$ ub(x, y, n)) fof(model_ub$_1$, axiom)
$\forall x, y, n$: (minsol_model_max(x, y, n) $\iff$ (model_max(x, y, n) and $\forall z$: (model_max(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_
$\forall x, y, n$: (minsol_model_ub(x, y, n) $\iff$ (model_ub(x, y, n) and $\forall z$: (model_ub(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_mode
$\forall x, y, z$: (minsol_model_ub(x, y, z) $\iff$ minsol_model_max(x, y, z)) fof(max_is_ub$_1$, conjecture)

NUM858=1.p Basic upper bound replace maximum
This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.
summation: \$int $\rightarrow$ \$int tff(summation_type, type)
ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(ub_type, type)
model_max: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(model_max_type, type)
model_ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(model_ub_type, type)
minsol_model_max: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(minsol_model_max_type, type)
minsol_model_ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(minsol_model_ub_type, type)
max: (\$int $\times$ \$int) $\rightarrow$ \$int tff(max_type, type)
$\forall x$: \$int, y: \$int: (\$lesseq(x, y) $\iff$ \$lesseq(summation(x), summation(y))) tff(summation_monotone, axiom)
$\forall x$: \$int, y: \$int: (max(x, y) = x or $\neg$ \$lesseq(y, x)) tff(max$_1$, axiom)
$\forall x$: \$int, y: \$int: (max(x, y) = y or $\neg$ \$lesseq(x, y)) tff(max$_2$, axiom)
$\forall x$: \$int, y: \$int, z: \$int: (ub(x, y, z) $\iff$ (\$lesseq(x, z) and \$lesseq(y, z))) tff(ub, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (model_max(x, y, n) $\iff$ n = max(x, y)) tff(model_max$_1$, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (model_ub(x, y, n) $\iff$ ub(x, y, n)) tff(model_ub$_1$, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (minsol_model_max(x, y, n) $\iff$ (model_max(x, y, n) and $\forall z$: \$int: (model_max(x, y, z) $\Rightarrow$ \$lesseq(n, z)))) tff(minsol_model_max, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (minsol_model_ub(x, y, n) $\iff$ (model_ub(x, y, n) and $\forall z$: \$int: (model_ub(x, y, z) $\Rightarrow$ \$lesseq(n, z)))) tff(minsol_model_ub, axiom)
$\forall x$: \$int, y: \$int, z: \$int: (minsol_model_ub(x, y, z) $\iff$ minsol_model_max(x, y, z)) tff(max_is_ub$_1$, conjecture)

NUM859+1.p Basic upper bound replace maximum with less-or-equal

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

$\forall x$: lesseq(x, x) fof(lesseq_ref, axiom)
$\forall x, y, z$: ((lesseq(x, y) and lesseq(y, z)) $\Rightarrow$ lesseq(x, z)) fof(lesseq_trans, axiom)
$\forall x, y$: ((lesseq(x, y) and lesseq(y, x)) $\Rightarrow$ $x = y$) fof(lesseq_antisymmetric, axiom)
$\forall x, y$: (lesseq(x, y) or lesseq(y, x)) fof(lesseq_total, axiom)
$\forall x, y, z$: (lesseq(x, y) $\iff$ lesseq$(z + x, z + y)$) fof(sum_monotone$_1$, axiom)
$\forall x, y$: (lesseq(x, y) $\iff$ lesseq(summation(x), summation(y))) fof(summation_monotone, axiom)
$\forall x, y$: (max$(x, y) = x$ or $\neg$ lesseq(y, x)) fof(max$_1$, axiom)
$\forall x, y$: (max$(x, y) = y$ or $\neg$ lesseq(x, y)) fof(max$_2$, axiom)
$\forall x, y, z$: (ub(x, y, z) $\iff$ (lesseq(x, z) and lesseq(y, z))) fof(ub, axiom)
$\forall x, y, n$: (model_max(x, y, n) $\iff$ lesseq(max(x, y), n)) fof(model_max$_2$, axiom)
$\forall x, y, n$: (model_ub(x, y, n) $\iff$ $\exists z$: (ub(x, y, z) and lesseq(z, n))) fof(model_ub$_2$, axiom)
$\forall x, y, n$: (minsol_model_max(x, y, n) $\iff$ (model_max(x, y, n) and $\forall z$: (model_max(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_
$\forall x, y, n$: (minsol_model_ub(x, y, n) $\iff$ (model_ub(x, y, n) and $\forall z$: (model_ub(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_mode
$\forall x, y, z$: (minsol_model_ub(x, y, z) $\iff$ minsol_model_max(x, y, z)) fof(max_is_ub$_1$, conjecture)

NUM859=1.p Basic upper bound replace maximum with less-or-equal

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

summation: \$int $\rightarrow$ \$int tff(summation_type, type)
ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(ub_type, type)
model_max: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(model_max_type, type)
model_ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(model_ub_type, type)
minsol_model_max: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(minsol_model_max_type, type)
minsol_model_ub: (\$int $\times$ \$int $\times$ \$int) $\rightarrow$ \$o tff(minsol_model_ub_type, type)
max: (\$int $\times$ \$int) $\rightarrow$ \$int tff(max_type, type)
$\forall x$: \$int, y: \$int: (\$lesseq(x, y) $\iff$ \$lesseq(summation(x), summation(y))) tff(summation_monotone, axiom)
$\forall x$: \$int, y: \$int: (max$(x, y) = x$ or $\neg$ \$lesseq(y, x)) tff(max$_1$, axiom)
$\forall x$: \$int, y: \$int: (max$(x, y) = y$ or $\neg$ \$lesseq(x, y)) tff(max$_2$, axiom)
$\forall x$: \$int, y: \$int, z: \$int: (ub(x, y, z) $\iff$ (\$lesseq(x, z) and \$lesseq(y, z))) tff(ub, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (model_max(x, y, n) $\iff$ \$lesseq(max(x, y), n)) tff(model_max$_2$, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (model_ub(x, y, n) $\iff$ $\exists z$: \$int: (ub(x, y, z) and \$lesseq(z, n))) tff(model_ub$_2$, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (minsol_model_max(x, y, n) $\iff$ (model_max(x, y, n) and $\forall z$: \$int: (model_max(x, y, z) $\Rightarrow$ \$lesseq(n, z)))) tff(minsol_model_max, axiom)
$\forall x$: \$int, y: \$int, n: \$int: (minsol_model_ub(x, y, n) $\iff$ (model_ub(x, y, n) and $\forall z$: \$int: (model_ub(x, y, z) $\Rightarrow$ \$lesseq(n, z)))) tff(minsol_model_ub, axiom)
$\forall x$: \$int, y: \$int, z: \$int: (minsol_model_ub(x, y, z) $\iff$ minsol_model_max(x, y, z)) tff(max_is_ub$_1$, conjecture)

NUM860+1.p Upper bound replace maximum embedded in a context (1)

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

$\forall x$: lesseq(x, x) fof(lesseq_ref, axiom)
$\forall x, y, z$: ((lesseq(x, y) and lesseq(y, z)) $\Rightarrow$ lesseq(x, z)) fof(lesseq_trans, axiom)
$\forall x, y$: ((lesseq(x, y) and lesseq(y, x)) $\Rightarrow$ $x = y$) fof(lesseq_antisymmetric, axiom)
$\forall x, y$: (lesseq(x, y) or lesseq(y, x)) fof(lesseq_total, axiom)
$\forall x, y, z$: (lesseq(x, y) $\iff$ lesseq$(z + x, z + y)$) fof(sum_monotone$_1$, axiom)
$\forall x, y$: (lesseq(x, y) $\iff$ lesseq(summation(x), summation(y))) fof(summation_monotone, axiom)
$\forall x, y$: (max$(x, y) = x$ or $\neg$ lesseq(y, x)) fof(max$_1$, axiom)
$\forall x, y$: (max$(x, y) = y$ or $\neg$ lesseq(x, y)) fof(max$_2$, axiom)
$\forall x, y, z$: (ub(x, y, z) $\iff$ (lesseq(x, z) and lesseq(y, z))) fof(ub, axiom)
$\forall x, y, n$: (model_max(x, y, n) $\iff$ lesseq(summation(max(x, y)), n)) fof(model_max$_3$, axiom)
$\forall x, y, n$: (model_ub(x, y, n) $\iff$ $\exists z$: (ub(x, y, z) and lesseq(summation(z), n))) fof(model_ub$_3$, axiom)
$\forall x, y, n$: (minsol_model_max(x, y, n) $\iff$ (model_max(x, y, n) and $\forall z$: (model_max(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_
$\forall x, y, n$: (minsol_model_ub(x, y, n) $\iff$ (model_ub(x, y, n) and $\forall z$: (model_ub(x, y, z) $\Rightarrow$ lesseq(n, z)))) fof(minsol_mode
$\forall x, y, z$: (minsol_model_ub(x, y, z) $\iff$ minsol_model_max(x, y, z)) fof(max_is_ub$_1$, conjecture)

NUM860=1.p Upper bound replace maximum embedded in a context (1)

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

$\text{summation}\colon \$\text{int} \to \$\text{int} \quad \text{tff(summation\_type, type)}$

$\text{ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(ub\_type, type)}$

$\text{model\_max}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(model\_max\_type, type)}$

$\text{model\_ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(model\_ub\_type, type)}$

$\text{minsol\_model\_max}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(minsol\_model\_max\_type, type)}$

$\text{minsol\_model\_ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(minsol\_model\_ub\_type, type)}$

$\text{max}\colon (\$\text{int} \times \$\text{int}) \to \$\text{int} \quad \text{tff(max\_type, type)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\$\text{lesseq}(x, y) \iff \$\text{lesseq}(\text{summation}(x), \text{summation}(y))) \quad \text{tff(summation\_monotone, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\max(x, y) = x \text{ or } \neg \$\text{lesseq}(y, x)) \quad \text{tff}(\text{max}_1, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\max(x, y) = y \text{ or } \neg \$\text{lesseq}(x, y)) \quad \text{tff}(\text{max}_2, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, z\colon \$\text{int}\colon (\text{ub}(x, y, z) \iff (\$\text{lesseq}(x, z) \text{ and } \$\text{lesseq}(y, z))) \quad \text{tff(ub, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{model\_max}(x, y, n) \iff \$\text{lesseq}(\text{summation}(\max(x, y)), n)) \quad \text{tff}(\text{model\_max}_3, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{model\_ub}(x, y, n) \iff \exists z\colon \$\text{int}\colon (\text{ub}(x, y, z) \text{ and } \$\text{lesseq}(\text{summation}(z), n))) \quad \text{tff}(\text{model\_ub}_3, \text{axio}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{minsol\_model\_max}(x, y, n) \iff (\text{model\_max}(x, y, n) \text{ and } \forall z\colon \$\text{int}\colon (\text{model\_max}(x, y, z) \Rightarrow \$\text{lesseq}(n, z)))) \quad \text{tff(minsol\_model\_max, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{minsol\_model\_ub}(x, y, n) \iff (\text{model\_ub}(x, y, n) \text{ and } \forall z\colon \$\text{int}\colon (\text{model\_ub}(x, y, z) \Rightarrow \$\text{lesseq}(n, z)))) \quad \text{tff(minsol\_model\_ub, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, z\colon \$\text{int}\colon (\text{minsol\_model\_ub}(x, y, z) \iff \text{minsol\_model\_max}(x, y, z)) \quad \text{tff}(\text{max\_is\_ub}_1, \text{conjecture})$

NUM861+1.p Upper bound replace maximum embedded in a context (2)

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

$\forall x\colon \text{lesseq}(x, x) \quad \text{fof(lesseq\_ref, axiom)}$

$\forall x, y, z\colon ((\text{lesseq}(x, y) \text{ and } \text{lesseq}(y, z)) \Rightarrow \text{lesseq}(x, z)) \quad \text{fof(lesseq\_trans, axiom)}$

$\forall x, y\colon ((\text{lesseq}(x, y) \text{ and } \text{lesseq}(y, x)) \Rightarrow x = y) \quad \text{fof(lesseq\_antisymmetric, axiom)}$

$\forall x, y\colon (\text{lesseq}(x, y) \text{ or } \text{lesseq}(y, x)) \quad \text{fof(lesseq\_total, axiom)}$

$\forall x, y, z\colon (\text{lesseq}(x, y) \iff \text{lesseq}(z + x, z + y)) \quad \text{fof}(\text{sum\_monotone}_1, \text{axiom})$

$\forall x, y\colon (\text{lesseq}(x, y) \iff \text{lesseq}(\text{summation}(x), \text{summation}(y))) \quad \text{fof(summation\_monotone, axiom)}$

$\forall x, y\colon (\max(x, y) = x \text{ or } \neg \text{lesseq}(y, x)) \quad \text{fof}(\text{max}_1, \text{axiom})$

$\forall x, y\colon (\max(x, y) = y \text{ or } \neg \text{lesseq}(x, y)) \quad \text{fof}(\text{max}_2, \text{axiom})$

$\forall x, y, z\colon (\text{ub}(x, y, z) \iff (\text{lesseq}(x, z) \text{ and } \text{lesseq}(y, z))) \quad \text{fof(ub, axiom)}$

$\forall x, y, n\colon (\text{model\_max}(x, y, n) \iff \text{lesseq}(c + \max(x, y), n)) \quad \text{fof}(\text{model\_max}_4, \text{axiom})$

$\forall x, y, n\colon (\text{model\_ub}(x, y, n) \iff \exists z\colon (\text{ub}(x, y, z) \text{ and } \text{lesseq}(c + z, n))) \quad \text{fof}(\text{model\_ub}_4, \text{axiom})$

$\forall x, y, n\colon (\text{minsol\_model\_max}(x, y, n) \iff (\text{model\_max}(x, y, n) \text{ and } \forall z\colon (\text{model\_max}(x, y, z) \Rightarrow \text{lesseq}(n, z)))) \quad \text{fof(minsol\_}$

$\forall x, y, n\colon (\text{minsol\_model\_ub}(x, y, n) \iff (\text{model\_ub}(x, y, n) \text{ and } \forall z\colon (\text{model\_ub}(x, y, z) \Rightarrow \text{lesseq}(n, z)))) \quad \text{fof(minsol\_mode}$

$\forall x, y, z\colon (\text{minsol\_model\_ub}(x, y, z) \iff \text{minsol\_model\_max}(x, y, z)) \quad \text{fof}(\text{max\_is\_ub}_1, \text{conjecture})$

NUM861=1.p Upper bound replace maximum embedded in a context (2)

This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.

$c\colon \$\text{int} \quad \text{tff(c\_type, type)}$

$\text{summation}\colon \$\text{int} \to \$\text{int} \quad \text{tff(summation\_type, type)}$

$\text{ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(ub\_type, type)}$

$\text{model\_max}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(model\_max\_type, type)}$

$\text{model\_ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(model\_ub\_type, type)}$

$\text{minsol\_model\_max}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(minsol\_model\_max\_type, type)}$

$\text{minsol\_model\_ub}\colon (\$\text{int} \times \$\text{int} \times \$\text{int}) \to \$\text{o} \quad \text{tff(minsol\_model\_ub\_type, type)}$

$\text{max}\colon (\$\text{int} \times \$\text{int}) \to \$\text{int} \quad \text{tff(max\_type, type)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\$\text{lesseq}(x, y) \iff \$\text{lesseq}(\text{summation}(x), \text{summation}(y))) \quad \text{tff(summation\_monotone, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\max(x, y) = x \text{ or } \neg \$\text{lesseq}(y, x)) \quad \text{tff}(\text{max}_1, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}\colon (\max(x, y) = y \text{ or } \neg \$\text{lesseq}(x, y)) \quad \text{tff}(\text{max}_2, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, z\colon \$\text{int}\colon (\text{ub}(x, y, z) \iff (\$\text{lesseq}(x, z) \text{ and } \$\text{lesseq}(y, z))) \quad \text{tff(ub, axiom)}$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{model\_max}(x, y, n) \iff \$\text{lesseq}(\$\text{sum}(c, \max(x, y)), n)) \quad \text{tff}(\text{model\_max}_4, \text{axiom})$

$\forall x\colon \$\text{int}, y\colon \$\text{int}, n\colon \$\text{int}\colon (\text{model\_ub}(x, y, n) \iff \exists z\colon \$\text{int}\colon (\text{ub}(x, y, z) \text{ and } \$\text{lesseq}(\$\text{sum}(c, z), n))) \quad \text{tff}(\text{model\_ub}_4, \text{axiom})$

$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_max}(x,y,n) \iff (\mathrm{model\_max}(x,y,n) \text{ and } \forall z\colon \$\mathrm{int}\colon (\mathrm{model\_max}(x,y,z) \Rightarrow \$\mathrm{lesseq}(n,z))))$ tff(minsol_model_max, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_ub}(x,y,n) \iff (\mathrm{model\_ub}(x,y,n) \text{ and } \forall z\colon \$\mathrm{int}\colon (\mathrm{model\_ub}(x,y,z) \Rightarrow \$\mathrm{lesseq}(n,z))))$ tff(minsol_model_ub, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_ub}(x,y,z) \iff \mathrm{minsol\_model\_max}(x,y,z))$ tff($\mathrm{max\_is\_ub}_1$, conjecture)

NUM862+1.p Upper bound replace maximum embedded in a context (1)+(2)
This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.
$\forall x\colon \mathrm{lesseq}(x,x)$ fof(lesseq_ref, axiom)
$\forall x,y,z\colon ((\mathrm{lesseq}(x,y) \text{ and } \mathrm{lesseq}(y,z)) \Rightarrow \mathrm{lesseq}(x,z))$ fof(lesseq_trans, axiom)
$\forall x,y\colon ((\mathrm{lesseq}(x,y) \text{ and } \mathrm{lesseq}(y,x)) \Rightarrow x=y)$ fof(lesseq_antisymmetric, axiom)
$\forall x,y\colon (\mathrm{lesseq}(x,y) \text{ or } \mathrm{lesseq}(y,x))$ fof(lesseq_total, axiom)
$\forall x,y,z\colon (\mathrm{lesseq}(x,y) \iff \mathrm{lesseq}(z+x,z+y))$ fof($\mathrm{sum\_monotone}_1$, axiom)
$\forall x,y\colon (\mathrm{lesseq}(x,y) \iff \mathrm{lesseq}(\mathrm{summation}(x),\mathrm{summation}(y)))$ fof(summation_monotone, axiom)
$\forall x,y\colon (\max(x,y)=x \text{ or } \neg\,\mathrm{lesseq}(y,x))$ fof(max_1, axiom)
$\forall x,y\colon (\max(x,y)=y \text{ or } \neg\,\mathrm{lesseq}(x,y))$ fof(max_2, axiom)
$\forall x,y,z\colon (\mathrm{ub}(x,y,z) \iff (\mathrm{lesseq}(x,z) \text{ and } \mathrm{lesseq}(y,z)))$ fof(ub, axiom)
$\forall x,y,n\colon (\mathrm{model\_max}(x,y,n) \iff \mathrm{lesseq}(c+\mathrm{summation}(\max(x,y)),n))$ fof($\mathrm{model\_max}_5$, axiom)
$\forall x,y,n\colon (\mathrm{model\_ub}(x,y,n) \iff \exists z\colon (\mathrm{ub}(x,y,z) \text{ and } \mathrm{lesseq}(c+\mathrm{summation}(z),n)))$ fof($\mathrm{model\_ub}_5$, axiom)
$\forall x,y,n\colon (\mathrm{minsol\_model\_max}(x,y,n) \iff (\mathrm{model\_max}(x,y,n) \text{ and } \forall z\colon (\mathrm{model\_max}(x,y,z) \Rightarrow \mathrm{lesseq}(n,z))))$ fof(minsol_
$\forall x,y,n\colon (\mathrm{minsol\_model\_ub}(x,y,n) \iff (\mathrm{model\_ub}(x,y,n) \text{ and } \forall z\colon (\mathrm{model\_ub}(x,y,z) \Rightarrow \mathrm{lesseq}(n,z))))$ fof(minsol_mode
$\forall x,y,z\colon (\mathrm{minsol\_model\_ub}(x,y,z) \iff \mathrm{minsol\_model\_max}(x,y,z))$ fof($\mathrm{max\_is\_ub}_1$, conjecture)

NUM862=1.p Upper bound replace maximum embedded in a context (1)+(2)
This is an abstraction of a problem to show equivalence of two given constraint models. More precisely, the task is to prove that the minimal solutions of a certain constraint model are preserved if the applications of the "maximum" function in it are replaced by "upper bounds" only.
$c\colon \$\mathrm{int}$ tff(c_type, type)
$\mathrm{summation}\colon \$\mathrm{int} \to \$\mathrm{int}$ tff(summation_type, type)
$\mathrm{ub}\colon (\$\mathrm{int} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(ub_type, type)
$\mathrm{model\_max}\colon (\$\mathrm{int} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(model_max_type, type)
$\mathrm{model\_ub}\colon (\$\mathrm{int} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(model_ub_type, type)
$\mathrm{minsol\_model\_max}\colon (\$\mathrm{int} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(minsol_model_max_type, type)
$\mathrm{minsol\_model\_ub}\colon (\$\mathrm{int} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(minsol_model_ub_type, type)
$\max\colon (\$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(max_type, type)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\$\mathrm{lesseq}(x,y) \iff \$\mathrm{lesseq}(\mathrm{summation}(x),\mathrm{summation}(y)))$ tff(summation_monotone, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\max(x,y)=x \text{ or } \neg\,\$\mathrm{lesseq}(y,x))$ tff(max_1, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\max(x,y)=y \text{ or } \neg\,\$\mathrm{lesseq}(x,y))$ tff(max_2, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}\colon (\mathrm{ub}(x,y,z) \iff (\$\mathrm{lesseq}(x,z) \text{ and } \$\mathrm{lesseq}(y,z)))$ tff(ub, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{model\_max}(x,y,n) \iff \$\mathrm{lesseq}(\$\mathrm{sum}(c,\mathrm{summation}(\max(x,y))),n))$ tff($\mathrm{model\_max}_5$, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{model\_ub}(x,y,n) \iff \exists z\colon \$\mathrm{int}\colon (\mathrm{ub}(x,y,z) \text{ and } \$\mathrm{lesseq}(\$\mathrm{sum}(c,\mathrm{summation}(z)),n)))$ tff(model_
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_max}(x,y,n) \iff (\mathrm{model\_max}(x,y,n) \text{ and } \forall z\colon \$\mathrm{int}\colon (\mathrm{model\_max}(x,y,z) \Rightarrow \$\mathrm{lesseq}(n,z))))$ tff(minsol_model_max, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, n\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_ub}(x,y,n) \iff (\mathrm{model\_ub}(x,y,n) \text{ and } \forall z\colon \$\mathrm{int}\colon (\mathrm{model\_ub}(x,y,z) \Rightarrow \$\mathrm{lesseq}(n,z))))$ tff(minsol_model_ub, axiom)
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}\colon (\mathrm{minsol\_model\_ub}(x,y,z) \iff \mathrm{minsol\_model\_max}(x,y,z))$ tff($\mathrm{max\_is\_ub}_1$, conjecture)

NUM863^1.p A property of cardinal numbers.
—A—=—A'— & —B—=—B'— & A' disjoint B', then —A U B— $\leq$ —A' U B'—.
include('Axioms/SET008^0.ax')
$\mathrm{is\_function}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{i}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(is_function_type, type)
$\mathrm{is\_function} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, f\colon \$\mathrm{i} \to \$\mathrm{i}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon \forall e\colon \$\mathrm{i}\colon ((x@e) \Rightarrow (y@(f@e))))$ thf(is_function, definition)
$\mathrm{injection}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{i}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(injection_type, type)
$\mathrm{injection} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, f\colon \$\mathrm{i} \to \$\mathrm{i}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon (\mathrm{is\_function}@x@f@y \text{ and } \forall e_1\colon \$\mathrm{i}, e_2\colon \$\mathrm{i}\colon ((x@e_1 \text{ and } x@e_2 \text{ and } (f@e_1) = (f@e_2)) \Rightarrow e_1 = e_2)))$ thf(injection, definition)
$\mathrm{surjection}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{i}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(surjection_type, type)
$\mathrm{surjection} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, f\colon \$\mathrm{i} \to \$\mathrm{i}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon (\mathrm{is\_function}@x@f@y \text{ and } \forall e_1\colon \$\mathrm{i}\colon ((y@e_1) \Rightarrow \exists e_2\colon \$\mathrm{i}\colon (x@e_2 \text{ and } (f@e_2) = e_1))))$ thf(surjection, definition)
$\mathrm{bijection}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{i}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(bijection_type, type)

$\mathrm{bijection} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, f\colon \$\mathrm{i} \to \$\mathrm{i}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon (\mathrm{injection}@x@f@y \text{ and } \mathrm{surjection}@x@f@y))$ thf(bijection, definition)
$\mathrm{equinumerous}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(equinumerous_type, type)
$\mathrm{equinumerous} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon \exists f\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\mathrm{bijection}@x@f@y))$ thf(equinumerous, definition)
$\mathrm{embedding}\colon (\$\mathrm{i} \to \$\mathrm{o}) \to (\$\mathrm{i} \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(embedding_type, type)
$\mathrm{embedding} = (\lambda x\colon \$\mathrm{i} \to \$\mathrm{o}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon \exists f\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\mathrm{injection}@x@f@y))$ thf(embedding, definition)
$\forall a\colon \$\mathrm{i} \to \$\mathrm{o}, \mathrm{ap}\colon \$\mathrm{i} \to \$\mathrm{o}, b\colon \$\mathrm{i} \to \$\mathrm{o}, \mathrm{bp}\colon \$\mathrm{i} \to \$\mathrm{o}\colon ((\mathrm{equinumerous}@a@\mathrm{ap} \text{ and } \mathrm{equinumerous}@b@\mathrm{bp} \text{ and } (\mathrm{intersection}@\mathrm{ap}@\mathrm{bp})$
$\mathrm{emptyset}) \Rightarrow (\mathrm{embedding}@(\mathrm{union}@a@b)@(\mathrm{union}@\mathrm{ap}@\mathrm{bp})))$ thf(prove, conjecture)

NUM864=1.p Sum idempotent element
$\exists x\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, x) = x$ tff(sum_idempotent_element, conjecture)

NUM865=1.p Associativity of sum
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}, z_1\colon \$\mathrm{int}, z_2\colon \$\mathrm{int}, z_3\colon \$\mathrm{int}, z_4\colon \$\mathrm{int}\colon ((\$\mathrm{sum}(x, y) = z_1 \text{ and } \$\mathrm{sum}(z_1, z) = z_2 \text{ and } \$\mathrm{sum}(y, z) =$
$z_3 \text{ and } \$\mathrm{sum}(x, z_3) = z_4) \Rightarrow z_2 = z_4)$ tff(associative_sum_forall, conjecture)

NUM866=1.p Prove sum with 0 is the identity
$\forall x\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, 0) = x$ tff(prove_sum_0_identity, conjecture)

NUM867=1.p Prove sum with 0 is the identity
$\forall x\colon \$\mathrm{int}\colon \$\mathrm{sum}(0, x) = x$ tff(prove_sum_0_identity_rev, conjecture)

NUM868=1.p Sum X and X is Y
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, x) = y$ tff(anti_sum_x_x_y, conjecture)

NUM869=1.p Sum X and Y is X
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, y) = x$ tff(anti_sum_x_y_x, conjecture)

NUM870=1.p Sum is not a function
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z_1\colon \$\mathrm{int}, z_2\colon \$\mathrm{int}\colon (\$\mathrm{sum}(x, y) = z_1 \text{ and } \$\mathrm{sum}(x, y) = z_2 \text{ and } z_1 \neq z_2)$ tff(anti_unique_sum, conjecture)

NUM871=1.p Sum is not associativity
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}, z_1\colon \$\mathrm{int}, z_2\colon \$\mathrm{int}, z_3\colon \$\mathrm{int}, z_4\colon \$\mathrm{int}\colon (\$\mathrm{sum}(x, y) = z_1 \text{ and } \$\mathrm{sum}(z_1, z) = z_2 \text{ and } \$\mathrm{sum}(z, x) =$
$z_3 \text{ and } \$\mathrm{sum}(z_3, y) = z_4 \text{ and } z_2 \neq z_4)$ tff(anti_associativity_sum, conjecture)

NUM872=1.p Sum something and 0 is not something
$\exists x\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, 0) \neq x$ tff(anti_sum_identity$_1$, conjecture)

NUM873=1.p Sum something and 0 is another thing
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\$\mathrm{sum}(x, 0) = y \text{ and } y \neq x)$ tff(anti_sum_identity$_2$, conjecture)

NUM874=1.p Sum idempotence
$\forall x\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, x) = x$ tff(anti_sum_idempotence, conjecture)

NUM875=1.p Sum not idempotence
$\forall x\colon \$\mathrm{int}\colon \$\mathrm{sum}(x, x) \neq x$ tff(anti_not_sum_idempotence, conjecture)

NUM876=1.p X minus X equals 0
$\forall x\colon \$\mathrm{int}\colon \$\mathrm{difference}(x, x) = 0$ tff(x_minus_x_equals$_0$, conjecture)

NUM877=1.p Difference identity
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\$\mathrm{difference}(x, y) = 0 \Rightarrow x = y)$ tff(diff_identity, conjecture)

NUM878=1.p Product idempotent element
$\exists x\colon \$\mathrm{int}\colon \$\mathrm{product}(x, x) = x$ tff(product_idempotent_element, conjecture)

NUM879=1.p Product X and X is not Y
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon \$\mathrm{product}(x, x) = y$ tff(anti_product_x_x_y, conjecture)

NUM880=1.p Product of X and Y is not X
$\forall x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon \$\mathrm{product}(x, y) = x$ tff(anti_product_x_y_x, conjecture)

NUM881=1.p Product is not a function
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z_1\colon \$\mathrm{int}, z_2\colon \$\mathrm{int}\colon (\$\mathrm{product}(x, y) = z_1 \text{ and } \$\mathrm{product}(x, y) = z_2 \text{ and } z_1 \neq z_2)$ tff(anti_unique_product, conj

NUM882=1.p Product is not associative
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}, z\colon \$\mathrm{int}, z_1\colon \$\mathrm{int}, z_2\colon \$\mathrm{int}, z_3\colon \$\mathrm{int}, z_4\colon \$\mathrm{int}\colon (\$\mathrm{product}(x, y) = z_1 \text{ and } \$\mathrm{product}(z_1, z) = z_2 \text{ and } \$\mathrm{product}(z, x) =$
$z_3 \text{ and } \$\mathrm{product}(z_3, y) = z_4 \text{ and } z_2 \neq z_4)$ tff(anti_associativity_product, conjecture)

NUM883=1.p Product of something and 1 is not that something
$\exists x\colon \$\mathrm{int}\colon \$\mathrm{product}(x, 1) \neq x$ tff(anti_product_identity$_1$, conjecture)

NUM884=1.p Not product identity
$\exists x\colon \$\mathrm{int}, y\colon \$\mathrm{int}\colon (\$\mathrm{product}(x, 1) = y \text{ and } y \neq x)$ tff(anti_product_identity$_2$, conjecture)

NUM885=1.p Product idempotence
$\forall x$: \$int: \$product$(x, x) = x$ tff(anti_product_idempotence, conjecture)

NUM886=1.p Product non-idempotence
$\forall x$: \$int: \$product$(x, x) \neq x$ tff(anti_not_product_idempotence, conjecture)

NUM887=1.p Product with 0 is identity
$\forall x$: \$int: \$product$(x, 0) = x$ tff(anti_product_0_identity, conjecture)

NUM888=1.p Product with 0 is identity
$\forall x$: \$int: \$product$(0, x) = x$ tff(anti_product_0_identity_rev, conjecture)

NUM889=1.p - - X is X
$\forall x$: \$int: \$uminus(\$uminus$(x)) = x$ tff(uminus_uminus, conjecture)

NUM890=1.p Sum of X and - X is 0
$\forall x$: \$int: \$sum$(x,$ \$uminus$(x)) = 0$ tff(sum_uminus_to$_0$, conjecture)

NUM891=1.p X = - X means X is 0
$\forall x$: \$int: $(x =$ \$uminus$(x) \iff x = 0)$ tff(uminus_equal, conjecture)

NUM892=1.p Definition of lesseq in terms of less and equality
$\forall x$: \$int, y: \$int: (\$lesseq$(x, y) \iff$ (\$less(x, y) or $x = y))$ tff(less_lesseq, conjecture)

NUM893=1.p Sum and difference
$\exists x$: \$int, y: \$int, z: \$int: (\$sum$(x, y) = z \iff$ (\$difference$(z, y) = x$ and \$difference$(z, x) = y))$ tff(sum_same_as_differenc

NUM894=1.p If Z is less than X + 1 then Z is less than or equal to X
$\forall x$: \$int, z: \$int: (\$less$(z,$ \$sum$(x, 1)) \Rightarrow$ \$lesseq$(z, x))$ tff(less_successor, conjecture)

NUM895=1.p Sum and difference
$\forall x$: \$int, y: \$int, z: \$int: (\$sum$(x, y) = z \Rightarrow$ \$difference$(z, x) = y)$ tff(sum_difference, conjecture)

NUM896=1.p Sum implies both less
$\forall x$: \$int, y: \$int, z: \$int: (\$sum$(x, y) = z \Rightarrow$ (\$less(x, z) and \$less$(y, z)))$ tff(anti_sum_larger, conjecture)

NUM897=1.p Sum less than difference
$\forall x$: \$int, y: \$int, z_1: \$int, z_2: \$int: (\$sum$(x, y) = z_1$ and \$difference$(x, y) = z_2$ and \$less$(z_1, z_2))$ tff(anti_sum_diff_less$_1$, conj

NUM898=1.p Sum and difference and less
$\forall x$: \$int, y: \$int, z_1: \$int, z_2: \$int: (\$sum$(x, y) = z_1$ and \$difference$(x, y) = z_2$ and \$less$(z_2, z_1))$ tff(anti_sum_diff_less$_2$, conj

NUM899=1.p Difference less than sum
$\forall x$: \$int, y: \$int, z_1: \$int, z_2: \$int: ((\$sum$(x, y) = z_1$ and \$difference$(x, y) = z_2) \Rightarrow$ \$less$(z_2, z_1))$ tff(anti_x_sum_y_greater_

NUM900=1.p Difference greater 0 implies less
$\forall x$: \$int, y: \$int, z: \$int: ((\$difference$(x, y) = z$ and \$less$(0, z)) \Rightarrow$ \$less$(y, x))$ tff(difference_greater, conjecture)

NUM901=1.p Difference something and itself is 0/1
$\forall x$: \$rat: \$difference$(x, x) = 0/1$ tff(rat_difference_problem$_{12}$, conjecture)

NUM902=1.p Difference is 0/1 implies equal
$\forall x$: \$rat, y: \$rat: (\$difference$(x, y) = 0/1 \Rightarrow x = y)$ tff(rat_difference_problem$_{13}$, conjecture)

NUM903=1.p - - something is something
$\forall x$: \$rat: \$uminus(\$uminus$(x)) = x$ tff(rat_uminus_problem$_7$, conjecture)

NUM904=1.p Sum something and - something is 0/1
$\forall x$: \$rat: \$sum$(x,$ \$uminus$(x)) = 0/1$ tff(rat_uminus_problem$_8$, conjecture)

NUM905=1.p X is - X only for 0
$\forall x$: \$rat: $(x =$ \$uminus$(x) \iff x = 0/1)$ tff(rat_uminus_problem$_9$, conjecture)

NUM906=1.p Definition of lesseq in terms of less and equality
$\forall x$: \$rat, y: \$rat: (\$lesseq$(x, y) \iff$ (\$less(x, y) or $x = y))$ tff(rat_combined_problem$_1$, conjecture)

NUM907=1.p Sum and difference
$\forall x$: \$rat, y: \$rat, z: \$rat: (\$sum$(x, y) = z \iff$ (\$difference$(z, y) = x$ and \$difference$(z, x) = y))$ tff(rat_combined_problem

NUM908=1.p Difference everything and iteself is 0.0
$\forall x$: \$real: \$difference$(x, x) = 0._0$ tff(real_difference_problem$_{12}$, conjecture)

NUM909=1.p Difference is 0.0 implies equality
$\forall x$: \$real, y: \$real: (\$difference$(x, y) = 0._0 \Rightarrow x = y)$ tff(real_difference_problem$_{13}$, conjecture)

NUM910=1.p - - something is something
$\forall x\colon \$\text{real}\colon \$\text{uminus}(\$\text{uminus}(x)) = x \qquad \text{tff}(\text{real\_uminus\_problem}_7, \text{conjecture})$

NUM911=1.p Sum something and - something is 0.0
$\forall x\colon \$\text{real}\colon \$\text{sum}(x, \$\text{uminus}(x)) = 0._0 \qquad \text{tff}(\text{real\_uminus\_problem}_8, \text{conjecture})$

NUM912=1.p X is - X only for 0.0
$\forall x\colon \$\text{real}\colon (x = \$\text{uminus}(x) \iff x = 0._0) \qquad \text{tff}(\text{real\_uminus\_problem}_9, \text{conjecture})$

NUM913=1.p Definition of lesseq in terms of less and equality
$\forall x\colon \$\text{real}, y\colon \$\text{real}\colon (\$\text{lesseq}(x, y) \iff (\$\text{less}(x, y) \text{ or } x = y)) \qquad \text{tff}(\text{real\_combined\_problem}_1, \text{conjecture})$

NUM914=1.p Sum and difference
$\exists x\colon \$\text{real}, y\colon \$\text{real}, z\colon \$\text{real}\colon (\$\text{sum}(x, y) = z \iff (\$\text{difference}(z, y) = x \text{ and } \$\text{difference}(z, x) = y)) \qquad \text{tff}(\text{real\_combined\_prob}$

NUM915=1.p Every sum right exists
$\forall u\colon \$\text{int}, v\colon \$\text{int}\colon \exists w\colon \$\text{int}\colon \$\text{sum}(u, w) = v \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM916=1.p Every sum left exists
$\forall u\colon \$\text{int}, v\colon \$\text{int}\colon \exists w\colon \$\text{int}\colon \$\text{sum}(w, u) = v \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM917=1.p Every difference right exists
$\forall u\colon \$\text{int}, v\colon \$\text{int}\colon \exists w\colon \$\text{int}\colon \$\text{difference}(u, w) = v \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM918=1.p Every difference left exists
$\forall u\colon \$\text{int}, v\colon \$\text{int}\colon \exists w\colon \$\text{int}\colon \$\text{difference}(w, u) = v \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM919=1.p No number inbetween
$\forall u\colon \$\text{int}\colon \exists v\colon \$\text{int}\colon (\$\text{less}(v, u) \text{ and } \neg \exists w\colon \$\text{int}\colon (\$\text{less}(v, w) \text{ and } \$\text{less}(w, u))) \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM920=1.p No such positive number
$\neg \exists u\colon \$\text{int}\colon (\$\text{less}(0, u) \text{ and } \forall v\colon \$\text{int}\colon (\$\text{less}(v, u) \Rightarrow \$\text{less}(\$\text{sum}(v, 1), u))) \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM921=1.p Increasing function property
$f\colon \$\text{int} \to \$\text{int} \qquad \text{tff}(\text{f\_type}, \text{type})$
$\forall u\colon \$\text{int}\colon \$\text{greater}(f(u), u) \Rightarrow \forall v\colon \$\text{int}\colon \$\text{less}(\$\text{difference}(v, f(v)), 0) \qquad \text{tff}(\text{co}_1, \text{conjecture})$

NUM922=1.p Universal predicate
$p\colon \$\text{int} \to \$\text{o} \qquad \text{tff}(\text{p\_type}, \text{type})$
$(p(0) \text{ and } \forall u\colon \$\text{int}\colon (p(u) \Rightarrow p(\$\text{sum}(u, 1))) \text{ and } \forall v\colon \$\text{int}\colon (p(v) \Rightarrow p(\$\text{difference}(v, 1)))) \Rightarrow \forall w\colon \$\text{int}\colon p(w) \qquad \text{tff}(\text{co}_1, \text{conjec}$

NUM927=1.p The Collatz Conjecture
f(X) = 3X + 1 if X is odd, X/2 if X is even. Prove this is cyclic. e.g., 3,10,5,16,8,4,2,1,4,2,1
$f\colon \$\text{int} \to \$\text{int} \qquad \text{tff}(\text{f\_type}, \text{type})$
$\text{iterate\_f}\colon (\$\text{int} \times \$\text{int}) \to \$\text{int} \qquad \text{tff}(\text{iterate\_f\_type}, \text{type})$
$\forall x\colon \$\text{int}\colon (\$\text{remainder\_t}(x, 2) = 1 \Rightarrow f(x) = \$\text{sum}(\$\text{product}(3, x), 1)) \qquad \text{tff}(\text{f\_odd}, \text{axiom})$
$\forall x\colon \$\text{int}\colon (\$\text{remainder\_t}(x, 2) = 0 \Rightarrow f(x) = \$\text{quotient\_t}(x, 2)) \qquad \text{tff}(\text{f\_even}, \text{axiom})$
$\forall i\colon \$\text{int}, x\colon \$\text{int}\colon (i = 1 \Rightarrow \text{iterate\_f}(i, x) = f(x)) \qquad \text{tff}(\text{iterate\_f\_base}, \text{axiom})$
$\forall i\colon \$\text{int}, x\colon \$\text{int}\colon (\$\text{greater}(i, 1) \Rightarrow \text{iterate\_f}(i, x) = \text{iterate\_f}(\$\text{difference}(i, 1), f(x))) \qquad \text{tff}(\text{iterate\_f}, \text{axiom})$
$\forall x\colon \$\text{int}\colon (\$\text{greatereq}(x, 1) \Rightarrow \exists i\colon \$\text{int}\colon \text{iterate\_f}(i, x) = 1) \qquad \text{tff}(\text{iterates\_to}_1, \text{conjecture})$

NUM927=2.p Related to the Collatz Conjecture
There are two sequences of different length that lead to the same value.
$f\colon \$\text{int} \to \$\text{int} \qquad \text{tff}(\text{f\_type}, \text{type})$
$\text{iterate\_f}\colon (\$\text{int} \times \$\text{int}) \to \$\text{int} \qquad \text{tff}(\text{iterate\_f\_type}, \text{type})$
$\forall x\colon \$\text{int}\colon (\$\text{remainder\_t}(x, 2) = 1 \Rightarrow f(x) = \$\text{sum}(\$\text{product}(3, x), 1)) \qquad \text{tff}(\text{f\_odd}, \text{axiom})$
$\forall x\colon \$\text{int}\colon (\$\text{remainder\_t}(x, 2) = 0 \Rightarrow f(x) = \$\text{quotient\_t}(x, 2)) \qquad \text{tff}(\text{f\_even}, \text{axiom})$
$\forall i\colon \$\text{int}, x\colon \$\text{int}\colon (i = 1 \Rightarrow \text{iterate\_f}(i, x) = f(x)) \qquad \text{tff}(\text{iterate\_f\_base}, \text{axiom})$
$\forall i\colon \$\text{int}, x\colon \$\text{int}\colon (\$\text{greater}(i, 1) \Rightarrow \text{iterate\_f}(i, x) = \text{iterate\_f}(\$\text{difference}(i, 1), f(x))) \qquad \text{tff}(\text{iterate\_f}, \text{axiom})$
$\forall x\colon \$\text{int}\colon (\$\text{greatereq}(x, 1) \Rightarrow \exists i_1\colon \$\text{int}, i_2\colon \$\text{int}\colon (\$\text{greatereq}(i_1, 1) \text{ and } \$\text{greater}(i_2, i_1) \text{ and } \text{iterate\_f}(i_1, x) = \text{iterate\_f}(i_2, x)))$