REL axioms

REL001+0.ax Relation Algebra
$\forall x_0, x_1\colon x_0 \vee x_1 = x_1 \vee x_0$ fof(maddux1_join_commutativity, axiom)
$\forall x_0, x_1, x_2\colon x_0 \vee (x_1 \vee x_2) = (x_0 \vee x_1) \vee x_2$ fof(maddux2_join_associativity, axiom)
$\forall x_0, x_1\colon x_0 = (x_0' \vee x_1')' \vee (x_0' \vee x_1)'$ fof(maddux3_a_kind_of_de_Morgan, axiom)
$\forall x_0, x_1\colon x_0 \wedge x_1 = (x_0' \vee x_1')'$ fof(maddux4_definiton_of_meet, axiom)
$\forall x_0, x_1, x_2\colon x_0; (x_1; x_2) = (x_0; x_1); x_2$ fof(composition_associativity, axiom)
$\forall x_0\colon x_0; 1 = x_0$ fof(composition_identity, axiom)
$\forall x_0, x_1, x_2\colon (x_0 \vee x_1); x_2 = x_0; x_2 \vee x_1; x_2$ fof(composition_distributivity, axiom)
$\forall x_0\colon (x_0^{\smile})^{\smile} = x_0$ fof(converse_idempotence, axiom)
$\forall x_0, x_1\colon (x_0 \vee x_1)^{\smile} = x_0^{\smile} \vee x_1^{\smile}$ fof(converse_additivity, axiom)
$\forall x_0, x_1\colon (x_0; x_1)^{\smile} = x_1^{\smile}; x_0^{\smile}$ fof(converse_multiplicativity, axiom)
$\forall x_0, x_1\colon x_0^{\smile}; (x_0; x_1)' \vee x_1' = x_1'$ fof(converse_cancellativity, axiom)
$\forall x_0\colon \top = x_0 \vee x_0'$ fof(def_top, axiom)
$\forall x_0\colon 0 = x_0 \wedge x_0'$ fof(def_zero, axiom)

REL001+1.ax Dedkind and two modular laws
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2) \vee (x_0 \wedge x_2; x_1^{\smile}); (x_1 \wedge x_0^{\smile}; x_2) = (x_0 \wedge x_2; x_1^{\smile}); (x_1 \wedge x_0^{\smile}; x_2)$ fof(dedekind_law, axiom)
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2) \vee (x_0; (x_1 \wedge x_0^{\smile}; x_2) \wedge x_2) = x_0; (x_1 \wedge x_0^{\smile}; x_2) \wedge x_2$ fof(modular_law$_1$, axiom)
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2) \vee ((x_0 \wedge x_2; x_1^{\smile}); x_1 \wedge x_2) = (x_0 \wedge x_2; x_1^{\smile}); x_1 \wedge x_2$ fof(modular_law$_2$, axiom)

REL001-0.ax Relation algebra
$a \vee b = b \vee a$ cnf(maddux1_join_commutativity$_1$, axiom)
$a \vee (b \vee c) = (a \vee b) \vee c$ cnf(maddux2_join_associativity$_2$, axiom)
$a = (a' \vee b')' \vee (a' \vee b)'$ cnf(maddux3_a_kind_of_de_Morgan$_3$, axiom)
$a \wedge b = (a' \vee b')'$ cnf(maddux4_definiton_of_meet$_4$, axiom)
$a; (b; c) = (a; b); c$ cnf(composition_associativity$_5$, axiom)
$a; 1 = a$ cnf(composition_identity$_6$, axiom)
$(a \vee b); c = a; c \vee b; c$ cnf(composition_distributivity$_7$, axiom)
$(a^{\smile})^{\smile} = a$ cnf(converse_idempotence$_8$, axiom)
$(a \vee b)^{\smile} = a^{\smile} \vee b^{\smile}$ cnf(converse_additivity$_9$, axiom)
$(a; b)^{\smile} = b^{\smile}; a^{\smile}$ cnf(converse_multiplicativity$_{10}$, axiom)
$a^{\smile}; (a; b)' \vee b' = b'$ cnf(converse_cancellativity$_{11}$, axiom)
$\top = a \vee a'$ cnf(def_top$_{12}$, axiom)
$0 = a \wedge a'$ cnf(def_zero$_{13}$, axiom)

REL001-1.ax Dedkind and two modular laws
$(a; b \wedge c) \vee (a \wedge c; b^{\smile}); (b \wedge a^{\smile}; c) = (a \wedge c; b^{\smile}); (b \wedge a^{\smile}; c)$ cnf(dedekind_law$_{14}$, axiom)
$(a; b \wedge c) \vee (a; (b \wedge a^{\smile}; c) \wedge c) = a; (b \wedge a^{\smile}; c) \wedge c$ cnf(modular_law_1$_{15}$, axiom)
$(a; b \wedge c) \vee ((a \wedge c; b^{\smile}); b \wedge c) = (a \wedge c; b^{\smile}); b \wedge c$ cnf(modular_law_2$_{16}$, axiom)

REL problems

REL001+1.p There is a (unique) least element, namely 0
include('Axioms/REL001+0.ax')
$\forall x_0\colon 0 \vee x_0 = x_0$ fof(goals, conjecture)

REL001-1.p There is a (unique) least element, namely 0
include('Axioms/REL001-0.ax')
$0 \vee \mathrm{sk}_1 \neq \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)

REL002+1.p There is a (unique) greatest element, namely x + x'
include('Axioms/REL001+0.ax')
$\forall x_0\colon x_0 \vee \top = \top$ fof(goals, conjecture)

REL002-1.p There is a (unique) greatest element, namely x + x'
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee \top \neq \top$ cnf(goals$_{14}$, negated_conjecture)

REL003+1.p Isotonicity of converse
x is less or equal than y iff the converse of x is less or equal than converse of y.
include('Axioms/REL001+0.ax')

$\forall x_0, x_1\colon ((x_0 \vee x_1 = x_1 \;\Rightarrow\; x_0^{\smile} \vee x_1^{\smile} = x_1^{\smile}) \text{ and } (x_0^{\smile} \vee x_1^{\smile} = x_1^{\smile} \;\Rightarrow\; x_0 \vee x_1 = x_1))$ fof(goals, conjecture)

REL003-1.p Isotonicity of converse
x is less or equal than y iff the converse of x is less or equal than converse of y.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2 \text{ or } \mathrm{sk}_1^{\smile} \vee \mathrm{sk}_2^{\smile} = \mathrm{sk}_2^{\smile}$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2 \;\Rightarrow\; \mathrm{sk}_1^{\smile} \vee \mathrm{sk}_2^{\smile} \neq \mathrm{sk}_2^{\smile}$ cnf(goals_{17}, negated_conjecture)

REL004+1.p Converse negation are interconvertible
include('Axioms/REL001+0.ax')
$\forall x_0\colon x_0^{\prime\smile} = (x_0^{\smile})'$ fof(goals, conjecture)

REL004+2.p Converse negation are interconvertible
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0\colon x_0^{\prime\smile} = (x_0^{\smile})'$ fof(goals, conjecture)

REL004+3.p Converse negation are interconvertible
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0\colon x_0^{\prime\smile} = (x_0^{\smile})'$ fof(goals, conjecture)

REL004-1.p Converse negation are interconvertible
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1^{\prime\smile} \neq (\mathrm{sk}_1^{\smile})'$ cnf(goals_{14}, negated_conjecture)

REL004-2.p Converse negation are interconvertible
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\prime\smile} \neq (\mathrm{sk}_1^{\smile})'$ cnf(goals_{17}, negated_conjecture)

REL004-3.p Converse negation are interconvertible
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\prime\smile} \neq (\mathrm{sk}_1^{\smile})'$ cnf(goals_{17}, negated_conjecture)

REL005+1.p Converse distributes over meet
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0 \wedge x_1)^{\smile} = x_0^{\smile} \wedge x_1^{\smile}$ fof(goals, conjecture)

REL005+2.p Converse distributes over meet
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon ((x_0 \wedge x_1)^{\smile} \vee (x_0^{\smile} \wedge x_1^{\smile}) = x_0^{\smile} \wedge x_1^{\smile} \text{ and } (x_0^{\smile} \wedge x_1^{\smile}) \vee (x_0 \wedge x_1)^{\smile} = (x_0 \wedge x_1)^{\smile})$ fof(goals, conjecture)

REL005+3.p Converse distributes over meet
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0 \wedge x_1)^{\smile} = x_0^{\smile} \wedge x_1^{\smile}$ fof(goals, conjecture)

REL005+4.p Converse distributes over meet
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon ((x_0 \wedge x_1)^{\smile} \vee (x_0^{\smile} \wedge x_1^{\smile}) = x_0^{\smile} \wedge x_1^{\smile} \text{ and } (x_0^{\smile} \wedge x_1^{\smile}) \vee (x_0 \wedge x_1)^{\smile} = (x_0 \wedge x_1)^{\smile})$ fof(goals, conjecture)

REL005-1.p Converse distributes over meet
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \neq \mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}$ cnf(goals_{14}, negated_conjecture)

REL005-2.p Converse distributes over meet
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \vee (\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}) = \mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile} \;\Rightarrow\; (\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile}$ cnf(goals_{14}, negated_conjecture)

REL005-3.p Converse distributes over meet
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \neq \mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}$ cnf(goals_{17}, negated_conjecture)

REL005-4.p Converse distributes over meet
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \vee (\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}) = \mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile} \Rightarrow (\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2^{\smile}) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile} \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2)^{\smile}$ cnf(goals_{17}, negated_conjecture)

REL006+1.p For empty meet the converse slides over meet
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0^{\smile} \wedge x_1 = 0 \Rightarrow x_0 \wedge x_1^{\smile} = 0)$ fof(goals, conjecture)

REL006-1.p For empty meet the converse slides over meet
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2 = 0$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1 \wedge \mathrm{sk}_2^{\smile} \neq 0$ cnf(goals_{15}, negated_conjecture)

REL007+1.p For empty meet the converse slides over meet
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0 \wedge x_1^{\smile} = 0 \Rightarrow x_0^{\smile} \wedge x_1 = 0)$ fof(goals, conjecture)

REL007-1.p For empty meet the converse slides over meet
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \wedge \mathrm{sk}_2^{\smile} = 0$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1^{\smile} \wedge \mathrm{sk}_2 \neq 0$ cnf(goals_{15}, negated_conjecture)

REL008+1.p Sequential composition distributes over addition
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon x_0; (x_1 \vee x_2) = x_0; x_1 \vee x_0; x_2$ fof(goals, conjecture)

REL008+2.p Sequential composition distributes over addition
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon ((x_0; (x_1 \vee x_2) \vee x_0; x_1) \vee x_0; x_2 = x_0; x_1 \vee x_0; x_2$ and $(x_0; x_1 \vee x_0; x_2) \vee x_0; (x_1 \vee x_2) = x_0; (x_1 \vee x_2))$ fof(goals, conjecture)

REL008+3.p Sequential composition distributes over addition
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon x_0; (x_1 \vee x_2) = x_0; x_1 \vee x_0; x_2$ fof(goals, conjecture)

REL008+4.p Sequential composition distributes over addition
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon ((x_0; (x_1 \vee x_2) \vee x_0; x_1) \vee x_0; x_2 = x_0; x_1 \vee x_0; x_2$ and $(x_0; x_1 \vee x_0; x_2) \vee x_0; (x_1 \vee x_2) = x_0; (x_1 \vee x_2))$ fof(goals, conjecture)

REL008-1.p Sequential composition distributes over addition
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{14}, negated_conjecture)

REL008-2.p Sequential composition distributes over addition
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3) \vee \mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) = \mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \Rightarrow (\mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \vee \mathrm{sk}_1; \mathrm{sk}_2) \vee \mathrm{sk}_1; \mathrm{sk}_3 \neq \mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{14}, negated_conjecture)

REL008-3.p Sequential composition distributes over addition
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{17}, negated_conjecture)

REL008-4.p Sequential composition distributes over addition
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3) \vee \mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) = \mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \Rightarrow (\mathrm{sk}_1; (\mathrm{sk}_2 \vee \mathrm{sk}_3) \vee \mathrm{sk}_1; \mathrm{sk}_2) \vee \mathrm{sk}_1; \mathrm{sk}_3 \neq \mathrm{sk}_1; \mathrm{sk}_2 \vee \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{17}, negated_conjecture)

REL009+1.p Sequential composition is isotone in both arguments
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0 \vee x_1 = x_1 \Rightarrow (x_0; x_2 \vee x_1; x_2 = x_1; x_2$ and $x_2; x_0 \vee x_2; x_1 = x_2; x_1))$ fof(goals, conjecture)

REL009+2.p Sequential composition is isotone in both arguments
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0 \vee x_1 = x_1 \;\Rightarrow\; (x_0; x_2 \vee x_1; x_2 = x_1; x_2 \text{ and } x_2; x_0 \vee x_2; x_1 = x_2; x_1))$ $\quad$ fof(goals, conjecture)

REL009-1.p Sequential composition is isotone in both arguments
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2$ $\quad$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_3 \vee \mathrm{sk}_2; \mathrm{sk}_3 = \mathrm{sk}_2; \mathrm{sk}_3 \;\Rightarrow\; \mathrm{sk}_3; \mathrm{sk}_1 \vee \mathrm{sk}_3; \mathrm{sk}_2 \neq \mathrm{sk}_3; \mathrm{sk}_2$ $\quad$ cnf(goals_{15}, negated_conjecture)

REL009-2.p Sequential composition is isotone in both arguments
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2$ $\quad$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_3 \vee \mathrm{sk}_2; \mathrm{sk}_3 = \mathrm{sk}_2; \mathrm{sk}_3 \;\Rightarrow\; \mathrm{sk}_3; \mathrm{sk}_1 \vee \mathrm{sk}_3; \mathrm{sk}_2 \neq \mathrm{sk}_3; \mathrm{sk}_2$ $\quad$ cnf(goals_{18}, negated_conjecture)

REL010+1.p Schroeder equivalence (first implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2 = 0 \;\Rightarrow\; x_1 \wedge x_0^{\smile}; x_2 = 0)$ $\quad$ fof(goals, conjecture)

REL010+2.p Schroeder equivalence (first implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2 = 0 \;\Rightarrow\; x_1 \wedge x_0^{\smile}; x_2 = 0)$ $\quad$ fof(goals, conjecture)

REL010-1.p Schroeder equivalence (first implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3 = 0$ $\quad$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3 \neq 0$ $\quad$ cnf(goals_{15}, negated_conjecture)

REL010-2.p Schroeder equivalence (first implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3 = 0$ $\quad$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3 \neq 0$ $\quad$ cnf(goals_{18}, negated_conjecture)

REL011+1.p Schroeder equivalence (second implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0 \wedge x_1^{\smile}; x_2 = 0 \;\Rightarrow\; x_1; x_0 \wedge x_2 = 0)$ $\quad$ fof(goals, conjecture)

REL011+2.p Schroeder equivalence (second implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0 \wedge x_1^{\smile}; x_2 = 0 \;\Rightarrow\; x_1; x_0 \wedge x_2 = 0)$ $\quad$ fof(goals, conjecture)

REL011-1.p Schroeder equivalence (second implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \wedge \mathrm{sk}_2^{\smile}; \mathrm{sk}_3 = 0$ $\quad$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_2; \mathrm{sk}_1 \wedge \mathrm{sk}_3 \neq 0$ $\quad$ cnf(goals_{15}, negated_conjecture)

REL011-2.p Schroeder equivalence (second implication)
Describes the interplay between composition, converse and complement, w.r.t. containment.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \wedge \mathrm{sk}_2^{\smile}; \mathrm{sk}_3 = 0$ $\quad$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_2; \mathrm{sk}_1 \wedge \mathrm{sk}_3 \neq 0$ $\quad$ cnf(goals_{18}, negated_conjecture)

REL012+1.p Cancelativity of converse

include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0; x_1)'; x_1^{\smile} \vee x_0' = x_0'$ fof(goals, conjecture)

REL012+2.p Cancelativity of converse
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0; x_1)'; x_1^{\smile} \vee x_0' = x_0'$ fof(goals, conjecture)

REL012-1.p Cancelativity of converse
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2)'; \mathrm{sk}_2^{\smile} \vee \mathrm{sk}_1' \neq \mathrm{sk}_1'$ cnf(goals_{14}, negated_conjecture)

REL012-2.p Cancelativity of converse
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2)'; \mathrm{sk}_2^{\smile} \vee \mathrm{sk}_1' \neq \mathrm{sk}_1'$ cnf(goals_{17}, negated_conjecture)

REL013+1.p Zero is annihilator
include('Axioms/REL001+0.ax')
$\forall x_0\colon (x_0; 0 = 0 \text{ and } 0; x_0 = 0)$ fof(goals, conjecture)

REL013-1.p Zero is annihilator
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; 0 = 0 \Rightarrow 0; \mathrm{sk}_1 \neq 0$ cnf(goals_{14}, negated_conjecture)

REL014+1.p One is neutral element
include('Axioms/REL001+0.ax')
$\forall x_0\colon (x_0; 1 = x_0 \text{ and } 1; x_0 = x_0)$ fof(goals, conjecture)

REL014-1.p One is neutral element
include('Axioms/REL001-0.ax')

REL015+1.p TOP is idempotent w.r.t. composition
include('Axioms/REL001+0.ax')
$\top; \top = \top$ fof(goals, conjecture)

REL015-1.p TOP is idempotent w.r.t. composition
include('Axioms/REL001-0.ax')
$\top; \top \neq \top$ cnf(goals_{14}, negated_conjecture)

REL016+1.p A modular law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon x_0; x_1 \wedge (x_0; x_2)' = x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)'$ fof(goals, conjecture)

REL016+2.p A modular law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon ((x_0; x_1 \wedge (x_0; x_2)') \vee (x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)') = x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)' \text{ and } (x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)') \vee (x_0; x_1 \wedge (x_0; x_2)') = x_0; x_1 \wedge (x_0; x_2)')$ fof(goals, conjecture)

REL016+3.p A modular law
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon x_0; x_1 \wedge (x_0; x_2)' = x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)'$ fof(goals, conjecture)

REL016+4.p A modular law
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon ((x_0; x_1 \wedge (x_0; x_2)') \vee (x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)') = x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)' \text{ and } (x_0; (x_1 \wedge x_2') \wedge (x_0; x_2)') \vee (x_0; x_1 \wedge (x_0; x_2)') = x_0; x_1 \wedge (x_0; x_2)')$ fof(goals, conjecture)

REL016-1.p A modular law
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)' \neq \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1; \mathrm{sk}_3)'$ cnf(goals_{14}, negated_conjecture)

REL016-2.p A modular law
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') \vee (\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') = \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1; \mathrm{sk}_3)' \Rightarrow (\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)'$ cnf(goals_{14}, negated_conjecture)

REL016-3.p A modular law
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1;\mathrm{sk}_2 \wedge (\mathrm{sk}_1;\mathrm{sk}_3)' \neq \mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1;\mathrm{sk}_3)'$ cnf(goals$_{17}$, negated_conjecture)

REL016-4.p A modular law
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1;\mathrm{sk}_2 \wedge (\mathrm{sk}_1;\mathrm{sk}_3)') \vee (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1;\mathrm{sk}_3)') = \mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1;\mathrm{sk}_3)' \Rightarrow (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3') \wedge (\mathrm{sk}_1;\mathrm{sk}_3)') \vee (\mathrm{sk}_1;\mathrm{sk}_2 \wedge (\mathrm{sk}_1;\mathrm{sk}_3)') \neq \mathrm{sk}_1;\mathrm{sk}_2 \wedge (\mathrm{sk}_1;\mathrm{sk}_3)'$ cnf(goals$_{17}$, negated_conjecture)

REL017+1.p Another modular law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0;x_1)' \vee x_0;x_2 = (x_0;(x_1 \wedge x_2'))' \vee x_0;x_2$ fof(goals, conjecture)

REL017+2.p Another modular law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon ((((x_0;x_1)' \vee x_0;x_2) \vee (x_0;(x_1 \wedge x_2'))') \vee x_0;x_2 = (x_0;(x_1 \wedge x_2'))' \vee x_0;x_2$ and $(((x_0;(x_1 \wedge x_2'))' \vee x_0;x_2) \vee (x_0;x_1)') \vee x_0;x_2 = (x_0;x_1)' \vee x_0;x_2)$ fof(goals, conjecture)

REL017+3.p Another modular law
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0;x_1)' \vee x_0;x_2 = (x_0;(x_1 \wedge x_2'))' \vee x_0;x_2$ fof(goals, conjecture)

REL017+4.p Another modular law
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon ((((x_0;x_1)' \vee x_0;x_2) \vee (x_0;(x_1 \wedge x_2'))') \vee x_0;x_2 = (x_0;(x_1 \wedge x_2'))' \vee x_0;x_2$ and $(((x_0;(x_1 \wedge x_2'))' \vee x_0;x_2) \vee (x_0;x_1)') \vee x_0;x_2 = (x_0;x_1)' \vee x_0;x_2)$ fof(goals, conjecture)

REL017-1.p Another modular law
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3 \neq (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3$ cnf(goals$_{14}$, negated_conjecture)

REL017-2.p Another modular law
include('Axioms/REL001-0.ax')
$(((\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3) \vee (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))') \vee \mathrm{sk}_1;\mathrm{sk}_3 = (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3 \Rightarrow (((\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3) \vee (\mathrm{sk}_1;\mathrm{sk}_2)') \vee \mathrm{sk}_1;\mathrm{sk}_3 \neq (\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3$ cnf(goals$_{14}$, negated_conjecture)

REL017-3.p Another modular law
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3 \neq (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3$ cnf(goals$_{17}$, negated_conjecture)

REL017-4.p Another modular law
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(((\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3) \vee (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))') \vee \mathrm{sk}_1;\mathrm{sk}_3 = (\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3 \Rightarrow (((\mathrm{sk}_1;(\mathrm{sk}_2 \wedge \mathrm{sk}_3'))' \vee \mathrm{sk}_1;\mathrm{sk}_3) \vee (\mathrm{sk}_1;\mathrm{sk}_2)') \vee \mathrm{sk}_1;\mathrm{sk}_3 \neq (\mathrm{sk}_1;\mathrm{sk}_2)' \vee \mathrm{sk}_1;\mathrm{sk}_3$ cnf(goals$_{17}$, negated_conjecture)

REL018+1.p Vectors are closed under complementation
If x is a vector then overlinex is a vector too.
include('Axioms/REL001+0.ax')
$\forall x_0\colon (x_0;\top = x_0 \Rightarrow x_0';\top = x_0')$ fof(goals, conjecture)

REL018-1.p Vectors are closed under complementation
If x is a vector then overlinex is a vector too.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1;\top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1';\top \neq \mathrm{sk}_1'$ cnf(goals$_{15}$, negated_conjecture)

REL019+1.p Vectors are closed under meet
If x and y are vectors then x meet y is a vector too.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon ((x_0;\top = x_0$ and $x_1;\top = x_1) \Rightarrow (x_0 \wedge x_1);\top = x_0 \wedge x_1)$ fof(goals, conjecture)

REL019+2.p Vectors are closed under meet
If x and y are vectors then x meet y is a vector too.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon ((x_0; \top = x_0 \text{ and } x_1; \top = x_1) \Rightarrow (x_0 \wedge x_1); \top = x_0 \wedge x_1)$ fof(goals, conjecture)

REL019-1.p Vectors are closed under meet
If x and y are vectors then x meet y is a vector too.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_2; \top = \mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \top \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals$_{16}$, negated_conjecture)

REL019-2.p Vectors are closed under meet
If x and y are vectors then x meet y is a vector too.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1$ cnf(goals$_{17}$, negated_conjecture)
$\mathrm{sk}_2; \top = \mathrm{sk}_2$ cnf(goals$_{18}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \top \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals$_{19}$, negated_conjecture)

REL020+1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge 1); x_1 = x_0 \wedge x_1)$ fof(goals, conjecture)

REL020+2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge 1); x_1 = x_0 \wedge x_1)$ fof(goals, conjecture)

REL020-1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge 1); \mathrm{sk}_2 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)

REL020-2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1$ cnf(goals$_{17}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge 1); \mathrm{sk}_2 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals$_{18}$, negated_conjecture)

REL021+1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge 1); x_1 \vee (x_0 \wedge x_1) = x_0 \wedge x_1)$ fof(goals, conjecture)

REL021+2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge 1); x_1 \vee (x_0 \wedge x_1) = x_0 \wedge x_1)$ fof(goals, conjecture)

REL021-1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge 1); \mathrm{sk}_2 \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2) \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)

REL021-2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')

include('Axioms/REL001-1.ax')
$sk_1;\top = sk_1$ cnf($goals_{17}$, negated_conjecture)
$(sk_1 \wedge 1); sk_2 \vee (sk_1 \wedge sk_2) \neq sk_1 \wedge sk_2$ cnf($goals_{18}$, negated_conjecture)

REL022+1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1 \colon (x_0;\top = x_0 \Rightarrow (x_0 \wedge x_1) \vee (x_0 \wedge 1); x_1 = (x_0 \wedge 1); x_1)$ fof(goals, conjecture)

REL022+2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1 \colon (x_0;\top = x_0 \Rightarrow (x_0 \wedge x_1) \vee (x_0 \wedge 1); x_1 = (x_0 \wedge 1); x_1)$ fof(goals, conjecture)

REL022-1.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')
$sk_1;\top = sk_1$ cnf($goals_{14}$, negated_conjecture)
$(sk_1 \wedge sk_2) \vee (sk_1 \wedge 1); sk_2 \neq (sk_1 \wedge 1); sk_2$ cnf($goals_{15}$, negated_conjecture)

REL022-2.p Restriction of subidentities
For vectors restriction of subidientities equals intersection with vectors.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$sk_1;\top = sk_1$ cnf($goals_{17}$, negated_conjecture)
$(sk_1 \wedge sk_2) \vee (sk_1 \wedge 1); sk_2 \neq (sk_1 \wedge 1); sk_2$ cnf($goals_{18}$, negated_conjecture)

REL023+1.p A simple consequence of isotonicity
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \wedge x_1^{\smile}); (x_1 \wedge x_2) \vee x_0; (x_1 \wedge x_2) = x_0; (x_1 \wedge x_2)$ fof(goals, conjecture)

REL023+2.p A simple consequence of isotonicity
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \wedge x_1^{\smile}); (x_1 \wedge x_2) \vee x_0; (x_1 \wedge x_2) = x_0; (x_1 \wedge x_2)$ fof(goals, conjecture)

REL023-1.p A simple consequence of isotonicity
include('Axioms/REL001-0.ax')
$(sk_1 \wedge sk_2^{\smile}); (sk_2 \wedge sk_3) \vee sk_1; (sk_2 \wedge sk_3) \neq sk_1; (sk_2 \wedge sk_3)$ cnf($goals_{14}$, negated_conjecture)

REL023-2.p A simple consequence of isotonicity
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(sk_1 \wedge sk_2^{\smile}); (sk_2 \wedge sk_3) \vee sk_1; (sk_2 \wedge sk_3) \neq sk_1; (sk_2 \wedge sk_3)$ cnf($goals_{17}$, negated_conjecture)

REL024+1.p A simple consequence of isotonicity
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \wedge x_1^{\smile}); (x_1 \wedge x_2) \vee (x_0 \wedge x_1^{\smile}); x_2 = (x_0 \wedge x_1^{\smile}); x_2$ fof(goals, conjecture)

REL024+2.p A simple consequence of isotonicity
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \wedge x_1^{\smile}); (x_1 \wedge x_2) \vee (x_0 \wedge x_1^{\smile}); x_2 = (x_0 \wedge x_1^{\smile}); x_2$ fof(goals, conjecture)

REL024-1.p A simple consequence of isotonicity
include('Axioms/REL001-0.ax')
$(sk_1 \wedge sk_2^{\smile}); (sk_2 \wedge sk_3) \vee (sk_1 \wedge sk_2^{\smile}); sk_3 \neq (sk_1 \wedge sk_2^{\smile}); sk_3$ cnf($goals_{14}$, negated_conjecture)

REL024-2.p A simple consequence of isotonicity
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(sk_1 \wedge sk_2^{\smile}); (sk_2 \wedge sk_3) \vee (sk_1 \wedge sk_2^{\smile}); sk_3 \neq (sk_1 \wedge sk_2^{\smile}); sk_3$ cnf($goals_{17}$, negated_conjecture)

REL025+1.p For subidentities converse is redundant
If x is a subidentity then the converse of x equals x.
include('Axioms/REL001+0.ax')

$\forall x_0\colon (x_0 \vee 1 = 1 \;\Rightarrow\; x_0^{\smile} = x_0)$ fof(goals, conjecture)

REL025+2.p For subidentities converse is redundant
If x is a subidentity then the converse of x equals x.
include('Axioms/REL001+0.ax')
$\forall x_0\colon ((x_0 \vee 1 = 1 \;\Rightarrow\; x_0^{\smile} \vee x_0 = x_0) \text{ and } (x_0 \vee 1 = 1 \;\Rightarrow\; x_0 \vee x_0^{\smile} = x_0^{\smile}))$ fof(goals, conjecture)

REL025-1.p For subidentities converse is redundant
If x is a subidentity then the converse of x equals x.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1^{\smile} \neq \mathrm{sk}_1$ cnf(goals$_{15}$, negated_conjecture)

REL025-2.p For subidentities converse is redundant
If x is a subidentity then the converse of x equals x.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1 \vee \mathrm{sk}_1^{\smile} = \mathrm{sk}_1^{\smile} \;\Rightarrow\; \mathrm{sk}_1^{\smile} \vee \mathrm{sk}_1 \neq \mathrm{sk}_1$ cnf(goals$_{17}$, negated_conjecture)

REL026+1.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0 \vee 1 = 1 \;\Rightarrow\; x_0;\top \wedge x_1 = x_0;x_1)$ fof(goals, conjecture)

REL026+2.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1\colon (x_0 \vee 1 = 1 \;\Rightarrow\; ((x_0;\top \wedge x_1) \vee x_0;x_1 = x_0;x_1 \text{ and } x_0;x_1 \vee (x_0;\top \wedge x_1) = x_0;\top \wedge x_1))$ fof(goals, conjecture)

REL026+3.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0 \vee 1 = 1 \;\Rightarrow\; x_0;\top \wedge x_1 = x_0;x_1)$ fof(goals, conjecture)

REL026+4.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1\colon (x_0 \vee 1 = 1 \;\Rightarrow\; ((x_0;\top \wedge x_1) \vee x_0;x_1 = x_0;x_1 \text{ and } x_0;x_1 \vee (x_0;\top \wedge x_1) = x_0;\top \wedge x_1))$ fof(goals, conjecture)

REL026-1.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1;\top \wedge \mathrm{sk}_2 \neq \mathrm{sk}_1;\mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)

REL026-2.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1;\mathrm{sk}_2 \vee (\mathrm{sk}_1;\top \wedge \mathrm{sk}_2) = \mathrm{sk}_1;\top \wedge \mathrm{sk}_2 \;\Rightarrow\; (\mathrm{sk}_1;\top \wedge \mathrm{sk}_2) \vee \mathrm{sk}_1;\mathrm{sk}_2 \neq \mathrm{sk}_1;\mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)

REL026-3.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{17}$, negated_conjecture)
$\mathrm{sk}_1;\top \wedge \mathrm{sk}_2 \neq \mathrm{sk}_1;\mathrm{sk}_2$ cnf(goals$_{18}$, negated_conjecture)

REL026-4.p Splitting rule for x;y if x is a subidentity
If x is a subidentity then the composition of x and y can be split into a meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{17}$, negated_conjecture)
$\mathrm{sk}_1;\mathrm{sk}_2 \vee (\mathrm{sk}_1;\top \wedge \mathrm{sk}_2) = \mathrm{sk}_1;\top \wedge \mathrm{sk}_2 \;\Rightarrow\; (\mathrm{sk}_1;\top \wedge \mathrm{sk}_2) \vee \mathrm{sk}_1;\mathrm{sk}_2 \neq \mathrm{sk}_1;\mathrm{sk}_2$ cnf(goals$_{18}$, negated_conjecture)

REL027+1.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001+0.ax')
$\forall x_0$: $(x_0 \vee 1 = 1 \;\Rightarrow\; (x_0; \top)' \wedge 1 = x_0' \wedge 1)$ fof(goals, conjecture)

REL027+2.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001+0.ax')
$\forall x_0$: $(x_0 \vee 1 = 1 \;\Rightarrow\; (((x_0; \top)' \wedge 1) \vee (x_0' \wedge 1) = x_0' \wedge 1 \text{ and } (x_0' \wedge 1) \vee ((x_0; \top)' \wedge 1) = (x_0; \top)' \wedge 1))$ fof(goals, conjecture)

REL027+3.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0$: $(x_0 \vee 1 = 1 \;\Rightarrow\; (x_0; \top)' \wedge 1 = x_0' \wedge 1)$ fof(goals, conjecture)

REL027+4.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0$: $(x_0 \vee 1 = 1 \;\Rightarrow\; (((x_0; \top)' \wedge 1) \vee (x_0' \wedge 1) = x_0' \wedge 1 \text{ and } (x_0' \wedge 1) \vee ((x_0; \top)' \wedge 1) = (x_0; \top)' \wedge 1))$ fof(goals, conjecture)

REL027-1.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$(\mathrm{sk}_1; \top)' \wedge 1 \neq \mathrm{sk}_1' \wedge 1$ cnf(goals_{15}, negated_conjecture)

REL027-2.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$(\mathrm{sk}_1' \wedge 1) \vee ((\mathrm{sk}_1; \top)' \wedge 1) = (\mathrm{sk}_1; \top)' \wedge 1 \;\Rightarrow\; ((\mathrm{sk}_1; \top)' \wedge 1) \vee (\mathrm{sk}_1' \wedge 1) \neq \mathrm{sk}_1' \wedge 1$ cnf(goals_{15}, negated_conjecture)

REL027-3.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$(\mathrm{sk}_1; \top)' \wedge 1 \neq \mathrm{sk}_1' \wedge 1$ cnf(goals_{18}, negated_conjecture)

REL027-4.p Complements of vectors and subidentities
The relative complement of subidentity x w.r.t. 1 can also be obtained by projecting the complement of the corresponding vector x;TOP to a subidentity.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$(\mathrm{sk}_1' \wedge 1) \vee ((\mathrm{sk}_1; \top)' \wedge 1) = (\mathrm{sk}_1; \top)' \wedge 1 \;\Rightarrow\; ((\mathrm{sk}_1; \top)' \wedge 1) \vee (\mathrm{sk}_1' \wedge 1) \neq \mathrm{sk}_1' \wedge 1$ cnf(goals_{18}, negated_conjecture)

REL028+1.p For subidentities meet and composition coincide
include('Axioms/REL001+0.ax')
$\forall x_0, x_1$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \;\Rightarrow\; x_0; x_1 = x_0 \wedge x_1)$ fof(goals, conjecture)

REL028+2.p For subidentities meet and composition coincide
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \;\Rightarrow\; x_0; x_1 = x_0 \wedge x_1)$ fof(goals, conjecture)

REL028-1.p For subidentities meet and composition coincide
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals_{15}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_2 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals_{16}, negated_conjecture)

REL028-2.p For subidentities meet and composition coincide
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals_{18}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_2 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2$ cnf(goals_{19}, negated_conjecture)

REL029+1.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \Rightarrow x_0; x_2 \wedge x_1; x_2 = (x_0 \wedge x_1); x_2)$ fof(goals, conjecture)

REL029+2.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \Rightarrow ((x_0; x_2 \wedge x_1; x_2) \vee (x_0 \wedge x_1); x_2 = (x_0 \wedge x_1); x_2 \text{ and } (x_0 \wedge x_1); x_2 \vee (x_0; x_2 \wedge x_1; x_2) = x_0; x_2 \wedge x_1; x_2))$ fof(goals, conjecture)

REL029+3.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
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$\forall x_0, x_1, x_2$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \Rightarrow x_0; x_2 \wedge x_1; x_2 = (x_0 \wedge x_1); x_2)$ fof(goals, conjecture)

REL029+4.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2$: $((x_0 \vee 1 = 1 \text{ and } x_1 \vee 1 = 1) \Rightarrow ((x_0; x_2 \wedge x_1; x_2) \vee (x_0 \wedge x_1); x_2 = (x_0 \wedge x_1); x_2 \text{ and } (x_0 \wedge x_1); x_2 \vee (x_0; x_2 \wedge x_1; x_2) = x_0; x_2 \wedge x_1; x_2))$ fof(goals, conjecture)

REL029-1.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals_{15}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3$ cnf(goals_{16}, negated_conjecture)

REL029-2.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals_{15}, negated_conjecture)
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \vee (\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3) = \mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3$ cnf(goals_{16}, negated_conjecture)

REL029-3.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals_{18}, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3$ cnf(goals_{19}, negated_conjecture)

REL029-4.p Distributivity of subidentities
For subidentities, sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)

$\mathrm{sk}_2 \vee 1 = 1$ cnf(goals$_{18}$, negated_conjecture)
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \vee (\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3) = \mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_3 \wedge \mathrm{sk}_2; \mathrm{sk}_3) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3$ cnf(goals$_{19}$, negated_conjecture)

REL030+1.p Propagation of subidentities
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \vee 1 = 1 \Rightarrow x_0; x_1 \wedge x_2' = x_0; x_1 \wedge (x_0; x_2)')$ fof(goals, conjecture)

REL030+2.p Propagation of subidentities
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \vee 1 = 1 \Rightarrow ((x_0; x_1 \wedge x_2') \vee (x_0; x_1 \wedge (x_0; x_2)') = x_0; x_1 \wedge (x_0; x_2)' \text{ and } (x_0; x_1 \wedge (x_0; x_2)') \vee (x_0; x_1 \wedge x_2') = x_0; x_1 \wedge x_2'))$ fof(goals, conjecture)

REL030+3.p Propagation of subidentities
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \vee 1 = 1 \Rightarrow x_0; x_1 \wedge x_2' = x_0; x_1 \wedge (x_0; x_2)')$ fof(goals, conjecture)

REL030+4.p Propagation of subidentities
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0 \vee 1 = 1 \Rightarrow ((x_0; x_1 \wedge x_2') \vee (x_0; x_1 \wedge (x_0; x_2)') = x_0; x_1 \wedge (x_0; x_2)' \text{ and } (x_0; x_1 \wedge (x_0; x_2)') \vee (x_0; x_1 \wedge x_2') = x_0; x_1 \wedge x_2'))$ fof(goals, conjecture)

REL030-1.p Propagation of subidentities
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3' \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)'$ cnf(goals$_{15}$, negated_conjecture)

REL030-2.p Propagation of subidentities
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$(\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3') \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') = \mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)' \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3') \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3'$ cnf(goals$_{15}$, negated_conjecture)

REL030-3.p Propagation of subidentities
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{17}$, negated_conjecture)
$\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3' \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)'$ cnf(goals$_{18}$, negated_conjecture)

REL030-4.p Propagation of subidentities
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{17}$, negated_conjecture)
$(\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3') \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') = \mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)' \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_2 \wedge (\mathrm{sk}_1; \mathrm{sk}_3)') \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3') \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3'$ cnf(goals$_{18}$, negated_conjecture)

REL031+1.p Partial functions are closed under composition
If x and y are partial functions then x;y is also a partial functions.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1 \colon ((x_0^{\smile}; x_0 \vee 1 = 1 \text{ and } x_1^{\smile}; x_1 \vee 1 = 1) \Rightarrow (x_0; x_1)^{\smile}; (x_0; x_1) \vee 1 = 1)$ fof(goals, conjecture)

REL031+2.p Partial functions are closed under composition
If x and y are partial functions then x;y is also a partial functions.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1 \colon ((x_0^{\smile}; x_0 \vee 1 = 1 \text{ and } x_1^{\smile}; x_1 \vee 1 = 1) \Rightarrow (x_0; x_1)^{\smile}; (x_0; x_1) \vee 1 = 1)$ fof(goals, conjecture)

REL031-1.p Partial functions are closed under composition
If x and y are partial functions then x;y is also a partial functions.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_2^{\smile}; \mathrm{sk}_2 \vee 1 = 1$ cnf(goals$_{15}$, negated_conjecture)
$(\mathrm{sk}_1; \mathrm{sk}_2)^{\smile}; (\mathrm{sk}_1; \mathrm{sk}_2) \vee 1 \neq 1$ cnf(goals$_{16}$, negated_conjecture)

REL031-2.p Partial functions are closed under composition
If x and y are partial functions then x;y is also a partial functions.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_2^{\smile}; \mathrm{sk}_2 \vee 1 = 1 \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_1; \mathrm{sk}_2)^{\smile}; (\mathrm{sk}_1; \mathrm{sk}_2) \vee 1 \neq 1 \qquad \mathrm{cnf}(\mathrm{goals}_{19}, \mathrm{negated\_conjecture})$

REL032+1.p Subdistributivity
Sequential composition subdistributes over meet, i.e. x;(y meet z) ≤ x;y meet x;z.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon x_0; (x_1 \wedge x_2) \vee (x_0; x_1 \wedge x_0; x_2) = x_0; x_1 \wedge x_0; x_2 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL032+2.p Subdistributivity
Sequential composition subdistributes over meet, i.e. x;(y meet z) ≤ x;y meet x;z.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon x_0; (x_1 \wedge x_2) \vee (x_0; x_1 \wedge x_0; x_2) = x_0; x_1 \wedge x_0; x_2 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL032-1.p Subdistributivity
Sequential composition subdistributes over meet, i.e. x;(y meet z) ≤ x;y meet x;z.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$

REL032-2.p Subdistributivity
Sequential composition subdistributes over meet, i.e. x;(y meet z) ≤ x;y meet x;z.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$

REL033+1.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge x_1); x_2 = x_0 \wedge x_1; x_2) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL033+2.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0; \top = x_0 \Rightarrow ((x_0 \wedge x_1); x_2 \vee (x_0 \wedge x_1; x_2) = x_0 \wedge x_1; x_2 \text{ and } (x_0 \wedge x_1; x_2) \vee (x_0 \wedge x_1); x_2 = (x_0 \wedge x_1); x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL033+3.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0; \top = x_0 \Rightarrow (x_0 \wedge x_1); x_2 = x_0 \wedge x_1; x_2) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL033+4.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0; \top = x_0 \Rightarrow ((x_0 \wedge x_1); x_2 \vee (x_0 \wedge x_1; x_2) = x_0 \wedge x_1; x_2 \text{ and } (x_0 \wedge x_1; x_2) \vee (x_0 \wedge x_1); x_2 = (x_0 \wedge x_1); x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL033-1.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL033-2.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3) = \mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conje}$

REL033-3.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq \mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL033-4.p Sequential composition distributes in each argument of meet
If x is a vector then sequential composition distributes over meet.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3) = \mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1 \wedge \mathrm{sk}_2; \mathrm{sk}_3) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_2); \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conje}$

REL034+1.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0; \top = x_0 \Rightarrow x_1; (x_0 \wedge x_2) \vee (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL034+2.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0; \top = x_0 \Rightarrow x_1; (x_0 \wedge x_2) \vee (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL034-1.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_2; (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL034-2.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_2; (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL035+1.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2 \colon (x_0; \top = x_0 \Rightarrow (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = x_1; (x_0 \wedge x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL035+2.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2 \colon (x_0; \top = x_0 \Rightarrow (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = x_1; (x_0 \wedge x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL035-1.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_2; (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL035-2.p Propagation of vectors
Pre-assertion x to z can be propagated as post-assertion x∧ to the left cofactor y.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1; \top = \mathrm{sk}_1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_2; (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL036+1.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001+0.ax')

$\forall x_0, x_1, x_2\colon (x_0;\top = x_0 \;\Rightarrow\; (x_1 \wedge x_0^{\smile}); x_2 \vee (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2))$ fof(goals, conjecture)

REL036+2.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0;\top = x_0 \;\Rightarrow\; (x_1 \wedge x_0^{\smile}); x_2 \vee (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2))$ fof(goals, conjecture)

REL036-1.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1;\top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); \mathrm{sk}_3 \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3)$ cnf(goals$_{15}$, negated_conjecture)

REL036-2.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1;\top = \mathrm{sk}_1$ cnf(goals$_{17}$, negated_conjecture)
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); \mathrm{sk}_3 \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3)$ cnf(goals$_{18}$, negated_conjecture)

REL037+1.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0;\top = x_0 \;\Rightarrow\; (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); x_2)$ fof(goals, conjecture)

REL037+2.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0;\top = x_0 \;\Rightarrow\; (x_1 \wedge x_0^{\smile}); (x_0 \wedge x_2) = (x_1 \wedge x_0^{\smile}); x_2)$ fof(goals, conjecture)

REL037-1.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1;\top = \mathrm{sk}_1$ cnf(goals$_{14}$, negated_conjecture)
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); \mathrm{sk}_3$ cnf(goals$_{15}$, negated_conjecture)

REL037-2.p Propagation of vectors
Post-assertion x∧ to y can be propagated as pre-assertion x to the right cofactor z.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1;\top = \mathrm{sk}_1$ cnf(goals$_{17}$, negated_conjecture)
$(\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); (\mathrm{sk}_1 \wedge \mathrm{sk}_3) \neq (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}); \mathrm{sk}_3$ cnf(goals$_{18}$, negated_conjecture)

REL038+1.p Modular law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2) \vee (x_0; (x_1 \wedge x_0^{\smile}; x_2) \wedge x_2) = x_0; (x_1 \wedge x_0^{\smile}; x_2) \wedge x_2$ fof(goals, conjecture)

REL038-1.p Modular law
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3) \wedge \mathrm{sk}_3) \neq \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3) \wedge \mathrm{sk}_3$ cnf(goals$_{14}$, negated_conjecture)

REL039+1.p Dedekind law
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0; x_1 \wedge x_2) \vee (x_0 \wedge x_2; x_1^{\smile}); (x_1 \wedge x_0^{\smile}; x_2) = (x_0 \wedge x_2; x_1^{\smile}); (x_1 \wedge x_0^{\smile}; x_2)$ fof(goals, conjecture)

REL039-1.p Dedekind law
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1 \wedge \mathrm{sk}_3; \mathrm{sk}_2^{\smile}); (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3) \neq (\mathrm{sk}_1 \wedge \mathrm{sk}_3; \mathrm{sk}_2^{\smile}); (\mathrm{sk}_2 \wedge \mathrm{sk}_1^{\smile}; \mathrm{sk}_3)$ cnf(goals$_{14}$, negated_conjecture)

REL040+1.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0^{\smile}; x_0 \vee 1 = 1 \;\Rightarrow\; x_0; (x_1 \wedge x_2) = x_0; x_1 \wedge x_0; x_2)$ fof(goals, conjecture)

REL040+2.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0^{\smile}; x_0 \vee 1 = 1 \Rightarrow (x_0; (x_1 \wedge x_2) \vee (x_0; x_1 \wedge x_0; x_2) = x_0; x_1 \wedge x_0; x_2$ and $(x_0; x_1 \wedge x_0; x_2) \vee x_0; (x_1 \wedge x_2) = x_0; (x_1 \wedge x_2)))$ fof(goals, conjecture)

REL040+3.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0^{\smile}; x_0 \vee 1 = 1 \Rightarrow x_0; (x_1 \wedge x_2) = x_0; x_1 \wedge x_0; x_2)$ fof(goals, conjecture)

REL040+4.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0^{\smile}; x_0 \vee 1 = 1 \Rightarrow (x_0; (x_1 \wedge x_2) \vee (x_0; x_1 \wedge x_0; x_2) = x_0; x_1 \wedge x_0; x_2$ and $(x_0; x_1 \wedge x_0; x_2) \vee x_0; (x_1 \wedge x_2) = x_0; (x_1 \wedge x_2)))$ fof(goals, conjecture)

REL040-1.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{15}, negated_conjecture)

REL040-2.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) = \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) \vee \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3)$ cnf(goals_{15}, negated_conjecture)

REL040-3.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3$ cnf(goals_{18}, negated_conjecture)

REL040-4.p Partial functions distribute over meet under sequential comp'n
If x is partial function then x;(y meet z) = x;y meet x;z.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\smile}; \mathrm{sk}_1 \vee 1 = 1$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \vee (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) = \mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3 \Rightarrow (\mathrm{sk}_1; \mathrm{sk}_2 \wedge \mathrm{sk}_1; \mathrm{sk}_3) \vee \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_1; (\mathrm{sk}_2 \wedge \mathrm{sk}_3)$ cnf(goals_{18}, negated_conjecture)

REL041+1.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001+0.ax')
$\forall x_0\colon (x_0^{\smile}; x_0 \vee 1 = 1 \Rightarrow \forall x_1\colon x_0; x_1 \wedge x_0; x_1' = 0)$ fof(goals, conjecture)

REL041+2.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0\colon (x_0^{\smile}; x_0 \vee 1 = 1 \Rightarrow \forall x_1\colon x_0; x_1 \wedge x_0; x_1' = 0)$ fof(goals, conjecture)

REL041-1.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001-0.ax')

$\mathrm{sk}_1^{\smile};\mathrm{sk}_1 \vee 1 = 1 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_1;\mathrm{sk}_2 \wedge \mathrm{sk}_1;\mathrm{sk}_2' \neq 0 \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL041-2.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1^{\smile};\mathrm{sk}_1 \vee 1 = 1 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_1;\mathrm{sk}_2 \wedge \mathrm{sk}_1;\mathrm{sk}_2' \neq 0 \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL042+1.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001+0.ax')
$\forall x_0\colon (\forall x_1\colon x_0;x_1 \wedge x_0;x_1' = 0 \Rightarrow x_0^{\smile};x_0 \vee 1 = 1) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL042+2.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0\colon (\forall x_1\colon x_0;x_1 \wedge x_0;x_1' = 0 \Rightarrow x_0^{\smile};x_0 \vee 1 = 1) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL042-1.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1;a \wedge \mathrm{sk}_1;a' = 0 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_1^{\smile};\mathrm{sk}_1 \vee 1 \neq 1 \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL042-2.p Equivalence of different definitions of partial functions
x is a partial function if x∧;x is a subidentity ([SS93]). x is a partial function if for all y x;y meet x;overliney = 0. These definitions are equivalent.
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1;a \wedge \mathrm{sk}_1;a' = 0 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_1^{\smile};\mathrm{sk}_1 \vee 1 \neq 1 \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL043+1.p Shunting rule
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0;x_1^{\smile} \vee x_2 = x_2 \Rightarrow x_2';x_1 \vee x_0' = x_0') \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL043+2.p Shunting rule
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0;x_1^{\smile} \vee x_2 = x_2 \Rightarrow x_2';x_1 \vee x_0' = x_0') \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL043-1.p Shunting rule
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1;\mathrm{sk}_2^{\smile} \vee \mathrm{sk}_3 = \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{14}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_3';\mathrm{sk}_2 \vee \mathrm{sk}_1' \neq \mathrm{sk}_1' \qquad \mathrm{cnf}(\mathrm{goals}_{15}, \mathrm{negated\_conjecture})$

REL043-2.p Shunting rule
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1;\mathrm{sk}_2^{\smile} \vee \mathrm{sk}_3 = \mathrm{sk}_3 \qquad \mathrm{cnf}(\mathrm{goals}_{17}, \mathrm{negated\_conjecture})$
$\mathrm{sk}_3';\mathrm{sk}_2 \vee \mathrm{sk}_1' \neq \mathrm{sk}_1' \qquad \mathrm{cnf}(\mathrm{goals}_{18}, \mathrm{negated\_conjecture})$

REL044+1.p Shunting rule
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0';x_1 \vee x_2' = x_2' \Rightarrow x_2;x_1^{\smile} \vee x_0 = x_0) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

REL044+2.p Shunting rule
include('Axioms/REL001+0.ax')

include('Axioms/REL001+1.ax')
$\forall x_0, x_1, x_2\colon (x_0'; x_1 \vee x_2' = x_2' \;\Rightarrow\; x_2; x_1^{\smile} \vee x_0 = x_0)$ fof(goals, conjecture)

REL044-1.p Shunting rule
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1'; \mathrm{sk}_2 \vee \mathrm{sk}_3' = \mathrm{sk}_3'$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_3; \mathrm{sk}_2^{\smile} \vee \mathrm{sk}_1 \neq \mathrm{sk}_1$ cnf(goals_{15}, negated_conjecture)

REL044-2.p Shunting rule
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1'; \mathrm{sk}_2 \vee \mathrm{sk}_3' = \mathrm{sk}_3'$ cnf(goals_{17}, negated_conjecture)
$\mathrm{sk}_3; \mathrm{sk}_2^{\smile} \vee \mathrm{sk}_1 \neq \mathrm{sk}_1$ cnf(goals_{18}, negated_conjecture)

REL045+1.p An unfold law
include('Axioms/REL001+0.ax')
$\forall x_0\colon x_0 \vee (x_0; x_0^{\smile}); x_0 = (x_0; x_0^{\smile}); x_0$ fof(goals, conjecture)

REL045+2.p An unfold law
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0\colon x_0 \vee (x_0; x_0^{\smile}); x_0 = (x_0; x_0^{\smile}); x_0$ fof(goals, conjecture)

REL045-1.p An unfold law
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee (\mathrm{sk}_1; \mathrm{sk}_1^{\smile}); \mathrm{sk}_1 \neq (\mathrm{sk}_1; \mathrm{sk}_1^{\smile}); \mathrm{sk}_1$ cnf(goals_{14}, negated_conjecture)

REL045-2.p An unfold law
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$\mathrm{sk}_1 \vee (\mathrm{sk}_1; \mathrm{sk}_1^{\smile}); \mathrm{sk}_1 \neq (\mathrm{sk}_1; \mathrm{sk}_1^{\smile}); \mathrm{sk}_1$ cnf(goals_{17}, negated_conjecture)

REL046+1.p Meet splitting
Meet can be split into 2 inequations iff the meet is on the right hand side of an inequation.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0 \vee (x_1 \wedge x_2) = x_1 \wedge x_2 \;\Rightarrow\; (x_0 \vee x_1 = x_1 \text{ and } x_0 \vee x_2 = x_2))$ fof(goals, conjecture)

REL046-1.p Meet splitting
Meet can be split into 2 inequations iff the meet is on the right hand side of an inequation.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_3) = \mathrm{sk}_2 \wedge \mathrm{sk}_3$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2 \;\Rightarrow\; \mathrm{sk}_1 \vee \mathrm{sk}_3 \neq \mathrm{sk}_3$ cnf(goals_{15}, negated_conjecture)

REL047+1.p Meet splitting
Meet can be split into 2 inequations iff the meet is on the right hand side of an inequation.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon ((x_0 \vee x_1 = x_1 \text{ and } x_0 \vee x_2 = x_2) \;\Rightarrow\; x_0 \vee (x_1 \wedge x_2) = x_1 \wedge x_2)$ fof(goals, conjecture)

REL047-1.p Meet splitting
Meet can be split into 2 inequations iff the meet is on the right hand side of an inequation.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1 \vee \mathrm{sk}_3 = \mathrm{sk}_3$ cnf(goals_{15}, negated_conjecture)
$\mathrm{sk}_1 \vee (\mathrm{sk}_2 \wedge \mathrm{sk}_3) \neq \mathrm{sk}_2 \wedge \mathrm{sk}_3$ cnf(goals_{16}, negated_conjecture)

REL048+1.p Join splitting
Join can be split into 2 inequations iff the join is on the left hand side of an inequation.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2\colon ((x_0 \vee x_1) \vee x_2 = x_2 \;\Rightarrow\; (x_0 \vee x_2 = x_2 \text{ and } x_1 \vee x_2 = x_2))$ fof(goals, conjecture)

REL048-1.p Join splitting
Join can be split into 2 inequations iff the join is on the left hand side of an inequation.
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1 \vee \mathrm{sk}_2) \vee \mathrm{sk}_3 = \mathrm{sk}_3$ cnf(goals_{14}, negated_conjecture)
$\mathrm{sk}_1 \vee \mathrm{sk}_3 = \mathrm{sk}_3 \;\Rightarrow\; \mathrm{sk}_2 \vee \mathrm{sk}_3 \neq \mathrm{sk}_3$ cnf(goals_{15}, negated_conjecture)

REL049+1.p Join splitting

Join can be split into 2 inequations iff the join is on the left hand side of an inequation.
include('Axioms/REL001+0.ax')
$\forall x_0, x_1, x_2$: $((x_0 \vee x_1 = x_1 \text{ and } x_2 \vee x_1 = x_1) \Rightarrow (x_0 \vee x_2) \vee x_1 = x_1)$ fof(goals, conjecture)

REL049-1.p Join splitting
Join can be split into 2 inequations iff the join is on the left hand side of an inequation.
include('Axioms/REL001-0.ax')
$\mathrm{sk}_1 \vee \mathrm{sk}_2 = \mathrm{sk}_2$ cnf(goals$_{14}$, negated_conjecture)
$\mathrm{sk}_3 \vee \mathrm{sk}_2 = \mathrm{sk}_2$ cnf(goals$_{15}$, negated_conjecture)
$(\mathrm{sk}_1 \vee \mathrm{sk}_3) \vee \mathrm{sk}_2 \neq \mathrm{sk}_2$ cnf(goals$_{16}$, negated_conjecture)

REL050+1.p The complement of x;TOP is left ideal
include('Axioms/REL001+0.ax')
$\forall x_0$: $(x_0; \top)' = (x_0; \top)'; \top$ fof(goals, conjecture)

REL050+2.p The complement of x;TOP is left ideal
include('Axioms/REL001+0.ax')
$\forall x_0$: $((x_0; \top)' \vee (x_0; \top)'; \top = (x_0; \top)'; \top \text{ and } (x_0; \top)'; \top \vee (x_0; \top)' = (x_0; \top)')$ fof(goals, conjecture)

REL050+3.p The complement of x;TOP is left ideal
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0$: $(x_0; \top)' = (x_0; \top)'; \top$ fof(goals, conjecture)

REL050+4.p The complement of x;TOP is left ideal
include('Axioms/REL001+0.ax')
include('Axioms/REL001+1.ax')
$\forall x_0$: $((x_0; \top)' \vee (x_0; \top)'; \top = (x_0; \top)'; \top \text{ and } (x_0; \top)'; \top \vee (x_0; \top)' = (x_0; \top)')$ fof(goals, conjecture)

REL050-1.p The complement of x;TOP is left ideal
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \top)' \neq (\mathrm{sk}_1; \top)'; \top$ cnf(goals$_{14}$, negated_conjecture)

REL050-2.p The complement of x;TOP is left ideal
include('Axioms/REL001-0.ax')
$(\mathrm{sk}_1; \top)' \vee (\mathrm{sk}_1; \top)'; \top = (\mathrm{sk}_1; \top)'; \top \Rightarrow (\mathrm{sk}_1; \top)'; \top \vee (\mathrm{sk}_1; \top)' \neq (\mathrm{sk}_1; \top)'$ cnf(goals$_{14}$, negated_conjecture)

REL050-3.p The complement of x;TOP is left ideal
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1; \top)' \neq (\mathrm{sk}_1; \top)'; \top$ cnf(goals$_{17}$, negated_conjecture)

REL050-4.p The complement of x;TOP is left ideal
include('Axioms/REL001-0.ax')
include('Axioms/REL001-1.ax')
$(\mathrm{sk}_1; \top)' \vee (\mathrm{sk}_1; \top)'; \top = (\mathrm{sk}_1; \top)'; \top \Rightarrow (\mathrm{sk}_1; \top)'; \top \vee (\mathrm{sk}_1; \top)' \neq (\mathrm{sk}_1; \top)'$ cnf(goals$_{17}$, negated_conjecture)

REL051+1.p Dense linear ordering
$\forall a$: $o(a, a)$ fof(f_{01}, axiom)
$\forall a, b$: $((a \neq b \text{ and } o(a, b)) \Rightarrow \neg\, o(b, a))$ fof(f_{02}, axiom)
$\forall a, b, c$: $((o(a, b) \text{ and } o(b, c)) \Rightarrow o(a, c))$ fof(f_{03}, axiom)
$\forall a, b$: $((a \neq b \text{ and } o(a, b)) \Rightarrow (o(a, f(a, b)) \text{ and } o(f(a, b), b)))$ fof(f_{04}, axiom)
$\forall a, b$: $(f(a, b) \neq a \text{ and } f(a, b) \neq b)$ fof(f_{05}, axiom)
$\forall a, b$: $(o(a, b) \text{ or } o(b, a))$ fof(f_{06}, axiom)

REL052+1.p Non-discrete dense ordering
$\forall a$: $o(a, a)$ fof(f_{01}, axiom)
$\forall a, b$: $((a \neq b \text{ and } o(a, b)) \Rightarrow \neg\, o(b, a))$ fof(f_{02}, axiom)
$\forall a, b, c$: $((o(a, b) \text{ and } o(b, c)) \Rightarrow o(a, c))$ fof(f_{03}, axiom)
$\forall a, b$: $((a \neq b \text{ and } o(a, b)) \Rightarrow (o(a, f(a, b)) \text{ and } o(f(a, b), b)))$ fof(f_{04}, axiom)
$\forall a, b$: $(f(a, b) \neq a \text{ and } f(a, b) \neq b)$ fof(f_{05}, axiom)
$\exists a, b$: $(o(a, b) \text{ and } a \neq b)$ fof(f_{06}, axiom)

REL053+1.p Relation Algebra
include('Axioms/REL001+0.ax')

REL053-1.p Relation Algebra

include('Axioms/REL001-0.ax')