QUA001∧0.ax Quantales
emptyset: \$i → \$o   thf(emptyset_type, type)
emptyset = (λx: \$i: \$false)   thf(emptyset_def, definition)
union: (\$i → \$o) → (\$i → \$o) → \$i → \$o   thf(union_type, type)
union = (λx: \$i → \$o, y: \$i → \$o, u: \$i: (x@u or y@u))   thf(union_def, definition)
singleton: \$i → \$i → \$o   thf(singleton_type, type)
singleton = (λx: \$i, u: \$i: $u = x$)   thf(singleton_def, definition)
0: \$i   thf(zero_type, type)
sup: (\$i → \$o) → \$i   thf(sup_type, type)
(sup@emptyset) = 0   thf(sup_es, axiom)
$\forall x$: \$i: (sup@(singleton@x)) = x thf(sup_singleset, axiom)
supset: ((\$i → \$o) → \$o) → \$i → \$o   thf(supset_type, type)
supset = (λf: (\$i → \$o) → \$o, x: \$i: $\exists y$: \$i → \$o: (f@y and (sup@y) = x))   thf(supset, definition)
unionset: ((\$i → \$o) → \$o) → \$i → \$o   thf(unionset_type, type)
unionset = (λf: (\$i → \$o) → \$o, x: \$i: $\exists y$: \$i → \$o: (f@y and y@x))   thf(unionset, definition)
$\forall x$: (\$i → \$o) → \$o: (sup@(supset@x)) = (sup@(unionset@x))   thf(sup_set, axiom)
addition: \$i → \$i → \$i   thf(addition_type, type)
addition = (λx: \$i, y: \$i: (sup@(union@(singleton@x)@(singleton@y))))   thf(addition_def, definition)
leq: \$i → \$i → \$o   thf(order_type, type)
$\forall x_1$: \$i, x_2: \$i: ((leq@x_1@x_2) $\iff$ (addition@x_1@x_2) = x_2)   thf(order_def, axiom)
multiplication: \$i → \$i → \$i   thf(multiplication_type, type)
crossmult: (\$i → \$o) → (\$i → \$o) → \$i → \$o   thf(crossmult_type, type)
crossmult = (λx: \$i → \$o, y: \$i → \$o, a: \$i: $\exists x_1$: \$i, y_1: \$i: (x@x_1 and y@y_1 and a = (multiplication@x_1@y_1)))   thf(crossmu
$\forall x$: \$i → \$o, y: \$i → \$o: (multiplication@(sup@x)@(sup@y)) = (sup@(crossmult@x@y))   thf(multiplication_def, axiom)
1: \$i   thf(one_type, type)
$\forall x$: \$i: (multiplication@x@1) = x thf(multiplication_neutral_right, axiom)
$\forall x$: \$i: (multiplication@1@x) = x thf(multiplication_neutral_left, axiom)

QUA001∧1.ax Tests for Quantales (Boolean sub-algebra below 1)
test: \$i → \$o   thf(tests, type)
$\forall x$: \$i: ((test@x) $\Rightarrow$ $\exists y$: \$i: ((addition@x@y) = 1 and (multiplication@x@y) = 0 and (multiplication@y@x) = 0))   thf(test_definition, axiom)

QUA014∧1.p Isotony with respect to multiplication
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o, y: \$i → \$o, z: \$i: ((leq@(sup@x)@(sup@y)) $\Rightarrow$ (leq@(multiplication@(sup@x)@z)@(multiplication@(sup@y)@z))

QUA002∧1.p Addition (Sumpremum) is commutative
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i, x_2: \$i: (addition@x_1@x_2) = (addition@x_2@x_1)   thf(addition_comm, conjecture)

QUA010∧1.p 0 is least element w.r.t. leq
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o: (sup@(union@x@(singleton@0))) = (sup@x)   thf(zero_least, conjecture)

QUA006∧1.p Zero is left-annihilator
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i: (multiplication@0@x_1) = 0   thf(multiplication_anni, conjecture)

QUA015∧1.p Isotony with respect to addition
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o, y: \$i → \$o, z: \$i: ((leq@(sup@x)@(sup@y)) $\Rightarrow$ (leq@(addition@z@(sup@x))@(addition@z@(sup@y))))   thf(a

QUA012∧1.p 0 annihilates arbitrary sums from the left
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o: (multiplication@0@(sup@x)) = 0   thf(multiplication_anni, conjecture)

QUA005∧1.p Zero is right-annihilator
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i: (multiplication@x_1@0) = 0   thf(multiplication_anni, conjecture)

QUA020∧1.p Addition splitting
An element is an upper bound of a sum iff it is an upper bound of : all its summands.
include('Axioms/QUA001^0.ax')
$\forall x$: \$i, y: \$i, z: \$i: ((leq@(addition@x@y)@z) $\iff$ (leq@x@z and leq@y@z))   thf(splitting, conjecture)

QUA016∧1.p Isotony with respect to addition
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o, y: \$i → \$o, z: \$i: ((leq@(sup@x)@(sup@y)) ⇒ (leq@(addition@(sup@x)@z)@(addition@(sup@y)@z))) thf(ad

QUA011∧1.p 0 annihilates arbitrary sums from the right
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o: (multiplication@(sup@x)@0) = 0 thf(multiplication_anni, conjecture)

QUA013∧1.p Isotony with respect to multiplication
include('Axioms/QUA001^0.ax')
$\forall x$: \$i → \$o, y: \$i → \$o, z: \$i: ((leq@(sup@x)@(sup@y)) ⇒ (leq@(multiplication@z@(sup@x))@(multiplication@z@(sup@y)))

QUA001∧1.p Addition is associative
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i, x_2: \$i, x_3: \$i: (addition@(addition@x_1@x_2)@x_3) = (addition@x_1@(addition@x_2@x_3)) thf(addition_asso, conjecture

QUA017∧1.p Tests are idempotent with respect to multiplication
include('Axioms/QUA001^0.ax')
include('Axioms/QUA001^1.ax')
$\forall x$: \$i: ((test@x) ⇒ (multiplication@x@x) = x) thf(test_idemp, conjecture)

QUA008∧1.p Left-distributivity of multiplication over addition
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i, x_2: \$i, x_3: \$i: (multiplication@(addition@x_1@x_2)@x_3) = (addition@(multiplication@x_1@x_3)@(multiplication@x_2@x_3))

QUA021∧1.p Quantales
include('Axioms/QUA001^0.ax')

QUA004∧1.p Addition is idempotent
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i: (addition@x_1@x_1) = x_1 thf(addition_idemp, conjecture)

QUA019∧1.p Infimums-property on tests
include('Axioms/QUA001^0.ax')
include('Axioms/QUA001^1.ax')
$\forall x$: \$i, y: \$i, z: \$i: ((test@x and test@y and test@z) ⇒ ((leq@x@(multiplication@y@z)) ⟺ (leq@x@y and leq@x@z)))

QUA003∧1.p Zero is neutral with respect to addition
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i: (addition@x_1@0) = x_1 thf(addition_neutral, conjecture)

QUA009∧1.p leq is an order
leq is an order. i.e., it is reflexive, transitive and antysymmetric
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i, x_2: \$i, x_3: \$i: (leq@x_1@x_1 and ((leq@x_1@x_2 and leq@x_2@x_3) ⇒ (leq@x_1@x_3)) and ((leq@x_1@x_2 and leq@x_2@x_1) ⇒
$x_1 = x_2$)) thf(multiplication_distrl, conjecture)

QUA007∧1.p Right-distributivity of multiplication over addition
include('Axioms/QUA001^0.ax')
$\forall x_1$: \$i, x_2: \$i, x_3: \$i: (multiplication@x_1@(addition@x_2@x_3)) = (addition@(multiplication@x_1@x_2)@(multiplication@x_1@x_3))

QUA018∧1.p Tests are commutative with respect to multiplication
include('Axioms/QUA001^0.ax')
include('Axioms/QUA001^1.ax')
$\forall x$: \$i, y: \$i: ((test@x and test@y) ⇒ (multiplication@x@y) = (multiplication@y@x)) thf(test_comm, conjecture)