

# PUZ axioms

## PUZ001-0.ax Mars and Venus axioms

$\text{from\_mars}(x)$  or  $\text{from\_venus}(x)$      $\text{cnf}(\text{from\_mars\_or\_venus}, \text{axiom})$   
 $\text{from\_mars}(x) \Rightarrow \neg \text{from\_venus}(x)$      $\text{cnf}(\text{not\_from\_mars\_and\_venus}, \text{axiom})$   
 $\text{male}(x)$  or  $\text{female}(x)$      $\text{cnf}(\text{male\_or\_female}, \text{axiom})$   
 $\text{male}(x) \Rightarrow \neg \text{female}(x)$      $\text{cnf}(\text{not\_male\_and\_female}, \text{axiom})$   
 $\text{truthteller}(x)$  or  $\text{liar}(x)$      $\text{cnf}(\text{truthteller\_or\_liar}, \text{axiom})$   
 $\text{truthteller}(x) \Rightarrow \neg \text{liar}(x)$      $\text{cnf}(\text{not\_truthteller\_and\_liar}, \text{axiom})$   
 $(\text{says}(x, y) \text{ and } \text{a\_truth}(y)) \Rightarrow \text{a\_truth}(y)$      $\text{cnf}(\text{statements\_are\_true\_or\_not}, \text{axiom})$   
 $\text{says}(x, y) \Rightarrow y = \text{statement\_by}(x)$      $\text{cnf}(\text{people\_say\_their\_statements}, \text{axiom})$   
 $\text{a\_truth}(\text{statement\_by}(x)) \Rightarrow \text{truthteller}(x)$      $\text{cnf}(\text{true\_statements\_made\_by\_truthtellers}, \text{axiom})$   
 $\text{a\_truth}(\text{statement\_by}(x))$  or  $\text{liar}(x)$      $\text{cnf}(\text{false\_statements\_made\_by\_liars}, \text{axiom})$   
 $(\text{from\_venus}(x) \text{ and } \text{female}(x)) \Rightarrow \text{truthteller}(x)$      $\text{cnf}(\text{venusian\_female\_are\_truthtellers}, \text{axiom})$   
 $(\text{from\_venus}(x) \text{ and } \text{male}(x)) \Rightarrow \text{liar}(x)$      $\text{cnf}(\text{venusian\_males\_are\_liars}, \text{axiom})$   
 $(\text{from\_mars}(x) \text{ and } \text{male}(x)) \Rightarrow \text{truthteller}(x)$      $\text{cnf}(\text{marsian\_males\_are\_truthtellers}, \text{axiom})$   
 $(\text{from\_mars}(x) \text{ and } \text{female}(x)) \Rightarrow \text{liar}(x)$      $\text{cnf}(\text{marsian\_females\_are\_liars}, \text{axiom})$   
 $(\text{truthteller}(x) \text{ and } \text{says}(x, y)) \Rightarrow \text{a\_truth}(y)$      $\text{cnf}(\text{truthtellers\_make\_true\_statements}, \text{axiom})$   
 $(\text{liar}(x) \text{ and } \text{says}(x, y)) \Rightarrow \neg \text{a\_truth}(y)$      $\text{cnf}(\text{liars\_make\_false\_statements}, \text{axiom})$

## PUZ002-0.ax Truthtellers and Liars axioms for two types of people

Axioms for two types of people; truthtellers and liars.

$\text{a\_truth}(\text{truthteller}(x))$  or  $\text{a\_truth}(\text{liar}(x))$      $\text{cnf}(\text{truthteller\_or\_liar}, \text{axiom})$   
 $\text{a\_truth}(\text{truthteller}(x)) \Rightarrow \neg \text{a\_truth}(\text{liar}(x))$      $\text{cnf}(\text{not\_both}, \text{axiom})$   
 $(\text{a\_truth}(\text{truthteller}(\text{truthteller})) \text{ and } \text{a\_truth}(\text{says}(\text{truthteller}, \text{statement}))) \Rightarrow \text{a\_truth}(\text{statement})$      $\text{cnf}(\text{truthtellers\_tell\_truth}, \text{axiom})$   
 $(\text{a\_truth}(\text{liar}(\text{liar})) \text{ and } \text{a\_truth}(\text{says}(\text{liar}, \text{statement}))) \Rightarrow \neg \text{a\_truth}(\text{statement})$      $\text{cnf}(\text{liars\_lie}, \text{axiom})$   
 $(\text{a\_truth}(\text{statement}) \text{ and } \text{a\_truth}(\text{says}(\text{truthteller}, \text{statement}))) \Rightarrow \text{a\_truth}(\text{truthteller}(\text{truthteller}))$      $\text{cnf}(\text{truths\_are\_told\_by\_truthtellers}, \text{axiom})$   
 $\text{a\_truth}(\text{says}(\text{liar}, \text{statement})) \Rightarrow (\text{a\_truth}(\text{statement}) \text{ or } \text{a\_truth}(\text{liar}(\text{liar})))$      $\text{cnf}(\text{liars\_are\_told\_by\_liars}, \text{axiom})$

## PUZ003-0.ax Truthtellers and Liars axioms for three types of people

Axioms for three types of people; truthtellers, liars and normal people.

$\text{a\_truth}(\text{truthteller}(x))$  or  $\text{a\_truth}(\text{liar}(x))$  or  $\text{a\_truth}(\text{normal}(x))$      $\text{cnf}(\text{person\_is\_one\_type}, \text{axiom})$   
 $\text{a\_truth}(\text{truthteller}(x)) \Rightarrow \neg \text{a\_truth}(\text{normal}(x))$      $\text{cnf}(\text{not\_truthteller\_and\_normal}, \text{axiom})$   
 $\text{a\_truth}(\text{truthteller}(x)) \Rightarrow \neg \text{a\_truth}(\text{liar}(x))$      $\text{cnf}(\text{not\_truthteller\_and\_liar}, \text{axiom})$   
 $\text{a\_truth}(\text{liar}(x)) \Rightarrow \neg \text{a\_truth}(\text{normal}(x))$      $\text{cnf}(\text{not\_liar\_and\_normal}, \text{axiom})$   
 $(\text{a\_truth}(\text{truthteller}(x)) \text{ and } \text{a\_truth}(\text{says}(x, y))) \Rightarrow \text{a\_truth}(y)$      $\text{cnf}(\text{truthtellers\_tell\_truth}, \text{axiom})$   
 $(\text{a\_truth}(\text{liar}(x)) \text{ and } \text{a\_truth}(\text{says}(x, y))) \Rightarrow \neg \text{a\_truth}(y)$      $\text{cnf}(\text{liars\_lie}, \text{axiom})$   
 $(\text{a\_truth}(x) \text{ and } \text{a\_truth}(\text{says}(y, x))) \Rightarrow (\text{a\_truth}(\text{truthteller}(y)) \text{ or } \text{a\_truth}(\text{normal}(y)))$      $\text{cnf}(\text{truthtellers\_and\_normal\_tell\_truth}, \text{axiom})$   
 $\text{a\_truth}(\text{says}(y, x)) \Rightarrow (\text{a\_truth}(x) \text{ or } \text{a\_truth}(\text{liar}(y)) \text{ or } \text{a\_truth}(\text{normal}(y)))$      $\text{cnf}(\text{liars\_and\_normal\_lie}, \text{axiom})$

# PUZ problems

## PUZ001+1.p Dreadbury Mansion

Someone who lives in Dreadbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Aunt Agatha hates. No one hates everyone. Agatha is not the butler. Therefore : Agatha killed herself.

$\exists x: (\text{lives}(x) \text{ and } \text{killed}(x, \text{agatha}))$      $\text{fof}(\text{pel55}_1, \text{axiom})$   
 $\text{lives}(\text{agatha})$      $\text{fof}(\text{pel55}_2_1, \text{axiom})$   
 $\text{lives}(\text{butler})$      $\text{fof}(\text{pel55}_2_2, \text{axiom})$   
 $\text{lives}(\text{charles})$      $\text{fof}(\text{pel55}_2_3, \text{axiom})$   
 $\forall x: (\text{lives}(x) \Rightarrow (x = \text{agatha} \text{ or } x = \text{butler} \text{ or } x = \text{charles}))$      $\text{fof}(\text{pel55}_3, \text{axiom})$   
 $\forall x, y: (\text{killed}(x, y) \Rightarrow \text{hates}(x, y))$      $\text{fof}(\text{pel55}_4, \text{axiom})$   
 $\forall x, y: (\text{killed}(x, y) \Rightarrow \neg \text{richer}(x, y))$      $\text{fof}(\text{pel55}_5, \text{axiom})$   
 $\forall x: (\text{hates}(\text{agatha}, x) \Rightarrow \neg \text{hates}(\text{charles}, x))$      $\text{fof}(\text{pel55}_6, \text{axiom})$   
 $\forall x: (x \neq \text{butler} \Rightarrow \text{hates}(\text{agatha}, x))$      $\text{fof}(\text{pel55}_7, \text{axiom})$   
 $\forall x: (\neg \text{richer}(x, \text{agatha}) \Rightarrow \text{hates}(\text{butler}, x))$      $\text{fof}(\text{pel55}_8, \text{axiom})$   
 $\forall x: (\text{hates}(\text{agatha}, x) \Rightarrow \text{hates}(\text{butler}, x))$      $\text{fof}(\text{pel55}_9, \text{axiom})$   
 $\forall x: \exists y: \neg \text{hates}(x, y)$      $\text{fof}(\text{pel55}_{10}, \text{axiom})$

agatha  $\neq$  butler      fof(pel55<sub>11</sub>, axiom)  
 killed(agatha, agatha)      fof(pel55, conjecture)

### PUZ001+2.p Dreadbury Mansion

Someone who lives in DreadburyMansion kills AuntAgatha. If somebody X lives in DreadburyMansion then X is AuntAgatha or X is the Butler or X is Charles. Everyone hates everyone that he kills. Noone is richer than someone that he kills. Charles hates noone who is hated by AuntAgatha. AuntAgatha does not hate the Butler. Everyone that is not the Butler is hated by AuntAgatha. The Butler hates everyone who is not richer than AuntAgatha. The Butler hates everyone who is hated by AuntAgatha. Noone hates everyone. AuntAgatha is not the Butler. Therefore, AuntAgatha kills AuntAgatha.

$\exists a, b, c$ : ( $\$true$  and predicate<sub>1</sub>( $b$ , live,  $a$ ) and modifier\_pp( $b$ , in, 'DreadburyMansion') and predicate<sub>2</sub>( $c$ , kill,  $a$ , 'AuntAgatha') and ( $d = 'AuntAgatha'$  or  $d = 'Butler'$  or  $d = 'Charles'$ )) and  $\forall f$ : ( $\$true \Rightarrow \forall g, h$ : (( $\$true$  and predicate<sub>2</sub>( $h$ , kill,  $f$ ,  $g$ ))  $\Rightarrow \exists i$ : predicate<sub>2</sub>( $i$ , hate,  $f$ ,  $g$ )) and  $\forall j$ : ( $\$true \Rightarrow \neg \exists k, l, m$ : ( $\$true$  and predicate<sub>2</sub>( $l$ , kill,  $j$ ,  $k$ ) and property<sub>2</sub>( $m$ , rich, comp\_than,  $m$ )) and  $\forall n, o$ : (( $\$true$  and predicate<sub>2</sub>( $o$ , hate, 'AuntAgatha',  $n$ ))  $\Rightarrow \neg \exists p$ : predicate<sub>2</sub>( $p$ , hate, 'Charles',  $n$ )) and  $\neg \exists q$ : predicate<sub>2</sub>( $q$ , hate, 'Butler',  $n$ ))  $\Rightarrow \exists s$ : predicate<sub>2</sub>( $s$ , hate, 'AuntAgatha',  $r$ ) and  $\forall t$ : (( $\$true$  and  $\neg \exists u$ : (property<sub>2</sub>( $u$ , rich, comp\_than, 'AuntAgatha'),  $u$ ))  $\Rightarrow \exists v$ : predicate<sub>2</sub>( $v$ , hate, 'Butler',  $t$ ) and  $\forall w, x$ : (( $\$true$  and predicate<sub>2</sub>( $x$ , hate, 'AuntAgatha',  $w$ ))  $\Rightarrow \exists y$ : predicate<sub>2</sub>( $y$ , kill, 'AuntAgatha',  $w$ )) and  $\neg \forall a_1$ : ( $\$true \Rightarrow \exists b_1$ : predicate<sub>2</sub>( $b_1$ , hate,  $z$ ,  $a_1$ ))) and 'AuntAgatha'  $\neq$  'Butler')      fof(background, axiom)  
 $\exists a$ : predicate<sub>2</sub>( $a$ , kill, 'AuntAgatha', 'AuntAgatha')      fof(prove, conjecture)

### PUZ001-1.p Dreadbury Mansion

Someone who lives in Dreadbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Aunt Agatha hates. No one hates everyone. Agatha is not the butler. Therefore : Agatha killed herself.

lives(agatha)      cnf(agatha, hypothesis)  
 lives(butler)      cnf(butler, hypothesis)  
 lives(charles)      cnf(charles, hypothesis)  
 killed( $x, y$ )  $\Rightarrow \neg$  richer( $x, y$ )      cnf(poorer\_killer, hypothesis)  
 hates(agatha,  $x$ )  $\Rightarrow \neg$  hates(charles,  $x$ )      cnf(different\_hates, hypothesis)  
 (hates( $x$ , agatha) and hates( $x$ , butler))  $\Rightarrow \neg$  hates( $x$ , charles)      cnf(no\_one\_hates\_everyone, hypothesis)  
 hates(agatha, agatha)      cnf(agatha\_hates\_agatha, hypothesis)  
 hates(agatha, charles)      cnf(agatha\_hates\_charles, hypothesis)  
 killed( $x, y$ )  $\Rightarrow$  hates( $x, y$ )      cnf(killer\_hates\_victim, hypothesis)  
 hates(agatha,  $x$ )  $\Rightarrow$  hates(butler,  $x$ )      cnf(same\_hates, hypothesis)  
 lives( $x$ )  $\Rightarrow$  (richer( $x$ , agatha) or hates(butler,  $x$ ))      cnf(butler\_hates\_poor, hypothesis)  
 killed(butler, agatha) or killed(charles, agatha)      cnf(prove\_neither\_charles\_nor\_butler\_did\_it, negated\_conjecture)

### PUZ001-2.p Dreadbury Mansion

Someone who lives in Dreadbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Aunt Agatha hates. No one hates everyone. Agatha is not the butler. Therefore : Agatha killed herself.

lives\_at\_dreadsbury(someone)      cnf(someone\_in\_mansion, axiom)  
 killed(someone, aunt\_agatha)      cnf(someone\_killed\_agatha, axiom)  
 lives\_at\_dreadsbury(aunt\_agatha)      cnf(agatha\_lives\_at\_mansion, axiom)  
 lives\_at\_dreadsbury(butler)      cnf(butler\_lives\_at\_mansion, axiom)  
 lives\_at\_dreadsbury(charles)      cnf(charles\_lives\_at\_mansion, axiom)  
 lives\_at\_dreadsbury(person)  $\Rightarrow$  (person = aunt\_agatha or person = butler or person = charles)      cnf(noone\_else\_lives\_at\_mansion, axiom)  
 killed(killer, victim)  $\Rightarrow$  hates(killer, victim)      cnf(killer\_hates\_victim, axiom)  
 killed(killer, victim)  $\Rightarrow \neg$  richer(killer, victim)      cnf(killer\_poorer\_than\_victim, axiom)  
 hates(aunt\_agatha, person)  $\Rightarrow \neg$  hates(charles, person)      cnf(charles\_and\_agatha\_hate\_different\_people, axiom)  
 person = butler or hates(aunt\_agatha, person)      cnf(agatha\_likes\_only\_butler, axiom)  
 richer(person, aunt\_agatha) or hates(butler, person)      cnf(butler\_hates\_poor\_people, axiom)  
 hates(aunt\_agatha, person)  $\Rightarrow$  hates(butler, person)      cnf(butler\_and\_agatha\_hate\_the\_same\_people, axiom)  
 $\neg$  hates(person, every\_one\_but(person))      cnf(noone\_hates\_everyone, axiom)  
 aunt\_agatha  $\neq$  butler      cnf(agatha\_is\_not\_the\_butler, axiom)  
 $\neg$  killed(aunt\_agatha, aunt\_agatha)      cnf(prove\_agatha\_killed\_herself, negated\_conjecture)

### PUZ001-3.p Dreadbury Mansion

Someone who lives in Dreadbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Aunt Agatha hates. No one hates everyone. Agatha is not the butler. Therefore : Agatha killed herself.

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lives(agatha)    cnf(agatha, axiom)
lives(butler)    cnf(butler, axiom)
lives(charles)   cnf(charles, axiom)
killed(x, y) => ¬richer(x, y)    cnf(poorer_killer, axiom)
hates(agatha, x) => ¬hates(charles, x)    cnf(different_hates, axiom)
(hates(x, agatha) and hates(x, butler)) => ¬hates(x, charles)    cnf(no_one_hates_everyone, axiom)
hates(agatha, agatha)    cnf(agatha_hates_agatha, axiom)
hates(agatha, charles)    cnf(agatha_hates_charles, axiom)
killed(x, y) => hates(x, y)    cnf(killer_hates_victim, axiom)
hates(agatha, x) => hates(butler, x)    cnf(same_hates, axiom)
lives(x) => (richer(x, agatha) or hates(butler, x))    cnf(butler_hates_poor, axiom)
killed(agatha, agatha) or killed(butler, agatha) or killed(charles, agatha)    cnf(somebody_did_it, negated_conjecture)

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### PUZ002-1.p The Animals Puzzle

1) The only animals in this house are cats. 2) Every animal is suitable for a pet, that loves to gaze at the moon. 3) When I detest an animal, I avoid it. 4) No animals are carnivorous, unless they prowl at night. 5) No cat fails to kill mice. 6) No animals ever take to me, except what are in this house. 7) Kangaroos are not suitable for pets. 8) None but carnivora kill mice. 9) I detest animals that do not take to me. 10) Animals that prowl at night always love to gaze at the moon. The problem is to prove that "I always avoid a kangaroo".

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in_house(cat) => cat(cat)    cnf(only_cats_in_house, axiom)
gazer(gazer) => suitable_pet(gazer)    cnf(gazers_are_suitable_pets, axiom)
detested(detested) => avoided(detested)    cnf(avoid_detested, axiom)
carnivore(carnivore) => prowler(carnivore)    cnf(carnivores_are_prowlers, axiom)
cat(cat) => mouse_killer(cat)    cnf(cats_are_mice_killers, axiom)
takes_to_me(taken_animal) => in_house(taken_animal)    cnf(in_house_if_takes_to_me, axiom)
kangaroo(kangaroo) => ¬suitable_pet(kangaroo)    cnf(kangaroos_are_not_pets, axiom)
mouse_killer(killer) => carnivore(killer)    cnf(mouse_killers_are_carnivores, axiom)
takes_to_me(animal) or detested(animal)    cnf(takes_to_me_or_detested, axiom)
prowler(prowler) => gazer(prowler)    cnf(prowlers_are_gazers, axiom)
kangaroo(the_kangaroo)    cnf(kangaroo_is_a_kangaroo, axiom)
¬avoided(the_kangaroo)    cnf(avoid_kangaroo, negated_conjecture)

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### PUZ003-1.p The Barber Puzzle

There is a barbers' club that obeys the following three conditions: (1) If any member A has shaved any other member B - whether himself or another - then all members have shaved A, though not necessarily at the same time. (2) Four of the members are named Guido, Lorenzo, Petrucio, and Cesare. (3) Guido has shaved Cesare. Prove Petrucio has shaved Lorenzo

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(member(x) and member(y) and shaved(x, y)) => shaved(members, x)    cnf(one_shaved_then_all_shaved, axiom)
(shaved(members, x) and member(y)) => shaved(y, x)    cnf(all_shaved_then_one_shaved, axiom)
member(guido)    cnf(guido, hypothesis)
member(lorenzo)    cnf(lorenzo, hypothesis)
member(petruchio)    cnf(petruchio, hypothesis)
member(cesare)    cnf(cesare, hypothesis)
shaved(guido, cesare)    cnf(guido_has_shaved_cesare, hypothesis)
¬shaved(petruchio, lorenzo)    cnf(prove_petruchio_has_shaved_lorenzo, negated_conjecture)

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### PUZ004-1.p The Letters Puzzle

(1) All the dated letters in this room are written on blue paper. (2) None of them are in black ink except those that are written in the third person. (3) I have not filed any of them that I can read. (4) None of them that are written on one sheet are undated. (5) All of them that are not crossed are in black ink. : (6) All of them written by Brown begin with "Dear Sir" : (7) All of them written on blue paper are filed. : (8) None of them written on more than one sheet are crossed. (9) None of them that begin with "Dear Sir" are written in third person. Prove that letters by Brown cannot be read.

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dated => on_blue_paper    cnf(dated_on_blue_paper, axiom)
in_third_person => in_black_ink    cnf(third_person_in_black_ink, axiom)
in_black_ink => in_third_person    cnf(black_ink_in_third_person, axiom)

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can_be_read ⇒ ¬filed      cnf(not_filed_if_read, axiom)
on_one_sheet ⇒ dated      cnf(dated_if_on_one_sheet, axiom)
crossed or in_black_ink   cnf(not_crossed_in_black_ink, axiom)
by_brown ⇒ begins_with_dear_sir   cnf(brown_starts_with_sir, axiom)
on_blue_paper ⇒ filed     cnf(filed_if_on_blue_paper, axiom)
crossed ⇒ on_one_sheet   cnf(on_one_sheet_if_crossed, axiom)
begins_with_dear_sir ⇒ ¬in_third_person   cnf(no_dear_sirs_in_third_person, axiom)
by_brown      cnf(letter_by_brown, hypothesis)
can_be_read   cnf(prove_it_cannot_be_read, negated_conjecture)

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### PUZ006-1.p Determine sex and race on Mars and Venus

Here's the situation: human observers in this exclusive club on Ganymede can't distinguish Martians from Venusians, males from females, except for the fact that Venusian women and Martian men always tell the truth and Venusian men and Martian women always lie. Ork says "Bog is from Venus." Bog says "Ork is from Mars." Ork says "Bog is male." Bog says "Ork is female." Who's what? (sex & race).

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include('Axioms/PUZ001-0.ax')
says(ork, bog_is_from_venus)   cnf(ork_says_bog_is_from_venus, hypothesis)
says(bog, ork_is_from_mars)    cnf(bog_says_ork_is_from_mar, hypothesis)
says(ork, bog_is_male)        cnf(ork_says_bog_is_male, hypothesis)
says(bog, ork_is_female)      cnf(bog_says_ork_is_female, hypothesis)
a_truth(bog_is_from_venus) ⇒ from_venus(bog)   cnf(bog_is_from_venus_1, hypothesis)
a_truth(ork_is_from_mars) ⇒ from_mars(ork)     cnf(ork_is_from_mars_1, hypothesis)
a_truth(bog_is_male) ⇒ male(bog)             cnf(bog_is_male_1, hypothesis)
a_truth(ork_is_female) ⇒ female(ork)         cnf(ork_is_female_1, hypothesis)
from_venus(bog) ⇒ a_truth(bog_is_from_venus)   cnf(bog_is_from_venus_2, hypothesis)
from_mars(ork) ⇒ a_truth(ork_is_from_mars)     cnf(ork_is_from_mars_2, hypothesis)
male(bog) ⇒ a_truth(bog_is_male)              cnf(bog_is_male_2, hypothesis)
female(ork) ⇒ a_truth(ork_is_female)          cnf(ork_is_female_2, hypothesis)
¬female(bog)   cnf(prove_bog_is_female, negated_conjecture)

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### PUZ007-1.p Mixed couples on Mars and Venus

Here's the situation: human observers in this exclusive club on Ganymede can't distinguish Martians from Venusians, males from females, except for the fact that Venusian women and Martian men always tell the truth and Venusian men and Martian women always lie. A says "I'm from Mars" B exclaims "That's not true!" A and B are married; are they a mixed couple?

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include('Axioms/PUZ001-0.ax')
says(a, a_from_mars)   cnf(a_says_hes_from_mars, hypothesis)
says(b, a_has_lied)   cnf(b_says_a_lies, hypothesis)
from_mars(a) ⇒ a_truth(a_from_mars)   cnf(a_from_mars_1, hypothesis)
a_truth(a_from_mars) ⇒ from_mars(a)   cnf(a_from_mars_2, hypothesis)
a_truth(a_from_mars) ⇒ ¬a_truth(statement_by(b))   cnf(a_from_mars_3, hypothesis)
statement_by(a) = a_from_mars   cnf(a_states, hypothesis)
statement_by(b) = a_has_lied   cnf(b_states, hypothesis)
a_truth(a_from_mars) or a_truth(statement_by(b))   cnf(truth_of_bs_statement, hypothesis)
female(a) ⇒ male(b)   cnf(different_sex_1, hypothesis)
male(a) ⇒ female(b)   cnf(different_sex_2, hypothesis)
from_mars(b) or from_mars(a)   cnf(one_from_mars, negated_conjecture)
from_venus(a) or from_venus(b)   cnf(one_from_venus, negated_conjecture)

```

### PUZ008-1.p Missionaries and Cannibals

There are 3 missionaries, 3 cannibals, and a boat on the west bank of a river. All wish to cross, but the boat holds at most 2 people. If the cannibals ever outnumber the missionaries on either bank of the river the outnumbered missionaries will be eaten. Can they all safely cross the river? If so, how? (The boat cannot cross empty.)

```

achievable(west(m(x), c(s(y))), boatonwest, east(m(z), c(w))) ⇒ achievable(west(m(x), c(y)), boatoneast, east(m(z), c(s(w))))
achievable(west(m(x), c(y)), boatoneast, east(m(z), c(s(w)))) ⇒ achievable(west(m(x), c(s(y))), boatonwest, east(m(z), c(w)))
achievable(west(m(x), c(s(s(y))))), boatonwest, east(m(z), c(w))) ⇒ achievable(west(m(x), c(y)), boatoneast, east(m(z), c(s(s(w))))
achievable(west(m(x), c(y)), boatoneast, east(m(z), c(s(s(w)))) ⇒ achievable(west(m(x), c(s(s(y))))), boatonwest, east(m(z), c(w)))
achievable(west(m(s(x)), c(y)), boatonwest, east(m(z), c(w))) ⇒ achievable(west(m(x), c(y)), boatoneast, east(m(s(z)), c(w)))
achievable(west(m(x), c(y)), boatoneast, east(m(s(z)), c(w))) ⇒ achievable(west(m(s(x)), c(y)), boatonwest, east(m(z), c(w)))
achievable(west(m(s(s(x))), c(y)), boatonwest, east(m(z), c(w))) ⇒ achievable(west(m(x), c(y)), boatoneast, east(m(s(s(z))))),
achievable(west(m(x), c(y)), boatoneast, east(m(s(s(z))), c(w))) ⇒ achievable(west(m(s(s(x))), c(y)), boatonwest, east(m(z), c(w)))

```

$\text{achievable}(\text{west}(m(s(x)), c(s(y))), \text{boatonwest}, \text{east}(m(z), c(w))) \Rightarrow \text{achievable}(\text{west}(m(x), c(y)), \text{boatoneast}, \text{east}(m(s(z)), c(s(w))))$   
 $\text{achievable}(\text{west}(m(x), c(y)), \text{boatoneast}, \text{east}(m(s(z)), c(s(w)))) \Rightarrow \text{achievable}(\text{west}(m(s(x)), c(s(y))), \text{boatonwest}, \text{east}(m(z), c(s(w))))$   
 $\text{achievable}(\text{west}(m(s(x)), c(s(s(x))))), y, \text{east}(z, w)) \quad \text{cnf}(\text{extra\_cannibal\_meal\_on\_west\_bank}, \text{axiom})$   
 $\text{achievable}(\text{west}(m(s(x)), c(s(s(s(x))))), y, \text{east}(z, w)) \quad \text{cnf}(\text{two\_extra\_cannibals\_meal\_on\_west\_bank}, \text{axiom})$   
 $\text{achievable}(\text{west}(x, y), z, \text{east}(m(s(w)), c(s(s(w)))))) \quad \text{cnf}(\text{extra\_cannibal\_meal\_on\_east\_bank}, \text{axiom})$   
 $\text{achievable}(\text{west}(x, y), z, \text{east}(m(s(w)), c(s(s(s(w)))))) \quad \text{cnf}(\text{two\_extra\_cannibals\_meal\_on\_east\_bank}, \text{axiom})$   
 $\text{achievable}(\text{west}(m(s(s(s(n_0))))), c(s(s(s(n_0))))), \text{boatonwest}, \text{east}(m(n_0), c(n_0))) \quad \text{cnf}(\text{start\_on\_west\_bank}, \text{hypothesis})$   
 $\neg \text{achievable}(\text{west}(m(n_0), c(n_0)), x, \text{east}(m(s(s(s(n_0))))), c(s(s(s(n_0)))))) \quad \text{cnf}(\text{prove\_can\_get\_to\_east\_bank}, \text{negated\_conjecture})$

### PUZ009-1.p Looking for Oona

In another curious incident, when the husband arrived on an island looking for Oona, he met 5 natives A,B,C,D,E who all guessed his purpose and grinned at meeting him. They said: A: Oona is on this island. B: Oona is not on this island. C: Oona was here yesterday. D: Oona is not here today, and she was not here yesterday. E: Either D is a knave or C is a knight. The logician thought for a while, but could get nowhere. ‘Won’t one of you please make another statement?’ the logician pleaded. At this point A said: Either E is a knave or C is a knight. Is Oona on the island?”

$\text{a\_is\_a\_knight} \Rightarrow \neg \text{b\_is\_a\_knight} \quad \text{cnf}(c_1, \text{axiom})$   
 $\text{a\_is\_a\_knight} \Rightarrow \neg \text{d\_is\_a\_knight} \quad \text{cnf}(c_2, \text{axiom})$   
 $\text{b\_is\_a\_knight} \Rightarrow \neg \text{a\_is\_a\_knight} \quad \text{cnf}(c_3, \text{axiom})$   
 $\text{b\_is\_a\_knight} \text{ or } \text{a\_is\_a\_knight} \quad \text{cnf}(c_4, \text{axiom})$   
 $\text{b\_is\_a\_knight} \Rightarrow (\text{c\_is\_a\_knight} \text{ or } \text{d\_is\_a\_knight}) \quad \text{cnf}(c_5, \text{axiom})$   
 $\text{c\_is\_a\_knight} \Rightarrow \neg \text{d\_is\_a\_knight} \quad \text{cnf}(c_6, \text{axiom})$   
 $\text{d\_is\_a\_knight} \Rightarrow \neg \text{a\_is\_a\_knight} \quad \text{cnf}(c_7, \text{axiom})$   
 $\text{d\_is\_a\_knight} \Rightarrow \text{b\_is\_a\_knight} \quad \text{cnf}(c_8, \text{axiom})$   
 $\text{d\_is\_a\_knight} \Rightarrow \neg \text{c\_is\_a\_knight} \quad \text{cnf}(c_9, \text{axiom})$   
 $(\text{e\_is\_a\_knight} \text{ and } \text{d\_is\_a\_knight}) \Rightarrow \text{c\_is\_a\_knight} \quad \text{cnf}(c_{10}, \text{axiom})$   
 $\text{e\_is\_a\_knight} \text{ or } \text{d\_is\_a\_knight} \quad \text{cnf}(c_{11}, \text{axiom})$   
 $\text{c\_is\_a\_knight} \Rightarrow \text{e\_is\_a\_knight} \quad \text{cnf}(c_{12}, \text{axiom})$   
 $\text{d\_is\_a\_knight} \text{ or } \text{a\_is\_a\_knight} \text{ or } \text{c\_is\_a\_knight} \quad \text{cnf}(c_{13}, \text{axiom})$   
 $\text{b\_is\_a\_knight} \Rightarrow (\text{d\_is\_a\_knight} \text{ or } \text{c\_is\_a\_knight}) \quad \text{cnf}(c_{14}, \text{axiom})$   
 $(\text{a\_is\_a\_knight} \text{ and } \text{e\_is\_a\_knight}) \Rightarrow \text{c\_is\_a\_knight} \quad \text{cnf}(c_{15}, \text{axiom})$   
 $\text{a\_is\_a\_knight} \text{ or } \text{e\_is\_a\_knight} \quad \text{cnf}(c_{16}, \text{axiom})$   
 $\text{c\_is\_a\_knight} \Rightarrow \text{a\_is\_a\_knight} \quad \text{cnf}(c_{17}, \text{axiom})$   
 $\text{b\_is\_a\_knight} \quad \text{cnf}(c_{18}, \text{negated\_conjecture})$

### PUZ011-1.p An ocean that borders on an African and an Asian country

There is a database of assertions about various countries and oceans and their relationships. Find which ocean borders on African and Asian countries.

$\text{ocean}(\text{atlantic}) \quad \text{cnf}(\text{atlantic}, \text{hypothesis})$   
 $\text{ocean}(\text{indian}) \quad \text{cnf}(\text{indian}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{brazil}) \quad \text{cnf}(\text{atlantic\_brazil}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{uruguay}) \quad \text{cnf}(\text{atlantic\_uruguay}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{venesuela}) \quad \text{cnf}(\text{atlantic\_venesuela}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{zaire}) \quad \text{cnf}(\text{atlantic\_zaire}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{nigeria}) \quad \text{cnf}(\text{atlantic\_nigeria}, \text{hypothesis})$   
 $\text{borders}(\text{atlantic}, \text{angola}) \quad \text{cnf}(\text{atlantic\_angola}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{india}) \quad \text{cnf}(\text{indian\_india}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{pakistan}) \quad \text{cnf}(\text{indian\_pakistan}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{iran}) \quad \text{cnf}(\text{indian\_iran}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{somalia}) \quad \text{cnf}(\text{indian\_somalia}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{kenya}) \quad \text{cnf}(\text{indian\_kenya}, \text{hypothesis})$   
 $\text{borders}(\text{indian}, \text{tanzania}) \quad \text{cnf}(\text{indian\_tanzania}, \text{hypothesis})$   
 $\text{south\_american}(\text{brazil}) \quad \text{cnf}(\text{brazil}, \text{hypothesis})$   
 $\text{south\_american}(\text{uruguay}) \quad \text{cnf}(\text{uruguay}, \text{hypothesis})$   
 $\text{south\_american}(\text{venesuela}) \quad \text{cnf}(\text{venezuela}, \text{hypothesis})$   
 $\text{african}(\text{zaire}) \quad \text{cnf}(\text{zaire}, \text{hypothesis})$   
 $\text{african}(\text{nigeria}) \quad \text{cnf}(\text{nigeria}, \text{hypothesis})$   
 $\text{african}(\text{angola}) \quad \text{cnf}(\text{angola}, \text{hypothesis})$   
 $\text{african}(\text{somalia}) \quad \text{cnf}(\text{somalia}, \text{hypothesis})$   
 $\text{african}(\text{kenya}) \quad \text{cnf}(\text{kenya}, \text{hypothesis})$

african(tanzania)    cnf(tanzania, hypothesis)  
 asian(india)        cnf(india, hypothesis)  
 asian(pakistan)    cnf(pakistan, hypothesis)  
 asian(iran)        cnf(iran, hypothesis)  
 (ocean(ocean) and borders(ocean, african) and african(african) and borders(ocean, asian))  $\Rightarrow$   $\neg$  asian(asian)    cnf(prove-true, axiom)

### PUZ012-1.p The Mislabeled Boxes

There are three boxes a, b, and c on a table. Each box contains apples or bananas or oranges. No two boxes contain the same thing. Each box has a label that says it contains apples or says it contains bananas or says it contains oranges. No box contains what it says on its label. The label on box a says "apples". The label on box b says "oranges". The label on box c says "bananas". You pick up box b and it contains apples. What do the other two boxes contain?

equal\_fruits( $x, x$ )    cnf(reflexivity\_for\_fruits, axiom)  
 equal\_boxes( $x, x$ )    cnf(reflexivity\_for\_boxes, axiom)  
 label( $x, y$ )  $\Rightarrow$   $\neg$  contains( $x, y$ )    cnf(label\_is\_wrong, axiom)  
 contains(boxa,  $x$ ) or contains(boxb,  $x$ ) or contains(boxc,  $x$ )    cnf(each\_thing\_is\_in\_a\_box, axiom)  
 contains( $x$ , apples) or contains( $x$ , bananas) or contains( $x$ , oranges)    cnf(each\_box\_contains\_something, axiom)  
 (contains( $x, y$ ) and contains( $x, z$ ))  $\Rightarrow$  equal\_fruits( $y, z$ )    cnf(contains\_is\_well\_defined<sub>1</sub>, axiom)  
 (contains( $x, y$ ) and contains( $z, y$ ))  $\Rightarrow$  equal\_boxes( $x, z$ )    cnf(contains\_is\_well\_defined<sub>2</sub>, axiom)  
 $\neg$  equal\_boxes(boxa, boxb)    cnf(boxa\_not\_boxb, axiom)  
 $\neg$  equal\_boxes(boxb, boxc)    cnf(boxb\_not\_boxc, axiom)  
 $\neg$  equal\_boxes(boxa, boxc)    cnf(boxa\_not\_boxc, axiom)  
 $\neg$  equal\_fruits(apples, bananas)    cnf(apples\_not\_bananas, axiom)  
 $\neg$  equal\_fruits(bananas, oranges)    cnf(bananas\_not\_oranges, axiom)  
 $\neg$  equal\_fruits(apples, oranges)    cnf(apples\_not\_oranges, axiom)  
 label(boxa, apples)    cnf(boxa\_labelled\_apples, hypothesis)  
 label(boxb, oranges)    cnf(boxb\_labelled\_oranges, hypothesis)  
 label(boxc, bananas)    cnf(boxc\_labelled\_bananas, hypothesis)  
 contains(boxb, apples)    cnf(boxb\_contains\_apples, hypothesis)  
 contains(boxa, bananas)  $\Rightarrow$   $\neg$  contains(boxc, oranges)    cnf(prove\_boxa\_contains\_bananas\_and\_boxc\_oranges, negated\_conj)

### PUZ012\_1.p The Mislabeled Boxes

There are three boxes a, b, and c on a table. Each box contains apples or bananas or oranges. No two boxes contain the same thing. Each box has a label that says it contains apples or says it contains bananas or says it contains oranges. No box contains what it says on its label. The label on box a says "apples". The label on box b says "oranges". The label on box c says "bananas". You pick up box b and it contains apples. What do the other two boxes contain?

box: \$tType    tff(box\_type, type)  
 fruit: \$tType    tff(fruit\_type, type)  
 boxa: box    tff(boxa\_type, type)  
 boxb: box    tff(boxb\_type, type)  
 boxc: box    tff(boxc\_type, type)  
 apples: fruit    tff(apples\_type, type)  
 bananas: fruit    tff(bananas\_type, type)  
 oranges: fruit    tff(oranges\_type, type)  
 equal\_fruits: (fruit  $\times$  fruit)  $\rightarrow$  \$o    tff(equal\_fruits\_type, type)  
 equal\_boxes: (box  $\times$  box)  $\rightarrow$  \$o    tff(equal\_boxes\_type, type)  
 contains: (box  $\times$  fruit)  $\rightarrow$  \$o    tff(contains\_type, type)  
 label: (box  $\times$  fruit)  $\rightarrow$  \$o    tff(label\_type, type)  
 $\forall x$ : fruit: equal\_fruits( $x, x$ )    tff(reflexivity\_for\_fruits, axiom)  
 $\forall x$ : box: equal\_boxes( $x, x$ )    tff(reflexivity\_for\_boxes, axiom)  
 $\forall x$ : box,  $y$ : fruit:  $\neg$  label( $x, y$ ) and contains( $x, y$ )    tff(label\_is\_wrong, axiom)  
 $\forall x$ : fruit: (contains(boxa,  $x$ ) or contains(boxb,  $x$ ) or contains(boxc,  $x$ ))    tff(each\_thing\_is\_in\_a\_box, axiom)  
 $\forall x$ : box: (contains( $x$ , apples) or contains( $x$ , bananas) or contains( $x$ , oranges))    tff(each\_box\_contains\_something, axiom)  
 $\forall x$ : box,  $y$ : fruit,  $z$ : fruit: ((contains( $x, y$ ) and contains( $x, z$ ))  $\Rightarrow$  equal\_fruits( $y, z$ ))    tff(contains\_is\_well\_defined<sub>1</sub>, axiom)  
 $\forall x$ : box,  $y$ : fruit,  $z$ : box: ((contains( $x, y$ ) and contains( $z, y$ ))  $\Rightarrow$  equal\_boxes( $x, z$ ))    tff(contains\_is\_well\_defined<sub>2</sub>, axiom)  
 $\neg$  equal\_boxes(boxa, boxb)    tff(boxa\_not\_boxb, axiom)  
 $\neg$  equal\_boxes(boxb, boxc)    tff(boxb\_not\_boxc, axiom)  
 $\neg$  equal\_boxes(boxa, boxc)    tff(boxa\_not\_boxc, axiom)  
 $\neg$  equal\_fruits(apples, bananas)    tff(apples\_not\_bananas, axiom)

$\neg \text{equal\_fruits}(\text{bananas}, \text{oranges}) \quad \text{tff}(\text{bananas\_not\_oranges}, \text{axiom})$   
 $\neg \text{equal\_fruits}(\text{apples}, \text{oranges}) \quad \text{tff}(\text{apples\_not\_oranges}, \text{axiom})$   
 $\text{label}(\text{boxa}, \text{apples}) \quad \text{tff}(\text{boxa\_labelled\_apples}, \text{hypothesis})$   
 $\text{label}(\text{boxb}, \text{oranges}) \quad \text{tff}(\text{boxb\_labelled\_oranges}, \text{hypothesis})$   
 $\text{label}(\text{boxc}, \text{bananas}) \quad \text{tff}(\text{boxc\_labelled\_bananas}, \text{hypothesis})$   
 $\text{contains}(\text{boxb}, \text{apples}) \quad \text{tff}(\text{boxb\_contains\_apples}, \text{hypothesis})$   
 $\text{contains}(\text{boxa}, \text{bananas}) \text{ and } \text{contains}(\text{boxc}, \text{oranges}) \quad \text{tff}(\text{prove\_boxa\_contains\_bananas\_and\_boxc\_oranges}, \text{conjecture})$

**PUZ015-1.p** Checkerboard and Dominoes : Opposing corners removed

There is a checker board whose upper left and lower right squares have been removed. There is a box of dominoes that are one square by two squares in size. Can you exactly cover the checker board with dominoes?

$\text{achievable}(\text{row}(x), \text{squares}(\text{not\_covered}, \text{not\_covered}, y_3, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(\text{covered}, \text{covered}, y_3, y_4, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, \text{not\_covered}, \text{not\_covered}, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, \text{covered}, \text{covered}, y_4, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, \text{not\_covered}, \text{not\_covered}, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, \text{covered}, \text{covered}, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, \text{not\_covered}, \text{not\_covered}, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, \text{covered}, \text{covered}, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, \text{not\_covered}, \text{not\_covered}, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, \text{covered}, \text{covered}, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, \text{not\_covered}, \text{not\_covered}, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, \text{covered}, \text{covered}, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, \text{not\_covered}, \text{not\_covered})) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, \text{covered}, \text{covered}))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(\text{successor}(x)), \text{squares}(y'_1, y'_2, y'_3, y'_4, y'_5, y'_6, y'_7, y'_8))$   
 $\text{successor}(n_1) = n_2 \quad \text{cnf}(\text{successor\_of\_1\_is2}, \text{axiom})$   
 $\text{successor}(n_2) = n_3 \quad \text{cnf}(\text{successor\_of\_2\_is3}, \text{axiom})$   
 $\text{successor}(n_3) = n_4 \quad \text{cnf}(\text{successor\_of\_3\_is4}, \text{axiom})$   
 $\text{successor}(n_4) = n_5 \quad \text{cnf}(\text{successor\_of\_4\_is5}, \text{axiom})$   
 $\text{successor}(n_5) = n_6 \quad \text{cnf}(\text{successor\_of\_5\_is6}, \text{axiom})$   
 $\text{successor}(n_6) = n_7 \quad \text{cnf}(\text{successor\_of\_6\_is7}, \text{axiom})$   
 $\text{successor}(n_7) = n_8 \quad \text{cnf}(\text{successor\_of\_7\_is8}, \text{axiom})$   
 $\text{successor}(n_8) = n_9 \quad \text{cnf}(\text{successor\_of\_8\_is9}, \text{axiom})$   
 $\text{covered}' = \text{not\_covered} \quad \text{cnf}(\text{complement\_of\_covered\_is\_not\_covered}, \text{axiom})$   
 $\text{not\_covered}' = \text{covered} \quad \text{cnf}(\text{complement\_of\_not\_covered\_is\_covered}, \text{axiom})$   
 $\text{removed}' = \text{not\_covered} \quad \text{cnf}(\text{complement\_of\_r\_is\_not\_covered}, \text{axiom})$   
 $\text{achievable}(\text{row}(n_1), \text{squares}(\text{removed}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}))$   
 $\neg \text{achievable}(\text{row}(n_8), \text{squares}(\text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{not\_covered})) \quad \text{cnf}(\text{try\_prove\_row\_8\_can\_be\_covered\_with\_dominoes})$

**PUZ016-1.p** Checkerboard and Dominoes : Row 1, columns 2 and 3 removed

There is a checker board whose second and third squares from the first row have been removed. There is a box of dominoes that are one square by two squares in size. Can you exactly cover the checker board with dominoes?

$\text{achievable}(\text{row}(x), \text{squares}(\text{not\_covered}, \text{not\_covered}, y_3, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(\text{covered}, \text{covered}, y_3, y_4, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, \text{not\_covered}, \text{not\_covered}, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, \text{covered}, \text{covered}, y_4, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, \text{not\_covered}, \text{not\_covered}, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, \text{covered}, \text{covered}, y_5, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, \text{not\_covered}, \text{not\_covered}, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, \text{covered}, \text{covered}, y_6, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, \text{not\_covered}, \text{not\_covered}, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, \text{covered}, \text{covered}, y_7, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, \text{not\_covered}, \text{not\_covered}, y_8)) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, \text{covered}, \text{covered}, y_8))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, \text{not\_covered}, \text{not\_covered})) \Rightarrow \text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, \text{covered}, \text{covered}))$   
 $\text{achievable}(\text{row}(x), \text{squares}(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8)) \Rightarrow \text{achievable}(\text{row}(\text{successor}(x)), \text{squares}(y'_1, y'_2, y'_3, y'_4, y'_5, y'_6, y'_7, y'_8))$   
 $\text{successor}(n_1) = n_2 \quad \text{cnf}(\text{successor\_of\_1\_is2}, \text{axiom})$   
 $\text{successor}(n_2) = n_3 \quad \text{cnf}(\text{successor\_of\_2\_is3}, \text{axiom})$   
 $\text{successor}(n_3) = n_4 \quad \text{cnf}(\text{successor\_of\_3\_is4}, \text{axiom})$   
 $\text{successor}(n_4) = n_5 \quad \text{cnf}(\text{successor\_of\_4\_is5}, \text{axiom})$   
 $\text{successor}(n_5) = n_6 \quad \text{cnf}(\text{successor\_of\_5\_is6}, \text{axiom})$   
 $\text{successor}(n_6) = n_7 \quad \text{cnf}(\text{successor\_of\_6\_is7}, \text{axiom})$   
 $\text{successor}(n_7) = n_8 \quad \text{cnf}(\text{successor\_of\_7\_is8}, \text{axiom})$   
 $\text{successor}(n_8) = n_9 \quad \text{cnf}(\text{successor\_of\_8\_is9}, \text{axiom})$   
 $\text{covered}' = \text{not\_covered} \quad \text{cnf}(\text{complement\_of\_covered\_is\_not\_covered}, \text{axiom})$   
 $\text{not\_covered}' = \text{covered} \quad \text{cnf}(\text{complement\_of\_not\_covered\_is\_covered}, \text{axiom})$   
 $\text{removed}' = \text{not\_covered} \quad \text{cnf}(\text{complement\_of\_removed\_is\_not\_covered}, \text{axiom})$   
 $\text{achievable}(\text{row}(n_1), \text{squares}(\text{not\_covered}, \text{removed}, \text{removed}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}, \text{not\_covered}))$   
 $\neg \text{achievable}(\text{row}(n_8), \text{squares}(\text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered}, \text{covered})) \quad \text{cnf}(\text{prove\_row\_8\_can\_be\_covered\_with\_dominoes})$

**PUZ020-1.p** A knights & knaves problem, if he's a knight, so is she

$\text{person}(x) \Rightarrow (\text{knight}(x) \text{ or } \text{knave}(x)) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $(\text{person}(x) \text{ and } \text{knight}(x)) \Rightarrow \neg \text{knave}(x) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$

$(\text{says}(x, y) \text{ and } \text{a\_truth}(y)) \Rightarrow \text{a\_truth}(y)$      $\text{cnf}(\text{statements\_are\_true\_or\_false}, \text{axiom})$   
 $\text{says}(x, y) \Rightarrow x \neq y$      $\text{cnf}(\text{people\_do\_not\_equal\_their\_statements}_1, \text{axiom})$   
 $\text{says}(x, y) \Rightarrow y = \text{statement\_by}(x)$      $\text{cnf}(\text{peoples\_statements}, \text{axiom})$   
 $\text{person}(x) \Rightarrow x \neq \text{statement\_by}(y)$      $\text{cnf}(\text{people\_do\_not\_equal\_their\_statement}_2, \text{axiom})$   
 $(\text{person}(x) \text{ and } \text{a\_truth}(\text{statement\_by}(x))) \Rightarrow \text{knight}(x)$      $\text{cnf}(\text{knight\_make\_true\_statements}, \text{axiom})$   
 $\text{person}(x) \Rightarrow (\text{a\_truth}(\text{statement\_by}(x)) \text{ or } \text{knave}(x))$      $\text{cnf}(\text{knaves\_make\_false\_statements}, \text{axiom})$   
 $(\text{knight}(x) \text{ and } \text{says}(x, y)) \Rightarrow \text{a\_truth}(y)$      $\text{cnf}(\text{knight\_say\_the\_truth}, \text{axiom})$   
 $(\text{knave}(x) \text{ and } \text{says}(x, y)) \Rightarrow \neg \text{a\_truth}(y)$      $\text{cnf}(\text{knaves\_do\_not\_say\_the\_truth}, \text{axiom})$   
 $\text{person}(\text{husband})$      $\text{cnf}(\text{husband}, \text{hypothesis})$   
 $\text{person}(\text{wife})$      $\text{cnf}(\text{wife}, \text{hypothesis})$   
 $\text{husband} \neq \text{wife}$      $\text{cnf}(\text{husband\_not\_wife}, \text{hypothesis})$   
 $\text{says}(\text{husband}, \text{statement\_by}(\text{husband}))$      $\text{cnf}(\text{husband\_makes\_statements}, \text{hypothesis})$   
 $(\text{a\_truth}(\text{statement\_by}(\text{husband})) \text{ and } \text{knight}(\text{husband})) \Rightarrow \text{knight}(\text{wife})$      $\text{cnf}(\text{truthful\_knight\_husband\_means\_knight\_wife})$   
 $\text{knight}(\text{husband}) \Rightarrow \text{a\_truth}(\text{statement\_by}(\text{husband}))$      $\text{cnf}(\text{knight\_husband\_makes\_true\_statements}, \text{hypothesis})$   
 $\text{a\_truth}(\text{statement\_by}(\text{husband})) \text{ or } \text{knight}(\text{wife})$      $\text{cnf}(\text{knight\_wife\_or\_truthful\_husband}, \text{hypothesis})$   
 $\text{knight}(\text{wife}) \Rightarrow \text{a\_truth}(\text{statement\_by}(\text{husband}))$      $\text{cnf}(\text{knight\_wife\_means\_truthful\_husband}, \text{hypothesis})$   
 $\neg \text{knight}(\text{husband})$      $\text{cnf}(\text{prove\_knight\_husband}, \text{negated\_conjecture})$

### PUZ021-1.p How to Win a Bride

Suppose you are an inhabitant of the island of 'knights' and 'knaves'. The knights always tell the truth and the knaves always lie. You fall in love with a girl there and wish to marry her. However, this girl has strange tastes; for some odd reason she does not wish to marry a knight; she wants to marry only a knave. But she wants a rich knave, not a poor one. (We assume for convenience that everyone is classified as either rich or poor.) Suppose, in fact, that you are a rich knave. You are allowed to make only one statement, can you convince her that you are a rich knave?

$\text{a\_truth}(\text{knight}(x), y) \Rightarrow \neg \text{a\_truth}(\text{knave}(x), y)$      $\text{cnf}(\text{not\_knight\_and\_knave}, \text{axiom})$   
 $\text{a\_truth}(\text{knight}(x), y) \text{ or } \text{a\_truth}(\text{knave}(x), y)$      $\text{cnf}(\text{knight\_or\_knave}, \text{axiom})$   
 $\text{a\_truth}(\text{rich}(x), y) \Rightarrow \neg \text{a\_truth}(\text{poor}(x), y)$      $\text{cnf}(\text{not\_rich\_and\_poor}, \text{axiom})$   
 $\text{a\_truth}(\text{rich}(x), y) \text{ or } \text{a\_truth}(\text{poor}(x), y)$      $\text{cnf}(\text{rich\_or\_poor}, \text{axiom})$   
 $(\text{a\_truth}(\text{knight}(x), z) \text{ and } \text{says}(x, y)) \Rightarrow \text{a\_truth}(y, z)$      $\text{cnf}(\text{knight\_tell\_truth}_1, \text{axiom})$   
 $(\text{a\_truth}(\text{knight}(x), z) \text{ and } \text{a\_truth}(y, z)) \Rightarrow \text{says}(x, y)$      $\text{cnf}(\text{knight\_tell\_truth}_2, \text{axiom})$   
 $(\text{a\_truth}(\text{knave}(x), z) \text{ and } \text{says}(x, y)) \Rightarrow \neg \text{a\_truth}(y, z)$      $\text{cnf}(\text{knaves\_lie}_1, \text{axiom})$   
 $\text{a\_truth}(\text{knave}(x), z) \Rightarrow (\text{says}(x, y) \text{ or } \text{a\_truth}(y, z))$      $\text{cnf}(\text{knaves\_lie}_2, \text{axiom})$   
 $\text{a\_truth}(\text{and}(x, y), z) \Rightarrow \text{a\_truth}(x, z)$      $\text{cnf}(\text{conjunction}_1, \text{axiom})$   
 $\text{a\_truth}(\text{and}(x, y), z) \Rightarrow \text{a\_truth}(y, z)$      $\text{cnf}(\text{conjunction}_2, \text{axiom})$   
 $(\text{a\_truth}(x, z) \text{ and } \text{a\_truth}(y, z)) \Rightarrow \text{a\_truth}(\text{and}(x, y), z)$      $\text{cnf}(\text{conjunction}_3, \text{axiom})$   
 $\text{says}(\text{me}, x) \Rightarrow \neg \text{a\_truth}(\text{and}(\text{knave}(\text{me}), \text{rich}(\text{me})), x)$      $\text{cnf}(\text{prove\_statement\_exists}_1, \text{negated\_conjecture})$   
 $\text{says}(\text{me}, x) \text{ or } \text{a\_truth}(\text{and}(\text{knave}(\text{me}), \text{rich}(\text{me})), x)$      $\text{cnf}(\text{prove\_statement\_exists}_2, \text{negated\_conjecture})$

### PUZ022-1.p An ocean that borders on two adjacent Australian states

There is a database of assertions about Australian states and oceans and their relationships. Find which ocean borders on two adjacent Australian states.

$\text{borders}(y, x) \Rightarrow \text{borders}(x, y)$      $\text{cnf}(\text{symmetry\_of\_borders}, \text{axiom})$   
 $\text{ocean}(\text{atlantic})$      $\text{cnf}(\text{atlantic}, \text{negated\_conjecture})$   
 $\text{ocean}(\text{indian})$      $\text{cnf}(\text{indian}, \text{negated\_conjecture})$   
 $\text{ocean}(\text{pacific})$      $\text{cnf}(\text{pacific}, \text{negated\_conjecture})$   
 $\text{ocean}(\text{southern})$      $\text{cnf}(\text{southern}, \text{negated\_conjecture})$   
 $\text{state}(\text{western\_australia})$      $\text{cnf}(\text{western\_australia}, \text{negated\_conjecture})$   
 $\text{state}(\text{northern\_territory})$      $\text{cnf}(\text{northern\_territory}, \text{negated\_conjecture})$   
 $\text{state}(\text{queensland})$      $\text{cnf}(\text{queensland}, \text{negated\_conjecture})$   
 $\text{state}(\text{south\_australia})$      $\text{cnf}(\text{south\_australia}, \text{negated\_conjecture})$   
 $\text{state}(\text{new\_south\_wales})$      $\text{cnf}(\text{new\_south\_wales}, \text{negated\_conjecture})$   
 $\text{state}(\text{victoria})$      $\text{cnf}(\text{victoria}, \text{negated\_conjecture})$   
 $\text{state}(\text{tasmania})$      $\text{cnf}(\text{tasmania}, \text{negated\_conjecture})$   
 $\text{borders}(\text{western\_australia}, \text{northern\_territory})$      $\text{cnf}(\text{wa\_nt}, \text{negated\_conjecture})$   
 $\text{borders}(\text{western\_australia}, \text{south\_australia})$      $\text{cnf}(\text{wa\_sa}, \text{negated\_conjecture})$   
 $\text{borders}(\text{south\_australia}, \text{northern\_territory})$      $\text{cnf}(\text{sa\_nt}, \text{negated\_conjecture})$   
 $\text{borders}(\text{south\_australia}, \text{queensland})$      $\text{cnf}(\text{sa\_qld}, \text{negated\_conjecture})$   
 $\text{borders}(\text{south\_australia}, \text{new\_south\_wales})$      $\text{cnf}(\text{sa\_nsw}, \text{negated\_conjecture})$   
 $\text{borders}(\text{south\_australia}, \text{victoria})$      $\text{cnf}(\text{sa\_vic}, \text{negated\_conjecture})$   
 $\text{borders}(\text{northern\_territory}, \text{queensland})$      $\text{cnf}(\text{nt\_qld}, \text{negated\_conjecture})$



```

borders(queensland, new_south_wales)    cnf(qld_nsw, negated_conjecture)
borders(new_south_wales, victoria)       cnf(nsw_vic, negated_conjecture)
borders(indian, western_australia)       cnf(indian_wa, negated_conjecture)
borders(indian, northern_territory)      cnf(indian_nt, negated_conjecture)
borders(indian, queensland)             cnf(indian_qld, negated_conjecture)
borders(southern, western_australia)     cnf(southern_wa, negated_conjecture)
borders(southern, south_australia)      cnf(southern_sa, negated_conjecture)
borders(southern, victoria)             cnf(southern_vic, negated_conjecture)
borders(southern, tasmania)            cnf(southern_tas, negated_conjecture)
borders(pacific, queensland)           cnf(pacific_qld, negated_conjecture)
borders(pacific, new_south_wales)      cnf(pacific_nsw, negated_conjecture)
borders(pacific, victoria)             cnf(pacific_vic, negated_conjecture)
borders(pacific, tasmania)            cnf(pacific_tas, negated_conjecture)
(state(state1) and state(state2) and borders(state1, state2) and borders(state1, ocean) and borders(state2, ocean)) ⇒
¬ocean(ocean)    cnf(prove_borders, negated_conjecture)

```

### PUZ023-1.p Knights and Knaves #27

There is an island with exactly two types of people : truth-tellers who always tell the truth and liars who always lie. There are a group of three people, A, B, and C on the island. A stranger passes by and asks A, "How many truth-tellers are among you ?" A answers indistinctly. So the stranger asks B, "what did A say?". B replies "A said that there is exactly one truth-teller among us." Then C says, "Don't believe B; he is lying!" What are B and C. Answer: B is a liar and C is a truth-teller.

```

include('Axioms/PUZ002-0.ax')
(people(x, y, z) and a_truth(one_truthteller)) ⇒ (a_truth(truthteller(x)) or a_truth(truthteller(y)) or a_truth(truthteller(z)))
(people(x, y, z) and a_truth(truthteller(x)) and a_truth(truthteller(y))) ⇒ ¬a_truth(one_truthteller)    cnf(two_truthtellers)
(people(x, y, z) and a_truth(truthteller(x)) and a_truth(truthteller(z))) ⇒ ¬a_truth(one_truthteller)    cnf(two_truthtellers)
(people(x, y, z) and a_truth(truthteller(y)) and a_truth(truthteller(z))) ⇒ ¬a_truth(one_truthteller)    cnf(two_truthtellers)
(people(x, y, z) and a_truth(truthteller(x))) ⇒ (a_truth(one_truthteller) or a_truth(truthteller(y)) or a_truth(truthteller(z)))
(people(x, y, z) and a_truth(truthteller(y))) ⇒ (a_truth(one_truthteller) or a_truth(truthteller(x)) or a_truth(truthteller(z)))
(people(x, y, z) and a_truth(truthteller(z))) ⇒ (a_truth(one_truthteller) or a_truth(truthteller(y)) or a_truth(truthteller(x)))
people(a, b, c)    cnf(a_b_and_c_are_people, hypothesis)
a_truth(says(a, garbage))    cnf(a_says_garbage, hypothesis)
a_truth(says(b, says(a, one_truthteller)))    cnf(b_says_a_says_one_truthteller, hypothesis)
a_truth(says(c, liar(b)))    cnf(c_says_b_lies, hypothesis)
(a_truth(liar(b)) and a_truth(liar(c))) ⇒ an_answer(b_and_c_liars)    cnf(b_and_c_liars, hypothesis)
(a_truth(liar(b)) and a_truth(truthteller(c))) ⇒ an_answer(b_liar_and_c_truthteller)    cnf(b_liar_and_c_truthteller, hypothesis)
(a_truth(truthteller(b)) and a_truth(liar(c))) ⇒ an_answer(b_truthteller_and_c_liar)    cnf(b_truthteller_and_c_liar, hypothesis)
(a_truth(truthteller(b)) and a_truth(truthteller(c))) ⇒ an_answer(b_and_c_truthtellers)    cnf(b_and_c_truthtellers, hypothesis)
¬an_answer(x)    cnf(prove_there_is_an_answer, negated_conjecture)

```

### PUZ024-1.p Knights and Knaves #31

There is an island with exactly two types of people - truth-tellers who always tell the truth and liars who always lie. There are a group of three people, A, B, and C on the island. A and B make the following statements. A: All of us are liars; B: Exactly one of us is a truth-teller. What are A, B, and C? Answer: A is a liar, B is a truth-teller, and C is a liar.

```

include('Axioms/PUZ002-0.ax')
(a_truth(says(x, one_truthteller)) and people(x, y, z) and a_truth(truthteller(x)) and a_truth(truthteller(y))) ⇒ ¬a_truth(truthteller(z))
(a_truth(one_truthteller) and people(x, y, z)) ⇒ (a_truth(truthteller(x)) or a_truth(truthteller(y)) or a_truth(truthteller(z)))
(a_truth(one_truthteller) and people(x, y, z) and a_truth(truthteller(y)) and a_truth(truthteller(x))) ⇒ ¬a_truth(truthteller(z))
(a_truth(one_truthteller) and people(x, y, z) and a_truth(liar(x)) and a_truth(truthteller(y))) ⇒ a_truth(liar(z))    cnf(one_truthteller)
(people(x, y, z) and a_truth(liar(x)) and a_truth(truthteller(y))) ⇒ (a_truth(one_truthteller) or a_truth(truthteller(z)))    cnf(one_truthteller)
(people(x, y, z) and a_truth(liar(x)) and a_truth(liar(y))) ⇒ (a_truth(one_truthteller) or a_truth(liar(z)))    cnf(three_liars)
(a_truth(says(x, all_are_liars)) and people(x, y, z)) ⇒ ¬a_truth(truthteller(x))    cnf(speaker_is_lying, axiom)
people(x, y, z) ⇒ (a_truth(all_are_liars) or a_truth(truthteller(x)) or a_truth(truthteller(y)) or a_truth(truthteller(z)))    cnf(one_truthteller)
people(b, c, a)    cnf(b_c_a_people, hypothesis)
people(a, c, b)    cnf(a_c_b_people, hypothesis)
people(c, b, a)    cnf(c_b_a_people, hypothesis)
a_truth(says(a, all_are_liars))    cnf(a_says_all_are_liars, hypothesis)
a_truth(says(b, one_truthteller))    cnf(b_says_one_truthteller, hypothesis)
¬a_truth(truthteller(truthteller))    cnf(prove_there_is_a_truthteller, negated_conjecture)

```

**PUZ025-1.p** Knights and Knaves #35

There is an island with exactly two types of people - truth-tellers who always tell the truth and liars who always lie. There are a group of three people, A, B, and C on the island. A and B make the following statements. A: "B and C are the same type". Someone asks "Are A and B of the same type?" What does C answer? Answer: "yes"

include('Axioms/PUZ002-0.ax')

```
(people(x, y, z) and a_truth(liar(x)) and a_truth(liar(y))) => a_truth(equal_type(x, y))    cnf(two_liars_are_equal, axiom)
(people(x, y, z) and a_truth(truthteller(x)) and a_truth(truthteller(y))) => a_truth(equal_type(x, y))    cnf(two_truthtellers, axiom)
(a_truth(equal_type(x, y)) and a_truth(truthteller(x))) => a_truth(truthteller(y))    cnf(truthteller_equals_truthteller, axiom)
(a_truth(equal_type(x, y)) and a_truth(liar(x))) => a_truth(liar(y))    cnf(liar_equals_liar, axiom)
a_truth(truthteller(x)) => (a_truth(equal_type(x, y)) or a_truth(liar(y)))    cnf(truthteller_not_equal_liar, axiom)
a_truth(liar(x)) => (a_truth(equal_type(x, y)) or a_truth(truthteller(y)))    cnf(liar_not_equal_truthteller, axiom)
a_truth(equal_type(x, y)) => a_truth(equal_type(y, x))    cnf(symmetry_of_equal_type, axiom)
(ask_1_if2(x, y) and a_truth(truthteller(x)) and a_truth(y)) => answer(yes)    cnf(truthteller_identifies_truths, axiom)
(ask_1_if2(x, y) and a_truth(truthteller(x))) => (a_truth(y) or answer(no))    cnf(truthteller_denies_lies, axiom)
(ask_1_if2(x, y) and a_truth(liar(x)) and a_truth(y)) => answer(no)    cnf(liar_denies_truths, axiom)
(ask_1_if2(x, y) and a_truth(liar(x))) => (a_truth(y) or answer(yes))    cnf(liar_supports_lies, axiom)
people(b, c, a)    cnf(b_c_a_are_people, hypothesis)
people(a, b, a)    cnf(a_b_c_are_people, hypothesis)
people(a, c, b)    cnf(a_c_b_are_people, hypothesis)
people(c, b, a)    cnf(c_b_a_are_people, hypothesis)
a_truth(says(a, equal_type(b, c)))    cnf(a_says_b_and_c_equal, hypothesis)
ask_1_if2(c, equal_type(a, b))    cnf(ask_c_if_a_and_b_equal, hypothesis)
¬ answer(answer)    cnf(prove_there_is_an_answer, negated_conjecture)
```

**PUZ026-1.p** Knights and Knaves #39

There is an island with exactly three types of people - truth-tellers who always tell the truth, and liars who always lie, and normals who sometimes tell the truth and sometimes lie. We are given three people, A, B, C, one of whom is a truth-teller, one a liar, and one a normal (but not necessarily in that order). They make the following statements. A: I am normal; B: That is true. C: I am not normal. What are A, B, and C? Answer: A is a liar, B is a normal, and C is a truth-teller.

include('Axioms/PUZ003-0.ax')

```
a_truth(not_normal(x)) => ¬ a_truth(normal(x))    cnf(not_normal_and_not_normal, axiom)
a_truth(not_normal(x)) or a_truth(normal(x))    cnf(normal_or_not_normal, axiom)
(people(x, y, z) and a_truth(truthteller(x))) => ¬ a_truth(truthteller(y))    cnf(not_two_truthtellers1, axiom)
(people(x, y, z) and a_truth(truthteller(x))) => ¬ a_truth(truthteller(z))    cnf(not_two_truthtellers2, axiom)
(people(x, y, z) and a_truth(liar(x))) => ¬ a_truth(liar(y))    cnf(not_two_liars1, axiom)
(people(x, y, z) and a_truth(liar(x))) => ¬ a_truth(liar(z))    cnf(not_two_liars2, axiom)
(people(x, y, z) and a_truth(normal(x))) => ¬ a_truth(normal(y))    cnf(not_two_normal1, axiom)
(people(x, y, z) and a_truth(normal(x))) => ¬ a_truth(normal(z))    cnf(not_two_normal2, axiom)
people(a, b, c)    cnf(a_b_c_are_people, hypothesis)
people(b, c, a)    cnf(b_c_a_are_people, hypothesis)
people(c, b, a)    cnf(c_b_a_are_people, hypothesis)
a_truth(says(a, normal(a)))    cnf(a_says_a_normal, hypothesis)
a_truth(says(b, normal(a)))    cnf(b_says_a_normal, hypothesis)
a_truth(says(c, not_normal(c)))    cnf(c_says_c_not_normal, hypothesis)
(a_truth(liar(liar)) and a_truth(normal(normal))) => ¬ a_truth(truthteller(truthteller))    cnf(prove_one_of_each, negated_conjecture)
```

**PUZ027-1.p** Knights and Knaves #42

There is an island with exactly three types of people - truth-tellers who always tell the truth, and liars who always lie, and normals who sometimes tell the truth and sometimes lie. Liars are said to be of the lowest rank, normals are middle rank, and truth-tellers of the highest rank. Two people A and B on the island make the following statements. A: I am of lower rank than B. B: That's not true! What are the ranks of A and B, and which of the two statements are true? Answer: A is a normal and B is a normal.

include('Axioms/PUZ003-0.ax')

```
a_truth(not_lower(x, x))    cnf(not_lower_is_irreflexive, axiom)
a_truth(not_lower(x, y)) => ¬ a_truth(lower(x, y))    cnf(not_not_lower_and_lower, axiom)
a_truth(not_lower(x, y)) or a_truth(lower(x, y))    cnf(not_lower_or_lower, axiom)
(a_truth(lower(x, y)) and a_truth(liar(x))) => (a_truth(normal(y)) or a_truth(truthteller(y)))    cnf(liars_lowest, axiom)
(a_truth(lower(x, y)) and a_truth(normal(x))) => a_truth(truthteller(y))    cnf(truthtellers_highest, axiom)
a_truth(lower(x, y)) => ¬ a_truth(truthteller(x))    cnf(truthtellers_lower_than_no_one, axiom)
```

$(a\_truth(lower(x, y)) \text{ and } a\_truth(truthteller(y))) \Rightarrow (a\_truth(normal(x)) \text{ or } a\_truth(liar(x)))$      $cnf(normal\_and\_liars\_lower)$   
 $(a\_truth(lower(x, y)) \text{ and } a\_truth(normal(y))) \Rightarrow a\_truth(liar(x))$      $cnf(liars\_lower\_than\_normal, axiom)$   
 $a\_truth(lower(x, y)) \Rightarrow \neg a\_truth(liar(y))$      $cnf(no\_one\_lower\_than\_liars, axiom)$   
 $(a\_truth(not\_lower(x, y)) \text{ and } a\_truth(truthteller(x))) \Rightarrow (a\_truth(truthteller(y)) \text{ or } a\_truth(lower(y, x)))$      $cnf(not\_lower\_t)$   
 $(a\_truth(not\_lower(x, y)) \text{ and } a\_truth(liar(x))) \Rightarrow (a\_truth(liar(y)) \text{ or } a\_truth(lower(y, x)))$      $cnf(not\_lower\_than\_liar, axiom)$   
 $(a\_truth(not\_lower(x, y)) \text{ and } a\_truth(normal(x))) \Rightarrow (a\_truth(normal(y)) \text{ or } a\_truth(lower(y, x)))$      $cnf(not\_lower\_than\_no)$   
 $a\_truth(says(a, lower(a, b)))$      $cnf(a\_says\_a\_lower\_than\_b, hypothesis)$   
 $a\_truth(says(b, not\_lower(a, b)))$      $cnf(b\_says\_a\_not\_lower\_than\_b, hypothesis)$   
 $(a\_truth(truthteller(a)) \text{ and } a\_truth(truthteller(b))) \Rightarrow answer(a\_and\_b\_truthteller)$      $cnf(a\_and\_b\_truthteller, hypothesis)$   
 $(a\_truth(truthteller(a)) \text{ and } a\_truth(normal(b))) \Rightarrow answer(a\_truthteller\_b\_normal)$      $cnf(a\_truthteller\_b\_normal, hypothesis)$   
 $(a\_truth(truthteller(a)) \text{ and } a\_truth(liar(b))) \Rightarrow answer(a\_truthteller\_b\_liar)$      $cnf(a\_truthteller\_b\_liar, hypothesis)$   
 $(a\_truth(normal(a)) \text{ and } a\_truth(truthteller(b))) \Rightarrow answer(a\_normal\_b\_truthteller)$      $cnf(a\_normal\_b\_truthteller, hypothesis)$   
 $(a\_truth(normal(a)) \text{ and } a\_truth(normal(b))) \Rightarrow answer(a\_and\_b\_normal)$      $cnf(a\_and\_b\_normal, hypothesis)$   
 $(a\_truth(normal(a)) \text{ and } a\_truth(liar(b))) \Rightarrow answer(a\_normal\_b\_liar)$      $cnf(a\_normal\_b\_liar, hypothesis)$   
 $(a\_truth(liar(a)) \text{ and } a\_truth(truthteller(b))) \Rightarrow answer(a\_liar\_b\_truthteller)$      $cnf(a\_liar\_b\_truthteller, hypothesis)$   
 $(a\_truth(liar(a)) \text{ and } a\_truth(normal(b))) \Rightarrow answer(a\_liar\_b\_normal)$      $cnf(a\_liar\_b\_normal, hypothesis)$   
 $(a\_truth(liar(a)) \text{ and } a\_truth(liar(b))) \Rightarrow answer(a\_and\_b\_liar)$      $cnf(a\_and\_b\_liar, hypothesis)$   
 $\neg answer(answer)$      $cnf(prove\_there\_is\_an\_answer, negated\_conjecture)$

### PUZ028-1.p People at a party

We can always choose 3 persons who are either familiar with each other or not familiar with each other, from 6 persons who meet at a party.

$person(1)$      $cnf(person_1, axiom)$   
 $person(two)$      $cnf(person_2, axiom)$   
 $person(three)$      $cnf(person_3, axiom)$   
 $person(four)$      $cnf(person_4, axiom)$   
 $person(five)$      $cnf(person_5, axiom)$   
 $person(six)$      $cnf(person_6, axiom)$   
 $after(1, two)$      $cnf(order_1, axiom)$   
 $after(two, three)$      $cnf(order_2, axiom)$   
 $after(three, four)$      $cnf(order_3, axiom)$   
 $after(four, five)$      $cnf(order_4, axiom)$   
 $after(five, six)$      $cnf(order_5, axiom)$   
 $(after(large, medium) \text{ and } after(medium, small)) \Rightarrow after(large, small)$      $cnf(transitivity\_of\_order, axiom)$   
 $(person(x) \text{ and } person(y) \text{ and } after(x, y)) \Rightarrow (familiar(x, y) \text{ or } not\_familiar(x, y))$      $cnf(familiar\_or\_not, axiom)$   
 $(familiar(x_1, x_2) \text{ and } familiar(x_2, x_3)) \Rightarrow \neg familiar(x_3, x_1)$      $cnf(three\_familiar, negated\_conjecture)$   
 $(not\_familiar(x_1, x_2) \text{ and } not\_familiar(x_2, x_3)) \Rightarrow \neg not\_familiar(x_3, x_1)$      $cnf(three\_not\_familiar, negated\_conjecture)$

### PUZ028-2.p People at a party

We can always choose 3 persons who are either familiar with each other or not familiar with each other, from 6 persons who meet at a party.

$person(n_1)$      $cnf(person_1, axiom)$   
 $person(n_2)$      $cnf(person_2, axiom)$   
 $person(n_3)$      $cnf(person_3, axiom)$   
 $person(n_4)$      $cnf(person_4, axiom)$   
 $person(n_5)$      $cnf(person_5, axiom)$   
 $person(n_6)$      $cnf(person_6, axiom)$   
 $not\_equal(n_1, n_2)$      $cnf(not\_equal\_1_2, axiom)$   
 $not\_equal(n_1, n_3)$      $cnf(not\_equal\_1_3, axiom)$   
 $not\_equal(n_1, n_4)$      $cnf(not\_equal\_1_4, axiom)$   
 $not\_equal(n_1, n_5)$      $cnf(not\_equal\_1_5, axiom)$   
 $not\_equal(n_1, n_6)$      $cnf(not\_equal\_1_6, axiom)$   
 $not\_equal(n_2, n_1)$      $cnf(not\_equal\_2_1, axiom)$   
 $not\_equal(n_2, n_3)$      $cnf(not\_equal\_2_3, axiom)$   
 $not\_equal(n_2, n_4)$      $cnf(not\_equal\_2_4, axiom)$   
 $not\_equal(n_2, n_5)$      $cnf(not\_equal\_2_5, axiom)$   
 $not\_equal(n_2, n_6)$      $cnf(not\_equal\_2_6, axiom)$   
 $not\_equal(n_3, n_1)$      $cnf(not\_equal\_3_1, axiom)$   
 $not\_equal(n_3, n_2)$      $cnf(not\_equal\_3_2, axiom)$   
 $not\_equal(n_3, n_4)$      $cnf(not\_equal\_3_4, axiom)$

$\text{not\_equal}(n_3, n_5)$        $\text{cnf}(\text{not\_equal.3}_5, \text{axiom})$   
 $\text{not\_equal}(n_3, n_6)$        $\text{cnf}(\text{not\_equal.3}_6, \text{axiom})$   
 $\text{not\_equal}(n_4, n_1)$        $\text{cnf}(\text{not\_equal.4}_1, \text{axiom})$   
 $\text{not\_equal}(n_4, n_2)$        $\text{cnf}(\text{not\_equal.4}_2, \text{axiom})$   
 $\text{not\_equal}(n_4, n_3)$        $\text{cnf}(\text{not\_equal.4}_3, \text{axiom})$   
 $\text{not\_equal}(n_4, n_5)$        $\text{cnf}(\text{not\_equal.4}_5, \text{axiom})$   
 $\text{not\_equal}(n_4, n_6)$        $\text{cnf}(\text{not\_equal.4}_6, \text{axiom})$   
 $\text{not\_equal}(n_5, n_1)$        $\text{cnf}(\text{not\_equal.5}_1, \text{axiom})$   
 $\text{not\_equal}(n_5, n_2)$        $\text{cnf}(\text{not\_equal.5}_2, \text{axiom})$   
 $\text{not\_equal}(n_5, n_3)$        $\text{cnf}(\text{not\_equal.5}_3, \text{axiom})$   
 $\text{not\_equal}(n_5, n_4)$        $\text{cnf}(\text{not\_equal.5}_4, \text{axiom})$   
 $\text{not\_equal}(n_5, n_6)$        $\text{cnf}(\text{not\_equal.5}_6, \text{axiom})$   
 $\text{not\_equal}(n_6, n_1)$        $\text{cnf}(\text{not\_equal.6}_1, \text{axiom})$   
 $\text{not\_equal}(n_6, n_2)$        $\text{cnf}(\text{not\_equal.6}_2, \text{axiom})$   
 $\text{not\_equal}(n_6, n_3)$        $\text{cnf}(\text{not\_equal.6}_3, \text{axiom})$   
 $\text{not\_equal}(n_6, n_4)$        $\text{cnf}(\text{not\_equal.6}_4, \text{axiom})$   
 $\text{not\_equal}(n_6, n_5)$        $\text{cnf}(\text{not\_equal.6}_5, \text{axiom})$   
 $(\text{person}(x) \text{ and } \text{person}(y) \text{ and } \text{not\_equal}(x, y)) \Rightarrow (\text{familiar}(x, y) \text{ or } \text{not\_familiar}(x, y))$        $\text{cnf}(\text{familiar\_or\_not}, \text{axiom})$   
 $(\text{familiar}(x_1, x_2) \text{ and } \text{familiar}(x_2, x_3)) \Rightarrow \neg \text{familiar}(x_3, x_1)$        $\text{cnf}(\text{three\_familiar}, \text{negated\_conjecture})$   
 $(\text{not\_familiar}(x_1, x_2) \text{ and } \text{not\_familiar}(x_2, x_3)) \Rightarrow \neg \text{not\_familiar}(x_3, x_1)$        $\text{cnf}(\text{three\_not\_familiar}, \text{negated\_conjecture})$

### PUZ028-5.p People at a party

We can always choose 3 persons who are either familiar with each other or not familiar with each other, from 6 persons who meet at a party.

$\text{person}(\text{one})$        $\text{cnf}(\text{person}_1, \text{axiom})$   
 $\text{person}(\text{two})$        $\text{cnf}(\text{person}_2, \text{axiom})$   
 $\text{person}(\text{three})$        $\text{cnf}(\text{person}_3, \text{axiom})$   
 $\text{person}(\text{four})$        $\text{cnf}(\text{person}_4, \text{axiom})$   
 $\text{person}(\text{five})$        $\text{cnf}(\text{person}_5, \text{axiom})$   
 $\text{person}(\text{six})$        $\text{cnf}(\text{person}_6, \text{axiom})$   
 $\text{after}(\text{one}, \text{two})$        $\text{cnf}(\text{order}_1, \text{axiom})$   
 $\text{after}(\text{two}, \text{three})$        $\text{cnf}(\text{order}_2, \text{axiom})$   
 $\text{after}(\text{three}, \text{four})$        $\text{cnf}(\text{order}_3, \text{axiom})$   
 $\text{after}(\text{four}, \text{five})$        $\text{cnf}(\text{order}_4, \text{axiom})$   
 $\text{after}(\text{five}, \text{six})$        $\text{cnf}(\text{order}_5, \text{axiom})$   
 $(\text{after}(\text{large}, \text{medium}) \text{ and } \text{after}(\text{medium}, \text{small})) \Rightarrow \text{after}(\text{large}, \text{small})$        $\text{cnf}(\text{transitivity\_of\_order}, \text{axiom})$   
 $(\text{person}(x) \text{ and } \text{person}(y) \text{ and } \text{after}(x, y)) \Rightarrow (\text{familiar}(x, y) \text{ or } \text{not\_familiar}(x, y))$        $\text{cnf}(\text{familiar\_or\_not}, \text{axiom})$   
 $(\text{familiar}(x_1, x_2) \text{ and } \text{familiar}(x_2, x_3)) \Rightarrow \neg \text{familiar}(x_1, x_3)$        $\text{cnf}(\text{three\_familiar}, \text{negated\_conjecture})$   
 $(\text{not\_familiar}(x_1, x_2) \text{ and } \text{not\_familiar}(x_2, x_3)) \Rightarrow \neg \text{not\_familiar}(x_1, x_3)$        $\text{cnf}(\text{three\_not\_familiar}, \text{negated\_conjecture})$

### PUZ028-6.p People at a party

We can always choose 3 persons who are either familiar with each other or not familiar with each other, from 6 persons who meet at a party.

$\text{person}(n_1)$        $\text{cnf}(\text{person}_1, \text{axiom})$   
 $\text{person}(n_2)$        $\text{cnf}(\text{person}_2, \text{axiom})$   
 $\text{person}(n_3)$        $\text{cnf}(\text{person}_3, \text{axiom})$   
 $\text{person}(n_4)$        $\text{cnf}(\text{person}_4, \text{axiom})$   
 $\text{person}(n_5)$        $\text{cnf}(\text{person}_5, \text{axiom})$   
 $\text{person}(n_6)$        $\text{cnf}(\text{person}_6, \text{axiom})$   
 $\text{not\_equal}(n_1, n_2)$        $\text{cnf}(\text{not\_equal.1}_2, \text{axiom})$   
 $\text{not\_equal}(n_1, n_3)$        $\text{cnf}(\text{not\_equal.1}_3, \text{axiom})$   
 $\text{not\_equal}(n_1, n_4)$        $\text{cnf}(\text{not\_equal.1}_4, \text{axiom})$   
 $\text{not\_equal}(n_1, n_5)$        $\text{cnf}(\text{not\_equal.1}_5, \text{axiom})$   
 $\text{not\_equal}(n_1, n_6)$        $\text{cnf}(\text{not\_equal.1}_6, \text{axiom})$   
 $\text{not\_equal}(n_2, n_1)$        $\text{cnf}(\text{not\_equal.2}_1, \text{axiom})$   
 $\text{not\_equal}(n_2, n_3)$        $\text{cnf}(\text{not\_equal.2}_3, \text{axiom})$   
 $\text{not\_equal}(n_2, n_4)$        $\text{cnf}(\text{not\_equal.2}_4, \text{axiom})$   
 $\text{not\_equal}(n_2, n_5)$        $\text{cnf}(\text{not\_equal.2}_5, \text{axiom})$   
 $\text{not\_equal}(n_2, n_6)$        $\text{cnf}(\text{not\_equal.2}_6, \text{axiom})$   
 $\text{not\_equal}(n_3, n_1)$        $\text{cnf}(\text{not\_equal.3}_1, \text{axiom})$

$\text{not\_equal}(n_3, n_2) \quad \text{cnf}(\text{not\_equal\_3}_2, \text{axiom})$   
 $\text{not\_equal}(n_3, n_4) \quad \text{cnf}(\text{not\_equal\_3}_4, \text{axiom})$   
 $\text{not\_equal}(n_3, n_5) \quad \text{cnf}(\text{not\_equal\_3}_5, \text{axiom})$   
 $\text{not\_equal}(n_3, n_6) \quad \text{cnf}(\text{not\_equal\_3}_6, \text{axiom})$   
 $\text{not\_equal}(n_4, n_1) \quad \text{cnf}(\text{not\_equal\_4}_1, \text{axiom})$   
 $\text{not\_equal}(n_4, n_2) \quad \text{cnf}(\text{not\_equal\_4}_2, \text{axiom})$   
 $\text{not\_equal}(n_4, n_3) \quad \text{cnf}(\text{not\_equal\_4}_3, \text{axiom})$   
 $\text{not\_equal}(n_4, n_5) \quad \text{cnf}(\text{not\_equal\_4}_5, \text{axiom})$   
 $\text{not\_equal}(n_4, n_6) \quad \text{cnf}(\text{not\_equal\_4}_6, \text{axiom})$   
 $\text{not\_equal}(n_5, n_1) \quad \text{cnf}(\text{not\_equal\_5}_1, \text{axiom})$   
 $\text{not\_equal}(n_5, n_2) \quad \text{cnf}(\text{not\_equal\_5}_2, \text{axiom})$   
 $\text{not\_equal}(n_5, n_3) \quad \text{cnf}(\text{not\_equal\_5}_3, \text{axiom})$   
 $\text{not\_equal}(n_5, n_4) \quad \text{cnf}(\text{not\_equal\_5}_4, \text{axiom})$   
 $\text{not\_equal}(n_5, n_6) \quad \text{cnf}(\text{not\_equal\_5}_6, \text{axiom})$   
 $\text{not\_equal}(n_6, n_1) \quad \text{cnf}(\text{not\_equal\_6}_1, \text{axiom})$   
 $\text{not\_equal}(n_6, n_2) \quad \text{cnf}(\text{not\_equal\_6}_2, \text{axiom})$   
 $\text{not\_equal}(n_6, n_3) \quad \text{cnf}(\text{not\_equal\_6}_3, \text{axiom})$   
 $\text{not\_equal}(n_6, n_4) \quad \text{cnf}(\text{not\_equal\_6}_4, \text{axiom})$   
 $\text{not\_equal}(n_6, n_5) \quad \text{cnf}(\text{not\_equal\_6}_5, \text{axiom})$   
 $(\text{person}(x) \text{ and } \text{person}(y) \text{ and } \text{not\_equal}(x, y)) \Rightarrow (\text{familiar}(x, y) \text{ or } \text{not\_familiar}(x, y)) \quad \text{cnf}(\text{familiar\_or\_not}, \text{axiom})$   
 $\text{familiar}(x_1, x_2) \Rightarrow \text{familiar}(x_2, x_1) \quad \text{cnf}(\text{symmetry\_of\_familiar}, \text{axiom})$   
 $\text{not\_familiar}(x_1, x_2) \Rightarrow \text{not\_familiar}(x_2, x_1) \quad \text{cnf}(\text{symmetry\_of\_not\_familiar}, \text{axiom})$   
 $(\text{familiar}(x_1, x_2) \text{ and } \text{familiar}(x_2, x_3)) \Rightarrow \neg \text{familiar}(x_3, x_1) \quad \text{cnf}(\text{three\_familiar}, \text{negated\_conjecture})$   
 $(\text{not\_familiar}(x_1, x_2) \text{ and } \text{not\_familiar}(x_2, x_3)) \Rightarrow \neg \text{not\_familiar}(x_3, x_1) \quad \text{cnf}(\text{three\_not\_familiar}, \text{negated\_conjecture})$

#### PUZ029-1.p The pigs and balloons puzzle

1) All, who neither dance on tight ropes nor eat penny-buns, are old. 2) Pigs, that are liable to giddiness, are treated with respect. 3) A wise balloonist takes an umbrella with him. 4) No one ought to lunch in public who looks ridiculous and eats penny-buns. 5) Young creatures, who go up in balloons, are liable to giddiness. 6) Fat creatures, who look ridiculous, may lunch in public, provided that they do not dance on tight ropes. 7) No wise creatures dance on tight ropes, if liable to giddiness. 8) A pig looks ridiculous, carrying an umbrella. 9) All, who do not dance on tight ropes, and who are treated with respect are fat. Show that no wise young pigs go up in balloons.

$\text{dances\_on\_tightropes}(x) \text{ or } \text{eats\_pennybuns}(x) \text{ or } \text{old}(x) \quad \text{cnf}(\text{boring\_old\_people}, \text{axiom})$   
 $(\text{pig}(x) \text{ and } \text{liable\_to\_giddiness}(x)) \Rightarrow \text{treated\_with\_respect}(x) \quad \text{cnf}(\text{giddy\_pigs\_reated\_with\_respect}, \text{axiom})$   
 $(\text{wise}(x) \text{ and } \text{balloonist}(x)) \Rightarrow \text{has\_umbrella}(x) \quad \text{cnf}(\text{wise\_ballonists\_have\_umbrellas}, \text{axiom})$   
 $(\text{looks\_ridiculous}(x) \text{ and } \text{eats\_pennybuns}(x)) \Rightarrow \neg \text{eats\_lunch\_in\_public}(x) \quad \text{cnf}(\text{dont\_look\_ridiculous\_eating\_buns\_in\_public}, \text{axiom})$   
 $(\text{balloonist}(x) \text{ and } \text{young}(x)) \Rightarrow \text{liable\_to\_giddiness}(x) \quad \text{cnf}(\text{young\_balloonists\_get\_giddy}, \text{axiom})$   
 $(\text{fat}(x) \text{ and } \text{looks\_ridiculous}(x)) \Rightarrow (\text{dances\_on\_tightropes}(x) \text{ or } \text{eats\_lunch\_in\_public}(x)) \quad \text{cnf}(\text{fat\_ridiculous\_off\_tightrope}, \text{axiom})$   
 $(\text{liable\_to\_giddiness}(x) \text{ and } \text{wise}(x)) \Rightarrow \neg \text{dances\_on\_tightropes}(x) \quad \text{cnf}(\text{wise\_giddy\_dont\_dance\_on\_tightrope}, \text{axiom})$   
 $(\text{pig}(x) \text{ and } \text{has\_umbrella}(x)) \Rightarrow \text{looks\_ridiculous}(x) \quad \text{cnf}(\text{pigs\_look\_ridiculous\_with\_umbrellas}, \text{axiom})$   
 $\text{treated\_with\_respect}(x) \Rightarrow (\text{dances\_on\_tightropes}(x) \text{ or } \text{fat}(x)) \quad \text{cnf}(\text{non\_dancers\_who\_are\_respected\_are\_fat}, \text{axiom})$   
 $\text{young}(x) \text{ or } \text{old}(x) \quad \text{cnf}(\text{young\_or\_old}, \text{axiom})$   
 $\text{young}(x) \Rightarrow \neg \text{old}(x) \quad \text{cnf}(\text{not\_young\_and\_old}, \text{axiom})$   
 $\text{wise}(\text{piggy}) \quad \text{cnf}(\text{piggy\_is\_wise}, \text{hypothesis})$   
 $\text{young}(\text{piggy}) \quad \text{cnf}(\text{piggy\_is\_young}, \text{hypothesis})$   
 $\text{pig}(\text{piggy}) \quad \text{cnf}(\text{piggy\_is\_a\_pig}, \text{hypothesis})$   
 $\text{balloonist}(\text{piggy}) \quad \text{cnf}(\text{prove\_piggy\_is\_no\_balloonist}, \text{negated\_conjecture})$

#### PUZ030-1.p Salt and Mustard Problem

$\text{both}(x) \Rightarrow \text{salt}(x) \quad \text{cnf}(\text{both}_1, \text{axiom})$   
 $\text{both}(x) \Rightarrow \text{mustard}(x) \quad \text{cnf}(\text{both}_2, \text{axiom})$   
 $(\text{salt}(x) \text{ and } \text{mustard}(x)) \Rightarrow \text{both}(x) \quad \text{cnf}(\text{both}_3, \text{axiom})$   
 $\text{oneof}(x) \Rightarrow (\text{salt}(x) \text{ or } \text{mustard}(x)) \quad \text{cnf}(\text{oneof}_1, \text{axiom})$   
 $\text{oneof}(x) \Rightarrow \neg \text{both}(x) \quad \text{cnf}(\text{oneof}_2, \text{axiom})$   
 $\text{oneof}(x) \Rightarrow \neg \text{neither}(x) \quad \text{cnf}(\text{oneof}_3, \text{axiom})$   
 $\text{both}(x) \text{ or } \text{neither}(x) \text{ or } \text{oneof}(x) \quad \text{cnf}(\text{one\_condition\_holds}_1, \text{axiom})$   
 $(\text{oneof}(x) \text{ and } \text{salt}(x)) \Rightarrow \neg \text{mustard}(x) \quad \text{cnf}(\text{oneof}_4, \text{axiom})$   
 $\text{both}(x) \Rightarrow \neg \text{neither}(x) \quad \text{cnf}(\text{neither}_1, \text{axiom})$   
 $\text{neither}(x) \Rightarrow \neg \text{salt}(x) \quad \text{cnf}(\text{neither}_2, \text{axiom})$   
 $\text{neither}(x) \Rightarrow \neg \text{mustard}(x) \quad \text{cnf}(\text{neither}_3, \text{axiom})$

$\text{salt}(x) \text{ or } \text{mustard}(x) \text{ or } \text{neither}(x) \quad \text{cnf}(\text{neither}_4, \text{axiom})$   
 $\text{salt}(\text{barry}) \Rightarrow (\text{oneof}(\text{cole}) \text{ or } \text{oneof}(\text{lang})) \quad \text{cnf}(\text{rule1}_1, \text{hypothesis})$   
 $\text{oneof}(\text{cole}) \Rightarrow \text{salt}(\text{barry}) \quad \text{cnf}(\text{rule1}_2, \text{hypothesis})$   
 $\text{oneof}(\text{lang}) \Rightarrow \text{salt}(\text{barry}) \quad \text{cnf}(\text{rule1}_3, \text{hypothesis})$   
 $\text{mustard}(\text{barry}) \Rightarrow (\text{neither}(\text{dix}) \text{ or } \text{both}(\text{mill})) \quad \text{cnf}(\text{rule2}_1, \text{hypothesis})$   
 $\text{neither}(\text{dix}) \Rightarrow \text{mustard}(\text{barry}) \quad \text{cnf}(\text{rule2}_2, \text{hypothesis})$   
 $\text{both}(\text{mill}) \Rightarrow \text{mustard}(\text{barry}) \quad \text{cnf}(\text{rule2}_3, \text{hypothesis})$   
 $\text{salt}(\text{cole}) \Rightarrow (\text{oneof}(\text{barry}) \text{ or } \text{neither}(\text{mill})) \quad \text{cnf}(\text{rule3}_1, \text{hypothesis})$   
 $\text{oneof}(\text{barry}) \Rightarrow \text{salt}(\text{cole}) \quad \text{cnf}(\text{rule3}_2, \text{hypothesis})$   
 $\text{neither}(\text{mill}) \Rightarrow \text{salt}(\text{cole}) \quad \text{cnf}(\text{rule3}_3, \text{hypothesis})$   
 $\text{mustard}(\text{cole}) \Rightarrow (\text{both}(\text{dix}) \text{ or } \text{both}(\text{lang})) \quad \text{cnf}(\text{rule4}_1, \text{hypothesis})$   
 $\text{both}(\text{dix}) \Rightarrow \text{mustard}(\text{cole}) \quad \text{cnf}(\text{rule4}_2, \text{hypothesis})$   
 $\text{both}(\text{lang}) \Rightarrow \text{mustard}(\text{cole}) \quad \text{cnf}(\text{rule4}_3, \text{hypothesis})$   
 $\text{salt}(\text{dix}) \Rightarrow (\text{neither}(\text{barry}) \text{ or } \text{both}(\text{cole})) \quad \text{cnf}(\text{rule5}_1, \text{hypothesis})$   
 $\text{neither}(\text{barry}) \Rightarrow \text{salt}(\text{dix}) \quad \text{cnf}(\text{rule5}_2, \text{hypothesis})$   
 $\text{both}(\text{cole}) \Rightarrow \text{salt}(\text{dix}) \quad \text{cnf}(\text{rule5}_3, \text{hypothesis})$   
 $\text{mustard}(\text{dix}) \Rightarrow (\text{neither}(\text{lang}) \text{ or } \text{neither}(\text{mill})) \quad \text{cnf}(\text{rule6}_1, \text{hypothesis})$   
 $\text{neither}(\text{lang}) \Rightarrow \text{mustard}(\text{dix}) \quad \text{cnf}(\text{rule6}_2, \text{hypothesis})$   
 $\text{neither}(\text{mill}) \Rightarrow \text{mustard}(\text{dix}) \quad \text{cnf}(\text{rule6}_3, \text{hypothesis})$   
 $\text{salt}(\text{lang}) \Rightarrow (\text{oneof}(\text{barry}) \text{ or } \text{oneof}(\text{dix})) \quad \text{cnf}(\text{rule7}_1, \text{hypothesis})$   
 $\text{oneof}(\text{barry}) \Rightarrow \text{salt}(\text{lang}) \quad \text{cnf}(\text{rule7}_2, \text{hypothesis})$   
 $\text{oneof}(\text{dix}) \Rightarrow \text{salt}(\text{lang}) \quad \text{cnf}(\text{rule7}_3, \text{hypothesis})$   
 $\text{mustard}(\text{lang}) \Rightarrow (\text{neither}(\text{cole}) \text{ or } \text{neither}(\text{mill})) \quad \text{cnf}(\text{rule8}_1, \text{hypothesis})$   
 $\text{neither}(\text{cole}) \Rightarrow \text{mustard}(\text{lang}) \quad \text{cnf}(\text{rule8}_2, \text{hypothesis})$   
 $\text{neither}(\text{mill}) \Rightarrow \text{mustard}(\text{lang}) \quad \text{cnf}(\text{rule8}_3, \text{hypothesis})$   
 $\text{salt}(\text{mill}) \Rightarrow (\text{both}(\text{barry}) \text{ or } \text{both}(\text{lang})) \quad \text{cnf}(\text{rule9}_1, \text{hypothesis})$   
 $\text{both}(\text{barry}) \Rightarrow \text{salt}(\text{mill}) \quad \text{cnf}(\text{rule9}_2, \text{hypothesis})$   
 $\text{both}(\text{lang}) \Rightarrow \text{salt}(\text{mill}) \quad \text{cnf}(\text{rule9}_3, \text{hypothesis})$   
 $\text{mustard}(\text{mill}) \Rightarrow (\text{oneof}(\text{cole}) \text{ or } \text{oneof}(\text{dix})) \quad \text{cnf}(\text{rule10}_1, \text{hypothesis})$   
 $\text{oneof}(\text{cole}) \Rightarrow \text{mustard}(\text{mill}) \quad \text{cnf}(\text{rule10}_2, \text{hypothesis})$   
 $\text{oneof}(\text{dix}) \Rightarrow \text{mustard}(\text{mill}) \quad \text{cnf}(\text{rule10}_3, \text{hypothesis})$   
 $(\text{neither}(\text{cole}) \text{ and } \text{neither}(\text{dix}) \text{ and } \text{both}(\text{barry}) \text{ and } \text{oneof}(\text{lang}) \text{ and } \text{salt}(\text{mill}) \text{ and } \text{mustard}(\text{lang})) \Rightarrow \neg \text{oneof}(\text{mill}) \quad \text{cnf}(\text{rule11}_1, \text{hypothesis})$

### PUZ031+1.p Schubert's Steamroller

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Therefore there is an animal that likes to eat a grain eating animal.

$\forall x: (\text{wolf}(x) \Rightarrow \text{animal}(x)) \quad \text{fof}(\text{pel47}_1, \text{axiom})$   
 $\exists x_1: \text{wolf}(x_1) \quad \text{fof}(\text{pel47}_1, \text{axiom})$   
 $\forall x: (\text{fox}(x) \Rightarrow \text{animal}(x)) \quad \text{fof}(\text{pel47}_2, \text{axiom})$   
 $\exists x_1: \text{fox}(x_1) \quad \text{fof}(\text{pel47}_2, \text{axiom})$   
 $\forall x: (\text{bird}(x) \Rightarrow \text{animal}(x)) \quad \text{fof}(\text{pel47}_3, \text{axiom})$   
 $\exists x_1: \text{bird}(x_1) \quad \text{fof}(\text{pel47}_3, \text{axiom})$   
 $\forall x: (\text{caterpillar}(x) \Rightarrow \text{animal}(x)) \quad \text{fof}(\text{pel47}_4, \text{axiom})$   
 $\exists x_1: \text{caterpillar}(x_1) \quad \text{fof}(\text{pel47}_4, \text{axiom})$   
 $\forall x: (\text{snail}(x) \Rightarrow \text{animal}(x)) \quad \text{fof}(\text{pel47}_5, \text{axiom})$   
 $\exists x_1: \text{snail}(x_1) \quad \text{fof}(\text{pel47}_5, \text{axiom})$   
 $\exists x: \text{grain}(x) \quad \text{fof}(\text{pel47}_6, \text{axiom})$   
 $\forall x_1: (\text{grain}(x_1) \Rightarrow \text{plant}(x_1)) \quad \text{fof}(\text{pel47}_6, \text{axiom})$   
 $\forall x: (\text{animal}(x) \Rightarrow (\forall y: (\text{plant}(y) \Rightarrow \text{eats}(x, y)) \text{ or } \forall y_1: ((\text{animal}(y_1) \text{ and } \text{much\_smaller}(y_1, x) \text{ and } \exists z: (\text{plant}(z) \text{ and } \text{eats}(y_1, \text{eats}(x, y_1)))))) \quad \text{fof}(\text{pel47}_7, \text{axiom})$   
 $\forall x, y: ((\text{bird}(y) \text{ and } (\text{snail}(x) \text{ or } \text{caterpillar}(x))) \Rightarrow \text{much\_smaller}(x, y)) \quad \text{fof}(\text{pel47}_8, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{fox}(y)) \Rightarrow \text{much\_smaller}(x, y)) \quad \text{fof}(\text{pel47}_9, \text{axiom})$   
 $\forall x, y: ((\text{fox}(x) \text{ and } \text{wolf}(y)) \Rightarrow \text{much\_smaller}(x, y)) \quad \text{fof}(\text{pel47}_{10}, \text{axiom})$   
 $\forall x, y: ((\text{wolf}(x) \text{ and } (\text{fox}(y) \text{ or } \text{grain}(y))) \Rightarrow \neg \text{eats}(x, y)) \quad \text{fof}(\text{pel47}_{11}, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{caterpillar}(y)) \Rightarrow \text{eats}(x, y)) \quad \text{fof}(\text{pel47}_{12}, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{snail}(y)) \Rightarrow \neg \text{eats}(x, y)) \quad \text{fof}(\text{pel47}_{13}, \text{axiom})$

$\forall x: ((\text{caterpillar}(x) \text{ or } \text{snail}(x)) \Rightarrow \exists y: (\text{plant}(y) \text{ and } \text{eats}(x, y)))$     fof(pel47<sub>14</sub>, axiom)  
 $\exists x, y: (\text{animal}(x) \text{ and } \text{animal}(y) \text{ and } \exists z: (\text{grain}(z) \text{ and } \text{eats}(y, z) \text{ and } \text{eats}(x, y)))$     fof(pel<sub>47</sub>, conjecture)

### PUZ031+2.p Schubert's Steamroller

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Therefore there is an animal that likes to eat a grain eating animal.

$\exists a: \text{wolf}(a)$     fof(wolf\_type, axiom)  
 $\exists a: \text{fox}(a)$     fof(fox\_type, axiom)  
 $\exists a: \text{bird}(a)$     fof(bird\_type, axiom)  
 $\exists a: \text{caterpillar}(a)$     fof(caterpillar\_type, axiom)  
 $\exists a: \text{snail}(a)$     fof(snail\_type, axiom)  
 $\forall x: (\text{wolf}(x) \Rightarrow \text{animal}(x))$     fof(pel47\_1\_1, axiom)  
 $\forall x: (\text{fox}(x) \Rightarrow \text{animal}(x))$     fof(pel47\_2\_1, axiom)  
 $\forall x: (\text{bird}(x) \Rightarrow \text{animal}(x))$     fof(pel47\_3\_1, axiom)  
 $\forall x: (\text{caterpillar}(x) \Rightarrow \text{animal}(x))$     fof(pel47\_4\_1, axiom)  
 $\forall x: (\text{snail}(x) \Rightarrow \text{animal}(x))$     fof(pel47\_4\_2, axiom)  
 $\exists a: \text{grain}(a)$     fof(grain\_type, axiom)  
 $\forall x: (\text{grain}(x) \Rightarrow \text{plant}(x))$     fof(pel47\_6\_2, axiom)  
 $\forall x: (\text{animal}(x) \Rightarrow (\forall y: (\text{plant}(y) \Rightarrow \text{eats}(x, y)) \text{ or } \forall y_1: ((\text{animal}(y_1) \text{ and } \text{much\_smaller}(y_1, x) \text{ and } \exists z: (\text{plant}(z) \text{ and } \text{eats}(y_1, \text{eats}(x, y_1))))))$     fof(pel47\_7, axiom)  
 $\forall y, x: ((\text{bird}(y) \text{ and } \text{snail}(x)) \Rightarrow \text{much\_smaller}(x, y))$     fof(pel47\_8, axiom)  
 $\forall y, x: ((\text{bird}(y) \text{ and } \text{caterpillar}(x)) \Rightarrow \text{much\_smaller}(x, y))$     fof(pel47\_8a, axiom)  
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{fox}(y)) \Rightarrow \text{much\_smaller}(x, y))$     fof(pel47\_9, axiom)  
 $\forall x, y: ((\text{fox}(x) \text{ and } \text{wolf}(y)) \Rightarrow \text{much\_smaller}(x, y))$     fof(pel47<sub>10</sub>, axiom)  
 $\forall x, y: ((\text{wolf}(x) \text{ and } \text{fox}(y)) \Rightarrow \neg \text{eats}(x, y))$     fof(pel47<sub>11</sub>, axiom)  
 $\forall x, y: ((\text{wolf}(x) \text{ and } \text{grain}(y)) \Rightarrow \neg \text{eats}(x, y))$     fof(pel47\_11a, axiom)  
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{caterpillar}(y)) \Rightarrow \text{eats}(x, y))$     fof(pel47<sub>12</sub>, axiom)  
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{snail}(y)) \Rightarrow \neg \text{eats}(x, y))$     fof(pel47<sub>13</sub>, axiom)  
 $\forall x: (\text{caterpillar}(x) \Rightarrow \exists y: (\text{plant}(y) \text{ and } \text{eats}(x, y)))$     fof(pel47<sub>14</sub>, axiom)  
 $\forall x: (\text{snail}(x) \Rightarrow \exists y: (\text{plant}(y) \text{ and } \text{eats}(x, y)))$     fof(pel47\_14a, axiom)  
 $\exists x, y: (\text{animal}(x) \text{ and } \text{animal}(y) \text{ and } \exists z: (\text{grain}(z) \text{ and } \text{eats}(y, z) \text{ and } \text{eats}(x, y)))$     fof(pel<sub>47</sub>, conjecture)

### PUZ031+3.p Schubert's Steamroller

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Therefore there is an animal that likes to eat a grain eating animal.

$\exists a: \text{edible}(a)$     fof(edible\_type, axiom)  
 $\exists a: \text{animal}(a)$     fof(animal\_type, axiom)  
 $\forall a: (\text{animal}(a) \Rightarrow \text{edible}(a))$     fof(animal\_is\_edible, axiom)  
 $\exists a: \text{wolf}(a)$     fof(wolf\_type, axiom)  
 $\forall a: (\text{wolf}(a) \Rightarrow \text{animal}(a))$     fof(wolf\_is\_animal, axiom)  
 $\exists a: \text{fox}(a)$     fof(fox\_type, axiom)  
 $\forall a: (\text{fox}(a) \Rightarrow \text{animal}(a))$     fof(fox\_is\_animal, axiom)  
 $\exists a: \text{bird}(a)$     fof(bird\_type, axiom)  
 $\forall a: (\text{bird}(a) \Rightarrow \text{animal}(a))$     fof(bird\_is\_animal, axiom)  
 $\exists a: \text{caterpillar}(a)$     fof(caterpillar\_type, axiom)  
 $\forall a: (\text{caterpillar}(a) \Rightarrow \text{animal}(a))$     fof(caterpillar\_is\_animal, axiom)  
 $\exists a: \text{snail}(a)$     fof(snail\_type, axiom)  
 $\forall a: (\text{snail}(a) \Rightarrow \text{animal}(a))$     fof(snail\_is\_animal, axiom)  
 $\exists a: \text{plant}(a)$     fof(plant\_type, axiom)  
 $\forall a: (\text{plant}(a) \Rightarrow \text{edible}(a))$     fof(plant\_is\_edible, axiom)  
 $\exists a: \text{grain}(a)$     fof(grain\_type, axiom)  
 $\forall a: (\text{grain}(a) \Rightarrow \text{plant}(a))$     fof(grain\_is\_plant, axiom)

$\forall x: (\text{animal}(x) \Rightarrow (\forall y: (\text{plant}(y) \Rightarrow \text{eats}(x, y)) \text{ or } \forall y_1: (\text{animal}(y_1) \Rightarrow ((\text{much\_smaller}(y_1, x) \text{ and } \exists z: (\text{plant}(z) \text{ and } \text{eats}(y_1, \text{eats}(x, y_1)))))) \text{ fof}(\text{pel47}_7, \text{axiom})$   
 $\forall y, x: ((\text{bird}(y) \text{ and } \text{snail}(x)) \Rightarrow \text{much\_smaller}(x, y)) \text{ fof}(\text{pel47}_8, \text{axiom})$   
 $\forall y, x: ((\text{bird}(y) \text{ and } \text{caterpillar}(x)) \Rightarrow \text{much\_smaller}(x, y)) \text{ fof}(\text{pel47\_8a}, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{fox}(y)) \Rightarrow \text{much\_smaller}(x, y)) \text{ fof}(\text{pel47}_9, \text{axiom})$   
 $\forall x, y: ((\text{fox}(x) \text{ and } \text{wolf}(y)) \Rightarrow \text{much\_smaller}(x, y)) \text{ fof}(\text{pel47}_{10}, \text{axiom})$   
 $\forall x, y: ((\text{wolf}(x) \text{ and } \text{fox}(y)) \Rightarrow \neg \text{eats}(x, y)) \text{ fof}(\text{pel47}_{11}, \text{axiom})$   
 $\forall x, y: ((\text{wolf}(x) \text{ and } \text{grain}(y)) \Rightarrow \neg \text{eats}(x, y)) \text{ fof}(\text{pel47\_11a}, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{caterpillar}(y)) \Rightarrow \text{eats}(x, y)) \text{ fof}(\text{pel47}_{12}, \text{axiom})$   
 $\forall x, y: ((\text{bird}(x) \text{ and } \text{snail}(y)) \Rightarrow \neg \text{eats}(x, y)) \text{ fof}(\text{pel47}_{13}, \text{axiom})$   
 $\forall x: (\text{caterpillar}(x) \Rightarrow \exists y: (\text{plant}(y) \text{ and } \text{eats}(x, y))) \text{ fof}(\text{pel47}_{14}, \text{axiom})$   
 $\forall x: (\text{snail}(x) \Rightarrow \exists y: (\text{plant}(y) \text{ and } \text{eats}(x, y))) \text{ fof}(\text{pel47\_14a}, \text{axiom})$   
 $\exists x, y, z: (\text{animal}(x) \text{ and } \text{animal}(y) \text{ and } \text{grain}(z) \text{ and } \text{eats}(y, z) \text{ and } \text{eats}(x, y)) \text{ fof}(\text{pel}_{47}, \text{conjecture})$

### PUZ031-1.p Schubert's Steamroller

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Therefore there is an animal that likes to eat a grain eating animal.

$\text{wolf}(x) \Rightarrow \text{animal}(x) \text{ cnf}(\text{wolf\_is\_an\_animal}, \text{axiom})$   
 $\text{fox}(x) \Rightarrow \text{animal}(x) \text{ cnf}(\text{fox\_is\_an\_animal}, \text{axiom})$   
 $\text{bird}(x) \Rightarrow \text{animal}(x) \text{ cnf}(\text{bird\_is\_an\_animal}, \text{axiom})$   
 $\text{caterpillar}(x) \Rightarrow \text{animal}(x) \text{ cnf}(\text{caterpillar\_is\_an\_animal}, \text{axiom})$   
 $\text{snail}(x) \Rightarrow \text{animal}(x) \text{ cnf}(\text{snail\_is\_an\_animal}, \text{axiom})$   
 $\text{wolf}(\text{a\_wolf}) \text{ cnf}(\text{there\_is\_a\_wolf}, \text{axiom})$   
 $\text{fox}(\text{a\_fox}) \text{ cnf}(\text{there\_is\_a\_fox}, \text{axiom})$   
 $\text{bird}(\text{a\_bird}) \text{ cnf}(\text{there\_is\_a\_bird}, \text{axiom})$   
 $\text{caterpillar}(\text{a\_caterpillar}) \text{ cnf}(\text{there\_is\_a\_caterpillar}, \text{axiom})$   
 $\text{snail}(\text{a\_snail}) \text{ cnf}(\text{there\_is\_a\_snail}, \text{axiom})$   
 $\text{grain}(\text{a\_grain}) \text{ cnf}(\text{there\_is\_a\_grain}, \text{axiom})$   
 $\text{grain}(x) \Rightarrow \text{plant}(x) \text{ cnf}(\text{grain\_is\_a\_plant}, \text{axiom})$   
 $(\text{animal}(\text{animal}) \text{ and } \text{plant}(\text{plant}) \text{ and } \text{animal}(\text{small\_animal}) \text{ and } \text{plant}(\text{other\_plant}) \text{ and } \text{much\_smaller}(\text{small\_animal}, \text{animal})$   
 $(\text{eats}(\text{animal}, \text{plant}) \text{ or } \text{eats}(\text{animal}, \text{small\_animal})) \text{ cnf}(\text{eating\_habits}, \text{axiom})$   
 $(\text{caterpillar}(\text{catapillar}) \text{ and } \text{bird}(\text{bird})) \Rightarrow \text{much\_smaller}(\text{catapillar}, \text{bird}) \text{ cnf}(\text{caterpillar\_smaller\_than\_bird}, \text{axiom})$   
 $(\text{snail}(\text{snail}) \text{ and } \text{bird}(\text{bird})) \Rightarrow \text{much\_smaller}(\text{snail}, \text{bird}) \text{ cnf}(\text{snail\_smaller\_than\_bird}, \text{axiom})$   
 $(\text{bird}(\text{bird}) \text{ and } \text{fox}(\text{fox})) \Rightarrow \text{much\_smaller}(\text{bird}, \text{fox}) \text{ cnf}(\text{bird\_smaller\_than\_fox}, \text{axiom})$   
 $(\text{fox}(\text{fox}) \text{ and } \text{wolf}(\text{wolf})) \Rightarrow \text{much\_smaller}(\text{fox}, \text{wolf}) \text{ cnf}(\text{fox\_smaller\_than\_wolf}, \text{axiom})$   
 $(\text{wolf}(\text{wolf}) \text{ and } \text{fox}(\text{fox})) \Rightarrow \neg \text{eats}(\text{wolf}, \text{fox}) \text{ cnf}(\text{wolf\_dont\_eat\_fox}, \text{axiom})$   
 $(\text{wolf}(\text{wolf}) \text{ and } \text{grain}(\text{grain})) \Rightarrow \neg \text{eats}(\text{wolf}, \text{grain}) \text{ cnf}(\text{wolf\_dont\_eat\_grain}, \text{axiom})$   
 $(\text{bird}(\text{bird}) \text{ and } \text{caterpillar}(\text{catapillar})) \Rightarrow \text{eats}(\text{bird}, \text{catapillar}) \text{ cnf}(\text{bird\_eats\_caterpillar}, \text{axiom})$   
 $(\text{bird}(\text{bird}) \text{ and } \text{snail}(\text{snail})) \Rightarrow \neg \text{eats}(\text{bird}, \text{snail}) \text{ cnf}(\text{bird\_dont\_eat\_snail}, \text{axiom})$   
 $\text{caterpillar}(\text{catapillar}) \Rightarrow \text{plant}(\text{caterpillar\_food\_of}(\text{catapillar})) \text{ cnf}(\text{caterpillar\_food\_is\_a\_plant}, \text{axiom})$   
 $\text{caterpillar}(\text{catapillar}) \Rightarrow \text{eats}(\text{catapillar}, \text{caterpillar\_food\_of}(\text{catapillar})) \text{ cnf}(\text{caterpillar\_eats\_caterpillar\_food}, \text{axiom})$   
 $\text{snail}(\text{snail}) \Rightarrow \text{plant}(\text{snail\_food\_of}(\text{snail})) \text{ cnf}(\text{snail\_food\_is\_a\_plant}, \text{axiom})$   
 $\text{snail}(\text{snail}) \Rightarrow \text{eats}(\text{snail}, \text{snail\_food\_of}(\text{snail})) \text{ cnf}(\text{snail\_eats\_snail\_food}, \text{axiom})$   
 $(\text{animal}(\text{animal}) \text{ and } \text{animal}(\text{grain\_eater}) \text{ and } \text{grain}(\text{grain}) \text{ and } \text{eats}(\text{animal}, \text{grain\_eater})) \Rightarrow \neg \text{eats}(\text{grain\_eater}, \text{grain}) \text{ cnf}(\text{grain\_eater\_eats\_grain}, \text{axiom})$

### PUZ031^5.p TPS problem from BASIC-FO-THMS

$\text{eats}: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(\text{eats}, \text{type})$   
 $\text{grain}: \$i \rightarrow \$o \text{ thf}(\text{grain}, \text{type})$   
 $\text{animal}: \$i \rightarrow \$o \text{ thf}(\text{animal}, \text{type})$   
 $\text{snail}: \$i \rightarrow \$o \text{ thf}(\text{snail}, \text{type})$   
 $\text{sf}: \$i \rightarrow \$i \text{ thf}(\text{sf}, \text{type})$   
 $\text{plant}: \$i \rightarrow \$o \text{ thf}(\text{plant}, \text{type})$   
 $\text{caterpillar}: \$i \rightarrow \$o \text{ thf}(\text{caterpillar}, \text{type})$   
 $\text{cf}: \$i \rightarrow \$i \text{ thf}(\text{cf}, \text{type})$   
 $\text{bird}: \$i \rightarrow \$o \text{ thf}(\text{bird}, \text{type})$   
 $\text{wolf}: \$i \rightarrow \$o \text{ thf}(\text{wolf}, \text{type})$



```

fox: $i → $o    thf(fox, type)
msmaller: $i → $i → $o    thf(msmaller, type)
a_grain: $i    thf(a_grain, type)
a_snail: $i    thf(a_snail, type)
a_caterpillar: $i    thf(a_caterpillar, type)
a_bird: $i    thf(a_bird, type)
a_fox: $i    thf(a_fox, type)
a_wolf: $i    thf(a_wolf, type)
¬∀x: $i: (animal@x or ¬wolf@x) and ∀x: $i: (animal@x or ¬fox@x) and ∀x: $i: (animal@x or ¬bird@x) and ∀x: $i: (animal@x or ¬grain@x)

```

### PUZ031\_1.p Schubert's Steamroller

Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them. Also there are some grains, and grains are plants. Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants. Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves. Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails. Caterpillars and snails like to eat some plants. Therefore there is an animal that likes to eat a grain eating animal.

```

animal: $tType    tff(animal_type, type)
wolf: $tType    tff(wolf_type, type)
wolf_to_animal: wolf → animal    tff(wolf_is_animal, type)
fox: $tType    tff(fox_type, type)
fox_to_animal: fox → animal    tff(fox_is_animal, type)
bird: $tType    tff(bird_type, type)
bird_to_animal: bird → animal    tff(bird_is_animal, type)
caterpillar: $tType    tff(caterpillar_type, type)
caterpillar_to_animal: caterpillar → animal    tff(caterpillar_is_animal, type)
snail: $tType    tff(snail_type, type)
snail_to_animal: snail → animal    tff(snail_is_animal, type)
plant: $tType    tff(plant_type, type)
grain: $tType    tff(grain_type, type)
grain_to_plant: grain → plant    tff(grain_is_plant, type)
edible: $tType    tff(edible_type, type)
animal_to_edible: animal → edible    tff(animal_is_edible, type)
plant_to_edible: plant → edible    tff(plant_is_edible, type)
eats: (animal × edible) → $o    tff(eats_type, type)
much_smaller: (animal × animal) → $o    tff(much_smaller_type, type)
∀x: animal: (∀y: plant: eats(x, plant_to_edible(y)) or ∀y1: animal: ((much_smaller(y1, x) and ∃z: plant: eats(y1, plant_to_edible(z))))    tff(pel477, axiom)
∀x: snail, y: bird: much_smaller(snail_to_animal(x), bird_to_animal(y))    tff(pel478, axiom)
∀x: caterpillar, y: bird: much_smaller(caterpillar_to_animal(x), bird_to_animal(y))    tff(pel47_8a, axiom)
∀x: bird, y: fox: much_smaller(bird_to_animal(x), fox_to_animal(y))    tff(pel479, axiom)
∀x: fox, y: wolf: much_smaller(fox_to_animal(x), wolf_to_animal(y))    tff(pel4710, axiom)
∀x: wolf, y: fox: ¬eats(wolf_to_animal(x), animal_to_edible(fox_to_animal(y)))    tff(pel4711, axiom)
∀x: wolf, y: grain: ¬eats(wolf_to_animal(x), plant_to_edible(grain_to_plant(y)))    tff(pel47_11a, axiom)
∀x: bird, y: caterpillar: eats(bird_to_animal(x), animal_to_edible(caterpillar_to_animal(y)))    tff(pel4712, axiom)
∀x: bird, y: snail: ¬eats(bird_to_animal(x), animal_to_edible(snail_to_animal(y)))    tff(pel4713, axiom)
∀x: caterpillar: ∃y: plant: eats(caterpillar_to_animal(x), plant_to_edible(y))    tff(pel4714, axiom)
∀x: snail: ∃y: plant: eats(snail_to_animal(x), plant_to_edible(y))    tff(pel47_14a, axiom)
∃x: animal, y: animal, z: grain: (eats(y, plant_to_edible(grain_to_plant(z))) and eats(x, animal_to_edible(y)))    tff(pel47, conj)

```

### PUZ032-1.p Knights and Knaves #26

On a certain island the inhabitants are partitioned into those who always tell the truth and those who always lie. I landed on the island and met three inhabitants A, B, and C. I asked A, 'Are you a truth teller or a liar?' He mumbled something which I couldn't make out. I asked B what A had said. B replied, 'A said he was a liar'. C then volunteered, 'Don't believe B, he's lying!' Prove C is a truth teller.

```

include('Axioms/PUZ002-0.ax')
a_truth(says(a, mumble))    cnf(a_mumbles, hypothesis)
a_truth(says(b, says(a, liar(a))))    cnf(b_says_a_says_hes_a_liar, hypothesis)
a_truth(says(c, liar(b)))    cnf(c_says_b_is_a_liar, hypothesis)
¬a_truth(truth teller(c))    cnf(prove_c_is_a_truth teller, negated_conjecture)

```

**PUZ033-1.p** The Winds and the Windows Puzzle

(1) There is always sunshine when the wind is in the East. (2) When it is cold and foggy, my neighbor practices the flute. (3) When my fire smokes, I set the door open. (4) When it is cold and I feel rheumatic, I light my fire. (5) When the wind is in the East and comes in gusts, my fire smokes. (6) When I keep the door open, I am free from headache. (7) Even when the sun is shining and it is not cold, I keep my window shut if it is foggy. (8) When the wind does not come in gusts, and when I have a fire and keep the door shut, I do not feel rheumatic. (9) Sunshine always brings on fog. (10) When my neighbor practices the flute, I shut the door, even if I have no headache. (11) When there is a fog and the wind is in the East, I feel rheumatic. Show that when the wind is in the East, I keep my windows shut.

```

wind_in_east ⇒ sunshine      cnf(c1, axiom)
(cold and foggy) ⇒ neighbor_practices_flute      cnf(c2, axiom)
fire_smokes ⇒ door_is_open    cnf(c3, axiom)
(cold and i_feel_rheumatic) ⇒ fire_is_lit      cnf(c4, axiom)
(wind_in_east and wind_in_gusts) ⇒ fire_smokes    cnf(c5, axiom)
door_is_open ⇒ ¬headache      cnf(c6, axiom)
(sunshine and foggy) ⇒ (cold or window_is_shut)  cnf(c7, axiom)
(fire_is_lit and i_feel_rheumatic) ⇒ (wind_in_gusts or door_is_open)  cnf(c8, axiom)
sunshine ⇒ foggy             cnf(c9, axiom)
neighbor_practices_flute ⇒ ¬door_is_open          cnf(c10, axiom)
(foggy and wind_in_east) ⇒ i_feel_rheumatic      cnf(c11, axiom)
wind_in_east      cnf(c12, hypothesis)
¬window_is_shut  cnf(prove_window_is_shut, negated_conjecture)

```

**PUZ034-1.003.p** N queens problem

The problem is to place 3 queens on an 3x3 chess board, so that no queen can attack another.

```

(less(low, high) and low+s(n0)=newLow and range(newLow, high, restOfNumbers)) ⇒ range(low, high, cons(low, restOfNumbers))
range(same, same, cons(same, empty_list))      cnf(make_list_of_numbers2, axiom)
less(n0, s(x))      cnf(less1, axiom)
less(x, y) ⇒ less(s(x), s(y))      cnf(less2, axiom)
x + n0 = x      cnf(add0, axiom)
x + y = z ⇒ x + s(y) = s(z)      cnf(add, axiom)
select(head, cons(head, tail), tail)      cnf(select1, axiom)
select(element, tail, newTail) ⇒ select(element, cons(head, tail), cons(head, newTail))      cnf(select2, axiom)
¬same(s(x), n0)      cnf(same_definition1, axiom)
¬same(n0, s(x))      cnf(same_definition2, axiom)
same(s(x), s(y)) ⇒ same(x, y)      cnf(same_definition3, axiom)
attack(queen, placedQueens) ⇒ diagonal_attack(queen, s(n0), placedQueens)      cnf(attack, axiom)
(diagonal_attack(queen, queenNumber, cons(placedQueen, otherPlacedQueens)) and diagonal1+queenNumber=placedQueen
queenNumber=diagonal2 and queenNumber+s(n0)=nextQueenNumber) ⇒ (same(diagonal1, queen) or same(diagonal2, queen))
¬diagonal_attack(queen, lastQueen, empty_list)      cnf(check_diagonals2, axiom)
(select(aQueen, unplacedQueens, restOfUnplacedQueens) and do_queens(restOfUnplacedQueens, cons(aQueen, safeQueens), placement)
(do_queens(unplacedQueens, safeQueens, placement) or attack(aQueen, safeQueens))      cnf(place_a_queen1, axiom)
do_queens(empty_list, placement, placement)      cnf(place_a_queen2, axiom)
(numberOfQueens+s(n0)=low and numberOfQueens+numberOfQueens=high and range(low, high, positions) and do_queens(
queens(numberOfQueens, placement)      cnf(set_up_queens, axiom)
¬queens(s(s(s(n0))), placement)      cnf(place_queens, negated_conjecture)

```

**PUZ034-1.004.p** N queens problem

The problem is to place 4 queens on an 4x4 chess board, so that no queen can attack another.

```

(less(low, high) and low+s(n0)=newLow and range(newLow, high, restOfNumbers)) ⇒ range(low, high, cons(low, restOfNumbers))
range(same, same, cons(same, empty_list))      cnf(make_list_of_numbers2, axiom)
less(n0, s(x))      cnf(less1, axiom)
less(x, y) ⇒ less(s(x), s(y))      cnf(less2, axiom)
x + n0 = x      cnf(add0, axiom)
x + y = z ⇒ x + s(y) = s(z)      cnf(add, axiom)
select(head, cons(head, tail), tail)      cnf(select1, axiom)
select(element, tail, newTail) ⇒ select(element, cons(head, tail), cons(head, newTail))      cnf(select2, axiom)
¬same(s(x), n0)      cnf(same_definition1, axiom)
¬same(n0, s(x))      cnf(same_definition2, axiom)
same(s(x), s(y)) ⇒ same(x, y)      cnf(same_definition3, axiom)

```

$\text{attack}(\text{queen}, \text{placedQueens}) \Rightarrow \text{diagonal\_attack}(\text{queen}, s(n_0), \text{placedQueens}) \quad \text{cnf}(\text{attack}, \text{axiom})$   
 $(\text{diagonal\_attack}(\text{queen}, \text{queenNumber}, \text{cons}(\text{placedQueen}, \text{otherPlacedQueens})) \text{ and } \text{diagonal}_1 + \text{queenNumber} = \text{placedQueen}$   
 $\text{queenNumber} = \text{diagonal}_2 \text{ and } \text{queenNumber} + s(n_0) = \text{nextQueenNumber}) \Rightarrow (\text{same}(\text{diagonal}_1, \text{queen}) \text{ or } \text{same}(\text{diagonal}_2, \text{queen})$   
 $\neg \text{diagonal\_attack}(\text{queen}, \text{lastQueen}, \text{empty\_list}) \quad \text{cnf}(\text{check\_diagonals}_2, \text{axiom})$   
 $(\text{select}(\text{aQueen}, \text{unplacedQueens}, \text{restOfUnplacedQueens}) \text{ and } \text{do\_queens}(\text{restOfUnplacedQueens}, \text{cons}(\text{aQueen}, \text{safeQueens}), \text{placement}))$   
 $(\text{do\_queens}(\text{unplacedQueens}, \text{safeQueens}, \text{placement}) \text{ or } \text{attack}(\text{aQueen}, \text{safeQueens})) \quad \text{cnf}(\text{place\_a\_queen}_1, \text{axiom})$   
 $\text{do\_queens}(\text{empty\_list}, \text{placement}, \text{placement}) \quad \text{cnf}(\text{place\_a\_queen}_2, \text{axiom})$   
 $(\text{numberOfQueens} + s(n_0) = \text{low} \text{ and } \text{numberOfQueens} + \text{numberOfQueens} = \text{high} \text{ and } \text{range}(\text{low}, \text{high}, \text{positions}) \text{ and } \text{do\_queens}$   
 $\text{queens}(\text{numberOfQueens}, \text{placement}) \quad \text{cnf}(\text{set\_up\_queens}, \text{axiom})$   
 $\neg \text{queens}(s(s(s(n_0))))), \text{placement}) \quad \text{cnf}(\text{place\_queens}, \text{negated\_conjecture})$

### PUZ035-1.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{person}(x) \Rightarrow (\text{isa}(x, \text{knight}) \text{ or } \text{isa}(x, \text{knave})) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{isa}(x, \text{knight}) \Rightarrow \neg \text{isa}(x, \text{knave}) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $\text{isa}(x, \text{knight}) \Rightarrow \text{tell\_the\_truth}(x) \quad \text{cnf}(\text{knights\_make\_true\_statements}, \text{axiom})$   
 $\text{isa}(x, \text{knave}) \Rightarrow \text{lies}(x) \quad \text{cnf}(\text{knaves\_make\_false\_statements}, \text{axiom})$   
 $\text{tell\_the\_truth}(x) \Rightarrow \neg \text{lies}(x) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}, \text{axiom})$   
 $(\text{response}(\text{yes}) \text{ and } \text{tell\_the\_truth}(\text{asked})) \Rightarrow (\text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{true\_one\_is\_a\_knight}, \text{axiom})$   
 $(\text{response}(\text{no}) \text{ and } \text{lies}(\text{asked})) \Rightarrow (\text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{lie\_one\_is\_a\_knight}, \text{axiom})$   
 $(\text{response}(\text{yes}) \text{ and } \text{lies}(\text{asked})) \Rightarrow \neg \text{isa}(\text{anyone}, \text{knight}) \quad \text{cnf}(\text{knight\_answers}_1, \text{axiom})$   
 $(\text{response}(\text{no}) \text{ and } \text{tell\_the\_truth}(\text{asked})) \Rightarrow \neg \text{isa}(\text{anyone}, \text{knight}) \quad \text{cnf}(\text{knight\_answers}_2, \text{axiom})$   
 $(\text{tell\_the\_truth}(\text{asked}) \text{ and } \text{isa}(\text{anyone}, \text{knight})) \Rightarrow \text{response}(\text{yes}) \quad \text{cnf}(\text{knight\_answers}_3, \text{axiom})$   
 $\text{tell\_the\_truth}(\text{asked}) \Rightarrow (\text{response}(\text{no}) \text{ or } \text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{truthful\_answer}, \text{axiom})$   
 $\text{response}(\text{yes}) \text{ or } \text{response}(\text{no}) \quad \text{cnf}(\text{two\_answers}, \text{axiom})$   
 $\text{person}(\text{asked}) \quad \text{cnf}(\text{asked\_person}, \text{axiom})$   
 $\text{person}(\text{other}) \quad \text{cnf}(\text{other\_person}, \text{axiom})$   
 $(\text{response}(a) \text{ and } \text{isa}(\text{asked}, x)) \Rightarrow \neg \text{isa}(\text{other}, y) \quad \text{cnf}(\text{prove\_answer}, \text{negated\_conjecture})$

### PUZ035-2.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{person}(x) \Rightarrow (\text{isa}(x, \text{knight}) \text{ or } \text{isa}(x, \text{knave})) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{isa}(x, \text{knight}) \Rightarrow \neg \text{isa}(x, \text{knave}) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $\text{isa}(x, \text{knight}) \Rightarrow \text{tell\_the\_truth}(x) \quad \text{cnf}(\text{knights\_make\_true\_statements}, \text{axiom})$   
 $\text{isa}(x, \text{knave}) \Rightarrow \text{lies}(x) \quad \text{cnf}(\text{knaves\_make\_false\_statements}, \text{axiom})$   
 $\text{tell\_the\_truth}(x) \Rightarrow \neg \text{lies}(x) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}, \text{axiom})$   
 $\text{person}(x) \Rightarrow (\text{tell\_the\_truth}(x) \text{ or } \text{lies}(x)) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}_2, \text{axiom})$   
 $(\text{response}(\text{yes}) \text{ and } \text{tell\_the\_truth}(\text{asked})) \Rightarrow (\text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{true\_one\_is\_a\_knight}, \text{axiom})$   
 $(\text{response}(\text{no}) \text{ and } \text{lies}(\text{asked})) \Rightarrow (\text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{lie\_one\_is\_a\_knight}, \text{axiom})$   
 $(\text{response}(\text{yes}) \text{ and } \text{lies}(\text{asked})) \Rightarrow \neg \text{isa}(\text{anyone}, \text{knight}) \quad \text{cnf}(\text{knight\_answers}_1, \text{axiom})$   
 $(\text{response}(\text{no}) \text{ and } \text{tell\_the\_truth}(\text{asked})) \Rightarrow \neg \text{isa}(\text{anyone}, \text{knight}) \quad \text{cnf}(\text{knight\_answers}_2, \text{axiom})$   
 $(\text{tell\_the\_truth}(\text{asked}) \text{ and } \text{isa}(\text{anyone}, \text{knight})) \Rightarrow \text{response}(\text{yes}) \quad \text{cnf}(\text{knight\_answers}_3, \text{axiom})$   
 $\text{tell\_the\_truth}(\text{asked}) \Rightarrow (\text{response}(\text{no}) \text{ or } \text{isa}(\text{asked}, \text{knight}) \text{ or } \text{isa}(\text{other}, \text{knight})) \quad \text{cnf}(\text{truthful\_answer}, \text{axiom})$   
 $\text{response}(\text{yes}) \text{ or } \text{response}(\text{no}) \quad \text{cnf}(\text{two\_answers}, \text{axiom})$   
 $\text{person}(\text{asked}) \quad \text{cnf}(\text{asked\_person}, \text{axiom})$   
 $\text{person}(\text{other}) \quad \text{cnf}(\text{other\_person}, \text{axiom})$   
 $(\text{response}(a) \text{ and } \text{isa}(\text{asked}, x)) \Rightarrow \neg \text{isa}(\text{other}, y) \quad \text{cnf}(\text{prove\_answer}, \text{negated\_conjecture})$

### PUZ035-3.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{person}(p) \Rightarrow (\text{truth}(\text{isa}(p, \text{knight})) \text{ or } \text{truth}(\text{isa}(p, \text{knave}))) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{truth}(\text{isa}(p, \text{knight})) \Rightarrow \neg \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $(\text{says}(p, s) \text{ and } \text{truth}(\text{isa}(p, \text{knight}))) \Rightarrow \text{truth}(s) \quad \text{cnf}(\text{knights\_make\_true\_statements}_1, \text{axiom})$   
 $(\text{says}(p, s) \text{ and } \text{truth}(s)) \Rightarrow \text{truth}(\text{isa}(p, \text{knight})) \quad \text{cnf}(\text{knights\_make\_true\_statements}_2, \text{axiom})$   
 $(\text{says}(p, s) \text{ and } \text{truth}(\text{isa}(p, \text{knave}))) \Rightarrow \text{truth}(\text{not}(s)) \quad \text{cnf}(\text{knaves\_make\_false\_statements}_1, \text{axiom})$

$(\text{says}(p, s) \text{ and } \text{truth}(\text{not}(s))) \Rightarrow \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{knaves\_make\_false\_statements}_2, \text{axiom})$   
 $\text{says}(\text{asked}, \text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight}))) \text{ or } \text{says}(\text{asked}, \text{not}(\text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight})))) \quad \text{cnf}(\text{one.})$   
 $\text{says}(\text{anyone}, s) \Rightarrow (\text{truth}(s) \text{ or } \text{truth}(\text{not}(s))) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}, \text{axiom})$   
 $\text{says}(\text{anyone}, \text{not}(s)) \Rightarrow (\text{truth}(s) \text{ or } \text{truth}(\text{not}(s))) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}_2, \text{axiom})$   
 $\text{truth}(s) \Rightarrow \neg \text{truth}(\text{not}(s)) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}_3, \text{axiom})$   
 $\text{truth}(\text{or}(s_1, s_2)) \Rightarrow (\text{truth}(s_1) \text{ or } \text{truth}(s_2)) \quad \text{cnf}(\text{or\_def}, \text{axiom})$   
 $\text{truth}(s_1) \Rightarrow \neg \text{truth}(\text{not}(\text{or}(s_1, \text{anything}))) \quad \text{cnf}(\text{not}_1, \text{axiom})$   
 $\text{truth}(s_2) \Rightarrow \neg \text{truth}(\text{not}(\text{or}(\text{anything}, s_2))) \quad \text{cnf}(\text{not}_2, \text{axiom})$   
 $\text{person}(\text{asked}) \quad \text{cnf}(\text{asked\_person}, \text{axiom})$   
 $\text{person}(\text{other}) \quad \text{cnf}(\text{other\_person}, \text{axiom})$   
 $(\text{truth}(\text{isa}(\text{asked}, p_1)) \text{ and } \text{truth}(\text{isa}(\text{other}, p_2))) \Rightarrow \neg \text{says}(\text{asked}, s) \quad \text{cnf}(\text{prove\_answer}, \text{negated\_conjecture})$

#### PUZ035-4.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{person}(p) \Rightarrow (\text{truth}(\text{isa}(p, \text{knight})) \text{ or } \text{truth}(\text{isa}(p, \text{knave}))) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{truth}(\text{isa}(p, \text{knight})) \Rightarrow \neg \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $(\text{says}(p, s) \text{ and } \text{truth}(\text{isa}(p, \text{knight}))) \Rightarrow \text{truth}(s) \quad \text{cnf}(\text{knights\_make\_true\_statements}_1, \text{axiom})$   
 $(\text{says}(p, s) \text{ and } \text{truth}(s)) \Rightarrow \text{truth}(\text{isa}(p, \text{knight})) \quad \text{cnf}(\text{knights\_make\_true\_statements}_2, \text{axiom})$   
 $\text{says}(\text{asked}, \text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight}))) \text{ or } \text{says}(\text{asked}, \text{not}(\text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight})))) \quad \text{cnf}(\text{one.})$   
 $\text{says}(\text{anyone}, s) \Rightarrow (\text{truth}(s) \text{ or } \text{truth}(\text{not}(s))) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}, \text{axiom})$   
 $\text{says}(\text{anyone}, \text{not}(s)) \Rightarrow (\text{truth}(s) \text{ or } \text{truth}(\text{not}(s))) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}_2, \text{axiom})$   
 $\text{truth}(s) \Rightarrow \neg \text{truth}(\text{not}(s)) \quad \text{cnf}(\text{statements\_are\_true\_or\_false}_3, \text{axiom})$   
 $\text{truth}(\text{or}(s_1, s_2)) \Rightarrow (\text{truth}(s_1) \text{ or } \text{truth}(s_2)) \quad \text{cnf}(\text{or\_def}, \text{axiom})$   
 $\text{truth}(s_1) \Rightarrow \neg \text{truth}(\text{not}(\text{or}(s_1, \text{anything}))) \quad \text{cnf}(\text{not}_1, \text{axiom})$   
 $\text{truth}(s_2) \Rightarrow \neg \text{truth}(\text{not}(\text{or}(\text{anything}, s_2))) \quad \text{cnf}(\text{not}_2, \text{axiom})$   
 $\text{person}(\text{asked}) \quad \text{cnf}(\text{asked\_person}, \text{axiom})$   
 $\text{person}(\text{other}) \quad \text{cnf}(\text{other\_person}, \text{axiom})$   
 $(\text{truth}(\text{isa}(\text{asked}, p_1)) \text{ and } \text{truth}(\text{isa}(\text{other}, p_2))) \Rightarrow \neg \text{says}(\text{asked}, s) \quad \text{cnf}(\text{prove\_answer}, \text{negated\_conjecture})$

#### PUZ035-5.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{truth}(\text{isa}(p, \text{knight})) \text{ or } \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{truth}(\text{isa}(p, \text{knight})) \Rightarrow \neg \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $(\text{truth}(\text{isa}(p, \text{knight})) \text{ and } \text{says}(p, s)) \Rightarrow \text{truth}(s) \quad \text{cnf}(\text{knights\_make\_true\_statements}_1, \text{axiom})$   
 $(\text{truth}(s) \text{ and } \text{says}(p, s)) \Rightarrow \text{truth}(\text{isa}(p, \text{knight})) \quad \text{cnf}(\text{knights\_make\_true\_statements}_2, \text{axiom})$   
 $\text{truth}(\text{or}(a, b)) \Rightarrow (\text{truth}(a) \text{ or } \text{truth}(b)) \quad \text{cnf}(\text{or}_1, \text{axiom})$   
 $\text{truth}(a) \Rightarrow \text{truth}(\text{or}(a, b)) \quad \text{cnf}(\text{or}_2, \text{axiom})$   
 $\text{truth}(b) \Rightarrow \text{truth}(\text{or}(a, b)) \quad \text{cnf}(\text{or}_3, \text{axiom})$   
 $\text{says}(\text{asked}, \text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight}))) \quad \text{cnf}(\text{says\_yes}, \text{axiom})$   
 $\text{truth}(\text{isa}(\text{asked}, x)) \Rightarrow \neg \text{truth}(\text{isa}(\text{other}, y)) \quad \text{cnf}(\text{query}, \text{negated\_conjecture})$

#### PUZ035-6.p Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

$\text{truth}(\text{isa}(p, \text{knight})) \text{ or } \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{everyone\_a\_knight\_or\_knave}, \text{axiom})$   
 $\text{truth}(\text{isa}(p, \text{knight})) \Rightarrow \neg \text{truth}(\text{isa}(p, \text{knave})) \quad \text{cnf}(\text{not\_both\_a\_knight\_and\_knave}, \text{axiom})$   
 $(\text{truth}(\text{isa}(p, \text{knight})) \text{ and } \text{says}(p, s)) \Rightarrow \text{truth}(s) \quad \text{cnf}(\text{knights\_make\_true\_statements}_1, \text{axiom})$   
 $(\text{truth}(s) \text{ and } \text{says}(p, s)) \Rightarrow \text{truth}(\text{isa}(p, \text{knight})) \quad \text{cnf}(\text{knights\_make\_true\_statements}_2, \text{axiom})$   
 $\text{truth}(\text{or}(a, b)) \Rightarrow (\text{truth}(a) \text{ or } \text{truth}(b)) \quad \text{cnf}(\text{or}_1, \text{axiom})$   
 $\text{truth}(a) \Rightarrow \text{truth}(\text{or}(a, b)) \quad \text{cnf}(\text{or}_2, \text{axiom})$   
 $\text{truth}(b) \Rightarrow \text{truth}(\text{or}(a, b)) \quad \text{cnf}(\text{or}_3, \text{axiom})$   
 $\text{truth}(c) \text{ or } \text{truth}(\text{not}(c)) \quad \text{cnf}(\text{not}_1, \text{axiom})$   
 $\text{truth}(c) \Rightarrow \neg \text{truth}(\text{not}(c)) \quad \text{cnf}(\text{not}_2, \text{axiom})$   
 $\text{says}(\text{asked}, \text{not}(\text{or}(\text{isa}(\text{asked}, \text{knight}), \text{isa}(\text{other}, \text{knight})))) \quad \text{cnf}(\text{says\_yes}, \text{axiom})$   
 $\text{truth}(\text{isa}(\text{asked}, x)) \Rightarrow \neg \text{truth}(\text{isa}(\text{other}, y)) \quad \text{cnf}(\text{query}, \text{negated\_conjecture})$

**PUZ035-7.p** Knights and Knaves #36

On an island, there live exactly two types of people: knights and knaves. Knights always tell the truth and knaves always lie. I landed on the island, met two inhabitants, asked one of them: "Is one of you a knight?" and he answered me. What can be said about the types of the asked and the other person depending on the answer I get?

```

truth(isa(p, knight)) or truth(isa(p, knave))    cnf(everyone_a_knight_or_knave, axiom)
truth(isa(p, knight)) => ~truth(isa(p, knave))   cnf(not_both_a_knight_and_knave, axiom)
(truth(isa(p, knight)) and reply(p, q, yes)) => truth(q)    cnf(knights_make_true_statements_1, axiom)
(truth(q) and reply(p, q, yes)) => truth(isa(p, knight))   cnf(knights_make_true_statements_2, axiom)
(truth(q) and truth(isa(p, knight))) => reply(p, q, yes)   cnf(knights_make_true_statements_3, axiom)
truth(q) or truth(isa(p, knight)) or reply(p, q, yes)    cnf(knights_make_true_statements_4, axiom)
reply(p, q, yes) or reply(p, q, no)    cnf(yes_or_no, axiom)
reply(p, q, yes) => ~reply(p, q, no)    cnf(not_yes_and_no, axiom)
truth(c) or truth(not(c))    cnf(true_1, axiom)
truth(c) => ~truth(not(c))    cnf(true_2, axiom)
truth(or(a, b)) => (truth(a) or truth(b))    cnf(or_1, axiom)
truth(a) => truth(or(a, b))    cnf(or_2, axiom)
truth(b) => truth(or(a, b))    cnf(or_3, axiom)
(reply(asked, or(isa(asked, knight), isa(other, knight)), r) and truth(isa(asked, x))) => ~truth(isa(other, y))    cnf(prove_and)

```

**PUZ036-1.005.p** TopSpin

TopSpin consists of a circular track with 20 pieces numbered 1..20 placed in the track, with a turnstile in the track that always holds four consecutive pieces. There are three legal moves in TopSpin: slide all the pieces round the track in either direction, or flip the turnstile. Given any initial board with scrambled pieces on the track, the problem is to find a sequence of moves that unscrambles the pieces.

```

~state(p1, p2, p3, p4, p5, p6, p7, p8, p9, p10, p11, p12, p13, p14, p15, p16, p17, p18, p19, p20)    cnf(make_like_this, negated_conjecture)
state(p5, p4, p3, p2, p1, p6, p7, p8, p9, p10, p11, p12, p13, p14, p15, p16, p17, p18, p19, p20)    cnf(initial_configuration, hypothesis)
state(x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20) => state(x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20)
state(x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20) => state(x20, x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20)
state(x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20) => state(x4, x3, x2, x1, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20)

```

**PUZ038-1.p** Quo vadis 1

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is a simple goal for testing.

```

include('Axioms/PUZ004-0.ax')
state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o)))
~state(b, v1, v2, v3(s(s(o))), o), v4, h, s1, s2, s3, s4, e1, e2)    cnf(goal_state, negated_conjecture)

```

**PUZ039-1.p** Quo vadis 2

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is an intermediate goal for testing.

```

include('Axioms/PUZ004-0.ax')
state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o)))
~state(b, v1, v2, v3, v4(s(s(o)), s(s(o))), h, s1, s2, s3, s4, e1, e2)    cnf(goal_state, negated_conjecture)

```

**PUZ040-1.p** Quo vadis 3

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is an intermediate goal for testing.

```

include('Axioms/PUZ004-0.ax')
state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o)))
~state(bb(o, o), v1, v2, v3, v4, h, s1, s2, s3, s4, e1, e2)    cnf(goal_state, negated_conjecture)

```

**PUZ041-1.p** Quo vadis 4

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is the true goal from the puzzle.

```

include('Axioms/PUZ004-0.ax')
state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o)))
~state(bb(s(s(o))), s(o), v1, v2, v3, v4, h, s1, s2, s3, s4, e1, e2)    cnf(goal_state, negated_conjecture)

```

**PUZ042-1.p** Quo vadis 5

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to  
the goal state. This is a medium goal for testing.

include('Axioms/PUZ004-0.ax')

state(bb( $o, s(o)$ ),  $v_1(o, o)$ ,  $v_2(o, s(s(o)))$ ),  $v_3(s(s(o)), o)$ ,  $v_4(s(s(o)), s(s(o)))$ ),  $h(s(s(o)), s(o))$ ,  $s_1(s(s(o)), s(o))$ ,  $s_2(s(s(o)), s(o))$ )  
 $\neg$ state( $b, v_1, v_2, v_3, v_4, h(s(s(o))), s(o), s_1, s_2, s_3, s_4, e_1, e_2$ )     cnf(goal\_state, negated\_conjecture)

**PUZ043-1.p** Mars and Venus axioms

include('Axioms/PUZ001-0.ax')

**PUZ044-1.p** Truth-tellers and Liars axioms for two types of people

Axioms for two types of people; truth-tellers and liars.

include('Axioms/PUZ002-0.ax')

**PUZ045-1.p** Truth-tellers and Liars axioms for three types of people

Axioms for three types of people; truth-tellers, liars and normal people.

include('Axioms/PUZ003-0.ax')

**PUZ046-1.p** Quo vadis axioms

include('Axioms/PUZ004-0.ax')

**PUZ047+1.p** Taking the wolf, goat, and cabbage across river

( $p(\text{south}, \text{south}, \text{south}, \text{south}, \text{start})$  and  $\forall t: (p(\text{south}, \text{north}, \text{south}, \text{north}, t) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{go\_alone}(t)))$  and  
 $p(\text{south}, \text{north}, \text{south}, \text{north}, \text{go\_alone}(t_1))$ ) and  $\forall t_2: (p(\text{south}, \text{south}, \text{north}, \text{south}, t_2) \Rightarrow p(\text{north}, \text{south}, \text{north}, \text{south}, \text{go\_alone}(t_2)))$  and  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, \text{go\_alone}(t_3))$ ) and  $\forall t_4: (p(\text{south}, \text{south}, \text{south}, \text{north}, t_4) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{take\_wolf}(t_4)))$  and  
 $p(\text{south}, \text{south}, \text{south}, \text{north}, \text{take\_wolf}(t_5))$ ) and  $\forall t_6: (p(\text{south}, \text{south}, \text{north}, \text{south}, t_6) \Rightarrow p(\text{north}, \text{north}, \text{north}, \text{south}, \text{take\_wolf}(t_6)))$  and  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, \text{take\_wolf}(t_7))$ ) and  $\forall x, y, u: (p(\text{south}, x, \text{south}, y, u) \Rightarrow p(\text{north}, x, \text{north}, y, \text{take\_goat}(u)))$  and  $\forall v: (p(\text{south}, x_1, \text{south}, y_1, \text{take\_goat}(v)))$  and  $\forall t_8: (p(\text{south}, \text{north}, \text{south}, \text{south}, t_8) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{take\_cabbage}(t_8)))$  and  
 $p(\text{south}, \text{north}, \text{south}, \text{south}, \text{take\_cabbage}(t_9))$ ) and  $\forall u_1: (p(\text{south}, \text{south}, \text{north}, \text{south}, u_1) \Rightarrow p(\text{north}, \text{south}, \text{north}, \text{north}, \text{take\_cabbage}(u_1)))$  and  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, \text{take\_cabbage}(v_1)) \Rightarrow \exists z: p(\text{north}, \text{north}, \text{north}, \text{north}, z)$      fof(thm100, conjecture)

**PUZ047-1.p** Taking the wolf, goat, and cabbage across river

$p(\text{south}, \text{south}, \text{south}, \text{south}, \text{start})$      cnf(thm100<sub>1</sub>, negated\_conjecture)  
 $p(\text{south}, \text{north}, \text{south}, \text{north}, a) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{go\_alone}(a))$      cnf(thm100<sub>2</sub>, negated\_conjecture)  
 $p(\text{north}, \text{north}, \text{south}, \text{north}, a) \Rightarrow p(\text{south}, \text{north}, \text{south}, \text{north}, \text{go\_alone}(a))$      cnf(thm100<sub>3</sub>, negated\_conjecture)  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, a) \Rightarrow p(\text{north}, \text{south}, \text{north}, \text{south}, \text{go\_alone}(a))$      cnf(thm100<sub>4</sub>, negated\_conjecture)  
 $p(\text{north}, \text{south}, \text{north}, \text{south}, a) \Rightarrow p(\text{south}, \text{south}, \text{north}, \text{south}, \text{go\_alone}(a))$      cnf(thm100<sub>5</sub>, negated\_conjecture)  
 $p(\text{south}, \text{south}, \text{south}, \text{north}, a) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{take\_wolf}(a))$      cnf(thm100<sub>6</sub>, negated\_conjecture)  
 $p(\text{north}, \text{north}, \text{south}, \text{north}, a) \Rightarrow p(\text{south}, \text{south}, \text{south}, \text{north}, \text{take\_wolf}(a))$      cnf(thm100<sub>7</sub>, negated\_conjecture)  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, a) \Rightarrow p(\text{north}, \text{north}, \text{north}, \text{south}, \text{take\_wolf}(a))$      cnf(thm100<sub>8</sub>, negated\_conjecture)  
 $p(\text{north}, \text{north}, \text{north}, \text{south}, a) \Rightarrow p(\text{south}, \text{south}, \text{north}, \text{south}, \text{take\_wolf}(a))$      cnf(thm100<sub>9</sub>, negated\_conjecture)  
 $p(\text{south}, a, \text{south}, b, c) \Rightarrow p(\text{north}, a, \text{north}, b, \text{take\_goat}(c))$      cnf(thm100<sub>10</sub>, negated\_conjecture)  
 $p(\text{north}, a, \text{north}, b, c) \Rightarrow p(\text{south}, a, \text{south}, b, \text{take\_goat}(c))$      cnf(thm100<sub>11</sub>, negated\_conjecture)  
 $p(\text{south}, \text{north}, \text{south}, \text{south}, a) \Rightarrow p(\text{north}, \text{north}, \text{south}, \text{north}, \text{take\_cabbage}(a))$      cnf(thm100<sub>12</sub>, negated\_conjecture)  
 $p(\text{north}, \text{north}, \text{south}, \text{north}, a) \Rightarrow p(\text{south}, \text{north}, \text{south}, \text{south}, \text{take\_cabbage}(a))$      cnf(thm100<sub>13</sub>, negated\_conjecture)  
 $p(\text{south}, \text{south}, \text{north}, \text{south}, a) \Rightarrow p(\text{north}, \text{south}, \text{north}, \text{north}, \text{take\_cabbage}(a))$      cnf(thm100<sub>14</sub>, negated\_conjecture)  
 $p(\text{north}, \text{south}, \text{north}, \text{north}, a) \Rightarrow p(\text{south}, \text{south}, \text{north}, \text{south}, \text{take\_cabbage}(a))$      cnf(thm100<sub>15</sub>, negated\_conjecture)  
 $\neg p(\text{north}, \text{north}, \text{north}, \text{north}, a)$      cnf(thm100<sub>16</sub>, negated\_conjecture)

**PUZ047^5.p** TPS problem THM100A

A naive formalization of the problem of moving man wolf goat cabbage from south to north side of river.

a: \$tType     thf(a\_type, type)

b: \$tType     thf(b\_type, type)

cN: a     thf(cN, type)

cP:  $a \rightarrow a \rightarrow a \rightarrow a \rightarrow b \rightarrow$  \$o     thf(cP, type)

cD:  $b \rightarrow b$      thf(cD, type)

cS: a     thf(cS, type)

cG:  $b \rightarrow b$      thf(cG, type)

cW:  $b \rightarrow b$      thf(cW, type)

cL:  $b \rightarrow b$      thf(cL, type)

cO: b     thf(cO, type)

(cP@cS@cS@cS@cS@cO and  $\forall t: b: ((cP@cS@cN@cS@cN@t) \Rightarrow (cP@cN@cN@cS@cN@(cL@t)))$  and  $\forall t_1: b: ((cP@cN@cN@cN@cN@cL@t_1) \Rightarrow (cP@cS@cS@cS@cS@cN@cL@cL@t_1))$  and  $\forall t_2: b: ((cP@cS@cS@cS@cS@cN@cS@cN@cS@cN@cL@t_2) \Rightarrow (cP@cN@cS@cN@cS@cN@cL@cL@t_2))$  and  $\forall t_3: b: ((cP@cS@cS@cS@cS@cN@cS@cN@cS@cN@cL@t_3) \Rightarrow (cP@cN@cS@cN@cS@cN@cL@cL@t_3))$ )

$(cP@cS@cS@cN@cS@(cL@t_3))$  and  $\forall t_4: b: ((cP@cS@cS@cS@cN@t_4) \Rightarrow (cP@cN@cN@cS@cN@(cW@t_4)))$  and  $\forall t_5: b: ((cP@cS@cS@cS@cS@cN@(cW@t_5))$  and  $\forall t_6: b: ((cP@cS@cS@cS@cN@cS@t_6) \Rightarrow (cP@cN@cN@cN@cS@(cW@t_6)))$  and  $\forall t_7: b: ((cP@cS@cS@cS@cN@cS@(cW@t_7))$  and  $\forall x: a, y: a, u: b: ((cP@cS@x@cS@y@u) \Rightarrow (cP@cN@x@cN@y@(cG@u)))$  and  $\forall x_1: a, y_1: b: ((cP@cS@x_1@cS@y_1@(cG@v))$  and  $\forall t_8: b: ((cP@cS@cN@cS@cS@t_8) \Rightarrow (cP@cN@cN@cS@cN@(cD@t_8)))$  and  $\forall t_9: b: ((cP@cS@cN@cS@cS@(cD@t_9))$  and  $\forall u_1: b: ((cP@cS@cS@cN@cS@u_1) \Rightarrow (cP@cN@cS@cN@cN@(cD@u_1)))$  and  $\forall v_1: b: ((cP@cS@cS@cN@cS@(cD@v_1)) \Rightarrow \exists z: b: (cP@cN@cN@cN@cN@z) \quad \text{thf}(cTHM100A, \text{conjecture}))$

**PUZ048-1.p** Quo vadis 6 - initial to intermediate

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is the true goal from the puzzle.

include('Axioms/PUZ004-0.ax')

state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o))  
¬state(bb(s(s(o)), s(s(o))), v1(o, o), v2(o, s(o)), v3(o, s(s(o))), v4(o, s(s(o))), h(s(s(s(o))), o), s1(s(s(o)), o), s2(s(s(o)), s(o)))

**PUZ049-1.p** Quo vadis 6 - intermediate to final

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is the true goal from the puzzle.

include('Axioms/PUZ004-0.ax')

state(bb(s(s(o)), s(s(o))), v1(o, o), v2(o, s(o)), v3(o, s(s(o))), v4(o, s(s(s(o))), h(s(s(s(s(o))), o), s1(s(s(o)), o), s2(s(s(o)), s(o))  
¬state(bb(s(s(s(o))), s(o)), v1, v2, v3, v4, h, s1, s2, s3, s4, e1, e2) \quad \text{cnf}(goal\\_state, \text{negated\\_conjecture})

**PUZ050-1.p** Quo vadis 6 - initial to intermediate

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is the true goal from the puzzle.

include('Axioms/PUZ004-0.ax')

state(bb(o, s(o)), v1(o, o), v2(o, s(s(o))), v3(s(s(o)), o), v4(s(s(o)), s(s(o))), h(s(s(o)), s(o)), s1(s(s(o))), s(o)), s2(s(s(o)), s(o))  
¬state(bb(o, o), v1(s(s(o)), o), v2(o, s(s(s(o))), v3(o, s(s(o))), v4(s(s(s(o))), s(s(o))), h(s(s(s(s(o))), o), s1(s(s(o)), s(o)), s2(s(s(s(o))), s(o)))

**PUZ051-1.p** Quo vadis 6 - intermediate to final

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is the true goal from the puzzle.

include('Axioms/PUZ004-0.ax')

state(bb(o, o), v1(s(s(o)), o), v2(o, s(s(s(o))), v3(o, s(s(o))), v4(s(s(s(o))), s(s(o))), h(s(s(s(s(o))), o), s1(s(s(o)), s(o)), s2(s(s(s(o))), s(o))  
¬state(bb(s(s(s(o))), s(o)), v1, v2, v3, v4, h, s1, s2, s3, s4, e1, e2) \quad \text{cnf}(goal\\_state, \text{negated\\_conjecture})

**PUZ054-1.p** Take black and white balls from a bag

Start with a bag with 10 white balls and 9 black balls. Take out two balls: if they have the same color, put a black ball back; if they have a different color, put a white ball back. The last ball left cannot be white.

$p(s(s(s(s(s(s(s(s(n_0))))))))), s(s(s(s(s(s(s(s(n_0)))))))) \quad \text{cnf}(\text{initial\_state}, \text{axiom})$

$p(s(s(x)), y) \Rightarrow p(x, s(y)) \quad \text{cnf}(\text{two\_whites\_out\_one\_black\_in}, \text{axiom})$

$p(x, s(s(y))) \Rightarrow p(x, s(y)) \quad \text{cnf}(\text{two\_blacks\_out\_one\_black\_in}, \text{axiom})$

$p(s(x), s(y)) \Rightarrow p(s(x), y) \quad \text{cnf}(\text{two\_different\_balls\_out\_one\_white\_in}, \text{axiom})$

$\neg p(s(n_0), n_0) \quad \text{cnf}(\text{goal\_state}, \text{negated\_conjecture})$

**PUZ055-1.p** Show that Sam Loyd's fifteen-puzzle is not solvable

The classic fifteen-puzzle with the 14 and 15 interchanged. Show that it is not solvable by showing satisfiability of the following problem.

state(l(n1, l(n2, l(n3, l(n4, l(end, l(n5, l(n6, l(n7, l(n8, l(end, l(n9, l(n10, l(n11, l(n12, l(end, l(n13, l(n15, l(n14, l(x, l(end, nil)))))))))))))  
¬state(l(n1, l(n2, l(n3, l(n4, l(end, l(n5, l(n6, l(n7, l(n8, l(end, l(n9, l(n10, l(n11, l(n12, l(end, l(n13, l(n14, l(n15, l(x, l(end, nil)))))))))))))

$l(x, l(a, r)) = l(a, l(x, r))$  or  $a = \text{end} \quad \text{cnf}(\text{shift\_sideways}, \text{axiom})$

$l(x, l(a, l(b, l(c, l(d, l(e, r)))))) = l(e, l(a, l(b, l(c, l(d, l(x, r)))))) \quad \text{cnf}(\text{shift\_up\_down}, \text{axiom})$

**PUZ056-1.p** Tower of Hanoi

state(nextto(on(small, on(medium, on(large, pin(n1))), nextto(pin(n2), pin(n3)))) \quad \text{cnf}(\text{initial\\_state}, \text{axiom})

¬state(nextto(pin(n1), nextto(pin(n2), on(small, on(medium, on(large, pin(n3)))))) \quad \text{cnf}(\text{final\\_state}, \text{negated\\_conjecture})

nextto(x, y) = nextto(y, x) \quad \text{cnf}(\text{nextto\\_commutative}, \text{axiom})

nextto(x, nextto(y, z)) = nextto(nextto(x, y), z) \quad \text{cnf}(\text{nextto\\_associative}, \text{axiom})

(fits(x, b) and fits(x, a))  $\Rightarrow$  nextto(on(x, a), b) = nextto(a, on(x, b)) \quad \text{cnf}(\text{a\\_move}, \text{axiom})

fits(small, pin(n)) \quad \text{cnf}(\text{fits}\_1, \text{axiom})

fits(medium, pin( $n$ ))      cnf(fits<sub>2</sub>, axiom)  
fits(large, pin( $n$ ))      cnf(fits<sub>3</sub>, axiom)  
fits(small, on(medium,  $a$ ))      cnf(fits<sub>4</sub>, axiom)  
fits(medium, on(large,  $a$ ))      cnf(fits<sub>5</sub>, axiom)  
fits(small, on(large,  $a$ ))      cnf(fits<sub>6</sub>, axiom)

**PUZ056-2.005.p** Towers of Hanoi k=05

Each instance encodes Tower of Hanoi with  $n$  discs as a reachability problem.

$p(i, t_1, t_2, t_3, t_4) \Rightarrow p(j, t_1, t_2, t_3, t_4)$       cnf(rule<sub>1</sub>, axiom)  
 $(p(t_0, i, t_2, t_3, t_4) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j)) \Rightarrow p(t_0, j, t_2, t_3, t_4)$       cnf(rule<sub>2</sub>, axiom)  
 $(p(t_0, t_1, i, t_3, t_4) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j)) \Rightarrow p(t_0, t_1, j, t_3, t_4)$       cnf(rule<sub>3</sub>, axiom)  
 $(p(t_0, t_1, t_2, i, t_4) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j)) \Rightarrow p(t_0, t_1, t_2, j, t_4)$   
 $(p(t_0, t_1, t_2, t_3, i) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j)) \Rightarrow p(t_0, t_1, t_2, t_3, j)$       cnf(rule<sub>5</sub>, axiom)  
 $\neg \text{neq}(s_0, s_0)$       cnf(neq<sub>1</sub>, axiom)  
 $\text{neq}(s_0, s_1)$       cnf(neq<sub>2</sub>, axiom)  
 $\text{neq}(s_0, s_2)$       cnf(neq<sub>3</sub>, axiom)  
 $\text{neq}(s_1, s_0)$       cnf(neq<sub>4</sub>, axiom)  
 $\neg \text{neq}(s_1, s_1)$       cnf(neq<sub>5</sub>, axiom)  
 $\text{neq}(s_1, s_2)$       cnf(neq<sub>6</sub>, axiom)  
 $\text{neq}(s_2, s_0)$       cnf(neq<sub>7</sub>, axiom)  
 $\text{neq}(s_2, s_1)$       cnf(neq<sub>8</sub>, axiom)  
 $\neg \text{neq}(s_2, s_2)$       cnf(neq<sub>9</sub>, axiom)  
 $p(s_0, s_0, s_0, s_0, s_0)$       cnf(init, axiom)  
 $\neg p(s_2, s_2, s_2, s_2, s_2)$       cnf(goal, negated\_conjecture)

**PUZ056-2.010.p** Towers of Hanoi k=10

Each instance encodes Tower of Hanoi with  $n$  discs as a reachability problem.

$p(i, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9) \Rightarrow p(j, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9)$       cnf(rule<sub>1</sub>, axiom)  
 $(p(t_0, i, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j)) \Rightarrow p(t_0, j, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9)$       cnf(rule<sub>2</sub>, axiom)  
 $(p(t_0, t_1, i, t_3, t_4, t_5, t_6, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j)) \Rightarrow p(t_0, t_1, j, t_3, t_4, t_5, t_6, t_7, t_8, t_9)$   
 $(p(t_0, t_1, t_2, i, t_4, t_5, t_6, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j)) \Rightarrow p(t_0, t_1, t_2, j, t_4, t_5, t_6, t_7, t_8, t_9)$       cnf(rule<sub>4</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, i, t_5, t_6, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j)) \Rightarrow p(t_0, t_1, t_2, t_3, j, t_5, t_6, t_7, t_8, t_9)$       cnf(rule<sub>5</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, t_4, i, t_6, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j) \text{ and } \text{neq}(t_4, i) \text{ and } \text{neq}(t_4, j)) \Rightarrow p(t_0, t_1, t_2, t_3, t_4, j, t_6, t_7, t_8, t_9)$       cnf(rule<sub>6</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, t_4, t_5, i, t_7, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j) \text{ and } \text{neq}(t_4, i) \text{ and } \text{neq}(t_4, j) \text{ and } \text{neq}(t_5, i) \text{ and } \text{neq}(t_5, j)) \Rightarrow p(t_0, t_1, t_2, t_3, t_4, t_5, j, t_7, t_8, t_9)$       cnf(rule<sub>7</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, i, t_8, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j) \text{ and } \text{neq}(t_4, i) \text{ and } \text{neq}(t_4, j) \text{ and } \text{neq}(t_5, i) \text{ and } \text{neq}(t_5, j) \text{ and } \text{neq}(t_6, i) \text{ and } \text{neq}(t_6, j)) \Rightarrow p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, j, t_8, t_9)$       cnf(rule<sub>8</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, i, t_9) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j) \text{ and } \text{neq}(t_4, i) \text{ and } \text{neq}(t_4, j) \text{ and } \text{neq}(t_5, i) \text{ and } \text{neq}(t_5, j) \text{ and } \text{neq}(t_6, i) \text{ and } \text{neq}(t_6, j) \text{ and } \text{neq}(t_7, i) \text{ and } \text{neq}(t_7, j)) \Rightarrow p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, j, t_9)$       cnf(rule<sub>9</sub>, axiom)  
 $(p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, i) \text{ and } \text{neq}(t_0, i) \text{ and } \text{neq}(t_0, j) \text{ and } \text{neq}(t_1, i) \text{ and } \text{neq}(t_1, j) \text{ and } \text{neq}(t_2, i) \text{ and } \text{neq}(t_2, j) \text{ and } \text{neq}(t_3, i) \text{ and } \text{neq}(t_3, j) \text{ and } \text{neq}(t_4, i) \text{ and } \text{neq}(t_4, j) \text{ and } \text{neq}(t_5, i) \text{ and } \text{neq}(t_5, j) \text{ and } \text{neq}(t_6, i) \text{ and } \text{neq}(t_6, j) \text{ and } \text{neq}(t_7, i) \text{ and } \text{neq}(t_7, j) \text{ and } \text{neq}(t_8, i) \text{ and } \text{neq}(t_8, j)) \Rightarrow p(t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, j)$       cnf(rule<sub>10</sub>, axiom)  
 $\neg \text{neq}(s_0, s_0)$       cnf(neq<sub>1</sub>, axiom)  
 $\text{neq}(s_0, s_1)$       cnf(neq<sub>2</sub>, axiom)  
 $\text{neq}(s_0, s_2)$       cnf(neq<sub>3</sub>, axiom)  
 $\text{neq}(s_1, s_0)$       cnf(neq<sub>4</sub>, axiom)  
 $\neg \text{neq}(s_1, s_1)$       cnf(neq<sub>5</sub>, axiom)  
 $\text{neq}(s_1, s_2)$       cnf(neq<sub>6</sub>, axiom)  
 $\text{neq}(s_2, s_0)$       cnf(neq<sub>7</sub>, axiom)  
 $\text{neq}(s_2, s_1)$       cnf(neq<sub>8</sub>, axiom)  
 $\neg \text{neq}(s_2, s_2)$       cnf(neq<sub>9</sub>, axiom)  
 $p(s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$       cnf(init, axiom)  
 $\neg p(s_2, s_2, s_2, s_2, s_2, s_2, s_2, s_2, s_2, s_2)$       cnf(goal, negated\_conjecture)

**PUZ057-1.p** Show the Hanoi problem is not solvable anymore

Move blocks from pin to pin, until a goal configuration is reached. Some rules are removed: one can only move a block to a pin next to it; one cannot place the small block on the large block.

state(nextto(on(small, on(medium, on(large, pin( $n_1$ ))), nextto(pin( $n_2$ ), pin( $n_3$ ))))      cnf(initial\_state, axiom)



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¬state(nexttto(pin( $n_1$ ), nexttto(pin( $n_2$ ), on(small, on(medium, on(large, pin( $n_3$ )))))))    cnf(final_state, negated_conjecture)
nexttto( $x$ , nexttto( $y$ ,  $z$ )) = nexttto(nexttto( $x$ ,  $y$ ),  $z$ )    cnf(nexttto_associative, axiom)
(fits( $x$ ,  $b$ ) and fits( $x$ ,  $a$ ))  $\Rightarrow$  nexttto(on( $x$ ,  $a$ ),  $b$ ) = nexttto( $a$ , on( $x$ ,  $b$ ))    cnf(a_move, axiom)
fits(small, pin( $n$ ))    cnf(fits1, axiom)
fits(medium, pin( $n$ ))    cnf(fits2, axiom)
fits(large, pin( $n$ ))    cnf(fits3, axiom)
fits(small, on(medium,  $a$ ))    cnf(fits4, axiom)
fits(medium, on(large,  $a$ ))    cnf(fits5, axiom)

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**PUZ058-1.p** Show the Hanoi problem is not solvable anymore

Move blocks from pin to pin, until a goal configuration is reached. Some rules are removed: one cannot move the medium block onto the second pin directly.

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state(nexttto(on(small, on(medium, on(large, pin( $n_1$ ))), nexttto(pin( $n_2$ ), pin( $n_3$ ))))    cnf(initial_state, axiom)
¬state(nexttto(pin( $n_1$ ), nexttto(pin( $n_2$ ), on(small, on(medium, on(large, pin( $n_3$ )))))))    cnf(final_state, negated_conjecture)
nexttto( $x$ ,  $y$ ) = nexttto( $y$ ,  $x$ )    cnf(nexttto_commutative, axiom)
nexttto( $x$ , nexttto( $y$ ,  $z$ )) = nexttto(nexttto( $x$ ,  $y$ ),  $z$ )    cnf(nexttto_associative, axiom)
(fits( $x$ ,  $b$ ) and fits( $x$ ,  $a$ ))  $\Rightarrow$  nexttto(on( $x$ ,  $a$ ),  $b$ ) = nexttto( $a$ , on( $x$ ,  $b$ ))    cnf(a_move, axiom)
fits(small, pin( $n$ ))    cnf(fits1, axiom)
fits(medium, pin( $n$ )) or  $n = n_2$     cnf(fits2, axiom)
fits(large, pin( $n$ ))    cnf(fits3, axiom)
fits(small, on(medium,  $a$ ))    cnf(fits4, axiom)
fits(medium, on(large,  $a$ ))    cnf(fits5, axiom)
fits(small, on(large,  $a$ ))    cnf(fits6, axiom)

```

**PUZ059-1.p** Quo vadis 7 - an unreachable state

bb is big block (square, size=4 tiles). s1-s4 : 4 small square blocks, size=1 tile v1-v4: 4 vertical blocks, size= 2 tiles  
b1: horizontal block, size= 2 tiles e1, e2 are the 2 blank tiles It's a 5x4 playing field to move from the start state to the goal state. This is a simple goal for testing.

include('Axioms/PUZ004-0.ax')

```

state(bb( $o$ ,  $s(o)$ ),  $v_1(o, o)$ ,  $v_2(o, s(s(o)))$ ,  $v_3(s(s(o)), o)$ ,  $v_4(s(s(o)), s(s(o)))$ ,  $h(s(s(o)), s(o))$ ,  $s_1(s(s(o)), s(o))$ ,  $s_2(s(s(o)), s(o))$ ,  $s_3(s(s(o)), s(o))$ ,  $s_4(s(s(o)), s(o))$ )
¬state(bb( $o$ ,  $s(o)$ ),  $v_1(o, o)$ ,  $v_2(s(s(o)), o)$ ,  $v_3(s(o), s(s(o)))$ ,  $v_4(s(s(o)), s(s(o)))$ ,  $s(s(o)), h(s(s(o)), s(o))$ ,  $s_1(s(s(o)), s(o))$ ,  $s_2(s(s(o)), s(o))$ ,  $s_3(s(s(o)), s(o))$ ,  $s_4(s(s(o)), s(o))$ )

```

**PUZ060+1.p** Food problems

$\forall$ peanuts, john, bill, sue, apples, chicken: ( $(\forall x: (\text{food}(x) \Rightarrow \text{likes}(\text{john}, x))$  and  $\text{food}(\text{apples})$  and  $\text{food}(\text{chicken})$  and  $\forall x: (\exists y: (\text{eats}(\text{bill}, x) \Rightarrow \text{eats}(\text{sue}, x))$  and  $\forall y: (\text{alive}(y) \Rightarrow \forall x: \text{not\_killed\_by}(y, x))$  and  $\text{likes}(\text{john}, \text{peanuts}))$  fof(prove\_this, conjecture)

**PUZ061+1.p** Food problems

$\forall$ peanuts, john, bill, sue, apples, chicken: ( $(\forall x, y: ((\text{alive}(y) \text{ and } \text{eats}(y, \text{chicken})) \Rightarrow \text{likes}(y, x))$  and  $\forall x: (\text{food}(x) \Rightarrow \text{likes}(\text{john}, x))$  and  $\text{food}(\text{apples})$  and  $\text{food}(\text{chicken})$  and  $\forall x: (\exists y: (\text{eats}(y, x) \text{ and } \text{not\_killed\_by}(y, x)) \Rightarrow \text{food}(x))$  and  $\text{eats}(\text{bill}, \text{peanuts})$  and  $\forall y: (\text{alive}(y) \Rightarrow \forall x: \text{not\_killed\_by}(y, x)) \Rightarrow \text{likes}(\text{john}, \text{peanuts}))$  fof(prove\_this, conjecture)

**PUZ062-2.p** Problem about mutilated chessboard problem

```

c_in( $v_A$ , c.Finite_Set_OFinites, tc_set( $t_a$ ))  $\Rightarrow$  (c_in( $v_x$ ,  $v_A$ ,  $t_a$ ) or c.Finite_Set_Ocard(c_insert( $v_x$ ,  $v_A$ ,  $t_a$ ),  $t_a$ ) = c.Suc(c.Finite_Set_Ocard( $v_A$ ,  $t_a$ )))    cnf(cls_Finite_Set_Ocard_insert_disjoint0, axiom)
c_in( $v_G$ , c.Finite_Set_OFinites, tc_set( $t_a$ ))  $\Rightarrow$  c_in(c_inter( $v_F$ ,  $v_G$ ,  $t_a$ ), c.Finite_Set_OFinites, tc_set( $t_a$ ))    cnf(cls_Finite_Set_Ocard_insert_disjoint1, axiom)
c_in( $v_t$ , c.Mutil_Otiling(c.Mutil_Odomino, tc_prod(tc_nat, tc_nat)), tc_set(tc_prod(tc_nat, tc_nat)))  $\Rightarrow$  c_in( $v_t$ , c.Finite_Set_OFinites, tc_set(tc_prod(tc_nat, tc_nat)))    cnf(cls_Mutil_Otiling_insert_disjoint0, axiom)
c_in( $v_c$ , c_inter( $v_A$ ,  $v_B$ ,  $t_a$ ),  $t_a$ )  $\Rightarrow$  c_in( $v_c$ ,  $v_B$ ,  $t_a$ )    cnf(cls_Set_OInt_iff1, axiom)
(c_in( $v_c$ ,  $v_B$ ,  $t_a$ ) and c_in( $v_c$ ,  $v_A$ ,  $t_a$ ))  $\Rightarrow$  c_in( $v_c$ , c_inter( $v_A$ ,  $v_B$ ,  $t_a$ ),  $t_a$ )    cnf(cls_Set_OInt_iff2, axiom)
c_in( $v_x$ , c_insert( $v_x$ ,  $v_A$ ,  $t_a$ ),  $t_a$ )    cnf(cls_Set_Oinsert_iff1, axiom)
¬c_in( $v_c$ , c.emptyset,  $t_a$ )    cnf(cls_Set_Oempty_iff0, axiom)
c_in( $v_t$ , c.Mutil_Otiling(c.Mutil_Odomino, tc_prod(tc_nat, tc_nat)), tc_set(tc_prod(tc_nat, tc_nat)))    cnf(cls_conjecture0, negated_conjecture)
c.Finite_Set_Ocard(c_inter(c.Mutil_Ocoloured( $c_0$ ),  $v_t$ , tc_prod(tc_nat, tc_nat)), tc_prod(tc_nat, tc_nat)) = c.Finite_Set_Ocard(c_inter( $v_a$ ,  $v_t$ , tc_prod(tc_nat, tc_nat)) = c.emptyset    cnf(cls_conjecture2, negated_conjecture)
c_inter(c.Mutil_Ocoloured( $c_0$ ),  $v_a$ , tc_prod(tc_nat, tc_nat)) = c_insert(c.Pair( $v_i$ ,  $v_j$ , tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat))    cnf(cls_conjecture1, negated_conjecture)
c_inter(c.Mutil_Ocoloured(c.Suc( $c_0$ )),  $v_a$ , tc_prod(tc_nat, tc_nat)) = c_insert(c.Pair( $v_m$ ,  $v_n$ , tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat))    cnf(cls_conjecture3, negated_conjecture)
c.Finite_Set_Ocard(c_insert(c.Pair( $v_i$ ,  $v_j$ , tc_nat, tc_nat), c_inter(c.Mutil_Ocoloured( $c_0$ ),  $v_t$ , tc_prod(tc_nat, tc_nat)), tc_prod(tc_nat, tc_nat)))    cnf(cls_conjecture4, negated_conjecture)
c.Finite_Set_Ocard(c_insert(c.Pair( $v_m$ ,  $v_n$ , tc_nat, tc_nat), c_inter(c.Mutil_Ocoloured(c.Suc( $c_0$ )),  $v_t$ , tc_prod(tc_nat, tc_nat))))    cnf(cls_conjecture5, negated_conjecture)

```

**PUZ063-1.p** Problem about mutilated chessboard problem

include('Axioms/MS001-1.ax')

include('Axioms/MS001-0.ax')

```

c_in(c_insert(c.Pair( $v_i$ ,  $v_j$ , tc_nat, tc_nat), c_insert(c.Pair( $v_i$ , c.Suc( $v_j$ ), tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat)), tc_prod(tc_nat, tc_nat)))    cnf(cls_conjecture6, negated_conjecture)

```

```

c.in(c.insert(c.Pair(v_i, v_j, tc_nat, tc_nat), c.insert(c.Pair(c.Suc(v_i), v_j, tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat)), t),
(c.in(v_a, v_A, tc_set(t_a)) and c.in(v_t, c.Mutil_Otiling(v_A, t_a), tc_set(t_a)) and c.inter(v_a, v_t, t_a) = c.emptyset) =>
c.in(c.union(v_a, v_t, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_Mutil_Otiling_OUn_0, axiom)
c.in(c.emptyset, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_Mutil_Otiling_Oempty_0, axiom)
c.union(c.union(v_A, v_B, t_a), v_C, t_a) = c.union(v_A, c.union(v_B, v_C, t_a), t_a)    cnf(cls_Set_OUn__assoc_0, axiom)
c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_0, negated_conjecture)
c.inter(c.emptyset, v_u, t_a) = c.emptyset    cnf(cls_conjecture_1, negated_conjecture)
¬ c.in(c.union(c.emptyset, v_u, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_2, negated_conjecture)

```

**PUZ063-2.p** Problem about mutilated chessboard problem

```

c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_0, negated_conjecture)
¬ c.in(c.union(c.emptyset, v_u, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_2, negated_conjecture)
c.union(c.emptyset, v_y, t_a) = v_y    cnf(cls_Set_OUn__empty__left_0, axiom)

```

**PUZ064-1.p** Problem about mutilated chessboard problem

```
include('Axioms/MS001-1.ax')
```

```
include('Axioms/MS001-0.ax')
```

```

c.in(c.insert(c.Pair(v_i, v_j, tc_nat, tc_nat), c.insert(c.Pair(v_i, c.Suc(v_j), tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat)), t),
c.in(c.insert(c.Pair(v_i, v_j, tc_nat, tc_nat), c.insert(c.Pair(c.Suc(v_i), v_j, tc_nat, tc_nat), c.emptyset, tc_prod(tc_nat, tc_nat)), t),
(c.in(v_a, v_A, tc_set(t_a)) and c.in(v_t, c.Mutil_Otiling(v_A, t_a), tc_set(t_a)) and c.inter(v_a, v_t, t_a) = c.emptyset) =>
c.in(c.union(v_a, v_t, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_Mutil_Otiling_OUn_0, axiom)
c.in(c.emptyset, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_Mutil_Otiling_Oempty_0, axiom)
c.union(c.union(v_A, v_B, t_a), v_C, t_a) = c.union(v_A, c.union(v_B, v_C, t_a), t_a)    cnf(cls_Set_OUn__assoc_0, axiom)
c.in(v_a, v_A, tc_set(t_a))    cnf(cls_conjecture_0, negated_conjecture)
c.in(v_t, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_1, negated_conjecture)
c.inter(v_a, v_t, t_a) = c.emptyset    cnf(cls_conjecture_2, negated_conjecture)
c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_3, negated_conjecture)
c.inter(c.union(v_a, v_t, t_a), v_u, t_a) = c.emptyset    cnf(cls_conjecture_4, negated_conjecture)
¬ c.in(c.union(c.union(v_a, v_t, t_a), v_u, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_5, negated_conjecture)
(c.inter(v_t, v_u, t_a) = c.emptyset and c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))) => c.in(c.union(v_t, v_u, t_a), c.Mutil.

```

**PUZ064-2.p** Problem about mutilated chessboard problem

```

c.in(v_a, v_A, tc_set(t_a))    cnf(cls_conjecture_0, negated_conjecture)
c.inter(v_a, v_t, t_a) = c.emptyset    cnf(cls_conjecture_2, negated_conjecture)
c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_3, negated_conjecture)
c.inter(c.union(v_a, v_t, t_a), v_u, t_a) = c.emptyset    cnf(cls_conjecture_4, negated_conjecture)
¬ c.in(c.union(c.union(v_a, v_t, t_a), v_u, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_conjecture_5, negated_conjecture)
(c.inter(v_t, v_u, t_a) = c.emptyset and c.in(v_u, c.Mutil_Otiling(v_A, t_a), tc_set(t_a))) => c.in(c.union(v_t, v_u, t_a), c.Mutil.
(c.in(v_a, v_A, tc_set(t_a)) and c.in(v_t, c.Mutil_Otiling(v_A, t_a), tc_set(t_a)) and c.inter(v_a, v_t, t_a) = c.emptyset) =>
c.in(c.union(v_a, v_t, t_a), c.Mutil_Otiling(v_A, t_a), tc_set(t_a))    cnf(cls_Mutil_Otiling_OUn_0, axiom)
c.Relation_OImage(c.Relation_Odiag(v_A, t_a), v_B, t_a, t_a) = c.inter(v_A, v_B, t_a)    cnf(cls_Relation_OImage__diag_0, axiom)
c.uminus(c.minus(v_A, v_B, tc_set(t_a)), tc_set(t_a)) = c.union(c.uminus(v_A, tc_set(t_a)), v_B, t_a)    cnf(cls_Set_OCompl__L_0, axiom)
c.uminus(c.UNIV, tc_set(t_a)) = c.emptyset    cnf(cls_Set_OCompl__UNIV__eq_0, axiom)
c.minus(v_A, c.uminus(v_B, tc_set(t_a)), tc_set(t_a)) = c.inter(v_A, v_B, t_a)    cnf(cls_Set_ODiff__Compl_0, axiom)
c.inter(c.UNIV, v_y, t_a) = v_y    cnf(cls_Set_OInt__UNIV__left_0, axiom)
c.in(v_c, c.inter(v_A, v_B, t_a), t_a) => c.in(v_c, v_B, t_a)    cnf(cls_Set_OInt__iff_1, axiom)
c.union(c.minus(v_B, v_A, tc_set(t_a)), v_A, t_a) = c.union(v_B, v_A, t_a)    cnf(cls_Set_OUn__Diff__cancel2_0, axiom)
c.union(v_A, c.minus(v_B, v_A, tc_set(t_a)), t_a) = c.union(v_A, v_B, t_a)    cnf(cls_Set_OUn__Diff__cancel_0, axiom)
c.union(c.UNIV, v_B, t_a) = c.UNIV    cnf(cls_Set_OUn__UNIV__left_0, axiom)
c.union(v_y, v_y, t_a) = v_y    cnf(cls_Set_OUn__absorb_0, axiom)
c.union(c.union(v_A, v_B, t_a), v_C, t_a) = c.union(v_A, c.union(v_B, v_C, t_a), t_a)    cnf(cls_Set_OUn__assoc_0, axiom)
c.union(c.emptyset, v_y, t_a) = v_y    cnf(cls_Set_OUn__empty__left_0, axiom)
c.union(v_y, c.emptyset, t_a) = v_y    cnf(cls_Set_OUn__empty__right_0, axiom)
c.lessequals(c.union(v_A, v_B, t_a), v_C, tc_set(t_a)) => c.lessequals(v_A, v_C, tc_set(t_a))    cnf(cls_Set_OUn__subset__iff_0, axiom)
c.lessequals(c.union(v_A, v_B, t_a), v_C, tc_set(t_a)) => c.lessequals(v_B, v_C, tc_set(t_a))    cnf(cls_Set_OUn__subset__iff_1, axiom)
(c.lessequals(v_B, v_C, tc_set(t_a)) and c.lessequals(v_A, v_C, tc_set(t_a))) => c.lessequals(c.union(v_A, v_B, t_a), v_C, tc_set(t_a))
c.uminus(c.uminus(v_y, tc_set(t_a)), tc_set(t_a)) = v_y    cnf(cls_Set_Odouble__complement_0, axiom)
¬ c.in(v_c, c.emptyset, t_a)    cnf(cls_Set_Oempty__iff_0, axiom)
c.lessequals(c.emptyset, v_A, tc_set(t_a))    cnf(cls_Set_Oempty__subsetI_0, axiom)
c.in(c.Main_OsubsetI_1(v_A, v_B, t_a), v_A, t_a) or c.lessequals(v_A, v_B, tc_set(t_a))    cnf(cls_Set_OsubsetI_0, axiom)
(c.lessequals(v_B, v_A, tc_set(t_a)) and c.lessequals(v_A, v_B, tc_set(t_a))) => v_A = v_B    cnf(cls_Set_Osubset__antisym_0, axiom)

```

c.lessequals(v\_A, v\_A, tc\_set(t\_a))      cnf(cls\_Set\_Osubset\_refl0, axiom)

**PUZ065+1.p** Sudoku 13273

```
include('Axioms/PUZ005+0.ax')
ssA(n1, n2) = n1      fof(ax353, axiom)
ssA(n1, n9) = n9      fof(ax354, axiom)
ssA(n2, n4) = n3      fof(ax355, axiom)
ssA(n2, n7) = n8      fof(ax356, axiom)
ssA(n3, n7) = n6      fof(ax357, axiom)
ssA(n4, n5) = n1      fof(ax358, axiom)
ssA(n4, n6) = n2      fof(ax359, axiom)
ssA(n4, n7) = n4      fof(ax360, axiom)
ssA(n5, n1) = n7      fof(ax361, axiom)
ssA(n5, n3) = n3      fof(ax362, axiom)
ssA(n6, n1) = n5      fof(ax363, axiom)
ssA(n7, n1) = n8      fof(ax364, axiom)
ssA(n7, n4) = n6      fof(ax365, axiom)
ssA(n8, n5) = n4      fof(ax366, axiom)
ssA(n8, n8) = n2      fof(ax367, axiom)
ssA(n9, n4) = n7      fof(ax368, axiom)
ssA(n9, n8) = n5      fof(ax369, axiom)
```

**PUZ066+1.p** Sudoku 19262

```
include('Axioms/PUZ005+0.ax')
ssA(n1, n2) = n6      fof(ax353, axiom)
ssA(n1, n4) = n2      fof(ax354, axiom)
ssA(n1, n7) = n5      fof(ax355, axiom)
ssA(n2, n2) = n1      fof(ax356, axiom)
ssA(n2, n3) = n3      fof(ax357, axiom)
ssA(n2, n5) = n6      fof(ax358, axiom)
ssA(n4, n2) = n7      fof(ax359, axiom)
ssA(n4, n8) = n3      fof(ax360, axiom)
ssA(n4, n9) = n1      fof(ax361, axiom)
ssA(n5, n1) = n8      fof(ax362, axiom)
ssA(n5, n4) = n5      fof(ax363, axiom)
ssA(n6, n4) = n4      fof(ax364, axiom)
ssA(n7, n5) = n1      fof(ax365, axiom)
ssA(n7, n8) = n6      fof(ax366, axiom)
ssA(n8, n1) = n9      fof(ax367, axiom)
ssA(n8, n7) = n2      fof(ax368, axiom)
ssA(n9, n1) = n4      fof(ax369, axiom)
```

**PUZ067+1.p** Sudoku 8618

```
include('Axioms/PUZ005+0.ax')
ssA(n1, n4) = n4      fof(ax353, axiom)
ssA(n1, n5) = n7      fof(ax354, axiom)
ssA(n1, n7) = n1      fof(ax355, axiom)
ssA(n2, n2) = n5      fof(ax356, axiom)
ssA(n2, n4) = n2      fof(ax357, axiom)
ssA(n3, n2) = n3      fof(ax358, axiom)
ssA(n3, n9) = n8      fof(ax359, axiom)
ssA(n4, n1) = n6      fof(ax360, axiom)
ssA(n4, n5) = n1      fof(ax361, axiom)
ssA(n4, n7) = n7      fof(ax362, axiom)
ssA(n5, n3) = n4      fof(ax363, axiom)
ssA(n5, n6) = n3      fof(ax364, axiom)
ssA(n6, n8) = n5      fof(ax365, axiom)
ssA(n7, n4) = n7      fof(ax366, axiom)
ssA(n7, n8) = n3      fof(ax367, axiom)
ssA(n8, n1) = n1      fof(ax368, axiom)
ssA(n9, n9) = n4      fof(ax369, axiom)
```

**PUZ068+1.p** Monday's Sudoku

47 1 5 2 3 8 6 1 7 4 3 5 7 3 1 4

include('Axioms/PUZ005+0.ax')

$ssA(n_1, n_4) = n_4$     fof(ax353, axiom)  
 $ssA(n_1, n_5) = n_7$     fof(ax354, axiom)  
 $ssA(n_1, n_7) = n_1$     fof(ax355, axiom)  
 $ssA(n_2, n_2) = n_5$     fof(ax356, axiom)  
 $ssA(n_2, n_4) = n_2$     fof(ax357, axiom)  
 $ssA(n_3, n_2) = n_3$     fof(ax358, axiom)  
 $ssA(n_3, n_9) = n_8$     fof(ax359, axiom)  
 $ssA(n_4, n_1) = n_6$     fof(ax360, axiom)  
 $ssA(n_4, n_5) = n_1$     fof(ax361, axiom)  
 $ssA(n_4, n_7) = n_7$     fof(ax362, axiom)  
 $ssA(n_5, n_3) = n_4$     fof(ax363, axiom)  
 $ssA(n_5, n_6) = n_3$     fof(ax364, axiom)  
 $ssA(n_6, n_8) = n_5$     fof(ax365, axiom)  
 $ssA(n_7, n_4) = n_7$     fof(ax366, axiom)  
 $ssA(n_7, n_8) = n_3$     fof(ax367, axiom)  
 $ssA(n_8, n_1) = n_1$     fof(ax368, axiom)  
 $ssA(n_9, n_9) = n_4$     fof(ax369, axiom)

**PUZ068+2.p** Monday's Sudoku

47 1 5 2 3 8 6 1 7 4 3 5 7 3 1 4

include('Axioms/PUZ006+0.ax')

$p(n_1, n_4, n_4)$     fof(ax353, axiom)  
 $p(n_1, n_5, n_7)$     fof(ax354, axiom)  
 $p(n_1, n_7, n_1)$     fof(ax355, axiom)  
 $p(n_2, n_2, n_5)$     fof(ax356, axiom)  
 $p(n_2, n_4, n_2)$     fof(ax357, axiom)  
 $p(n_3, n_2, n_3)$     fof(ax358, axiom)  
 $p(n_3, n_9, n_8)$     fof(ax359, axiom)  
 $p(n_4, n_1, n_6)$     fof(ax360, axiom)  
 $p(n_4, n_5, n_1)$     fof(ax361, axiom)  
 $p(n_4, n_7, n_7)$     fof(ax362, axiom)  
 $p(n_5, n_3, n_4)$     fof(ax363, axiom)  
 $p(n_5, n_6, n_3)$     fof(ax364, axiom)  
 $p(n_6, n_8, n_5)$     fof(ax365, axiom)  
 $p(n_7, n_4, n_7)$     fof(ax366, axiom)  
 $p(n_7, n_8, n_3)$     fof(ax367, axiom)  
 $p(n_8, n_1, n_1)$     fof(ax368, axiom)  
 $p(n_9, n_9, n_4)$     fof(ax369, axiom)

**PUZ068-1.p** Monday's Sudoku

47 1 5 2 3 8 6 1 7 4 3 5 7 3 1 4

include('Axioms/PUZ005-0.ax')

$el(s(n_0), s(s(s(n_0))))$ ,  $s(s(s(n_0)))$     cnf(c01, axiom)  
 $el(s(n_0), s(s(s(s(s(n_0))))))$ ,  $s(s(s(s(s(n_0))))))$     cnf(c02, axiom)  
 $el(s(n_0), s(s(s(s(s(s(n_0))))))$ ,  $s(n_0)$     cnf(c03, axiom)  
 $el(s(s(n_0)), s(s(n_0)), s(s(s(s(n_0))))$     cnf(c04, axiom)  
 $el(s(s(n_0)), s(s(s(n_0))), s(s(n_0)))$     cnf(c05, axiom)  
 $el(s(s(s(n_0))), s(s(n_0)), s(s(s(n_0))))$     cnf(c06, axiom)  
 $el(s(s(s(n_0))), s(s(s(s(s(s(s(n_0))))))$ ,  $s(s(s(s(s(s(s(n_0))))))$     cnf(c07, axiom)  
 $el(s(s(s(s(n_0))))$ ,  $s(n_0)$ ,  $s(s(s(s(s(n_0))))$     cnf(c08, axiom)  
 $el(s(s(s(s(n_0))))$ ,  $s(s(s(s(n_0))))$ ,  $s(n_0)$     cnf(c09, axiom)  
 $el(s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(n_0))))$     cnf(c10, axiom)  
 $el(s(s(s(s(s(n_0))))$ ,  $s(s(s(n_0)))$ ,  $s(s(s(s(n_0))))$     cnf(c11, axiom)  
 $el(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(n_0))))$ ,  $s(s(s(n_0)))$     cnf(c12, axiom)  
 $el(s(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(n_0))))$     cnf(c13, axiom)  
 $el(s(s(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(n_0))))$     cnf(c14, axiom)  
 $el(s(s(s(s(s(s(s(s(n_0))))$ ,  $s(s(s(s(s(s(s(n_0))))$ ,  $s(s(s(n_0)))$     cnf(c15, axiom)





$ssA(n_4, n_8) = n_3$      $fof(ax_{361}, axiom)$   
 $ssA(n_5, n_1) = n_5$      $fof(ax_{362}, axiom)$   
 $ssA(n_5, n_5) = n_4$      $fof(ax_{363}, axiom)$   
 $ssA(n_5, n_7) = n_2$      $fof(ax_{364}, axiom)$   
 $ssA(n_7, n_1) = n_2$      $fof(ax_{365}, axiom)$   
 $ssA(n_7, n_7) = n_8$      $fof(ax_{366}, axiom)$   
 $ssA(n_8, n_4) = n_1$      $fof(ax_{367}, axiom)$   
 $ssA(n_8, n_6) = n_3$      $fof(ax_{368}, axiom)$   
 $ssA(n_9, n_4) = n_7$      $fof(ax_{369}, axiom)$

**PUZ071-1.p** Thursday's Sudoku

45 8 71 2 16 3 5 4 2

2 8 1 3

include('Axioms/PUZ005-0.ax')

$el(s(n_0), s(n_0), s(s(s(s(n_0))))))$      $cnf(c_{01}, axiom)$   
 $el(s(n_0), s(s(n_0)), s(s(s(s(s(n_0))))))$      $cnf(c_{02}, axiom)$   
 $el(s(n_0), s(s(s(s(s(s(n_0)))))), s(s(s(s(s(s(s(n_0))))))))$      $cnf(c_{03}, axiom)$   
 $el(s(s(n_0)), s(s(s(s(s(s(s(s(n_0))))))))$      $cnf(c_{04}, axiom)$   
 $el(s(s(n_0)), s(s(s(s(s(s(s(s(n_0))))))))$      $cnf(c_{05}, axiom)$   
 $el(s(s(s(n_0))), s(s(n_0)), s(s(n_0)))$      $cnf(c_{06}, axiom)$   
 $el(s(s(s(s(n_0))), s(s(s(n_0))), s(n_0))$      $cnf(c_{07}, axiom)$   
 $el(s(s(s(s(s(n_0))), s(s(s(s(n_0))))), s(s(s(s(s(n_0))))))$      $cnf(c_{08}, axiom)$   
 $el(s(s(s(s(s(n_0))), s(s(s(s(s(s(n_0))))))))$      $cnf(c_{09}, axiom)$   
 $el(s(s(s(s(s(n_0))))), s(n_0), s(s(s(s(s(n_0))))))$      $cnf(c_{10}, axiom)$   
 $el(s(s(s(s(s(n_0))))), s(s(s(s(s(n_0))))), s(s(s(s(n_0))))$      $cnf(c_{11}, axiom)$   
 $el(s(s(s(s(s(n_0))))), s(s(s(s(s(s(n_0))))))$      $cnf(c_{12}, axiom)$   
 $el(s(s(s(s(s(s(n_0)))))), s(n_0), s(s(n_0)))$      $cnf(c_{13}, axiom)$   
 $el(s(s(s(s(s(s(s(n_0)))))), s(s(s(s(s(s(s(n_0))))))$      $cnf(c_{14}, axiom)$   
 $el(s(s(s(s(s(s(s(n_0)))))), s(s(s(s(n_0))))$      $cnf(c_{15}, axiom)$   
 $el(s(s(s(s(s(s(s(s(n_0)))))), s(s(s(s(s(n_0))))$      $cnf(c_{16}, axiom)$   
 $el(s(s(s(s(s(s(s(s(s(n_0)))))), s(s(s(s(s(s(n_0))))))$      $cnf(c_{17}, axiom)$

**PUZ072+1.p** Friday's Sudoku

1 9 3 8 6 124 7 3 5 8 6 4 2 7 5

include('Axioms/PUZ005+0.ax')

$ssA(n_1, n_2) = n_1$      $fof(ax_{353}, axiom)$   
 $ssA(n_1, n_9) = n_9$      $fof(ax_{354}, axiom)$   
 $ssA(n_2, n_4) = n_3$      $fof(ax_{355}, axiom)$   
 $ssA(n_2, n_7) = n_8$      $fof(ax_{356}, axiom)$   
 $ssA(n_3, n_7) = n_6$      $fof(ax_{357}, axiom)$   
 $ssA(n_4, n_5) = n_1$      $fof(ax_{358}, axiom)$   
 $ssA(n_4, n_6) = n_2$      $fof(ax_{359}, axiom)$   
 $ssA(n_4, n_7) = n_4$      $fof(ax_{360}, axiom)$   
 $ssA(n_5, n_1) = n_7$      $fof(ax_{361}, axiom)$   
 $ssA(n_5, n_3) = n_3$      $fof(ax_{362}, axiom)$   
 $ssA(n_6, n_1) = n_5$      $fof(ax_{363}, axiom)$   
 $ssA(n_7, n_1) = n_8$      $fof(ax_{364}, axiom)$   
 $ssA(n_7, n_4) = n_6$      $fof(ax_{365}, axiom)$   
 $ssA(n_8, n_5) = n_4$      $fof(ax_{366}, axiom)$   
 $ssA(n_8, n_8) = n_2$      $fof(ax_{367}, axiom)$   
 $ssA(n_9, n_4) = n_7$      $fof(ax_{368}, axiom)$   
 $ssA(n_9, n_8) = n_5$      $fof(ax_{369}, axiom)$

**PUZ072-1.p** Friday's Sudoku

1 9 3 8 6 124 7 3 5 8 6 4 2 7 5

include('Axioms/PUZ005-0.ax')

$el(s(n_0), s(s(n_0)), s(n_0))$      $cnf(c_{01}, axiom)$   
 $el(s(n_0), s(s(s(s(s(s(s(s(n_0))))))))$      $cnf(c_{02}, axiom)$   
 $el(s(s(n_0)), s(s(s(n_0))), s(s(s(n_0))))$      $cnf(c_{03}, axiom)$   
 $el(s(s(n_0)), s(s(s(s(s(s(s(n_0))))))$      $cnf(c_{04}, axiom)$   
 $el(s(s(s(n_0))), s(s(s(s(s(s(n_0))))))$      $cnf(c_{05}, axiom)$

```

el(s(s(s(s(n0))))), s(s(s(s(s(n0))))), s(n0))    cnf(c06, axiom)
el(s(s(s(s(n0))))), s(s(s(s(s(s(n0)))))), s(s(n0))    cnf(c07, axiom)
el(s(s(s(s(n0))))), s(s(s(s(s(s(s(n0))))))), s(s(s(s(n0))))    cnf(c08, axiom)
el(s(s(s(s(s(n0))))), s(n0), s(s(s(s(s(s(s(n0))))))))    cnf(c09, axiom)
el(s(s(s(s(s(n0))))), s(s(s(n0))), s(s(s(n0))))    cnf(c10, axiom)
el(s(s(s(s(s(s(n0))))), s(n0), s(s(s(s(s(s(n0)))))))    cnf(c11, axiom)
el(s(s(s(s(s(s(s(n0))))), s(n0), s(s(s(s(s(s(s(n0))))))))    cnf(c12, axiom)
el(s(s(s(s(s(s(s(n0))))), s(s(s(s(n0))))), s(s(s(s(s(s(n0)))))))    cnf(c13, axiom)
el(s(s(s(s(s(s(s(n0))))), s(s(s(s(n0))))), s(s(s(s(n0))))    cnf(c14, axiom)
el(s(s(s(s(s(s(s(s(n0))))), s(s(s(s(s(s(s(n0))))))))), s(s(n0))    cnf(c15, axiom)
el(s(s(s(s(s(s(s(s(s(n0))))), s(s(s(s(n0))))), s(s(s(s(s(s(s(n0))))))))    cnf(c16, axiom)
el(s(s(s(s(s(s(s(s(s(s(n0))))), s(s(s(s(s(s(s(s(n0))))))))), s(s(s(s(n0))))    cnf(c17, axiom)

```

### PUZ078+1.p Leo the Liar

Leo the Liar is a strange liar. On six days of the week, he lies. On the seventh day he always tells the truth. He made the following statements on three consecutive days: Day 1. I lie on Monday and Tuesday. Day 2. Today it is Thursday, Saturday, or Sunday. Day 3. I lie on Wednesday and Friday. Prove that Leo tells the truth on Tuesday.

```

∀x: (x = monday or x = tuesday or x = wednesday or x = thursday or x = friday or x = saturday or x = sunday)    fof(days_of_week, axiom)

```

```

monday ≠ tuesday and monday ≠ wednesday and monday ≠ thursday and monday ≠ friday and monday ≠ saturday and monday ≠ sunday
and tuesday ≠ wednesday and tuesday ≠ thursday and tuesday ≠ friday and tuesday ≠ saturday and tuesday ≠ sunday
and wednesday ≠ thursday and wednesday ≠ friday and wednesday ≠ saturday and wednesday ≠ sunday and thursday ≠ friday
and thursday ≠ saturday and thursday ≠ sunday and friday ≠ saturday and friday ≠ sunday and saturday ≠ sunday    fof(uniqunames, axiom)

```

```

∀x, y: ((truthday(x) and truthday(y)) ⇒ x = y)    fof(truth_unique, axiom)

```

```

∀x, y: ((day1(x) and day1(y)) ⇒ x = y)    fof(day1_unique, axiom)

```

```

∀x, y: ((day2(x) and day2(y)) ⇒ x = y)    fof(day2_unique, axiom)

```

```

∀x, y: ((day3(x) and day3(y)) ⇒ x = y)    fof(day3_unique, axiom)

```

```

∃x: truthday(x)    fof(truth_once, axiom)

```

```

∃x: day1(x)    fof(day1_once, axiom)

```

```

∃x: day2(x)    fof(day2_once, axiom)

```

```

∃x: day3(x)    fof(day3_once, axiom)

```

```

consecutive(sunday) = monday    fof(monday_follows_sunday, axiom)

```

```

consecutive(monday) = tuesday    fof(tuesday_follows_monday, axiom)

```

```

consecutive(tuesday) = wednesday    fof(wednesday_follows_tuesday, axiom)

```

```

consecutive(wednesday) = thursday    fof(thursday_follows_wednesday, axiom)

```

```

consecutive(thursday) = friday    fof(friday_follows_thursday, axiom)

```

```

consecutive(friday) = saturday    fof(saturday_follows_friday, axiom)

```

```

consecutive(saturday) = sunday    fof(sunday_follows_saturday, axiom)

```

```

∀x: (day1(x) ⇔ day2(consecutive(x)))    fof(day2_follows_day1, axiom)

```

```

∀x: (day2(x) ⇔ day3(consecutive(x)))    fof(day3_follows_day2, axiom)

```

```

∀x: (day1(x) ⇒ (truthday(x) ⇔ (¬truthday(monday) and ¬truthday(tuesday))))    fof(statement1, axiom)

```

```

∀x: (day2(x) ⇒ (truthday(x) ⇔ (day2(thursday) or day2(saturday) or day2(sunday))))    fof(statement2, axiom)

```

```

∀x: (day3(x) ⇒ (truthday(x) ⇔ (¬truthday(wednesday) and ¬truthday(friday))))    fof(statement3, axiom)

```

```

truthday(tuesday)    fof(tuesday_leo_tells_truths, conjecture)

```

### PUZ079+2.p Another Sudoku

```

1 6 4 9 5 7 4 9 5 47 53 3 1 91 68 7 5 2 9 3 2 1 6 4

```

```

include('Axioms/PUZ006+0.ax')

```

```

p(n1, n1, n1)    fof(ax353, axiom)

```

```

p(n1, n4, n6)    fof(ax354, axiom)

```

```

p(n1, n6, n4)    fof(ax355, axiom)

```

```

p(n1, n9, n9)    fof(ax356, axiom)

```

```

p(n2, n4, n5)    fof(ax357, axiom)

```

```

p(n2, n6, n7)    fof(ax358, axiom)

```

```

p(n3, n3, n4)    fof(ax359, axiom)

```

```

p(n3, n5, n9)    fof(ax360, axiom)

```

```

p(n3, n7, n5)    fof(ax369, axiom)

```

```

p(n4, n1, n4)    fof(ax361, axiom)

```

```

p(n4, n2, n7)    fof(ax362, axiom)

```



$p(n_4, n_8, n_5)$     fof(ax363, axiom)  
 $p(n_4, n_9, n_3)$     fof(ax364, axiom)  
 $p(n_5, n_3, n_3)$     fof(ax365, axiom)  
 $p(n_5, n_7, n_1)$     fof(ax366, axiom)  
 $p(n_6, n_1, n_9)$     fof(ax367, axiom)  
 $p(n_6, n_2, n_1)$     fof(ax368, axiom)  
 $p(n_6, n_8, n_6)$     fof(ax369<sub>1</sub>, axiom)  
 $p(n_6, n_9, n_8)$     fof(ax369<sub>2</sub>, axiom)  
 $p(n_7, n_3, n_7)$     fof(ax369<sub>3</sub>, axiom)  
 $p(n_7, n_5, n_5)$     fof(ax369<sub>4</sub>, axiom)  
 $p(n_7, n_7, n_2)$     fof(ax369<sub>5</sub>, axiom)  
 $p(n_8, n_4, n_9)$     fof(ax369<sub>6</sub>, axiom)  
 $p(n_8, n_6, n_3)$     fof(ax369<sub>7</sub>, axiom)  
 $p(n_9, n_1, n_2)$     fof(ax369<sub>8</sub>, axiom)  
 $p(n_9, n_4, n_1)$     fof(ax369<sub>9</sub>, axiom)  
 $p(n_9, n_6, n_6)$     fof(ax369<sub>10</sub>, axiom)  
 $p(n_9, n_9, n_4)$     fof(ax369<sub>11</sub>, axiom)

### PUZ080+2.p Another Sudoku

3 1 7 9 6 9 4 4 8 6 2 4 7 1 67 21 3 1 9 7 8 3

include('Axioms/PUZ006+0.ax')

$p(n_1, n_4, n_3)$     fof(ax143, axiom)  
 $p(n_1, n_6, n_1)$     fof(ax161, axiom)  
 $p(n_2, n_2, n_7)$     fof(ax227, axiom)  
 $p(n_2, n_5, n_9)$     fof(ax259, axiom)  
 $p(n_2, n_8, n_6)$     fof(ax286, axiom)  
 $p(n_3, n_1, n_9)$     fof(ax319, axiom)  
 $p(n_3, n_9, n_4)$     fof(ax394, axiom)  
 $p(n_4, n_1, n_4)$     fof(ax414, axiom)  
 $p(n_4, n_4, n_8)$     fof(ax448, axiom)  
 $p(n_4, n_6, n_6)$     fof(ax466, axiom)  
 $p(n_4, n_9, n_2)$     fof(ax492, axiom)  
 $p(n_5, n_4, n_4)$     fof(ax544, axiom)  
 $p(n_5, n_6, n_7)$     fof(ax567, axiom)  
 $p(n_6, n_5, n_1)$     fof(ax651, axiom)  
 $p(n_7, n_3, n_6)$     fof(ax736, axiom)  
 $p(n_7, n_4, n_7)$     fof(ax747, axiom)  
 $p(n_7, n_6, n_2)$     fof(ax762, axiom)  
 $p(n_7, n_7, n_1)$     fof(ax771, axiom)  
 $p(n_8, n_1, n_3)$     fof(ax813, axiom)  
 $p(n_8, n_3, n_1)$     fof(ax831, axiom)  
 $p(n_8, n_7, n_9)$     fof(ax879, axiom)  
 $p(n_8, n_9, n_7)$     fof(ax897, axiom)  
 $p(n_9, n_3, n_8)$     fof(ax938, axiom)  
 $p(n_9, n_7, n_3)$     fof(ax973, axiom)

### PUZ081^1.p 1 of <http://philosophy.hku.hk/think/logic/knight.php>

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, 'Neither Zoey nor I are knaves'. Who is a knight and who is a knave?

islander: \$i    thf(islander, type)

knight: \$i    thf(knight, type)

knave: \$i    thf(knave, type)

says: \$i → \$o → \$o    thf(says, type)

zoey: \$i    thf(zoey, type)

mel: \$i    thf(mel, type)

is\_a: \$i → \$i → \$o    thf(is\_a, type)

$\forall x: \$i: ((\text{is\_a}@x@\text{islander}) \Rightarrow (\text{is\_a}@x@\text{knight} \text{ or } \text{is\_a}@x@\text{knave}))$     thf(kk\_6<sub>1</sub>, axiom)

$\forall x: \$i: ((\text{is\_a}@x@\text{knight}) \Rightarrow \forall a: \$o: ((\text{says}@x@a) \Rightarrow a))$     thf(kk\_6<sub>2</sub>, axiom)

$\forall x: \$i: ((\text{is\_a}@x@\text{knave}) \Rightarrow \forall a: \$o: ((\text{says}@x@a) \Rightarrow \neg a))$     thf(kk\_6<sub>3</sub>, axiom)

```

is_a@zoey@islander and is_a@mel@islander    thf(kk_64, axiom)
says@zoey@(is_a@mel@knave)    thf(kk_65, axiom)
says@mel@¬is_a@zoey@knave or is_a@mel@knave    thf(kk_66, axiom)
∃y: $i, z: $i: ((y = knight < > y = knave) and (z = knight < > z = knave) and is_a@mel@y and is_a@zoey@z)    thf(query,

```

**PUZ081^2.p** 1 of <http://philosophy.hku.hk/think/logic/knight.php>

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, ‘Neither Zoey nor I are knaves’. Who is a knight and who is a knave?

```

mel: $i    thf(mel_type, type)
zoey: $i    thf(zoey_type, type)
knight: $i → $o    thf(knight_type, type)
knave: $i → $o    thf(knave_type, type)
says: $i → $o → $o    thf(says_type, type)
∀p: $i: (knight@p) < > (knave@p)    thf(knights_xor_knaves, axiom)
∀p: $i, s: $o: ((knight@p and says@p@s) ⇒ s)    thf(knights_tell_truth, axiom)
∀p: $i, s: $o: ((knave@p and says@p@s) ⇒ ¬s)    thf(knaves_lie, axiom)
says@zoey@(knave@mel)    thf(zoey_speaks, axiom)
says@mel@(¬knave@zoey and ¬knave@mel)    thf(mel_speaks, axiom)
∃tZ: $i → $o, tM: $i → $o: (tZ@zoey and tM@mel)    thf(what_are_zoey_and_mel, conjecture)

```

**PUZ081^3.p** 1 of <http://philosophy.hku.hk/think/logic/knight.php>

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, ‘Neither Zoey nor I are knaves’. Who is a knight and who is a knave?

```

mel: $i    thf(mel_type, type)
zoey: $i    thf(zoey_type, type)
knight: $i → $o    thf(knight_type, type)
knave: $i → $o    thf(knave_type, type)
says: $i → $o → $o    thf(says_type, type)
∀p: $i: (knight@p) < > (knave@p)    thf(knights_xor_knaves, axiom)
∀p: $i, s: $o: ((knight@p and says@p@s) ⇒ s)    thf(knights_tell_truth, axiom)
∀p: $i, s: $o: ((knave@p and says@p@s) ⇒ ¬s)    thf(knaves_lie, axiom)
says@zoey@(knave@mel)    thf(zoey_speaks, axiom)
says@mel@(¬knave@zoey and ¬knave@mel)    thf(mel_speaks, axiom)
∃knight: $i, knave: $i: (knight@knight and knave@knave)    thf(who_is_knight_and_knave, conjecture)

```

**PUZ082^1.p** Peter the liar

Peter says that everything he says is false. Show that not everything Peter says is false.

```

peter: $i    thf(peter, type)
says: $i → $o → $o    thf(says, type)
says@peter@∀x: $o: ((says@peter@x) ⇒ ¬x)    thf(ax1, axiom)
¬∀x: $o: ((says@peter@x) ⇒ ¬x)    thf(thm, conjecture)

```

**PUZ083^1.p** Peter the untruthful

Peter says that everything he says is false. Show that not everything Peter says is true.

```

peter: $i    thf(peter, type)
says: $i → $o → $o    thf(says, type)
says@peter@∀x: $o: ((says@peter@x) ⇒ ¬x)    thf(ax1, axiom)
¬∀x: $o: ((says@peter@x) ⇒ x)    thf(thm, conjecture)

```

**PUZ084^1.p** The friends puzzle - reflexivity for Peter’s wife

(i) Peter is a friend of John, so if Peter knows that John knows something then John knows that Peter knows the same thing. (ii) Peter is married, so if Peter’s wife knows something, then Peter knows the same thing. John and Peter have an appointment, let us consider the following situation: (a) Peter knows the time of their appointment. (b) Peter also knows that John knows the place of their appointment. Moreover, (c) Peter’s wife knows that if Peter knows the time of their appointment, then John knows that too (since John and Peter are friends). Finally, (d) Peter knows that if John knows the place and the time of their appointment, then John knows that he has an appointment. From this situation we want to prove (e) that each of the two friends knows that the other one knows that he has an appointment.

```

include('Axioms/LCL013^0.ax')
peter: $i → $i → $o    thf(peter, type)

```

```

john: $i → $i → $o    thf(john, type)
wife: ($i → $i → $o) → $i → $i → $o    thf(wife, type)
mreflexive@peter    thf(refl_peter, axiom)
mreflexive@john    thf(refl_john, axiom)
mreflexive@(wife@peter)    thf(refl_wife_peter, axiom)
mtransitive@peter    thf(trans_peter, axiom)
mtransitive@john    thf(trans_john, axiom)
mtransitive@(wife@peter)    thf(trans_wife_peter, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@(wife@peter)@a)@a))    thf(conj, conjecture)

```

**PUZ085**<sup>1.p</sup> The friends puzzle - transitivity for Peter's wife

(i) Peter is a friend of John, so if Peter knows that John knows something then John knows that Peter knows the same thing. (ii) Peter is married, so if Peter's wife knows something, then Peter knows the same thing. John and Peter have an appointment, let us consider the following situation: (a) Peter knows the time of their appointment. (b) Peter also knows that John knows the place of their appointment. Moreover, (c) Peter's wife knows that if Peter knows the time of their appointment, then John knows that too (since John and Peter are friends). Finally, (d) Peter knows that if John knows the place and the time of their appointment, then John knows that he has an appointment. From this situation we want to prove (e) that each of the two friends knows that the other one knows that he has an appointment.

```

include('Axioms/LCL013^0.ax')
peter: $i → $i → $o    thf(peter, type)
john: $i → $i → $o    thf(john, type)
wife: ($i → $i → $o) → $i → $i → $o    thf(wife, type)
mreflexive@peter    thf(refl_peter, axiom)
mreflexive@john    thf(refl_john, axiom)
mreflexive@(wife@peter)    thf(refl_wife_peter, axiom)
mtransitive@peter    thf(trans_peter, axiom)
mtransitive@john    thf(trans_john, axiom)
mtransitive@(wife@peter)    thf(trans_wife_peter, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@(wife@peter)@a)@(mbox@(wife@peter)@(mbox@(wife@peter)@a))))

```

**PUZ086**<sup>1.p</sup> The friends puzzle - they both know

(i) Peter is a friend of John, so if Peter knows that John knows something then John knows that Peter knows the same thing. (ii) Peter is married, so if Peter's wife knows something, then Peter knows the same thing. John and Peter have an appointment, let us consider the following situation: (a) Peter knows the time of their appointment. (b) Peter also knows that John knows the place of their appointment. Moreover, (c) Peter's wife knows that if Peter knows the time of their appointment, then John knows that too (since John and Peter are friends). Finally, (d) Peter knows that if John knows the place and the time of their appointment, then John knows that he has an appointment. From this situation we want to prove (e) that each of the two friends knows that the other one knows that he has an appointment.

```

include('Axioms/LCL013^0.ax')
peter: $i → $i → $o    thf(peter, type)
john: $i → $i → $o    thf(john, type)
wife: ($i → $i → $o) → $i → $i → $o    thf(wife, type)
mreflexive@peter    thf(refl_peter, axiom)
mreflexive@john    thf(refl_john, axiom)
mreflexive@(wife@peter)    thf(refl_wife_peter, axiom)
mtransitive@peter    thf(trans_peter, axiom)
mtransitive@john    thf(trans_john, axiom)
mtransitive@(wife@peter)    thf(trans_wife_peter, axiom)
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@peter@(mbox@john@a)@(mbox@john@(mbox@peter@a))))    thf(
mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@(wife@peter)@a)@(mbox@peter@a))    thf(ax_ii, axiom)
time: $i → $o    thf(time, type)
place: $i → $o    thf(place, type)
appointment: $i → $o    thf(appointment, type)
mvalid@(mbox@peter@time)    thf(ax_a, axiom)
mvalid@(mbox@peter@(mbox@john@place))    thf(ax_b, axiom)
mvalid@(mbox@(wife@peter)@(mimplies@(mbox@peter@time)@(mbox@john@time)))    thf(ax_c, axiom)
mvalid@(mbox@peter@(mbox@john@(mimplies@(mand@place@time)@appointment)))    thf(ax_d, axiom)
mvalid@(mand@(mbox@peter@(mbox@john@appointment))@(mbox@john@(mbox@peter@appointment)))    thf(conj, conj)

```

**PUZ088**<sup>5.p</sup> TPS problem THM68

If everyone likes Bruce and Lyle likes everyone who likes someone then someone likes everyone.

cLIKES: \$i → \$i → \$o    thf(cLIKES, type)

cLYLE: \$i    thf(cLYLE, type)

cBRUCE: \$i    thf(cBRUCE, type)

$(\forall x: \$i: (cLIKES@x@cBRUCE) \text{ and } \forall y: \$i: (\exists z: \$i: (cLIKES@y@z) \Rightarrow (cLIKES@cLYLE@y))) \Rightarrow \exists u: \$i: \forall v: \$i: (cLIKES@u@v)$

**PUZ090**<sup>5.p</sup> TPS problem THM210

Lewis Carroll's problem of the winds and windows; from the Ninth Paper on Logic, November 1892.

cOPEN\_WINDOW: \$o    thf(cOPEN\_WINDOW, type)

cEAST: \$o    thf(cEAST, type)

cRHEUMATIC: \$o    thf(cRHEUMATIC, type)

cFOGGY: \$o    thf(cFOGGY, type)

cOPEN\_DOOR: \$o    thf(cOPEN\_DOOR, type)

cFLUTE: \$o    thf(cFLUTE, type)

cSUNSHINE: \$o    thf(cSUNSHINE, type)

cFIRE: \$o    thf(cFIRE, type)

cGUSTY: \$o    thf(cGUSTY, type)

cHEADACHE: \$o    thf(cHEADACHE, type)

cSMOKES: \$o    thf(cSMOKES, type)

cCOLD: \$o    thf(cCOLD, type)

cSMOKE: \$o    thf(cSMOKE, type)

$((cEAST \Rightarrow cSUNSHINE) \text{ and } ((cCOLD \text{ and } cFOGGY) \Rightarrow cFLUTE) \text{ and } ((cFIRE \text{ and } cSMOKE) \Rightarrow cOPEN\_DOOR) \text{ and } cFIRE) \text{ and } ((cEAST \text{ and } cGUSTY) \Rightarrow cSMOKES) \text{ and } (cOPEN\_DOOR \Rightarrow \neg cHEADACHE) \text{ and } (cFOGGY \Rightarrow \neg cOPEN\_WINDOW) \text{ and } ((\neg cGUSTY \text{ and } cFIRE \text{ and } \neg cOPEN\_DOOR) \Rightarrow \neg cRHEUMATIC) \text{ and } (cSUNSHINE \Rightarrow cFOGGY) \text{ and } (cFLUTE \Rightarrow \neg cOPEN\_DOOR) \text{ and } ((cFOGGY \text{ and } cEAST) \Rightarrow cRHEUMATIC)) \Rightarrow (cEAST \Rightarrow \neg cOPEN\_WINDOW)$     thf(cTHM<sub>210</sub>, conjecture)

**PUZ091**<sup>5.p</sup> TPS problem from BASIC-FO-THMS

cIN\_BED\_BEFORE\_FOUR: \$o    thf(cIN\_BED\_BEFORE\_FOUR, type)

cRISES\_AT\_FIVE: \$o    thf(cRISES\_AT\_FIVE, type)

cLOGICIAN: \$o    thf(cLOGICIAN, type)

cEARNEST: \$o    thf(cEARNEST, type)

cGAMBLER: \$o    thf(cGAMBLER, type)

cLIKELY\_LOSE\_MONEY: \$o    thf(cLIKELY\_LOSE\_MONEY, type)

cLIVELY: \$o    thf(cLIVELY, type)

cRAVENOUS: \$o    thf(cRAVENOUS, type)

cBETTER\_TAKE\_TO\_CAB\_DRIVING: \$o    thf(cBETTER\_TAKE\_TO\_CAB\_DRIVING, type)

cEATS\_PORKCHOPS: \$o    thf(cEATS\_PORKCHOPS, type)

cHAS\_LOST\_MONEY: \$o    thf(cHAS\_LOST\_MONEY, type)

$((cLOGICIAN \text{ and } cEATS\_PORKCHOPS) \Rightarrow cLIKELY\_LOSE\_MONEY) \text{ and } ((cGAMBLER \text{ and } \neg cRAVENOUS) \Rightarrow cLIKELY\_LOSE\_MONEY) \text{ and } ((\neg cLIVELY \text{ and } cHAS\_LOST\_MONEY \text{ and } cLIKELY\_LOSE\_MONEY) \Rightarrow cRISES\_AT\_FIVE) \text{ and } (cRAVENOUS) \text{ and } ((cLIVELY \text{ and } cIN\_BED\_BEFORE\_FOUR) \Rightarrow cBETTER\_TAKE\_TO\_CAB\_DRIVING) \text{ and } ((cRAVENOUS \text{ and } cEATS\_PORKCHOPS) \text{ and } ((cLOGICIAN \text{ and } cLIKELY\_LOSE\_MONEY) \Rightarrow cBETTER\_TAKE\_TO\_CAB\_DRIVING) \text{ and } \neg cLIKELY\_LOSE\_MONEY) \text{ and } ((\neg cGAMBLER \text{ and } \neg cRAVENOUS) \Rightarrow cLIVELY) \text{ and } ((cLIVELY \text{ and } cLOGICIAN \text{ and } \neg cLIKELY\_LOSE\_MONEY) \text{ and } ((cRAVENOUS \text{ and } cEARNEST) \Rightarrow \neg cBETTER\_TAKE\_TO\_CAB\_DRIVING) \text{ and } ((cGAMBLER \text{ and } \neg cIN\_BED\_BEFORE\_FOUR) \text{ and } ((cHAS\_LOST\_MONEY \text{ and } \neg cEATS\_PORKCHOPS \text{ and } \neg cRISES\_AT\_FIVE) \Rightarrow \neg cBETTER\_TAKE\_TO\_CAB\_DRIVING) \text{ and } ((cGAMBLER \text{ and } cIN\_BED\_BEFORE\_FOUR \text{ and } \neg cRAVENOUS) \Rightarrow \neg cBETTER\_TAKE\_TO\_CAB\_DRIVING) \text{ and } ((cRAVENOUS \text{ and } \neg cLIVELY \text{ and } \neg cLIKELY\_LOSE\_MONEY) \Rightarrow cGAMBLER)) \Rightarrow ((cEARNEST \text{ and } cLOGICIAN) \Rightarrow (cRISES\_AT\_FIVE \text{ and } \neg cIN\_BED\_BEFORE\_FOUR))$     thf(cPO

**PUZ092**<sup>5.p</sup> TPS problem from BASIC-HO-EQ-THMS

$\forall aCRES: \$i, mRSACRES: \$i, bBARRY: \$i, mRSBARRY: \$i, cOLE: \$i, mRSCOLE: \$i, dDIX: \$i, mRSDIX: \$i, eEDEN: \$i, mRSEDEDEN, mRSACRES \text{ and } bBARRY = mRSBARRY \text{ and } eEDEN = mRSHALL) \Rightarrow cOLE = mRSDIX) \text{ and } ((aCRES = mRSACRES \text{ and } hHALL = mRSHALL \text{ and } bBARRY = mRSCOLE) \Rightarrow dDIX \neq mRSEDEDEN) \text{ and } ((cOLE = mRSCOLE \text{ and } dDIX \neq mRSDIX \text{ and } dDIX = cOLE \text{ and } aCRES \neq mRSBARRY) \Rightarrow eEDEN \neq mRSHALL) \text{ and } ((aCRES = mRSACRES \text{ and } dDIX = mRSDIX \text{ and } bBARRY \neq mRSCOLE) \Rightarrow eEDEN = mRSHALL) \text{ and } ((eEDEN = mRSEDEDEN \text{ and } hHALL = mRSHALL \text{ and } cOLE = mRSCOLE) \Rightarrow aCRES \neq mRSBARRY) \text{ and } ((bBARRY = mRSBARRY \text{ and } cOLE = mRSCOLE \text{ and } cOLE = bBARRY \text{ and } eEDEN \neq mRSHALL) \Rightarrow dDIX = mRSEDEDEN)) \Rightarrow (aCRES \neq mRSACRES \text{ or } bBARRY \neq mRSBARRY \text{ or } cOLE = mRSCOLE \text{ or } dDIX \neq mRSDIX \text{ or } eEDEN \neq mRSEDEDEN \text{ or } hHALL \neq mRSHALL))$     thf(cSIXFRIENDS\_EASIER, conjecture)

**PUZ093**<sup>5.p</sup> TPS problem from BASIC-HO-EQ-THMS

$\exists x a: \$i, x a a: \$i, x b: \$i, x b b: \$i, x c: \$i, x c c: \$i, x d: \$i, x d d: \$i, x e: \$i, x e e: \$i, x h: \$i, x h h: \$i: \forall p: \$i \rightarrow \$i: (((p@x a) = (p@x a a) \text{ and } (p@x b) = (p@x b b) \text{ and } (p@x e) = (p@x h h)) \Rightarrow (p@x c) = (p@x d d)) \text{ and } (((p@x a) = (p@x a a) \text{ and } (p@x h) = (p@x h h) \text{ and } (p@x b) = (p@x c c)) \Rightarrow (p@x d) \neq (p@x e e)) \text{ and } (((p@x c) = (p@x c c) \text{ and } (p@x c c) = (p@x d) \text{ and } (p@x d) = (p@x d d) \text{ and } (p@x a) \neq (p@x b b)) \Rightarrow (p@x e) \neq (p@x h h)) \text{ and } (((p@x a) = (p@x a a) \text{ and } (p@x d) = (p@x d d) \text{ and } (p@x b) \neq (p@x c c)) \Rightarrow (p@x e) = (p@x h h)) \text{ and } (((p@x e) = (p@x e e) \text{ and } (p@x h) = (p@x h h) \text{ and } (p@x c) = (p@x d d)) \Rightarrow (p@x a) \neq (p@x b b)) \text{ and } (((p@x b) = (p@x b b) \text{ and } (p@x b b) = (p@x c) \text{ and } (p@x c) = (p@x c c) \text{ and } (p@x e) \neq (p@x h h)) \Rightarrow (p@x d) = (p@x e e))) \Rightarrow ((p@x a) \neq (p@x a a) \text{ or } (p@x b) \neq (p@x b b) \text{ or } (p@x c) \neq (p@x c c) \text{ or } (p@x d) \neq (p@x d d) \text{ or } (p@x e) \neq (p@x e e) \text{ or } (p@x h) \neq (p@x h h))) \quad \text{thf(cSIXFRIENDS_PTH, conjecture)}$

**PUZ094**<sup>5.p</sup> TPS problem from BASIC-FO-THMS

Reduced version of Schubert's Steamroller.

eats:  $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(eats, type)}$

grain:  $\$i \rightarrow \$o \quad \text{thf(grain, type)}$

animal:  $\$i \rightarrow \$o \quad \text{thf(animal, type)}$

snail:  $\$i \rightarrow \$o \quad \text{thf(snail, type)}$

sf:  $\$i \rightarrow \$i \quad \text{thf(sf, type)}$

plant:  $\$i \rightarrow \$o \quad \text{thf(plant, type)}$

bird:  $\$i \rightarrow \$o \quad \text{thf(bird, type)}$

wolf:  $\$i \rightarrow \$o \quad \text{thf(wolf, type)}$

fox:  $\$i \rightarrow \$o \quad \text{thf(fox, type)}$

msmaller:  $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(msmaller, type)}$

a\_grain:  $\$i \quad \text{thf(a\_grain, type)}$

a\_snail:  $\$i \quad \text{thf(a\_snail, type)}$

a\_bird:  $\$i \quad \text{thf(a\_bird, type)}$

a\_fox:  $\$i \quad \text{thf(a\_fox, type)}$

a\_wolf:  $\$i \quad \text{thf(a\_wolf, type)}$

$\neg \forall x: \$i: (\text{animal}@x \text{ or } \neg \text{wolf}@x) \text{ and } \forall x: \$i: (\text{animal}@x \text{ or } \neg \text{fox}@x) \text{ and } \forall x: \$i: (\text{animal}@x \text{ or } \neg \text{bird}@x) \text{ and } \forall x: \$i: (\text{animal}@x \text{ or } \neg \text{wolf}@x)$

**PUZ095**<sup>5.p</sup> TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$

$s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$

$cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_BLACK\_type, type)$

$cCKB\_INF: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_INF\_type, type)$

$cCKB\_INJ: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_INJ\_type, type)$

$cCKB\_XPL: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_XPL\_type, type)$

$cCKB6\_BLACK = (\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w@_c1@c1 \text{ and } \forall x j: \$i, x k: \$i: ((x w@x j@x k) \Rightarrow (x w@(s@(s@x j)))@x k) (x w@x u@x v))) \quad \text{thf}(cCKB6\_BLACK\_def, definition)$

$cCKB\_INJ = (\lambda x h: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall x x_1: \$i, x y_1: \$i, x x_2: \$i, x y_2: \$i, x u: \$i, x v: \$i: ((x h@x x_1@x y_1@x u@x v \text{ and } x h@x x_2@x y_2@x u@x v) \text{ and } x y_1 = x y_2)) \quad \text{thf}(cCKB\_INJ\_def, definition)$

$cCKB\_XPL = (\lambda x h: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, x k: \$i \rightarrow \$i \rightarrow \$o, x m: \$i, x n: \$i: (x k@x m@x n \text{ and } \forall x x: \$i, x y: \$i: ((x k@x x@x y) = (x k@x m@x n) \text{ and } \exists x u: \$i, x v: \$i: (x h@x x@x y@x u@x v \text{ and } x k@x u@x v \text{ and } \neg x u = x m \text{ and } x v = x n)))) \quad \text{thf}(cCKB\_XPL\_def, definition)$

$cCKB\_INF = (\lambda x k: \$i \rightarrow \$i \rightarrow \$o: \exists x h: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, x m: \$i, x n: \$i: (cCKB\_INJ@x h \text{ and } cCKB\_XPL@x h@x k@x m@x n)) \quad \text{thf}(cCKB\_INF@cCKB6\_BLACK)$

$cCKB6\_L_{50000}, conjecture)$

**PUZ096**<sup>5.p</sup> TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$

$c_2: \$i \quad \text{thf}(c2\_type, type)$

$c_3: \$i \quad \text{thf}(c3\_type, type)$

$c_4: \$i \quad \text{thf}(c4\_type, type)$

$g: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(g\_type, type)$

$s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$

$cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_BLACK\_type, type)$

$cCKB6\_H: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_H\_type, type)$

$cCKB\_INJ: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_INJ\_type, type)$

$cCKB6\_BLACK = (\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w@_c1@c1 \text{ and } \forall x j: \$i, x k: \$i: ((x w@x j@x k) \Rightarrow (x w@(s@(s@x j)))@x k) (x w@x u@x v))) \quad \text{thf}(cCKB6\_BLACK\_def, definition)$

$cCKB6\_H = (\lambda x x: \$i, x y: \$i, x u: \$i, x v: \$i: (cCKB6\_BLACK@x x@x y \text{ and } (((g@(s@(s@x x)))@x u@x v) = c_1 \text{ and } x u = (s@(s@(s@x x))) \text{ and } x v = (s@x y)) \text{ or } ((g@(s@(s@x x)))@x u@x v) = c_2 \text{ and } x u = (s@(s@x x)) \text{ and } x v = (s@(s@x y))) \text{ or } ((g@x u@x v) = c_3 \text{ and } x u = (s@x x) \text{ and } x v = (s@x y)) \text{ or } ((g@(s@(s@x x)))@x u@x v) = c_4 \text{ and } x u = (s@(s@x x)) \text{ and } x v = x y)))) \quad \text{thf}(cCKB6\_H\_def, definition)$

cCKB.INJ = ( $\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv \text{ and } xh@xx_2@xy_2@xu@xv \text{ and } xy_1 = xy_2)))$ ) thf(cCKB.INJ\_def, definition)  
cCKB.INJ@cCKB6.H thf(cCKB6.L25000, conjecture)

**PUZ097** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $c_2: \$i$  thf(c2\_type, type)  
 $c_3: \$i$  thf(c3\_type, type)  
 $c_4: \$i$  thf(c4\_type, type)  
 $g: \$i \rightarrow \$i \rightarrow \$i$  thf(g\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB.BLACK:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_BLACK\_type, type)  
cCKB.EVEN:  $\$i \rightarrow \$o$  thf(cCKB\_EVEN\_type, type)  
cCKB.H:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_H\_type, type)  
cCKB.INJ:  $(\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cCKB.INJ\_type, type)  
cCKB.ODD:  $\$i \rightarrow \$o$  thf(cCKB\_ODD\_type, type)  
cCKB.INJ = ( $\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv \text{ and } xh@xx_2@xy_2@xu@xv \text{ and } xy_1 = xy_2)))$ ) thf(cCKB.INJ\_def, definition)  
cCKB.EVEN = ( $\lambda xx: \$i: (xx = (s@c_1) \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1))))))$ )  
cCKB.ODD = ( $\lambda xx: \$i: (xx = c_1 \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1))))))$ )  
cCKB.BLACK = ( $\lambda xu: \$i, xv: \$i: ((cCKB.ODD@xu \text{ and } cCKB.ODD@xv) \text{ or } (cCKB.EVEN@xu \text{ and } cCKB.EVEN@xv))$ )  
cCKB.H = ( $\lambda xx: \$i, xy: \$i, xu: \$i, xv: \$i: (cCKB.BLACK@xx@xy \text{ and } ((g@(s@(s@xx))@s@xy) = c_1 \text{ and } xu = (s@(s@(s@xx))) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@s@xy) = c_2 \text{ and } xu = (s@(s@xx)) \text{ and } xv = (s@(s@xy))) \text{ or } ((g@(s@(s@xx))@s@xy) = c_3 \text{ and } xu = (s@xx) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@s@xy) = c_4 \text{ and } xu = (s@(s@xx)) \text{ and } xv = xy)))$ ) thf(cCKB.H\_type, type)  
cCKB.INJ@cCKB.H thf(cL2500, conjecture)

**PUZ098** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB.BLACK:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_BLACK\_type, type)  
cCKB.EVEN:  $\$i \rightarrow \$o$  thf(cCKB\_EVEN\_type, type)  
cCKB.FIN:  $(\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cCKB\_FIN\_type, type)  
cCKB.INF:  $(\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cCKB\_INF\_type, type)  
cCKB.INJ:  $(\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cCKB.INJ\_type, type)  
cCKB.ODD:  $\$i \rightarrow \$o$  thf(cCKB\_ODD\_type, type)  
cCKB.XPL:  $(\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_XPL\_type, type)  
cCKB.INJ = ( $\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv \text{ and } xh@xx_2@xy_2@xu@xv \text{ and } xy_1 = xy_2)))$ ) thf(cCKB.INJ\_def, definition)  
cCKB.XPL = ( $\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xk: \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (xk@xm@xn \text{ and } \forall xx: \$i, xy: \$i: ((xk@xx@xy) = xm \text{ and } xn = xy)) \text{ and } \exists xu: \$i, xv: \$i: (xh@xx@xy@xu@xv \text{ and } xk@xu@xv \text{ and } \neg xu = xm \text{ and } xv = xn))$ ) thf(cCKB.XPL\_def, definition)  
cCKB.INF = ( $\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \exists xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (cCKB.INJ@xh \text{ and } cCKB.XPL@xh@xk@xm@xn)$ )  
cCKB.FIN = ( $\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \neg cCKB.INF@xk$ ) thf(cCKB.FIN\_def, definition)  
cCKB.ODD = ( $\lambda xx: \$i: (xx = c_1 \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1))))))$ )  
cCKB.EVEN = ( $\lambda xx: \$i: (xx = (s@c_1) \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1))))))$ )  
cCKB.BLACK = ( $\lambda xu: \$i, xv: \$i: ((cCKB.ODD@xu \text{ and } cCKB.ODD@xv) \text{ or } (cCKB.EVEN@xu \text{ and } cCKB.EVEN@xv))$ )  
cCKB.FIN@cCKB.BLACK thf(cL7000, conjecture)

**PUZ099** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $c_2: \$i$  thf(c2\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB.E2:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_E2\_type, type)  
cCKB.E2 = ( $\lambda xx: \$i, xy: \$i: \forall xp: \$i \rightarrow \$o: ((xp@xx \text{ and } \forall xu: \$i: ((xp@xu) \Rightarrow (xp@(s@(s@xu)))) \Rightarrow (xp@xy)))$ ) thf(cCKB.E2\_def, definition)  
 $\neg cCKB.E2@c_1@c_2$  thf(cCKB.L38000, conjecture)

**PUZ100** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB.E2:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB\_E2\_type, type)  
cCKB.E2 = ( $\lambda xx: \$i, xy: \$i: \forall xp: \$i \rightarrow \$o: ((xp@xx \text{ and } \forall xu: \$i: ((xp@xu) \Rightarrow (xp@(s@(s@xu)))) \Rightarrow (xp@xy)))$ ) thf(cCKB.E2\_def, definition)  
 $\forall xx: \$i: (cCKB.E2@xx@xx)$  thf(cCKB.L32000, conjecture)

**PUZ101** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $c_2: \$i \quad \text{thf}(c2\_type, type)$   
 $c_3: \$i \quad \text{thf}(c3\_type, type)$   
 $c_4: \$i \quad \text{thf}(c4\_type, type)$   
 $g: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(g\_type, type)$   
 $s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_BLACK\_type, type)$   
 $cCKB6\_H: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_H\_type, type)$   
 $cCKB\_XPL: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_XPL\_type, type)$   
 $cCKB6\_BLACK = (\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w @ c_1 @ c_1 \text{ and } \forall x j: \$i, x k: \$i: ((x w @ x_j @ x_k) \Rightarrow (x w @ (s @ (s @ x_j))) @ x_k) (x w @ x u @ x v)))) \quad \text{thf}(cCKB6\_BLACK\_def, definition)$   
 $cCKB6\_H = (\lambda x x: \$i, x y: \$i, x u: \$i, x v: \$i: (cCKB6\_BLACK @ x x @ x y \text{ and } (((g @ (s @ (s @ x x))) @ (s @ x y)) = c_1 \text{ and } x u = (s @ (s @ (s @ x x)))) \text{ and } x v = (s @ x y)) \text{ or } ((g @ (s @ (s @ x x))) @ (s @ x y)) = c_2 \text{ and } x u = (s @ (s @ x x)) \text{ and } x v = (s @ (s @ x y))) \text{ or } ((g @ (s @ (s @ x x))) @ (s @ x y)) = c_3 \text{ and } x u = (s @ x x) \text{ and } x v = (s @ x y)) \text{ or } ((g @ (s @ (s @ x x))) @ (s @ x y)) = c_4 \text{ and } x u = (s @ (s @ x x)) \text{ and } x v = x y)))) \quad \text{thf}(cCKB6\_H\_def, definition)$   
 $cCKB\_XPL = (\lambda x h: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, x k: \$i \rightarrow \$i \rightarrow \$o, x m: \$i, x n: \$i: (x k @ x m @ x n \text{ and } \forall x x: \$i, x y: \$i: ((x k @ x x @ x y) = x m \text{ and } x n))) \quad \text{thf}(cCKB\_XPL\_def, definition)$   
 $cCKB\_XPL @ cCKB6\_H @ cCKB6\_BLACK @ c_1 @ c_1 \quad \text{thf}(cCKB6\_L_{40000}, conjecture)$

**PUZ102^5.p** TPS problem from CHECKERBOARD-THMS

$s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB\_E_2: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_E2\_type, type)$   
 $cCKB\_E_2 = (\lambda x x: \$i, x y: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ x x \text{ and } \forall x u: \$i: ((x p @ x u) \Rightarrow (x p @ (s @ (s @ x u)))) \Rightarrow (x p @ x y)))) \quad \text{thf}(cCKB\_E2\_def, definition)$   
 $\forall x x: \$i: (cCKB\_E_2 @ x x @ (s @ (s @ x x))) \quad \text{thf}(cCKB\_L_{33000}, conjecture)$

**PUZ103^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB6\_NUM: \$i \rightarrow \$o \quad \text{thf}(cCKB6\_NUM\_type, type)$   
 $cCKB6\_NUM = (\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))) \quad \text{thf}(cCKB6\_NUM\_def, definition)$   
 $\forall x x: \$i: ((cCKB6\_NUM @ x x) \Rightarrow (cCKB6\_NUM @ (s @ (s @ x x)))) \quad \text{thf}(cCKB6\_L_{3000}, conjecture)$

**PUZ104^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB6\_NUM: \$i \rightarrow \$o \quad \text{thf}(cCKB6\_NUM\_type, type)$   
 $cCKB6\_NUM = (\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))) \quad \text{thf}(cCKB6\_NUM\_def, definition)$   
 $\forall x x: \$i: ((cCKB6\_NUM @ x x) \Rightarrow (cCKB6\_NUM @ (s @ (s @ x x)))) \quad \text{thf}(cCKB6\_L_{4000}, conjecture)$

**PUZ105^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB6\_NUM: \$i \rightarrow \$o \quad \text{thf}(cCKB6\_NUM\_type, type)$   
 $cCKB\_E_2: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_E2\_type, type)$   
 $cCKB6\_NUM = (\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))) \quad \text{thf}(cCKB6\_NUM\_def, definition)$   
 $cCKB\_E_2 = (\lambda x x: \$i, x y: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ x x \text{ and } \forall x u: \$i: ((x p @ x u) \Rightarrow (x p @ (s @ (s @ x u)))) \Rightarrow (x p @ x y)))) \quad \text{thf}(cCKB\_E2\_def, definition)$   
 $\forall x x: \$i: ((cCKB6\_NUM @ x x) \Rightarrow (cCKB\_E_2 @ (s @ (s @ x x)))) \quad \text{thf}(cCKB\_L_{36000}, conjecture)$

**PUZ106^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $s: \$i \rightarrow \$i \quad \text{thf}(s\_type, type)$   
 $cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB6\_BLACK\_type, type)$   
 $cCKB\_E_2: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_E2\_type, type)$   
 $cCKB6\_BLACK = (\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w @ c_1 @ c_1 \text{ and } \forall x j: \$i, x k: \$i: ((x w @ x_j @ x_k) \Rightarrow (x w @ (s @ (s @ x_j))) @ x_k) (x w @ x u @ x v)))) \quad \text{thf}(cCKB6\_BLACK\_def, definition)$   
 $cCKB\_E_2 = (\lambda x x: \$i, x y: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ x x \text{ and } \forall x u: \$i: ((x p @ x u) \Rightarrow (x p @ (s @ (s @ x u)))) \Rightarrow (x p @ x y)))) \quad \text{thf}(cCKB\_E2\_def, definition)$   
 $\forall x x: \$i, x y: \$i: ((cCKB6\_BLACK @ x x @ x y) \Rightarrow (cCKB\_E_2 @ x x @ x y)) \quad \text{thf}(cCKB\_L_{37000}, conjecture)$

**PUZ107^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad \text{thf}(c1\_type, type)$   
 $cCKB\_FIN: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_FIN\_type, type)$   
 $cCKB\_INF: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_INF\_type, type)$   
 $cCKB\_INJ: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cCKB\_INJ\_type, type)$   
 $cCKB\_XPL: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cCKB\_XPL\_type, type)$

$cCKB\_INJ = (\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv \text{ and } xh@xx_2@xy_2@xu@xv \text{ and } xx_1 = xx_2 \text{ and } xy_1 = xy_2))) \quad thf(cCKB\_INJ\_def, \text{definition})$   
 $cCKB\_XPL = (\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xk: \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (xk@xm@xn \text{ and } \forall xx: \$i, xy: \$i: ((xk@xx@xy) \Rightarrow \exists xu: \$i, xv: \$i: (xh@xx@xy@xu@xv \text{ and } xk@xu@xv \text{ and } \neg xu = xm \text{ and } xv = xn)))) \quad thf(cCKB\_XPL\_def, \text{definition})$   
 $cCKB\_INF = (\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \exists xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (cCKB\_INJ@xh \text{ and } cCKB\_XPL@xh@xk@xm))$   
 $cCKB\_FIN = (\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \neg cCKB\_INF@xk) \quad thf(cCKB\_FIN\_def, \text{definition})$   
 $cCKB\_FIN@xk: \$i, xv: \$i: (xu = c_1 \text{ and } xv = c_1) \quad thf(cL_{3000}, \text{conjecture})$

**PUZ108^5.p** TPS problem from CHECKERBOARD-THMS

$s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB\_E_2: \$i \rightarrow \$i \rightarrow \$o \quad thf(cCKB\_E2\_type, \text{type})$   
 $cCKB\_E_2 = (\lambda xx: \$i, xy: \$i: \forall xp: \$i \rightarrow \$o: ((xp@xx \text{ and } \forall xu: \$i: ((xp@xu) \Rightarrow (xp@(s@(s@xu)))) \Rightarrow (xp@xy))) \quad thf(cCKB\_E2\_def, \text{definition})$   
 $\forall xx: \$i, xy: \$i: ((cCKB\_E_2@xx@xy) \Rightarrow (cCKB\_E_2@(s@xx)@(s@xy))) \quad thf(cCKB\_L_{35000}, \text{conjecture})$

**PUZ109^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad thf(cCKB6\_BLACK\_type, \text{type})$   
 $cCKB6\_BLACK = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj))@xk))) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $(xw@xu@xv)) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $\forall xj: \$i, xk: \$i: ((cCKB6\_BLACK@xj@xk) \Rightarrow (cCKB6\_BLACK@(s@xj)@(s@xk))) \quad thf(cCKB6\_L_{9000}, \text{conjecture})$

**PUZ110^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad thf(cCKB6\_BLACK\_type, \text{type})$   
 $cCKB6\_BLACK = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj))@xk))) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $(xw@xu@xv)) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $\forall xj: \$i, xk: \$i: ((cCKB6\_BLACK@xj@xk) \Rightarrow (cCKB6\_BLACK@(s@(s@xj))@xk)) \quad thf(cCKB6\_L_{8000}, \text{conjecture})$

**PUZ111^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $cS: \$i \rightarrow \$i \quad thf(cS\_type, \text{type})$   
 $cX: \$i \quad thf(cX\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB\_EVEN: \$i \rightarrow \$o \quad thf(cCKB\_EVEN\_type, \text{type})$   
 $cCKB\_ODD: \$i \rightarrow \$o \quad thf(cCKB\_ODD\_type, \text{type})$   
 $cCKB\_EVEN = (\lambda xx: \$i: (xx = (s@c_1) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1)))))) \quad thf(cCKB\_EVEN\_def, \text{definition})$   
 $cCKB\_ODD = (\lambda xx: \$i: (xx = c_1 \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1)))))) \quad thf(cCKB\_ODD\_def, \text{definition})$   
 $\forall xx: \$i: ((cCKB\_EVEN@xx) \Rightarrow (cCKB\_EVEN@(s@(s@xx)) \text{ and } cCKB\_ODD@(cS@cX))) \quad thf(cCKB\_L_{10000}, \text{conjecture})$

**PUZ112^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $cS: \$i \rightarrow \$i \quad thf(cS\_type, \text{type})$   
 $cX: \$i \quad thf(cX\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB\_EVEN: \$i \rightarrow \$o \quad thf(cCKB\_EVEN\_type, \text{type})$   
 $cCKB\_ODD: \$i \rightarrow \$o \quad thf(cCKB\_ODD\_type, \text{type})$   
 $cCKB\_EVEN = (\lambda xx: \$i: (xx = (s@c_1) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1)))))) \quad thf(cCKB\_EVEN\_def, \text{definition})$   
 $cCKB\_ODD = (\lambda xx: \$i: (xx = c_1 \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1)))) \text{ or } xx = (s@(s@(s@(s@(s@c_1)))))) \quad thf(cCKB\_ODD\_def, \text{definition})$   
 $\forall xx: \$i: ((cCKB\_ODD@xx) \Rightarrow (cCKB\_ODD@(s@(s@xx)) \text{ and } cCKB\_EVEN@(cS@cX))) \quad thf(cCKB\_L_{9000}, \text{conjecture})$

**PUZ113^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$   
 $cCKB6\_BLACK: \$i \rightarrow \$i \rightarrow \$o \quad thf(cCKB6\_BLACK\_type, \text{type})$   
 $cCKB6\_BLACK = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj))@xk))) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $(xw@xu@xv)) \quad thf(cCKB6\_BLACK\_def, \text{definition})$   
 $\forall xj: \$i, xk: \$i: ((cCKB6\_BLACK@xj@xk) \Rightarrow (cCKB6\_BLACK@(s@(s@xj))@(s@(s@xk)))) \quad thf(cCKB6\_L_{11000}, \text{conjecture})$

**PUZ114^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \quad thf(c1\_type, \text{type})$   
 $s: \$i \rightarrow \$i \quad thf(s\_type, \text{type})$



cCKB6\_BLACK:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB6\_BLACK\_type, type)  
cCKB6\_BLACK =  $(\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w @ c_1 @ c_1 \text{ and } \forall x j: \$i, x k: \$i: ((x w @ x j @ x k) \Rightarrow (x w @ (s @ (s @ x j))) @ x k) (x w @ x u @ x v))))$  thf(cCKB6\_BLACK\_def, definition)  
 $\forall x j: \$i, x k: \$i: ((cCKB6\_BLACK @ x j @ x k) \Rightarrow (cCKB6\_BLACK @ (s @ (s @ (s @ x j))) @ (s @ x k)))$  thf(cCKB6.L10000, conjecture)

**PUZ115** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_NUM:  $\$i \rightarrow \$o$  thf(cCKB6\_NUM\_type, type)  
cCKB6\_NUM =  $(\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))$  thf(cCKB6\_NUM\_def, definition)  
 $\forall x x: \$i: ((cCKB6\_NUM @ x x) \Rightarrow (s @ (s @ (s @ (s @ (s @ (s @ (s @ x x)))))))) = x x$  thf(cCKB6.L1000, conjecture)

**PUZ116** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $c_5: \$i$  thf(c5\_type, type)  
 $g: \$i \rightarrow \$i \rightarrow \$i$  thf(g\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_BLACK:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB6\_BLACK\_type, type)  
cCKB6\_BLACK =  $(\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w @ c_1 @ c_1 \text{ and } \forall x j: \$i, x k: \$i: ((x w @ x j @ x k) \Rightarrow (x w @ (s @ (s @ x j))) @ x k) (x w @ x u @ x v))))$  thf(cCKB6\_BLACK\_def, definition)  
 $\forall x x: \$i, x y: \$i: ((cCKB6\_BLACK @ x x @ x y) \Rightarrow (g @ (s @ (s @ x x)) @ (s @ x y)) \neq c_5)$  thf(cCKB.L39100, conjecture)

**PUZ117** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_NUM:  $\$i \rightarrow \$o$  thf(cCKB6\_NUM\_type, type)  
cCKB6\_NUM =  $(\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))$  thf(cCKB6\_NUM\_def, definition)  
 $\forall x x: \$i, x y: \$i: ((cCKB6\_NUM @ x x \text{ and } cCKB6\_NUM @ x y \text{ and } (s @ x x) = (s @ x y)) \Rightarrow x x = x y)$  thf(cCKB6.L2000, conjecture)

**PUZ118** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $c_2: \$i$  thf(c2\_type, type)  
 $c_3: \$i$  thf(c3\_type, type)  
 $c_4: \$i$  thf(c4\_type, type)  
 $c_5: \$i$  thf(c5\_type, type)  
 $g: \$i \rightarrow \$i \rightarrow \$i$  thf(g\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_BLACK:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cCKB6\_BLACK\_type, type)  
cCKB6\_H:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cCKB6\_H\_type, type)  
cCKB6\_BLACK =  $(\lambda x u: \$i, x v: \$i: \forall x w: \$i \rightarrow \$i \rightarrow \$o: ((x w @ c_1 @ c_1 \text{ and } \forall x j: \$i, x k: \$i: ((x w @ x j @ x k) \Rightarrow (x w @ (s @ (s @ x j))) @ x k) (x w @ x u @ x v))))$  thf(cCKB6\_BLACK\_def, definition)  
cCKB6\_H =  $(\lambda x x: \$i, x y: \$i, x u: \$i, x v: \$i: (cCKB6\_BLACK @ x x @ x y \text{ and } (((g @ (s @ (s @ x x)) @ (s @ x y)) = c_1 \text{ and } x u = (s @ (s @ (s @ x x)))) \text{ and } x v = (s @ x y)) \text{ or } ((g @ (s @ (s @ x x)) @ (s @ x y)) = c_2 \text{ and } x u = (s @ (s @ x x)) \text{ and } x v = (s @ (s @ x y))) \text{ or } ((g @ (s @ (s @ x x)) @ (s @ x y)) = c_3 \text{ and } x u = (s @ x x) \text{ and } x v = (s @ x y)) \text{ or } ((g @ (s @ (s @ x x)) @ (s @ x y)) = c_4 \text{ and } x u = (s @ (s @ x x)) \text{ and } x v = x y))))$  thf(cCKB6\_H\_def, definition)  
 $\forall x x: \$i, x y: \$i, x u: \$i, x v: \$i: ((cCKB6\_H @ x x @ x y @ x u @ x v) \Rightarrow (g @ x u @ x v) \neq c_5)$  thf(cCKB6.L31000, conjecture)

**PUZ119** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_NUM:  $\$i \rightarrow \$o$  thf(cCKB6\_NUM\_type, type)  
cCKB6\_NUM =  $(\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))$  thf(cCKB6\_NUM\_def, definition)  
 $\forall x x: \$i, x y: \$i: ((cCKB6\_NUM @ x x \text{ and } cCKB6\_NUM @ x y \text{ and } (s @ (s @ x x)) = (s @ (s @ x y))) \Rightarrow x x = x y)$  thf(cCKB6.L2500, conjecture)

**PUZ120** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)  
 $s: \$i \rightarrow \$i$  thf(s\_type, type)  
cCKB6\_NUM:  $\$i \rightarrow \$o$  thf(cCKB6\_NUM\_type, type)  
cCKB6\_NUM =  $(\lambda x x: \$i: \forall x p: \$i \rightarrow \$o: ((x p @ c_1 \text{ and } \forall x w: \$i: ((x p @ x w) \Rightarrow (x p @ (s @ x w)))) \Rightarrow (x p @ x x))$  thf(cCKB6\_NUM\_def, definition)  
 $\forall x x: \$i, x y: \$i: ((cCKB6\_NUM @ x x \text{ and } cCKB6\_NUM @ x y \text{ and } (s @ (s @ (s @ x x))) = (s @ (s @ (s @ x y)))) \Rightarrow x x = x y)$  thf(cCKB6.L26000, conjecture)

**PUZ121** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$  thf(c1\_type, type)

$s: \$i \rightarrow \$i$      $\text{thf}(s\_type, \text{type})$   
 $\text{cCKB\_BLACK}: \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB\_BLACK\_type}, \text{type})$   
 $\text{cCKB\_EVEN}: \$i \rightarrow \$o$      $\text{thf}(\text{cCKB\_EVEN\_type}, \text{type})$   
 $\text{cCKB\_ODD}: \$i \rightarrow \$o$      $\text{thf}(\text{cCKB\_ODD\_type}, \text{type})$   
 $\text{cCKB\_EVEN} = (\lambda xx: \$i: (xx = (s@c_1) \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1))))))$   
 $\text{cCKB\_ODD} = (\lambda xx: \$i: (xx = c_1 \text{ or } xx = (s@(s@c_1)) \text{ or } xx = (s@(s@(s@c_1))) \text{ or } xx = (s@(s@(s@(s@c_1))))))$   
 $\text{cCKB\_BLACK} = (\lambda xu: \$i, xv: \$i: ((\text{cCKB\_ODD}@xu \text{ and } \text{cCKB\_ODD}@xv) \text{ or } (\text{cCKB\_EVEN}@xu \text{ and } \text{cCKB\_EVEN}@xv)))$   
 $\forall xu: \$i, xv: \$i: ((\text{cCKB\_BLACK}@xu@xv) \Rightarrow (\text{cCKB\_BLACK}@ (s@xu)@(s@xv) \text{ and } \text{cCKB\_BLACK}@xu@(s@(s@xv)) \text{ and } c_1))$

**PUZ122** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$\text{cCKB\_FIN}: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cCKB\_FIN\_type}, \text{type})$   
 $\text{cCKB\_INF}: (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cCKB\_INF\_type}, \text{type})$   
 $\text{cCKB\_INJ}: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cCKB\_INJ\_type}, \text{type})$   
 $\text{cCKB\_XPL}: (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB\_XPL\_type}, \text{type})$   
 $\text{cCKB\_INJ} = (\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: \forall xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv \text{ and } xh@xx_2@xy_2@xu@xv \text{ and } xx_1 = xx_2 \text{ and } xy_1 = xy_2)))$      $\text{thf}(\text{cCKB\_INJ\_def}, \text{definition})$   
 $\text{cCKB\_XPL} = (\lambda xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xk: \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (xk@xm@xn \text{ and } \forall xx: \$i, xy: \$i: ((xk@xx@xy) = c_1 \text{ or } \exists xu: \$i, xv: \$i: (xh@xx@xy@xu@xv \text{ and } xk@xu@xv \text{ and } \neg xu = xm \text{ and } xv = xn))))$      $\text{thf}(\text{cCKB\_XPL\_def}, \text{definition})$   
 $\text{cCKB\_INF} = (\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \exists xh: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o, xm: \$i, xn: \$i: (\text{cCKB\_INJ}@xh \text{ and } \text{cCKB\_XPL}@xh@xk@xm@xn))$   
 $\text{cCKB\_FIN} = (\lambda xk: \$i \rightarrow \$i \rightarrow \$o: \neg \text{cCKB\_INF}@xk)$      $\text{thf}(\text{cCKB\_FIN\_def}, \text{definition})$   
 $\forall xk: \$i \rightarrow \$i \rightarrow \$o, xz: \$i, xw: \$i: ((\text{cCKB\_FIN}@xk) \Rightarrow (\text{cCKB\_FIN}@ \lambda xu: \$i, xv: \$i: (xk@xu@xv \text{ or } (xu = xz \text{ and } xv = xw))))$      $\text{thf}(\text{cL2000\_pme}, \text{conjecture})$

**PUZ123** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$      $\text{thf}(c1\_type, \text{type})$   
 $c_8: \$i$      $\text{thf}(c8\_type, \text{type})$   
 $s: \$i \rightarrow \$i$      $\text{thf}(s\_type, \text{type})$   
 $\text{cCKB6\_BLACK}: \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB6\_BLACK\_type}, \text{type})$   
 $\text{cCKB6\_BLACK} = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj)))@xk@xw@xu@xv)))$      $\text{thf}(\text{cCKB6\_BLACK\_def}, \text{definition})$   
 $\forall xx: \$i, xy: \$i: ((\text{cCKB6\_BLACK}@xx@xy) \Rightarrow \neg((s@(s@xx)) = c_1 \text{ and } (s@xy) = c_1) \text{ or } ((s@(s@xx)) = c_8 \text{ and } (s@xy) = c_8))$      $\text{thf}(\text{cCKB\_L39000}, \text{conjecture})$

**PUZ124** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$      $\text{thf}(c1\_type, \text{type})$   
 $c_2: \$i$      $\text{thf}(c2\_type, \text{type})$   
 $c_3: \$i$      $\text{thf}(c3\_type, \text{type})$   
 $c_4: \$i$      $\text{thf}(c4\_type, \text{type})$   
 $g: \$i \rightarrow \$i \rightarrow \$i$      $\text{thf}(g\_type, \text{type})$   
 $s: \$i \rightarrow \$i$      $\text{thf}(s\_type, \text{type})$   
 $\text{cCKB6\_BLACK}: \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB6\_BLACK\_type}, \text{type})$   
 $\text{cCKB6\_H}: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB6\_H\_type}, \text{type})$   
 $\text{cCKB6\_BLACK} = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj)))@xk@xw@xu@xv)))$      $\text{thf}(\text{cCKB6\_BLACK\_def}, \text{definition})$   
 $\text{cCKB6\_H} = (\lambda xx: \$i, xy: \$i, xu: \$i, xv: \$i: (\text{cCKB6\_BLACK}@xx@xy \text{ and } (((g@(s@(s@xx))@(s@xy)) = c_1 \text{ and } xu = (s@(s@(s@xx))) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_2 \text{ and } xu = (s@(s@xx)) \text{ and } xv = (s@(s@xy))) \text{ or } ((g@xy) = c_3 \text{ and } xu = (s@xx) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_4 \text{ and } xu = (s@(s@xx)) \text{ and } xv = xy))))$      $\text{thf}(\text{cCKB6\_H\_def}, \text{definition})$   
 $\forall xx: \$i, xy: \$i, xu: \$i, xv: \$i: ((\text{cCKB6\_H}@xx@xy@xu@xv \text{ and } (g@xu@xv) = c_3) \Rightarrow (xu = (s@(s@(s@xx))) \text{ and } xv = (s@xy)))$      $\text{thf}(\text{cCKB\_L43000}, \text{conjecture})$

**PUZ125** $\wedge$ **5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i$      $\text{thf}(c1\_type, \text{type})$   
 $c_2: \$i$      $\text{thf}(c2\_type, \text{type})$   
 $c_3: \$i$      $\text{thf}(c3\_type, \text{type})$   
 $c_4: \$i$      $\text{thf}(c4\_type, \text{type})$   
 $g: \$i \rightarrow \$i \rightarrow \$i$      $\text{thf}(g\_type, \text{type})$   
 $s: \$i \rightarrow \$i$      $\text{thf}(s\_type, \text{type})$   
 $\text{cCKB6\_BLACK}: \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB6\_BLACK\_type}, \text{type})$   
 $\text{cCKB6\_H}: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cCKB6\_H\_type}, \text{type})$   
 $\text{cCKB6\_BLACK} = (\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj)))@xk@xw@xu@xv)))$      $\text{thf}(\text{cCKB6\_BLACK\_def}, \text{definition})$

cCKB6.H =  $(\lambda xx: \$i, xy: \$i, xu: \$i, xv: \$i: (cCKB6\_BLACK@xx@xy \text{ and } (((g@(s@(s@xx))@(s@xy)) = c_1 \text{ and } xu = (s@(s@(s@xx))) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_2 \text{ and } xu = (s@(s@xx)) \text{ and } xv = (s@(s@xy))) \text{ or } ((g@ c_3 \text{ and } xu = (s@xx) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_4 \text{ and } xu = (s@(s@xx)) \text{ and } xv = xy)))) \text{ thf}(cCKB6\_L_{30000}, \text{conjecture})$

**PUZ126^5.p** TPS problem from CHECKERBOARD-THMS

$c_1: \$i \text{ thf}(c1\_type, type)$   
 $c_2: \$i \text{ thf}(c2\_type, type)$   
 $c_3: \$i \text{ thf}(c3\_type, type)$   
 $c_4: \$i \text{ thf}(c4\_type, type)$   
 $g: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(g\_type, type)$   
 $s: \$i \rightarrow \$i \text{ thf}(s\_type, type)$   
cCKB6.BLACK:  $\$i \rightarrow \$i \rightarrow \$o \text{ thf}(cCKB6\_BLACK\_type, type)$   
cCKB6.H:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cCKB6\_H\_type, type)$   
cCKB6.BLACK =  $(\lambda xu: \$i, xv: \$i: \forall xw: \$i \rightarrow \$i \rightarrow \$o: ((xw@c_1@c_1 \text{ and } \forall xj: \$i, xk: \$i: ((xw@xj@xk) \Rightarrow (xw@(s@(s@xj))@xk) \text{ and } (xw@xu@xv))) \text{ thf}(cCKB6\_BLACK\_def, definition)$   
cCKB6.H =  $(\lambda xx: \$i, xy: \$i, xu: \$i, xv: \$i: (cCKB6\_BLACK@xx@xy \text{ and } (((g@(s@(s@xx))@(s@xy)) = c_1 \text{ and } xu = (s@(s@(s@xx))) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_2 \text{ and } xu = (s@(s@xx)) \text{ and } xv = (s@(s@xy))) \text{ or } ((g@ c_3 \text{ and } xu = (s@xx) \text{ and } xv = (s@xy)) \text{ or } ((g@(s@(s@xx))@(s@xy)) = c_4 \text{ and } xu = (s@(s@xx)) \text{ and } xv = xy)))) \text{ thf}(cCKB6\_L_{43100}, \text{conjecture})$   
 $\forall xx: \$i, xy: \$i, xu: \$i, xv: \$i: ((cCKB6\_H@xx@xy@xu@xv \text{ and } (g@xu@xv) = c_3) \Rightarrow ((s@(s@(s@(s@(s@xu)))) = xx \text{ and } (s@(s@(s@(s@(s@(s@xv)))))) = xy)) \text{ thf}(cCKB6\_L_{43100}, \text{conjecture})$

**PUZ127^5.p** TPS problem from CHECKERBOARD-THMS

$c_5: \$i \text{ thf}(c_5, type)$   
 $g: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(g, type)$   
 $c_4: \$i \text{ thf}(c_4, type)$   
 $c_3: \$i \text{ thf}(c_3, type)$   
 $c_2: \$i \text{ thf}(c_2, type)$   
 $c_1: \$i \text{ thf}(c_1, type)$   
 $s: \$i \rightarrow \$i \text{ thf}(s, type)$   
 $c_8: \$i \text{ thf}(c_8, type)$   
 $\neg (s@(s@(s@(s@(s@(s@(s@c_8)))))) = c_8 \text{ and } \forall xx: \$i: (s@(s@(s@(s@xx)))) \neq xx \text{ and } \forall xx: \$i, xy: \$i: ((g@xx@xy) = c_5 \iff ((xx = c_8 \text{ and } xy = c_8) \text{ or } (xx = c_1 \text{ and } xy = c_1))) \text{ and } \forall xx: \$i, xy: \$i: ((g@xx@xy) = c_1 \iff (g@(s@xx@xy) = c_3) \text{ and } \forall xx: \$i, xy: \$i: ((g@xx@xy) = c_2 \iff (g@xx@(s@xy) = c_4) \text{ and } \forall xx: \$i, xy: \$i: ((g@c_1@xy) \neq c_3 \text{ and } (g@c_8@xy) \neq c_1 \text{ and } (g@xx@c_1) \neq c_4 \text{ and } (g@xx@c_8) \neq c_2) \text{ and } c_1 = (s@c_8) \text{ and } c_2 = (s@c_1) \text{ and } c_3 = (s@c_2) \text{ and } c_4 = (s@c_3) \text{ and } c_5 = (s@c_4) \text{ and } \forall xx: \$i, xy: \$i: ((g@xx@xy) = c_1 \text{ or } (g@xx@xy) = c_2 \text{ or } (g@xx@xy) = c_3 \text{ or } (g@xx@xy) = c_4 \text{ or } (g@xx@xy) = c_5)) \text{ thf}(cTOUGHNUT_2, \text{conjecture})$

**PUZ128+1.p** Iokaste patricide triangle

Iokaste is a parent of Oedipus. Iokaste is a parent of Polyneikes. Oedipus is a parent of Polyneikes. Polyneikes is a parent of Thersandros. Oedipus is a patricide. Thersandros is not a patricide. Therefore, Iokaste is a parent of a patricide who is a parent of somebody who is not a patricide.

parent\_of(iokaste, oedipus) fof(iokaste\_oedipus, axiom)  
parent\_of(iokaste, polyneikes) fof(iokaste\_polyneikes, axiom)  
parent\_of(oedipus, polyneikes) fof(oedipus\_polyneikes, axiom)  
parent\_of(polyneikes, thersandros) fof(polyneikes\_thersandros, axiom)  
patricide(oedipus) fof(oedipus\_patricidal, axiom)  
 $\neg \text{patricide}(\text{thersandros}) \text{ fof}(\text{thersandros\_not\_patricidal}, \text{axiom})$   
 $\exists p, nP: (\text{parent\_of}(\text{iokaste}, p) \text{ and } \text{patricide}(p) \text{ and } \text{parent\_of}(p, nP) \text{ and } \neg \text{patricide}(nP)) \text{ fof}(\text{iokaste\_parent\_patricide\_parent\_not\_patricide}, \text{axiom})$

**PUZ128+2.p** Iokaste patricide triangle

Iokaste is a parent of Oedipus. Iokaste is a parent of Polyneikes. Oedipus is a parent of Polyneikes. Polyneikes is a parent of Thersandros. Oedipus is a patricide. Thersandros is not a patricide. Therefore, Iokaste is a parent of a patricide that is a parent of somebody who is not a patricide.

$\exists a, b, c, d, e: (\text{parent}(a) \text{ and } \text{relation}(a, \text{of}, \text{'Oedipus'}) \text{ and } \text{'Iokaste'} = a \text{ and } \text{parent}(b) \text{ and } \text{relation}(b, \text{of}, \text{'Polyneikes'}) \text{ and } \text{'Iokaste'} = b \text{ and } \text{parent}(c) \text{ and } \text{relation}(c, \text{of}, \text{'Polyneikes'}) \text{ and } \text{'Oedipus'} = c \text{ and } \text{parent}(d) \text{ and } \text{relation}(d, \text{of}, \text{'Thersandros'}) \text{ and } \text{'Polyneikes'} = d \text{ and } \text{patricide}(e) \text{ and } \text{'Oedipus'} = e \text{ and } \neg \exists f: (\text{patricide}(f) \text{ and } \text{'Thersandros'} = f)) \text{ fof}(\text{background}, \text{axiom})$   
 $\exists a, b, c, d: (\text{parent}(a) \text{ and } \text{patricide}(b) \text{ and } \text{parent}(c) \text{ and } \$\text{true} \text{ and } \neg \exists e: (\text{patricide}(e) \text{ and } d = e) \text{ and } \text{relation}(c, \text{of}, d) \text{ and } b = c \text{ and } \text{relation}(a, \text{of}, b) \text{ and } \text{'Iokaste'} = a) \text{ fof}(\text{prove}, \text{conjecture})$

**PUZ129+2.p** The grocer is not a cyclist

If every honest and industrious person is healthy, and no grocer is healthy, and every industrious grocer is honest, and every cyclist is industrious, and every unhealthy cyclist is dishonest, and no healthy person is unhealthy, and no honest person is dishonest, and every grocer is a person, and every cyclist is a person then no grocer is a cyclist.  
 $(\forall a: ((\text{person}(a) \text{ and } \text{property}_1(a, \text{honest}, \text{pos})) \text{ and } \text{property}_1(a, \text{industrious}, \text{pos})) \Rightarrow \exists b: (\text{property}_1(b, \text{healthy}, \text{pos}) \text{ and } a = b)) \text{ and } \forall c: (\text{grocer}(c) \Rightarrow \neg \exists d: (\text{property}_1(d, \text{healthy}, \text{pos}) \text{ and } c = d)) \text{ and } \forall e: ((\text{grocer}(e) \text{ and } \text{property}_1(e, \text{industrious}, \text{pos})) \Rightarrow \exists f: (\text{property}_1(f, \text{honest}, \text{pos}) \text{ and } e = f)) \text{ and } \forall g: (\text{cyclist}(g) \Rightarrow \exists h: (\text{property}_1(h, \text{industrious}, \text{pos}) \text{ and } g = h)) \text{ and } \forall i: ((\text{cyclist}(i) \text{ and } \text{property}_1(i, \text{unhealthy}, \text{pos})) \Rightarrow \exists j: (\text{property}_1(j, \text{dishonest}, \text{pos}) \text{ and } i = j)) \text{ and } \forall k: ((\text{person}(k) \text{ and } \neg \exists l: (\text{property}_1(l, \text{unhealthy}, \text{pos}) \text{ and } k = l)) \text{ and } \forall m: ((\text{person}(m) \text{ and } \text{property}_1(m, \text{honest}, \text{pos})) \Rightarrow \neg \exists n: (\text{property}_1(n, \text{dishonest}, \text{pos}) \text{ and } m = n)) \text{ and } \forall o: (\text{grocer}(o) \Rightarrow \exists p: (\text{person}(p) \text{ and } o = p)) \text{ and } \forall q: (\text{cyclist}(q) \Rightarrow \exists r: (\text{person}(r) \text{ and } q = r))) \Rightarrow \forall s: (\text{grocer}(s) \Rightarrow \neg \exists t: (\text{cyclist}(t) \text{ and } s = t)) \quad \text{fof(prove, conjecture)}$

### PUZ130+1.p Garfield and Odie

Garfield is a cat and Odie is a dog. Cats and dogs are pets. John is a human. Every pet has a human owner. Jon owns Garfield and Odie, and they are the only cat and dog he owns. If a dog chases a cat, then the cat's owner hates the dog's owner. Odie has chased Garfield. Therefore, Jon hates himself.

```

∃a: cat(a)      fof(cat_type, axiom)
cat(garfield)  fof(garfield_type, axiom)
∃a: dog(a)      fof(dog_type, axiom)
dog(odie)      fof(odie_type, axiom)
∃a: pet(a)      fof(pet_type, axiom)
∀a: (dog(a) ⇒ pet(a))    fof(dog_pet_type, axiom)
∀a: (cat(a) ⇒ pet(a))    fof(cat_pet_type, axiom)
∃a: human(a)    fof(human_type, axiom)
human(jon)      fof(jon_type, axiom)
∀a: (pet(a) ⇒ human(owner_of(a)))    fof(owner_of_type, axiom)
∀x: (pet(x) ⇒ ∃y: (human(y) and owner(x, y)))    fof(pet_owner_axiom, axiom)
owner(jon, odie)    fof(jon_o_owner_axiom, axiom)
owner(jon, garfield)    fof(jon_g_owner_axiom, axiom)
∀x: (∀x: (cat(x) ⇒ owner(jon, x)) ⇒ x = garfield)    fof(jon_only_g_owner_axiom, axiom)
∀x: (∀x: (dog(x) ⇒ owner(jon, x)) ⇒ x = odie)    fof(jon_only_o_owner_axiom, axiom)
∀x, y: ((cat(x) and dog(y)) ⇒ (chased(y, x) ⇒ hates(owner_of(x), owner_of(y))))    fof(cat_chase_axiom, axiom)
∀x, y: ((human(x) and pet(y)) ⇒ (owner(x, y) ⇔ x = owner_of(y)))    fof(owner_def, axiom)
chased(odie, garfield)    fof(odie_chase_axiom, axiom)
hates(jon, jon)    fof(jon_conjecture, conjecture)

```

### PUZ130\_1.p Garfield and Odie

Garfield is a cat and Odie is a dog. Cats and dogs are pets. John is a human. Every pet has a human owner. Jon owns Garfield and Odie, and they are the only cat and dog he owns. If a dog chases a cat, then the cat's owner hates the dog's owner. Odie has chased Garfield. Therefore, Jon hates himself.

```

animal: $tType    tff(animal_type, type)
cat: $tType       tff(cat_type, type)
dog: $tType       tff(dog_type, type)
human: $tType     tff(human_type, type)
cat_to_animal: cat → animal    tff(cat_to_animal_type, type)
dog_to_animal: dog → animal    tff(dog_to_animal_type, type)
garfield: cat     tff(garfield_type, type)
odie: dog         tff(odie_type, type)
jon: human        tff(jon_type, type)
owner_of: animal → human    tff(owner_of_type, type)
chased: (dog × cat) → $o    tff(chased_type, type)
hates: (human × human) → $o    tff(hates_type, type)
∀a: animal: ∃h: human: h = owner_of(a)    tff(human_owner, axiom)
jon = owner_of(cat_to_animal(garfield))    tff(jon_owns_garfield, axiom)
jon = owner_of(dog_to_animal(odie))    tff(jon_owns_odie, axiom)
∀a: animal: (jon = owner_of(a) ⇒ (a = cat_to_animal(garfield) or a = dog_to_animal(odie)))    tff(jon_owns_only, axiom)
∀c: cat, d: dog: (chased(d, c) ⇒ hates(owner_of(cat_to_animal(c)), owner_of(dog_to_animal(d))))    tff(dog_chase_cat, axiom)
chased(odie, garfield)    tff(odie_chased_garfield, axiom)
hates(jon, jon)    tff(jon_hates_jon, conjecture)

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### PUZ131+1.p Victor teaches Michael

Every student is enrolled in at least one course. Every professor teaches at least one course. Every course has at least one student enrolled. Every course has at least one professor teaching. The coordinator of a course teaches the course. If a student is enrolled in a course then the student is taught by every professor who teaches the course. Michael is enrolled in CSC410. Victor is the coordinator of CSC410. Therefore, Michael is taught by Victor.

$\exists a: \text{student}(a) \quad \text{fof}(\text{student\_type}, \text{axiom})$   
 $\exists a: \text{professor}(a) \quad \text{fof}(\text{professor\_type}, \text{axiom})$   
 $\exists a: \text{course}(a) \quad \text{fof}(\text{course\_type}, \text{axiom})$   
 $\text{student}(\text{michael}) \quad \text{fof}(\text{michael\_type}, \text{axiom})$   
 $\text{professor}(\text{victor}) \quad \text{fof}(\text{victor\_type}, \text{axiom})$   
 $\text{course}(\text{csc410}) \quad \text{fof}(\text{csc410\_type}, \text{axiom})$   
 $\forall a: (\text{course}(a) \Rightarrow \text{professor}(\text{coordinatorof}(a))) \quad \text{fof}(\text{coordinator\_of\_type}, \text{axiom})$   
 $\forall x: (\text{student}(x) \Rightarrow \exists y: (\text{course}(y) \text{ and } \text{enrolled}(x, y))) \quad \text{fof}(\text{student\_enrolled\_axiom}, \text{axiom})$   
 $\forall x: (\text{professor}(x) \Rightarrow \exists y: (\text{course}(y) \text{ and } \text{teaches}(x, y))) \quad \text{fof}(\text{professor\_teaches}, \text{axiom})$   
 $\forall x: (\text{course}(x) \Rightarrow \exists y: (\text{student}(y) \text{ and } \text{enrolled}(y, x))) \quad \text{fof}(\text{course\_enrolled}, \text{axiom})$   
 $\forall x: (\text{course}(x) \Rightarrow \exists y: (\text{professor}(y) \text{ and } \text{teaches}(y, x))) \quad \text{fof}(\text{course\_teaches}, \text{axiom})$   
 $\forall x: (\text{course}(x) \Rightarrow \text{teaches}(\text{coordinatorof}(x), x)) \quad \text{fof}(\text{coordinator\_teaches}, \text{axiom})$   
 $\forall x, y: ((\text{student}(x) \text{ and } \text{course}(y)) \Rightarrow (\text{enrolled}(x, y) \Rightarrow \forall z: (\text{professor}(z) \Rightarrow (\text{teaches}(z, y) \Rightarrow \text{taughtby}(x, z)))))) \quad \text{fof}(\text{student\_enrolled\_taught\_axiom}, \text{axiom})$   
 $\text{enrolled}(\text{michael}, \text{csc410}) \quad \text{fof}(\text{michael\_enrolled\_csc410\_axiom}, \text{axiom})$   
 $\text{coordinatorof}(\text{csc410}) = \text{victor} \quad \text{fof}(\text{victor\_coordinator\_csc410\_axiom}, \text{axiom})$   
 $\text{taughtby}(\text{michael}, \text{victor}) \quad \text{fof}(\text{teaching\_conjecture}, \text{conjecture})$

### PUZ131.1.p Victor teaches Michael

Every student is enrolled in at least one course. Every professor teaches at least one course. Every course has at least one student enrolled. Every course has at least one professor teaching. The coordinator of a course teaches the course. If a student is enrolled in a course then the student is taught by every professor who teaches the course. Michael is enrolled in CSC410. Victor is the coordinator of CSC410. Therefore, Michael is taught by Victor.

$\text{student}: \$\text{Type} \quad \text{tff}(\text{student\_type}, \text{type})$   
 $\text{professor}: \$\text{Type} \quad \text{tff}(\text{professor\_type}, \text{type})$   
 $\text{course}: \$\text{Type} \quad \text{tff}(\text{course\_type}, \text{type})$   
 $\text{michael}: \text{student} \quad \text{tff}(\text{michael\_type}, \text{type})$   
 $\text{victor}: \text{professor} \quad \text{tff}(\text{victor\_type}, \text{type})$   
 $\text{csc410}: \text{course} \quad \text{tff}(\text{csc410\_type}, \text{type})$   
 $\text{enrolled}: (\text{student} \times \text{course}) \rightarrow \$\text{o} \quad \text{tff}(\text{enrolled\_type}, \text{type})$   
 $\text{teaches}: (\text{professor} \times \text{course}) \rightarrow \$\text{o} \quad \text{tff}(\text{teaches\_type}, \text{type})$   
 $\text{taughtby}: (\text{student} \times \text{professor}) \rightarrow \$\text{o} \quad \text{tff}(\text{taught\_by\_type}, \text{type})$   
 $\text{coordinatorof}: \text{course} \rightarrow \text{professor} \quad \text{tff}(\text{coordinator\_of\_type}, \text{type})$   
 $\forall x: \text{student}: \exists y: \text{course}: \text{enrolled}(x, y) \quad \text{tff}(\text{student\_enrolled\_axiom}, \text{axiom})$   
 $\forall x: \text{professor}: \exists y: \text{course}: \text{teaches}(x, y) \quad \text{tff}(\text{professor\_teaches}, \text{axiom})$   
 $\forall x: \text{course}: \exists y: \text{student}: \text{enrolled}(y, x) \quad \text{tff}(\text{course\_enrolled}, \text{axiom})$   
 $\forall x: \text{course}: \exists y: \text{professor}: \text{teaches}(y, x) \quad \text{tff}(\text{course\_teaches}, \text{axiom})$   
 $\forall x: \text{course}: \text{teaches}(\text{coordinatorof}(x), x) \quad \text{tff}(\text{coordinator\_teaches}, \text{axiom})$   
 $\forall x: \text{student}, y: \text{course}: (\text{enrolled}(x, y) \Rightarrow \forall z: \text{professor}: (\text{teaches}(z, y) \Rightarrow \text{taughtby}(x, z))) \quad \text{tff}(\text{student\_enrolled\_taught\_axiom}, \text{axiom})$   
 $\text{enrolled}(\text{michael}, \text{csc410}) \quad \text{tff}(\text{michael\_enrolled\_csc410\_axiom}, \text{axiom})$   
 $\text{coordinatorof}(\text{csc410}) = \text{victor} \quad \text{tff}(\text{victor\_coordinator\_csc410\_axiom}, \text{axiom})$   
 $\text{taughtby}(\text{michael}, \text{victor}) \quad \text{tff}(\text{teaching\_conjecture}, \text{conjecture})$

### PUZ132+1.p Crime in beautiful Washington

A capital is a city. USA is a country. Every city has crime. Washington is the capital of the USA. Every country has a beautiful capital. Therefore, Washington is beautiful but has crime.

$\exists a: \text{capital}(a) \quad \text{fof}(\text{capital\_type}, \text{axiom})$   
 $\exists a: \text{city}(a) \quad \text{fof}(\text{city\_type}, \text{axiom})$   
 $\forall a: (\text{capital}(a) \Rightarrow \text{city}(a)) \quad \text{fof}(\text{capital\_city\_type}, \text{axiom})$   
 $\exists a: \text{country}(a) \quad \text{fof}(\text{country\_type}, \text{axiom})$   
 $\text{capital}(\text{washington}) \quad \text{fof}(\text{washington\_type}, \text{axiom})$   
 $\text{country}(\text{usa}) \quad \text{fof}(\text{usa\_type}, \text{axiom})$   
 $\forall a: (\text{country}(a) \Rightarrow \text{capital}(\text{capital\_city}(a))) \quad \text{fof}(\text{country\_capital\_type}, \text{axiom})$   
 $\forall x: (\text{city}(x) \Rightarrow \text{has\_crime}(x)) \quad \text{fof}(\text{crime\_axiom}, \text{axiom})$   
 $\text{capital\_city}(\text{usa}) = \text{washington} \quad \text{fof}(\text{usa\_capital\_axiom}, \text{axiom})$   
 $\forall x: (\text{country}(x) \Rightarrow \text{beautiful}(\text{capital\_city}(x))) \quad \text{fof}(\text{beautiful\_capital\_axiom}, \text{axiom})$   
 $\text{beautiful}(\text{washington}) \text{ and } \text{has\_crime}(\text{washington}) \quad \text{fof}(\text{washington\_conjecture}, \text{conjecture})$

**PUZ133+1.p** N queens problem has the variable symmetry property

queens\_p  $\Rightarrow \forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n)) \Rightarrow (p(i) \neq p(j) \text{ and } p(i) + i \neq p(j) + j \text{ and } -p(i) \neq -p(j)))$  fof(queens\_p, axiom)  
 $\forall i: \text{perm}(i) = -s(n)$  fof(permutation, axiom)  
 $\forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n) \text{ and } (\text{le}(s(i), j) \iff \text{le}(s(\text{perm}(j)), \text{perm}(i)))) \Rightarrow (q(i) \neq q(j) \text{ and } q(i) + i \neq q(j) + j \text{ and } -q(i) \neq -q(j))) \Rightarrow \text{queens}_q$  fof(queens\_q, axiom)  
 $(\text{queens}_p \text{ and } \forall i: q(i) = p(\text{perm}(i))) \Rightarrow \text{queens}_q$  fof(queens\_sym, conjecture)  
 $\forall i: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n)) \Rightarrow (\text{le}(s(n_0), \text{perm}(i)) \text{ and } \text{le}(\text{perm}(i), n)))$  fof(permutation\_range, axiom)  
 $\forall j, i: -i = -\text{perm}(j)$  fof(permutation\_another\_one, axiom)  
 $\forall x, y, z: ((\text{le}(x, y) \text{ and } \text{le}(y, z)) \Rightarrow \text{le}(x, z))$  fof(le\_trans, axiom)  
 $\forall x: \text{le}(x, s(x))$  fof(succ\_le, axiom)  
 $\forall i, j, k, l: (i + j = k + l \iff -i = -l)$  fof(plus\_1, axiom)  
 $\forall i, j, k, l: (-i = -k \iff -i = -j)$  fof(minus\_1, axiom)

**PUZ133+2.p** N queens problem has the variable symmetry property

queens\_p  $\Rightarrow \forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n)) \Rightarrow (p(i) \neq p(j) \text{ and } p(i) + i \neq p(j) + j \text{ and } -p(i) \neq -p(j)))$  fof(queens\_p, axiom)  
 $\forall i: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n)) \Rightarrow \text{perm}(i) = -s(n))$  fof(permutation, axiom)  
 $\forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n) \text{ and } (\text{le}(s(i), j) \iff \text{le}(s(\text{perm}(j)), \text{perm}(i)))) \Rightarrow (q(i) \neq q(j) \text{ and } q(i) + i \neq q(j) + j \text{ and } -q(i) \neq -q(j))) \Rightarrow \text{queens}_q$  fof(queens\_q, axiom)  
 $(\text{queens}_p \text{ and } \forall i: q(i) = p(\text{perm}(i))) \Rightarrow \text{queens}_q$  fof(queens\_sym, conjecture)  
 $\forall i: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n)) \Rightarrow (\text{le}(s(n_0), \text{perm}(i)) \text{ and } \text{le}(\text{perm}(i), n)))$  fof(permutation\_range, axiom)  
 $\forall j, i: -i = -\text{perm}(j)$  fof(permutation\_another\_one, axiom)  
 $\forall x, y, z: ((\text{le}(x, y) \text{ and } \text{le}(y, z)) \Rightarrow \text{le}(x, z))$  fof(le\_trans, axiom)  
 $\forall x: \text{le}(x, s(x))$  fof(succ\_le, axiom)  
 $\forall i, j, k, l: (i + j = k + l \iff -i = -l)$  fof(plus\_1, axiom)  
 $\forall i, j, k, l: (-i = -k \iff -i = -j)$  fof(minus\_1, axiom)

**PUZ133+3.p** N queens problem has the variable symmetry property

queens\_p  $\Rightarrow \forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n)) \Rightarrow (p(i) \neq p(j) \text{ and } p(i) + i \neq p(j) + j \text{ and } -p(i) \neq -p(j)))$  fof(queens\_p, axiom)  
 $\forall i: \text{perm}(i) = -s(n)$  fof(permutation, axiom)  
 $\forall i, j: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n) \text{ and } \text{le}(s(i), j) \text{ and } \text{le}(j, n)) \Rightarrow (q(i) \neq q(j) \text{ and } q(i) + i \neq q(j) + j \text{ and } -q(i) \neq -q(j))) \Rightarrow \text{queens}_q$  fof(queens\_q, axiom)  
 $(\text{queens}_p \text{ and } \forall i: q(i) = p(\text{perm}(i))) \Rightarrow \text{queens}_q$  fof(queens\_sym, conjecture)  
 $\forall i: ((\text{le}(s(n_0), i) \text{ and } \text{le}(i, n)) \Rightarrow (\text{le}(s(n_0), \text{perm}(i)) \text{ and } \text{le}(\text{perm}(i), n)))$  fof(permutation\_range, axiom)  
 $\forall i, j: (\text{lt}(i, j) \iff \text{lt}(\text{perm}(j), \text{perm}(i)))$  fof(permutation\_anti\_monotone, axiom)  
 $\forall j, i: -i = -\text{perm}(j)$  fof(permutation\_another\_one, axiom)  
 $\forall x, y, z: ((\text{le}(x, y) \text{ and } \text{le}(y, z)) \Rightarrow \text{le}(x, z))$  fof(le\_trans, axiom)  
 $\forall x: \text{le}(x, s(x))$  fof(succ\_le, axiom)  
 $\forall i, j, k, l: (i + j = k + l \iff -i = -l)$  fof(plus\_1, axiom)  
 $\forall i, j, k, l: (-i = -k \iff -i = -j)$  fof(minus\_1, axiom)

**PUZ133=2.p** N queens problem has the variable symmetry property

queens\_q: \$o tff(queens\_q\_type, type)  
 queens\_p: \$o tff(queens\_p\_type, type)  
 q: \$int  $\rightarrow$  \$int tff(q\_type, type)  
 p: \$int  $\rightarrow$  \$int tff(p\_type, type)  
 n: \$int tff(n\_type, type)  
 perm: \$int  $\rightarrow$  \$int tff(perm\_type, type)  
 queens\_p  $\Rightarrow \forall i: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i)$  tff(permutation, axiom)  
 $\forall i: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i)$  tff(permutation, axiom)  
 $\forall i: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i)$  tff(permutation, axiom)  
 $\forall i, j: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i) \text{ and } \text{perm}(j) = \text{difference}(\text{sum}(1, n), j) \iff (\text{le}(\text{sum}(1, i), j) \iff \text{le}(\text{sum}(1, \text{perm}(j)), \text{perm}(i))) \Rightarrow (q(i) \neq q(j) \text{ and } \text{sum}(q(i), i) \neq \text{sum}(q(j), j) \text{ and } \text{difference}(q(i), i) \neq \text{difference}(q(j), j))) \Rightarrow \text{queens}_q$  tff(queens\_q, axiom)  
 $(\text{queens}_p \text{ and } \forall i: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i)) \Rightarrow \text{queens}_q$  tff(queens\_sym, conjecture)  
 $\forall i: \text{perm}(i) = \text{difference}(\text{sum}(1, n), i) \Rightarrow (\text{le}(1, \text{perm}(i)) \text{ and } \text{le}(\text{perm}(i), n))$  tff(permutation\_range, axiom)  
 $\forall j: \text{perm}(j) = \text{difference}(\text{sum}(1, n), j) \iff \text{difference}(i, j) = \text{difference}(\text{perm}(j), \text{perm}(i))$  tff(permutation\_another\_one, axiom)

**PUZ136^1.p** Under two assumptions there are at least two individuals.

parent:  $\$i \rightarrow \$i \rightarrow \$o$  thf(parent, type)  
kronus:  $\$i$  thf(kronus, type)  
zeus:  $\$i$  thf(zeus, type)  
parent@kronus@zeus thf(ax<sub>1</sub>, axiom)  
sutekh:  $\$i$  thf(sutekh, type)  
horus:  $\$i$  thf(horus, type)  
¬parent@sutekh@horus thf(ax<sub>2</sub>, axiom)  
 $\exists p: \$i \rightarrow \$o, x: \$i, y: \$i: (p@x \text{ and } \neg p@y)$  thf(hotwo, conjecture)

**PUZ137^1.p** Peter the liar says everything

Peter says that everything he says is false. Show that Peter says everything.

peter:  $\$i$  thf(peter, type)  
says:  $\$i \rightarrow \$o \rightarrow \$o$  thf(says, type)  
says@peter@ $\forall x: \$o: ((\text{says@peter}@x) \Rightarrow \neg x)$  thf(ax<sub>1</sub>, axiom)  
 $\forall x: \$o: (\text{says@peter}@x)$  thf(thm, conjecture)

**PUZ138+2.p** Platinum Blonde

+ + + + + - - - 12 - - - - 3 - - 2-3 -4 - + + + + + - 1-8 - 5 - - 6 - 7 - 8 - - - 9 -  
- + + + + + - 8 - 5 - - - 9 - 4 - 5 - - 47 - 6 - - + + + + +

include('Axioms/PUZ006+0.ax')

$p(n_1, n_8, n_1)$  fof(ax<sub>181</sub>, axiom)  
 $p(n_1, n_9, n_2)$  fof(ax<sub>192</sub>, axiom)  
 $p(n_2, n_9, n_3)$  fof(ax<sub>293</sub>, axiom)  
 $p(n_3, n_3, n_2)$  fof(ax<sub>332</sub>, axiom)  
 $p(n_3, n_4, n_3)$  fof(ax<sub>343</sub>, axiom)  
 $p(n_3, n_7, n_4)$  fof(ax<sub>374</sub>, axiom)  
 $p(n_4, n_3, n_1)$  fof(ax<sub>431</sub>, axiom)  
 $p(n_4, n_4, n_8)$  fof(ax<sub>448</sub>, axiom)  
 $p(n_4, n_9, n_5)$  fof(ax<sub>495</sub>, axiom)  
 $p(n_5, n_2, n_6)$  fof(ax<sub>526</sub>, axiom)  
 $p(n_5, n_5, n_7)$  fof(ax<sub>557</sub>, axiom)  
 $p(n_5, n_7, n_8)$  fof(ax<sub>578</sub>, axiom)  
 $p(n_6, n_6, n_9)$  fof(ax<sub>669</sub>, axiom)  
 $p(n_7, n_3, n_8)$  fof(ax<sub>738</sub>, axiom)  
 $p(n_7, n_4, n_5)$  fof(ax<sub>745</sub>, axiom)  
 $p(n_8, n_1, n_9)$  fof(ax<sub>819</sub>, axiom)  
 $p(n_8, n_5, n_4)$  fof(ax<sub>854</sub>, axiom)  
 $p(n_8, n_7, n_5)$  fof(ax<sub>875</sub>, axiom)  
 $p(n_9, n_1, n_4)$  fof(ax<sub>914</sub>, axiom)  
 $p(n_9, n_2, n_7)$  fof(ax<sub>927</sub>, axiom)  
 $p(n_9, n_6, n_6)$  fof(ax<sub>966</sub>, axiom)

**PUZ139\_1.p** Caramel vanilla coffee helps people stay awake

beverage:  $\$tType$  tff(beverage\_type, type)  
syrup:  $\$tType$  tff(syrup\_type, type)  
coffee: beverage tff(coffee\_type, type)  
vanilla\_syrup: syrup tff(vanilla\_syrup\_type, type)  
caramel\_syrup: syrup tff(caramel\_syrup\_type, type)  
mixture:  $\forall > \text{beverageOrSyrup}: \$tType: (\text{beverageOrSyrup} \times \text{syrup}) \rightarrow \text{beverageOrSyrup}$  tff(mixture\_type, type)  
help\_people\_stay\_awake: beverage  $\rightarrow \$o$  tff(help\_people\_stay\_awake\_type, type)  
 $\forall s: \text{syrup}: \text{help\_people\_stay\_awake}(\text{mixture}(\text{beverage}, \text{coffee}, s))$  tff(mixture\_of\_coffee\_help\_people\_stay\_awake, axiom)  
help\_people\_stay\_awake(mixture(beverage, coffee, mixture(syrup, caramel\_syrup, vanilla\_syrup))) tff(caramel\_vanilla\_coffee)

**PUZ140^1.p** A mixture of coffee and syrup that is hot

syrup:  $\$tType$  thf(syrup\_type, type)  
beverage:  $\$tType$  thf(beverage\_type, type)  
coffee: beverage thf(coffee\_type, type)  
heat: beverage  $\rightarrow \text{beverage}$  thf(heat\_type, type)  
hot: beverage  $\rightarrow \$o$  thf(hot\_type, type)  
mix: beverage  $\rightarrow \text{syrup} \rightarrow \text{beverage}$  thf(mix\_type, type)  
hot\_mixture: beverage  $\rightarrow \text{syrup} \rightarrow \text{beverage}$  thf(hot\_mixture\_type, type)

cold\_mixture: beverage  $\rightarrow$  syrup  $\rightarrow$  beverage    thf(cold\_mixture\_type, type)  
 hot\_mixture = ( $\lambda b$ : beverage,  $s$ : syrup: (heat@(mix@b@s)))    thf(hot\_mixture\_definition, definition)  
 cold\_mixture = ( $\lambda b$ : beverage,  $s$ : syrup: (mix@b@s))    thf(cold\_mixture\_definition, definition)  
 $\forall b$ : beverage: (hot@(heat@b))    thf(its\_hot, axiom)  
 $\exists$ mixture: beverage  $\rightarrow$  syrup  $\rightarrow$  beverage:  $\forall s$ : syrup:  $\exists b$ : beverage: ((mixture@coffee@s) = b and hot@b)    thf(hot\_coffee, cor)

**PUZ140^2.p** A mixture of coffee and syrup that is hot

syrup: \$tType    thf(syrup\_type, type)  
 beverage: \$tType    thf(beverage\_type, type)  
 coffee: beverage    thf(coffee\_type, type)  
 mix: beverage  $\rightarrow$  syrup  $\rightarrow$  beverage    thf(mix\_type, type)  
 coffee\_mixture: syrup  $\rightarrow$  beverage    thf(coffee\_mixture\_type, type)  
 hot: beverage  $\rightarrow$  \$o    thf(hot\_type, type)  
 coffee\_mixture = (mix@coffee)    thf(coffee\_mixture\_definition, definition)  
 $\forall s$ : syrup: ((coffee\_mixture@s) = coffee and hot@(coffee\_mixture@s))    thf(coffee\_and\_syrup\_is\_hot\_coffee, axiom)  
 $\exists$ syrupMixer: syrup  $\rightarrow$  beverage:  $\forall s$ : syrup:  $\exists b$ : beverage: (b = (syrupMixer@s) and b = coffee and hot@b)    thf(there\_is\_hot)

**PUZ141^1.p** Labyrinth1

Move 2 to the right.

position: \$tType    thf(position\_type, type)  
 direction: \$tType    thf(direction\_type, type)  
 left: direction    thf(left\_type, type)  
 right: direction    thf(right\_type, type)  
 $\top$ : direction    thf(top\_type, type)  
 bottom: direction    thf(bottom\_type, type)  
 next: position  $\rightarrow$  direction  $\rightarrow$  position    thf(next\_type, type)  
 $\forall d_1$ : direction,  $d_2$ : direction,  $p$ : position: (next@(next@p@d<sub>1</sub>)@d<sub>2</sub>) = (next@(next@p@d<sub>2</sub>)@d<sub>1</sub>)    thf(next\_comm, axiom)  
 $\forall p$ : position: (next@(next@p@left)@right) = p    thf(left\_right, axiom)  
 $\forall p$ : position: (next@(next@p@ $\top$ )@bottom) = p    thf(top\_bottom, axiom)  
 wall: position  $\rightarrow$  \$o    thf(wall\_type, type)  
 movelist: \$tType    thf(movelist\_type, type)  
 nomove: movelist    thf(nomove\_type, type)  
 movedir: movelist  $\rightarrow$  direction  $\rightarrow$  movelist    thf(movedir\_type, type)  
 playerpos: movelist  $\rightarrow$  position    thf(playerpos\_type, type)  
 $\forall pO$ : position,  $m$ : movelist,  $d$ : direction: ((playerpos@m) = pO  $\Rightarrow$  ( $\neg$  wall@(next@pO@d)  $\Rightarrow$  (playerpos@(movedir@m@d) = (next@pO@d)))    thf(player\_move\_legal, axiom)  
 $\forall pO$ : position,  $m$ : movelist,  $d$ : direction: ((playerpos@m) = pO  $\Rightarrow$  ((wall@(next@pO@d)  $\Rightarrow$  (playerpos@(movedir@m@d) = pO)))    thf(player\_move\_illegal, axiom)  
 $c_{00}$ : position    thf(c00\_type, type)  
 $c_{10}$ : position    thf(c10\_type, type)  
 $c_{20}$ : position    thf(c20\_type, type)  
 $c_{10}$  = (next@c<sub>00</sub>@right)    thf(c10\_defin, definition)  
 $c_{20}$  = (next@c<sub>10</sub>@right)    thf(c20\_defin, definition)  
 (wall@c<sub>00</sub>) = \$false    thf(c00\_axiom, axiom)  
 (wall@c<sub>10</sub>) = \$false    thf(c10\_axiom, axiom)  
 (wall@c<sub>20</sub>) = \$false    thf(c20\_axiom, axiom)  
 (playerpos@nomove) = c<sub>00</sub>    thf(start\_axiom, axiom)  
 $\exists m$ : movelist: (playerpos@m) = c<sub>20</sub>    thf(exercise, conjecture)

**PUZ146^1.p** Peter and Mary have different hobbies

hobby: \$tType    thf(hobby\_type, type)  
 earthling: \$tType    thf(earthling\_type, type)  
 peter: earthling    thf(peter, type)  
 mary: earthling    thf(mary, type)  
 beer\_drinking: hobby    thf(beer\_drinking, type)  
 belly\_dancing: hobby    thf(belly\_dancing, type)  
 weight\_lifting: hobby    thf(weight\_lifting, type)  
 has\_hobby: earthling  $\rightarrow$  hobby  $\rightarrow$  \$o    thf(has\_hobby, type)  
 peters\_hobbies: hobby  $\rightarrow$  \$o    thf(peters\_hobbies, type)  
 marys\_hobbies: hobby  $\rightarrow$  \$o    thf(marys\_hobbies, type)  
 peter  $\neq$  mary    thf(not\_the\_same<sub>1</sub>, axiom)



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beer_drinking ≠ belly_dancing and belly_dancing ≠ weight_lifting and beer_drinking ≠ weight_lifting    thf(not_the_same2, a
peters_hobbies = (has_hobby@peter)    thf(peters_hobbies001, definition)
marys_hobbies = (has_hobby@mary)    thf(marys_hobbies002, definition)
marys_hobbies@belly_dancing    thf(mary_does_belly_dancing, axiom)
¬ marys_hobbies@beer_drinking    thf(mary_does_not_do_beer_drinking, axiom)
peters_hobbies@beer_drinking    thf(peter_does_beer_drinking, axiom)
peters_hobbies@weight_lifting    thf(peter_does_weight_lifting, axiom)
peters_hobbies ≠ marys_hobbies    thf(peter_and_mary_have_different_hobbies, conjecture)

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