

# PHI axioms

**PHI001** $\wedge$ **0.ax** Axioms for Goedel's Ontological Proof of the Existence of God

positive:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(positive\_tp, type)

god:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(god\_tp, type)

essence:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(essence\_tp, type)

necessary\_existence:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(necessary\_existence\_tp, type)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mequiv@(positive@ $\lambda$ x:  $\mu$ : (mnot@(phi@x))@(mnot@(positive@phi))))

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mforall\_indset@ $\lambda$ psi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(mand@(positive@phi)@(mb

god =  $(\lambda$ x:  $\mu$ : (mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(phi@x)))) thf(defD<sub>1</sub>, definition)

mvalid@(positive@god) thf(axA<sub>3</sub>, axiom)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(mbox@(positive@phi)))) thf(axA<sub>4</sub>, axiom)

essence =  $(\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ ,  $x$ :  $\mu$ : (mand@(phi@x)@(mforall\_indset@ $\lambda$ psi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(psi@x)@(mbox@

necessary\_existence =  $(\lambda$ x:  $\mu$ : (mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(essence@phi@x)@(mbox@(mexists\_ind@ $\lambda$ y

mvalid@(positive@necessary\_existence) thf(axA<sub>5</sub>, axiom)

# PHI problems

**PHI001** $\wedge$ **1.p** Axioms for Goedel's Ontological Proof of the Existence of God

include('Axioms/LCL016^0.ax')

include('Axioms/LCL016^1.ax')

include('Axioms/PHI001^0.ax')

**PHI002** $\wedge$ **1.p** Positive properties are possibly exemplified

include('Axioms/LCL016^0.ax')

include('Axioms/PHI001^0.ax')

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(mdia@(mexists\_ind@ $\lambda$ x:  $\mu$ : (phi@x)))) thf(

**PHI002** $\wedge$ **2.p** Positive properties are possibly exemplified

include('Axioms/LCL016^0.ax')

positive:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(positive\_tp, type)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mequiv@(positive@ $\lambda$ x:  $\mu$ : (mnot@(phi@x))@(mnot@(positive@phi))))

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mforall\_indset@ $\lambda$ psi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(mand@(positive@phi)@(mb

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(mdia@(mexists\_ind@ $\lambda$ x:  $\mu$ : (phi@x)))) thf(

**PHI003** $\wedge$ **1.p** Possibly, God exists

include('Axioms/LCL016^0.ax')

include('Axioms/PHI001^0.ax')

mvalid@(mdia@(mexists\_ind@ $\lambda$ x:  $\mu$ : (god@x))) thf(corC, conjecture)

**PHI003** $\wedge$ **2.p** Possibly, God exists

include('Axioms/LCL016^0.ax')

positive:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(positive\_tp, type)

god:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(god\_tp, type)

god =  $(\lambda$ x:  $\mu$ : (mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(phi@x)))) thf(defD<sub>1</sub>, definition)

mvalid@(positive@god) thf(axA<sub>3</sub>, axiom)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(mdia@(mexists\_ind@ $\lambda$ x:  $\mu$ : (phi@x)))) thf(

mvalid@(mdia@(mexists\_ind@ $\lambda$ x:  $\mu$ : (god@x))) thf(corC, conjecture)

**PHI004** $\wedge$ **1.p** Being God-like is an essence of any God-like being

include('Axioms/LCL016^0.ax')

include('Axioms/PHI001^0.ax')

mvalid@(mforall\_ind@ $\lambda$ x:  $\mu$ : (mimplies@(god@x)@(essence@god@x))) thf(thmT<sub>2</sub>, conjecture)

**PHI004** $\wedge$ **2.p** Being God-like is an essence of any God-like being

include('Axioms/LCL016^0.ax')

positive:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(positive\_tp, type)

god:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(god\_tp, type)

essence:  $(\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o) \rightarrow \mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$  thf(essence\_tp, type)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mequiv@(positive@ $\lambda$ x:  $\mu$ : (mnot@(phi@x))@(mnot@(positive@phi))))

god =  $(\lambda$ x:  $\mu$ : (mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(phi@x)))) thf(defD<sub>1</sub>, definition)

mvalid@(mforall\_indset@ $\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(positive@phi)@(mbox@(positive@phi)))) thf(axA<sub>4</sub>, axiom)

essence =  $(\lambda$ phi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ ,  $x$ :  $\mu$ : (mand@(phi@x)@(mforall\_indset@ $\lambda$ psi:  $\mu \rightarrow \mathcal{S}i \rightarrow \mathcal{S}o$ : (mimplies@(psi@x)@(mbox@

mvalid@(mforall\_ind@ $\lambda x$ : mu: (mimplies@(god@ $x$ )@(essence@god@ $x$ ))) thf(thmT<sub>2</sub>, conjecture)

**PHI004^4.p** Being God-like is an essence of any God-like being

qML\_mu: \$tType thf(ty\_n.t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

qML\_i: \$tType thf(ty\_n.t\_QML\_\_\_S5U\_\_Oi, type)

scott\_G: qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_OG, type)

scott\_NE.QML\_mu: qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_ONE.001t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

scott\_P: (qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o)  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_OP, type)

scott\_ess\_QML\_mu: (qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o)  $\rightarrow$  qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_Oess.001t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

scott\_G = ( $\lambda x$ : qML\_mu:  $\lambda w$ : qML\_i:  $\forall y$ : qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o: ((scott\_P@ $y@w$ )  $\Rightarrow$  ( $y@x@w$ ))) thf(fact\_0\_G\_def, axiom)

**PHI005^1.p** Necessarily, God exists

include('Axioms/LCL016^0.ax')

include('Axioms/LCL016^1.ax')

include('Axioms/PHI001^0.ax')

mvalid@(mbox@(mexists\_ind@ $\lambda x$ : mu: (god@ $x$ ))) thf(thmT<sub>3</sub>, conjecture)

**PHI005^2.p** Necessarily, God exists

include('Axioms/LCL016^0.ax')

include('Axioms/LCL016^1.ax')

positive: (mu  $\rightarrow$  \$i  $\rightarrow$  \$o)  $\rightarrow$  \$i  $\rightarrow$  \$o thf(positive\_tp, type)

god: mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(god\_tp, type)

essence: (mu  $\rightarrow$  \$i  $\rightarrow$  \$o)  $\rightarrow$  mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(essence\_tp, type)

necessary\_existence: mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(necessary\_existence\_tp, type)

god = ( $\lambda x$ : mu: (mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(positive@phi)@(phi@ $x$ )))) thf(defD<sub>1</sub>, definition)

necessary\_existence = ( $\lambda x$ : mu: (mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(essence@phi@ $x$ )@(mbox@(mexists\_ind@ $\lambda y$ : mu: (positive@necessary\_existence)))))) thf(axA<sub>5</sub>, axiom)

mvalid@(mdia@(mexists\_ind@ $\lambda x$ : mu: (god@ $x$ ))) thf(corC, lemma)

mvalid@(mforall\_ind@ $\lambda x$ : mu: (mimplies@(god@ $x$ )@(essence@god@ $x$ ))) thf(thmT<sub>2</sub>, lemma)

mvalid@(mbox@(mexists\_ind@ $\lambda x$ : mu: (god@ $x$ ))) thf(thmT<sub>3</sub>, conjecture)

**PHI005^3.p** Necessarily, God exists

Trying to prove T3 with only K - not possible.

include('Axioms/LCL016^0.ax')

include('Axioms/PHI001^0.ax')

mvalid@(mbox@(mexists\_ind@ $\lambda x$ : mu: (god@ $x$ ))) thf(thmT<sub>3</sub>, conjecture)

**PHI005^4.p** Necessarily, God exists

qML\_mu: \$tType thf(ty\_n.t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

qML\_i: \$tType thf(ty\_n.t\_QML\_\_\_S5U\_\_Oi, type)

scott\_G: qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_OG, type)

scott\_NE.QML\_mu: qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_ONE.001t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

scott\_P: (qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o)  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_OP, type)

scott\_ess\_QML\_mu: (qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o)  $\rightarrow$  qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o thf(sy\_c.Scott\_\_S5U\_\_Oess.001t\_QML\_\_\_S5U\_\_O\_\_092\_\_060mu\_062, type)

scott\_G = ( $\lambda x$ : qML\_mu:  $\lambda w$ : qML\_i:  $\forall y$ : qML\_mu  $\rightarrow$  qML\_i  $\rightarrow$  \$o: ((scott\_P@ $y@w$ )  $\Rightarrow$  ( $y@x@w$ ))) thf(fact\_0\_G\_def, axiom)

**PHI006^4.p** Inconsistency of the axioms in Goedel's original manuscript

include('Axioms/LCL017^0.ax')

positive: (mu  $\rightarrow$  \$i  $\rightarrow$  \$o)  $\rightarrow$  \$i  $\rightarrow$  \$o thf(positive\_tp, type)

god: mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(god\_tp, type)

essence: (mu  $\rightarrow$  \$i  $\rightarrow$  \$o)  $\rightarrow$  mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(essence\_tp, type)

necessary\_existence: mu  $\rightarrow$  \$i  $\rightarrow$  \$o thf(necessary\_existence\_tp, type)

mvalid@(mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(positive@ $\lambda x$ : mu: (mnot@(phi@ $x$ )))@(mnot@(positive@phi)))) thf(axA<sub>3</sub>, axiom)

mvalid@(mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(mnot@(positive@phi))@(positive@ $\lambda x$ : mu: (mnot@(phi@ $x$ )))) thf(axA<sub>4</sub>, axiom)

mvalid@(mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mforall\_indset@ $\lambda$ psi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(mand@(positive@phi)@(mbox@(mexists\_ind@ $\lambda y$ : mu: (positive@necessary\_existence)))))) thf(axA<sub>5</sub>, axiom)

god = ( $\lambda x$ : mu: (mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(positive@phi)@(phi@ $x$ )))) thf(defD<sub>1</sub>, definition)

mvalid@(positive@god) thf(axA<sub>3</sub>, axiom)

mvalid@(mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(positive@phi)@(mbox@(positive@phi)))) thf(axA<sub>4</sub>, axiom)

essence = ( $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o,  $x$ : mu: (mforall\_indset@ $\lambda$ psi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(psi@ $x$ )@(mbox@(mforall\_ind@ $\lambda y$ : mu: (positive@necessary\_existence)))))) thf(axA<sub>5</sub>, axiom)

necessary\_existence = ( $\lambda x$ : mu: (mforall\_indset@ $\lambda$ phi: mu  $\rightarrow$  \$i  $\rightarrow$  \$o: (mimplies@(essence@phi@ $x$ )@(mbox@(mexists\_ind@ $\lambda y$ : mu: (positive@necessary\_existence)))))) thf(axA<sub>5</sub>, axiom)

mvalid@(positive@necessary\_existence) thf(axA<sub>5</sub>, axiom)

\$false thf(conj, conjecture)

**PHI006^5.p** Inconsistency of the axioms in Goedel's original manuscript

include('Axioms/LCL017'0.ax')

positive: (mu → \$i → \$o) → \$i → \$o    thf(positive\_tp, type)

god: mu → \$i → \$o    thf(god\_tp, type)

essence: (mu → \$i → \$o) → mu → \$i → \$o    thf(essence\_tp, type)

necessary\_existence: mu → \$i → \$o    thf(necessary\_existence\_tp, type)

mvalid@(mforall\_indset@λphi: mu → \$i → \$o: (mimplies@(positive@λx: mu: (mnot@(phi@x))@(mnot@(positive@phi))))

mvalid@(mforall\_indset@λphi: mu → \$i → \$o: (mimplies@(mnot@(positive@phi))@(positive@λx: mu: (mnot@(phi@x))))

mvalid@(mforall\_indset@λphi: mu → \$i → \$o: (mforall\_indset@λpsi: mu → \$i → \$o: (mimplies@(mand@(positive@phi))@(mbox@

essence = (λphi: mu → \$i → \$o, x: mu: (mforall\_indset@λpsi: mu → \$i → \$o: (mimplies@(psi@x))@(mbox@(mforall\_ind@λy: r

necessary\_existence = (λx: mu: (mforall\_indset@λphi: mu → \$i → \$o: (mimplies@(essence@phi@x))@(mbox@(mexists\_ind@λy: r

mvalid@(positive@necessary\_existence)    thf(axA<sub>5</sub>, axiom)

\$false    thf(conj, conjecture)

**PHI007^5.p** Inconsistency of the axioms in Goedel's original manuscript

Scott's variant without the conjunct in the definition of essence.

qML\_mu: \$tType    thf(ty\_n.t\_QML\_S5\_O\_092\_060mu\_062, type)

qML\_i: \$tType    thf(ty\_n.t\_QML\_S5\_Oi, type)

inconsistency\_G: qML\_mu → qML\_i → \$o    thf(sy\_c\_Inconsistency\_S5\_OG, type)

incons1905966852QML\_mu: qML\_mu → qML\_i → \$o    thf(sy\_c\_Inconsistency\_S5\_ONE\_001t\_QML\_S5\_O\_092\_060mu\_062,

inconsistency\_P: (qML\_mu → qML\_i → \$o) → qML\_i → \$o    thf(sy\_c\_Inconsistency\_S5\_OP, type)

incons1389517216QML\_mu: (qML\_mu → qML\_i → \$o) → qML\_mu → qML\_i → \$o    thf(sy\_c\_Inconsistency\_S5\_Oess\_001t\_QML\_mu,

qML\_r: qML\_i → qML\_i → \$o    thf(sy\_c\_QML\_S5\_Or, type)

∀w: qML\_i, x: qML\_mu → qML\_i → \$o: ((inconsistency\_P@λy: qML\_mu: λz: qML\_i: ¬x@y@z@w) ⇒ ¬inconsistency\_P@x@w)

**PHI008^4.p** Modal Collapse of Goedel's ontological argument in Scott's variant

qML\_mu: \$tType    thf(ty\_n.t\_QML\_S5U\_O\_092\_060mu\_062, type)

qML\_i: \$tType    thf(ty\_n.t\_QML\_S5U\_Oi, type)

scott\_G: qML\_mu → qML\_i → \$o    thf(sy\_c\_Scott\_S5U\_OG, type)

scott\_NE\_QML\_mu: qML\_mu → qML\_i → \$o    thf(sy\_c\_Scott\_S5U\_ONE\_001t\_QML\_S5U\_O\_092\_060mu\_062, type)

scott\_P: (qML\_mu → qML\_i → \$o) → qML\_i → \$o    thf(sy\_c\_Scott\_S5U\_OP, type)

scott\_ess\_QML\_mu: (qML\_mu → qML\_i → \$o) → qML\_mu → qML\_i → \$o    thf(sy\_c\_Scott\_S5U\_Oess\_001t\_QML\_S5U\_O\_092\_060mu\_062,

scott\_G = (λx: qML\_mu: λw: qML\_i: ∀y: qML\_mu → qML\_i → \$o: ((scott\_P@y@w) ⇒ (y@x@w)))    thf(fact\_0\_G\_def, axiom)