RNG axioms

RNG001-0.ax Ring theory axioms
$0 + x{=}x$ cnf(additive_identity$_1$, axiom)
$x + 0{=}x$ cnf(additive_identity$_2$, axiom)
$x \cdot y{=}x \cdot y$ cnf(closure_of_multiplication, axiom)
$x + y{=}x + y$ cnf(closure_of_addition, axiom)
$-x + x{=}0$ cnf(left_inverse, axiom)
$x + -x{=}0$ cnf(right_inverse, axiom)
$(x + y{=}u \text{ and } y + z{=}v \text{ and } u + z{=}w) \Rightarrow x + v{=}w$ cnf(associativity_of_addition$_1$, axiom)
$(x + y{=}u \text{ and } y + z{=}v \text{ and } x + v{=}w) \Rightarrow u + z{=}w$ cnf(associativity_of_addition$_2$, axiom)
$x + y{=}z \Rightarrow y + x{=}z$ cnf(commutativity_of_addition, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } u \cdot z{=}w) \Rightarrow x \cdot v{=}w$ cnf(associativity_of_multiplication$_1$, axiom)
$(x \cdot y{=}u \text{ and } y \cdot z{=}v \text{ and } x \cdot v{=}w) \Rightarrow u \cdot z{=}w$ cnf(associativity_of_multiplication$_2$, axiom)
$(x \cdot y{=}v_1 \text{ and } x \cdot z{=}v_2 \text{ and } y + z{=}v_3 \text{ and } x \cdot v_3{=}v_4) \Rightarrow v_1 + v_2{=}v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y{=}v_1 \text{ and } x \cdot z{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_1 + v_2{=}v_4) \Rightarrow x \cdot v_3{=}v_4$ cnf(distributivity$_2$, axiom)
$(y \cdot x{=}v_1 \text{ and } z \cdot x{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_3 \cdot x{=}v_4) \Rightarrow v_1 + v_2{=}v_4$ cnf(distributivity$_3$, axiom)
$(y \cdot x{=}v_1 \text{ and } z \cdot x{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_1 + v_2{=}v_4) \Rightarrow v_3 \cdot x{=}v_4$ cnf(distributivity$_4$, axiom)
$(x + y{=}u \text{ and } x + y{=}v) \Rightarrow u = v$ cnf(addition_is_well_defined, axiom)
$(x \cdot y{=}u \text{ and } x \cdot y{=}v) \Rightarrow u = v$ cnf(multiplication_is_well_defined, axiom)

RNG002-0.ax Ring theory (equality) axioms
$0 + x = x$ cnf(left_identity, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-0 = 0$ cnf(additive_inverse_identity, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot 0 = 0$ cnf(multiply_additive_id$_1$, axiom)
$0 \cdot x = 0$ cnf(multiply_additive_id$_2$, axiom)
$-(x + y) = -x + -y$ cnf(distribute_additive_inverse, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(multiply_additive_inverse$_1$, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(multiply_additive_inverse$_2$, axiom)
$(x + y) + z = x + (y + z)$ cnf(associative_addition, axiom)
$x + y = y + x$ cnf(commutative_addition, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associative_multiplication, axiom)

RNG003-0.ax Alternative ring theory (equality) axioms
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$(x \cdot x) \cdot y = x \cdot (x \cdot y)$ cnf(left_alternative, axiom)
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)

RNG004-0.ax Alternative ring theory (equality) axioms
$0 + x = x$ cnf(left_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(add_inverse, axiom)
$-(x + y) = -x + -y$ cnf(sum_of_inverses, axiom)

$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(multiply_over_add$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(multiply_over_add$_2$, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$(x \cdot x) \cdot y = x \cdot (x \cdot y)$ cnf(left_alternative, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$-0 = 0$ cnf(inverse_additive_identity, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$x + z = y + z \Rightarrow x = y$ cnf(left_cancellation_for_addition, axiom)
$z + x = z + y \Rightarrow x = y$ cnf(right_cancellation_for_addition, axiom)

RNG005-0.ax Ring theory (equality) axioms
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x \cdot (y \cdot z) = (x \cdot y) \cdot z$ cnf(associativity_for_multiplication, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)

RNG problems

RNG001-1.p X.additive_identity = additive_identity for any X
include('Axioms/RNG001-0.ax')
$\neg\, a \cdot 0 = 0$ cnf(prove_a_times_0_is$_0$, negated_conjecture)

RNG001-2.p X.additive_identity = additive_identity for any X
$(a = b \text{ and } c + d = a) \Rightarrow c + d = b$ cnf(sum_substitution$_3$, axiom)
$0 + x = x$ cnf(additive_identity$_1$, axiom)
$x + 0 = x$ cnf(additive_identity$_2$, axiom)
$x \cdot y = x \cdot y$ cnf(closure_of_multiplication, axiom)
$x + y = x + y$ cnf(closure_of_addition, axiom)
$-x + x = 0$ cnf(additive_inverse$_1$, axiom)
$x + -x = 0$ cnf(additive_inverse$_2$, axiom)
$(x + y = u \text{ and } y + z = v \text{ and } u + z = w) \Rightarrow x + v = w$ cnf(associativity_of_addition$_1$, axiom)
$(x + y = u \text{ and } y + z = v \text{ and } x + v = w) \Rightarrow u + z = w$ cnf(associativity_of_addition$_2$, axiom)
$x + y = z \Rightarrow y + x = z$ cnf(commutativity_of_addition, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity_of_multiplication$_1$, axiom)
$(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity_of_multiplication$_2$, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } x \cdot v_3 = v_4) \Rightarrow v_1 + v_2 = v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } v_1 + v_2 = v_4) \Rightarrow x \cdot v_3 = v_4$ cnf(distributivity$_2$, axiom)
$(y \cdot x = v_1 \text{ and } z \cdot x = v_2 \text{ and } y + z = v_3 \text{ and } v_3 \cdot x = v_4) \Rightarrow v_1 + v_2 = v_4$ cnf(distributivity$_3$, axiom)
$(y \cdot x = v_1 \text{ and } z \cdot x = v_2 \text{ and } y + z = v_3 \text{ and } v_1 + v_2 = v_4) \Rightarrow v_3 \cdot x = v_4$ cnf(distributivity$_4$, axiom)
$(x + y = u \text{ and } x + y = v) \Rightarrow u = v$ cnf(addition_is_well_defined, axiom)
$\neg\, a \cdot 0 = 0$ cnf(theorem, negated_conjecture)

RNG001-3.p X.additive_identity = additive_identity for any X
$0 + x = x$ cnf(additive_identity$_1$, axiom)
$-x + x = 0$ cnf(additive_inverse$_1$, axiom)
$(x + y = u \text{ and } y + z = v \text{ and } u + z = w) \Rightarrow x + v = w$ cnf(associativity_of_addition$_1$, axiom)
$(x + y = u \text{ and } y + z = v \text{ and } x + v = w) \Rightarrow u + z = w$ cnf(associativity_of_addition$_2$, axiom)
$x \cdot y = x \cdot y$ cnf(closure_of_multiplication, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } x \cdot v_3 = v_4) \Rightarrow v_1 + v_2 = v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } v_1 + v_2 = v_4) \Rightarrow x \cdot v_3 = v_4$ cnf(distributivity$_2$, axiom)
$\neg\, a \cdot 0 = 0$ cnf(prove_a_times_additive_id_is_additive_id, negated_conjecture)

RNG001-4.p X.additive_identity = additive_identity for any X
include('Axioms/RNG001-0.ax')

$(x+y{=}z \text{ and } x+w{=}z) \ \Rightarrow\ y=w$ cnf(cancellation$_1$, axiom)
$(x+y{=}z \text{ and } w+y{=}z) \ \Rightarrow\ x=w$ cnf(cancellation$_2$, axiom)
$\neg\, a\cdot 0{=}0$ cnf(prove_a_times_additive_id_is_additive_id, negated_conjecture)

RNG002-1.p Right cancellation for addition
include('Axioms/RNG001-0.ax')
$a+b{=}d$ cnf(a_plus_b_is_d, hypothesis)
$a+c{=}d$ cnf(a_plus_c_is_d, hypothesis)
$b\neq c$ cnf(prove_b_equals_c, negated_conjecture)

RNG003-1.p Left cancellation for addition
include('Axioms/RNG001-0.ax')
$a+c{=}d$ cnf(a_plus_c_is_d, hypothesis)
$b+c{=}d$ cnf(b_plus_c_is_d, hypothesis)
$a\neq b$ cnf(prove_a_equals_b, negated_conjecture)

RNG004-1.p X*Y = -X*-Y
include('Axioms/RNG001-0.ax')
$a\cdot b{=}c$ cnf(a_times_b, hypothesis)
$(-a)\cdot(-b){=}d$ cnf(a_inverse_times_b_inverse, hypothesis)
$c\neq d$ cnf(prove_c_equals_d, negated_conjecture)

RNG004-2.p X*Y = -X*-Y
include('Axioms/RNG001-0.ax')
$(x+y{=}z \text{ and } x+w{=}z) \ \Rightarrow\ y=w$ cnf(cancellation$_1$, axiom)
$(x+y{=}z \text{ and } w+y{=}z) \ \Rightarrow\ x=w$ cnf(cancellation$_2$, axiom)
$a\cdot b{=}c$ cnf(a_times_b, hypothesis)
$(-a)\cdot(-b){=}d$ cnf(a_inverse_times_b_inverse, hypothesis)
$c\neq d$ cnf(prove_c_equals_d, negated_conjecture)

RNG005-1.p (-X*Y) + (X*Y) = additive_identity
include('Axioms/RNG001-0.ax')
$a\cdot b{=}d$ cnf(a_times_b, hypothesis)
$(-a)\cdot b{=}c$ cnf(a_inverse_times_b, hypothesis)
$\neg\, c+d{=}0$ cnf(prove_sum_is_additive_id, negated_conjecture)

RNG006-1.p X*(Y+ -Z) = (X*Y) + -(X*Z)
include('Axioms/RNG001-0.ax')
$a\cdot b{=}c \ \Rightarrow\ a\cdot(-b){=}-c$ cnf(product_lemma$_1$, axiom)
$a\cdot b{=}c \ \Rightarrow\ (-a)\cdot b{=}-c$ cnf(product_lemma$_2$, axiom)
$a\cdot b{=}c \ \Rightarrow\ (-a)\cdot(-b){=}c$ cnf(product_lemma$_3$, axiom)
$b+-c{=}\text{bS\_Ic}$ cnf(b_plus_inverse_c, hypothesis)
$a\cdot b{=}\text{aPb}$ cnf(a_times_b, hypothesis)
$a\cdot c{=}\text{aPc}$ cnf(a_times_c, hypothesis)
$\text{aPb}+-\text{aPc}{=}\text{aPb\_S\_IaPc}$ cnf(aPb_plus_IaPc, hypothesis)
$\neg\, a\cdot\text{bS\_Ic}{=}\text{aPb\_S\_IaPc}$ cnf(prove_a_times_bS_Ic_is_aPb_S__IaPc, negated_conjecture)

RNG006-3.p X*(Y+ -Z) = (X*Y) + -(X*Z)
include('Axioms/RNG001-0.ax')
$b+-c{=}\text{bS\_Ic}$ cnf(b_plus_inverse_c, hypothesis)
$a\cdot b{=}\text{aPb}$ cnf(a_times_b, hypothesis)
$a\cdot c{=}\text{aPc}$ cnf(a_times_c, hypothesis)
$\text{aPb}+-\text{aPc}{=}\text{aPb\_S\_IaPc}$ cnf(aPb_plus_IaPc, hypothesis)
$\neg\, a\cdot\text{bS\_Ic}{=}\text{aPb\_S\_IaPc}$ cnf(prove_a_times_bS_Ic_is_aPb_S__IaPc, negated_conjecture)

RNG007-1.p In Boolean rings, X is its own inverse
Given a ring in which for all x, x * x = x, prove that for all x, x + x = additive_identity
include('Axioms/RNG001-0.ax')
$x\cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$\neg\, a+a{=}0$ cnf(prove_a_plus_a_is_id, negated_conjecture)

RNG007-4.p In Boolean rings, X is its own inverse
Given a ring in which for all x, x * x = x, prove that for all x, x + x = additive_identity
include('Axioms/RNG002-0.ax')
$x\cdot x=x$ cnf(boolean_ring, hypothesis)

$a + a \neq 0$ cnf(prove_inverse, negated_conjecture)

RNG007-5.p In Boolean rings, X is its own inverse
Given a ring in which for all x, x * x = x, prove that for all x, x + x = additive_identity.
include('Axioms/RNG001-0.ax')
$-0 + 0{=}0$ cnf(additive_inverse_identity, axiom)
$-(-x) + 0{=}x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot 0{=}0$ cnf(multiply_additive_id$_1$, axiom)
$0 \cdot x{=}0$ cnf(multiply_additive_id$_2$, axiom)
$-x + -y{=}-(x + y)$ cnf(distribute_additive_inverse, axiom)
$x \cdot (-y){=}-x \cdot y$ cnf(multiply_additive_inverse, axiom)
$x \cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$a \cdot a \neq 0$ cnf(prove_a_plus_a_is_id, negated_conjecture)

RNG008-1.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG001-0.ax')
$x \cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

RNG008-2.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG001-0.ax')
$(x + y{=}z \text{ and } x + w{=}z) \Rightarrow y = w$ cnf(cancellation$_1$, axiom)
$(x + y{=}z \text{ and } w + y{=}z) \Rightarrow x = w$ cnf(cancellation$_2$, axiom)
$x \cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

RNG008-3.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG002-0.ax')
$x + 0 = x$ cnf(right_identity, axiom)
$x + -x = 0$ cnf(right_inverse, axiom)
$x \cdot x = x$ cnf(boolean_ring, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG008-4.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG002-0.ax')
$x \cdot x = x$ cnf(boolean_ring, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG008-5.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG001-0.ax')
$-0 + 0{=}0$ cnf(additive_inverse_identity, axiom)
$-(-x) + 0{=}x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot 0{=}0$ cnf(multiply_additive_id$_1$, axiom)
$0 \cdot x{=}0$ cnf(multiply_additive_id$_2$, axiom)
$-x + -y{=}-(x + y)$ cnf(distribute_additive_inverse, axiom)
$x \cdot (-y){=}-x \cdot y$ cnf(multiply_additive_inverse, axiom)
$x \cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

RNG008-6.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG001-0.ax')
$x \cdot 0{=}0$ cnf(x_times_identity_x_is_identity, axiom)

$0 \cdot x{=}0$ cnf(identity_times_x_is_identity, axiom)
$x \cdot x{=}x$ cnf(x_squared_is_x, hypothesis)
$a \cdot b{=}c$ cnf(a_times_b_is_c, hypothesis)
$\neg\, b \cdot a{=}c$ cnf(prove_b_times_a_is_c, negated_conjecture)

RNG008-7.p Boolean rings are commutative
Given a ring in which for all x, x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG005-0.ax')
$x \cdot x = x$ cnf(boolean_ring, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG009-5.p If X*X*X = X then the ring is commutative
Given a ring in which for all x, x * x * x = x, prove that for all x and y, x * y = y * x.
$x + 0 = x$ cnf(right_identity, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$(x + y) + z = x + (y + z)$ cnf(associative_addition, axiom)
$x + y = y + x$ cnf(commutative_addition, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associative_multiplication, axiom)
$x \cdot (x \cdot x) = x$ cnf(x_cubed_is_x, hypothesis)
$a \cdot b \neq b \cdot a$ cnf(prove_commutativity, negated_conjecture)

RNG009-7.p If X*X*X = X then the ring is commutative
Given a ring in which for all x, x * x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG005-0.ax')
$x \cdot (x \cdot x) = x$ cnf(x_cubed_is_x, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG010-1.p Skew symmetry of the auxilliary function
The left and right Moufang identities imply the skew symmetry of s(W,X,Y,Z) = (W*X,Y,Z) - X*(W,Y,Z) - (X,Y,Z)*W. Recall that skew symmetry means that the function sign changes when any two arguments are swapped. This problem proves the case for swapping the first two arguments.
include('Axioms/RNG004-0.ax')
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
$(x \cdot (y \cdot x)) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(left_moufang, hypothesis)
$\text{associator}(\text{cx}{\cdot}\text{cx}, \text{cy}, \text{cz}) \neq \text{associator}(\text{cx}, \text{cy}, \text{cz}){\cdot}\text{cx}{+}\text{cx}{\cdot}\text{associator}(\text{cx}, \text{cy}, \text{cz})$ cnf(prove_skew_symmetry, negated_conjecture)

RNG010-2.p Skew symmetry of the auxilliary function
The left and right Moufang identities imply the skew symmetry of s(W,X,Y,Z) = (W*X,Y,Z) - X*(W,Y,Z) - (X,Y,Z)*W. Recall that skew symmetry means that the function sign changes when any two arguments are swapped. This problem proves the case for swapping the first two arguments.
include('Axioms/RNG004-0.ax')
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$(y \cdot x) \cdot y \neq y \cdot (x \cdot y)$ cnf(middle_law, axiom)
$\text{associator}(y, x, z) \neq -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_1$, axiom)
$\text{associator}(z, y, x) \neq -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_2$, axiom)
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
$(x \cdot (y \cdot x)) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(left_moufang, hypothesis)
$\text{associator}(\text{cx}{\cdot}\text{cx}, \text{cy}, \text{cz}) \neq \text{associator}(\text{cx}, \text{cy}, \text{cz}){\cdot}\text{cx}{+}\text{cx}{\cdot}\text{associator}(\text{cx}, \text{cy}, \text{cz})$ cnf(prove_skew_symmetry, negated_conjecture)

RNG010-6.p Skew symmetry of the auxilliary function
The three Moufang identities imply the skew symmetry of s(W,X,Y,Z) = (W*X,Y,Z) - X*(W,Y,Z) - (X,Y,Z)*W. Recall that skew symmetry means that the function sign changes when any two arguments are swapped. This problem proves the case for swapping the first two arguments.
include('Axioms/RNG003-0.ax')
$s(w, x, y, z) = (\text{associator}(w \cdot x, y, z) + -x \cdot \text{associator}(w, y, z)) + -\text{associator}(x, y, z) \cdot w$ cnf(defines_s, axiom)
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
$(x \cdot (y \cdot x)) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(left_moufang, hypothesis)
$(x \cdot y) \cdot (z \cdot x) = (x \cdot (y \cdot z)) \cdot x$ cnf(middle_moufang, hypothesis)

$s(a,b,c,d) \neq -s(b,a,c,d)$ cnf(prove_skew_symmetry, negated_conjecture)

RNG010-7.p Skew symmetry of the auxilliary function
The three Moufang identities imply the skew symmetry of s(W,X,Y,Z) = (W*X,Y,Z) - X*(W,Y,Z) - (X,Y,Z)*W. Recall that skew symmetry means that the function sign changes when any two arguments are swapped. This problem proves the case for swapping the first two arguments.
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$s(w,x,y,z) = (\text{associator}(w \cdot x, y, z) + -x \cdot \text{associator}(w, y, z)) + -\text{associator}(x, y, z) \cdot w$ cnf(defines_s, axiom)
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
$(x \cdot (y \cdot x)) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(left_moufang, hypothesis)
$(x \cdot y) \cdot (z \cdot x) = (x \cdot (y \cdot z)) \cdot x$ cnf(middle_moufang, hypothesis)
$s(a,b,c,d) \neq -s(b,a,c,d)$ cnf(prove_skew_symmetry, negated_conjecture)

RNG011-5.p In a right alternative ring (((X,X,Y)*X)*(X,X,Y)) = Add Id
$x + y = y + x$ cnf(commutative_addition, axiom)
$(x + y) + z = x + (y + z)$ cnf(associative_addition, axiom)
$x + 0 = x$ cnf(right_identity, axiom)
$0 + x = x$ cnf(left_identity, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$-0 = 0$ cnf(additive_inverse_identity, axiom)
$x + (-x + y) = y$ cnf(property_of_inverse_and_add, axiom)
$-(x + y) = -x + -y$ cnf(distribute_additive_inverse, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$x \cdot 0 = 0$ cnf(multiply_additive_id$_1$, axiom)
$0 \cdot x = 0$ cnf(multiply_additive_id$_2$, axiom)
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverse, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(multiply_additive_inverse$_1$, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(multiply_additive_inverse$_2$, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$(\text{associator}(x, x, y) \cdot x) \cdot \text{associator}(x, x, y) = 0$ cnf(middle_associator, axiom)
$(\text{associator}(a, a, b) \cdot a) \cdot \text{associator}(a, a, b) \neq 0$ cnf(prove_equality, negated_conjecture)

RNG012-6.p Product of inverses equal product
include('Axioms/RNG003-0.ax')
$(-a) \cdot (-b) \neq a \cdot b$ cnf(prove_equation, negated_conjecture)

RNG013-6.p -X*Y = -(X*Y)
include('Axioms/RNG003-0.ax')
$(-a) \cdot b \neq -a \cdot b$ cnf(prove_equation, negated_conjecture)

RNG014-6.p -X*Y = -(X*Y)
include('Axioms/RNG003-0.ax')
$a \cdot (-b) \neq -a \cdot b$ cnf(prove_equation, negated_conjecture)

RNG015-6.p X*(Y+ -Z) = (X*Y) + -(X*Z)
include('Axioms/RNG003-0.ax')
$x \cdot (y + -z) \neq x \cdot y + -x \cdot z$ cnf(prove_distributivity, negated_conjecture)

RNG016-6.p (X+ -Y)*Z = (X*Z) + -(Y*Z)
include('Axioms/RNG003-0.ax')
$(x + -y) \cdot z \neq x \cdot z + -y \cdot z$ cnf(prove_distributivity, negated_conjecture)

RNG017-6.p -X*(Y+Z) = -(X*Y) + -(X*Z)
include('Axioms/RNG003-0.ax')
$(-x) \cdot (y + z) \neq -x \cdot y + -x \cdot z$ cnf(prove_distributivity, negated_conjecture)

RNG018-6.p (X+Y)* -Z = -(X*Z) + -(Y*Z)
include('Axioms/RNG003-0.ax')
$(x + y) \cdot (-z) \neq -x \cdot z + -y \cdot z$ cnf(prove_distributivity, negated_conjecture)

RNG019-6.p First part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$\text{associator}(x, y, u + v) \neq \text{associator}(x, y, u) + \text{associator}(x, y, v)$ cnf(prove_linearised_form$_1$, negated_conjecture)

RNG019-7.p First part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x, y, u + v) \neq \text{associator}(x, y, u) + \text{associator}(x, y, v)$ cnf(prove_linearised_form$_1$, negated_conjecture)

RNG020-6.p Second part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$\text{associator}(x, u + v, y) \neq \text{associator}(x, u, y) + \text{associator}(x, v, y)$ cnf(prove_linearised_form$_2$, negated_conjecture)

RNG020-7.p Second part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x, u + v, y) \neq \text{associator}(x, u, y) + \text{associator}(x, v, y)$ cnf(prove_linearised_form$_2$, negated_conjecture)

RNG021-6.p Third part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$\text{associator}(u + v, x, y) \neq \text{associator}(u, x, y) + \text{associator}(v, x, y)$ cnf(prove_linearised_form$_3$, negated_conjecture)

RNG021-7.p Third part of the linearised form of the associator
The associator can be expressed in another form called a linearised form. There are three clauses to be proved to establish the equivalence of the two forms.
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)

$\text{associator}(u+v,x,y) \neq \text{associator}(u,x,y) + \text{associator}(v,x,y)$ cnf(prove_linearised_form$_3$, negated_conjecture)

RNG023-6.p Left alternative
include('Axioms/RNG003-0.ax')
$\text{associator}(x,x,y) \neq 0$ cnf(prove_left_alternative, negated_conjecture)

RNG023-7.p Left alternative
include('Axioms/RNG003-0.ax')
$(-x)\cdot(-y) = x\cdot y$ cnf(product_of_inverses, axiom)
$(-x)\cdot y = -x\cdot y$ cnf(inverse_product$_1$, axiom)
$x\cdot(-y) = -x\cdot y$ cnf(inverse_product$_2$, axiom)
$x\cdot(y+-z) = x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x+-y)\cdot z = x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x)\cdot(y+z) = -x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x+y)\cdot(-z) = -x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x,x,y) \neq 0$ cnf(prove_left_alternative, negated_conjecture)

RNG024-6.p Right alternative
include('Axioms/RNG003-0.ax')
$\text{associator}(x,y,y) \neq 0$ cnf(prove_right_alternative, negated_conjecture)

RNG024-7.p Right alternative
include('Axioms/RNG003-0.ax')
$(-x)\cdot(-y) = x\cdot y$ cnf(product_of_inverses, axiom)
$(-x)\cdot y = -x\cdot y$ cnf(inverse_product$_1$, axiom)
$x\cdot(-y) = -x\cdot y$ cnf(inverse_product$_2$, axiom)
$x\cdot(y+-z) = x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x+-y)\cdot z = x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x)\cdot(y+z) = -x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x+y)\cdot(-z) = -x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x,y,y) \neq 0$ cnf(prove_right_alternative, negated_conjecture)

RNG025-1.p Middle or Flexible Law
include('Axioms/RNG004-0.ax')
$(\text{cy}\cdot\text{cx})\cdot\text{cy} \neq \text{cy}\cdot(\text{cx}\cdot\text{cy})$ cnf(prove_middle_law, negated_conjecture)

RNG025-4.p Middle or Flexible Law
include('Axioms/RNG003-0.ax')
$\text{associator}(x,y,z) + \text{associator}(x,z,y) \neq 0$ cnf(prove_equation, negated_conjecture)

RNG025-5.p Middle or Flexible Law
include('Axioms/RNG003-0.ax')
$(-x)\cdot(-y) = x\cdot y$ cnf(product_of_inverses, axiom)
$(-x)\cdot y = -x\cdot y$ cnf(inverse_product$_1$, axiom)
$x\cdot(-y) = -x\cdot y$ cnf(inverse_product$_2$, axiom)
$x\cdot(y+-z) = x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x+-y)\cdot z = x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x)\cdot(y+z) = -x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x+y)\cdot(-z) = -x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x,y,z) + \text{associator}(x,z,y) \neq 0$ cnf(prove_equation, negated_conjecture)

RNG025-6.p Middle or Flexible Law
include('Axioms/RNG003-0.ax')
$\text{associator}(x,y,x) \neq 0$ cnf(prove_flexible_law, negated_conjecture)

RNG025-7.p Middle or Flexible Law
include('Axioms/RNG003-0.ax')
$(-x)\cdot(-y) = x\cdot y$ cnf(product_of_inverses, axiom)
$(-x)\cdot y = -x\cdot y$ cnf(inverse_product$_1$, axiom)
$x\cdot(-y) = -x\cdot y$ cnf(inverse_product$_2$, axiom)
$x\cdot(y+-z) = x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x+-y)\cdot z = x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x)\cdot(y+z) = -x\cdot y + -x\cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x+y)\cdot(-z) = -x\cdot z + -y\cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x,y,x) \neq 0$ cnf(prove_flexible_law, negated_conjecture)

RNG025-8.p Middle or Flexible Law
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$(x \cdot x) \cdot y = x \cdot (x \cdot y)$ cnf(left_alternative, axiom)
$\text{associator}(x, y, u + v) = \text{associator}(x, y, u) + \text{associator}(x, y, v)$ cnf(linearised_associator$_1$, axiom)
$\text{associator}(x, u + v, y) = \text{associator}(x, u, y) + \text{associator}(x, v, y)$ cnf(linearised_associator$_2$, axiom)
$\text{associator}(u + v, x, y) = \text{associator}(u, x, y) + \text{associator}(v, x, y)$ cnf(linearised_associator$_3$, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$\text{associator}(a, b, c) + \text{associator}(a, c, b) \neq 0$ cnf(prove_flexible_law, negated_conjecture)

RNG025-9.p Middle or Flexible Law
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$(x \cdot x) \cdot y = x \cdot (x \cdot y)$ cnf(left_alternative, axiom)
$\text{associator}(x, y, u + v) = \text{associator}(x, y, u) + \text{associator}(x, y, v)$ cnf(linearised_associator$_1$, axiom)
$\text{associator}(x, u + v, y) = \text{associator}(x, u, y) + \text{associator}(x, v, y)$ cnf(linearised_associator$_2$, axiom)
$\text{associator}(u + v, x, y) = \text{associator}(u, x, y) + \text{associator}(v, x, y)$ cnf(linearised_associator$_3$, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$\text{associator}(a, b, c) + \text{associator}(a, c, b) \neq 0$ cnf(prove_flexible_law, negated_conjecture)

RNG026-6.p Teichmuller Identity
include('Axioms/RNG003-0.ax')
$(\text{associator}(a \cdot b, c, d) + \text{associator}(a, b, c \cdot d)) + -((\text{associator}(a, b \cdot c, d) + a \cdot \text{associator}(b, c, d)) + \text{associator}(a, b, c) \cdot d) \neq 0$ cnf(prove_teichmuller_identity, negated_conjecture)

RNG026-7.p Teichmuller Identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)

$(\text{associator}(a \cdot b, c, d) + \text{associator}(a, b, c \cdot d)) + -((\text{associator}(a, b \cdot c, d) + a \cdot \text{associator}(b, c, d)) + \text{associator}(a, b, c) \cdot d) \neq 0$ cnf(prove_teichmuller_identity, negated_conjecture)

RNG027-1.p Right Moufang identity
include('Axioms/RNG004-0.ax')
$\text{cz} \cdot (\text{cx} \cdot (\text{cy} \cdot \text{cx})) \neq ((\text{cz} \cdot \text{cx}) \cdot \text{cy}) \cdot \text{cx}$ cnf(prove_right_moufang, negated_conjecture)

RNG027-2.p Right Moufang identity
include('Axioms/RNG004-0.ax')
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$(y \cdot x) \cdot y = y \cdot (x \cdot y)$ cnf(middle_law, axiom)
$\text{associator}(y, x, z) = -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_1$, axiom)
$\text{associator}(z, y, x) = -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_2$, axiom)
$\text{cz} \cdot (\text{cx} \cdot (\text{cy} \cdot \text{cx})) \neq ((\text{cz} \cdot \text{cx}) \cdot \text{cy}) \cdot \text{cx}$ cnf(prove_right_moufang, negated_conjecture)

RNG027-5.p Right Moufang identity
include('Axioms/RNG003-0.ax')
$\text{cz} \cdot (\text{cx} \cdot (\text{cy} \cdot \text{cx})) \neq ((\text{cz} \cdot \text{cx}) \cdot \text{cy}) \cdot \text{cx}$ cnf(prove_right_moufang, negated_conjecture)

RNG027-7.p Right Moufang identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{cz} \cdot (\text{cx} \cdot (\text{cy} \cdot \text{cx})) \neq ((\text{cz} \cdot \text{cx}) \cdot \text{cy}) \cdot \text{cx}$ cnf(prove_right_moufang, negated_conjecture)

RNG027-8.p Right Moufang identity
include('Axioms/RNG003-0.ax')
$\text{associator}(x, x \cdot y, z) \neq \text{associator}(x, y, z) \cdot x$ cnf(prove_right_moufang, negated_conjecture)

RNG027-9.p Right Moufang identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\text{associator}(x, x \cdot y, z) \neq \text{associator}(x, y, z) \cdot x$ cnf(prove_right_moufang, negated_conjecture)

RNG028-1.p Left Moufang identity
include('Axioms/RNG004-0.ax')
$(\text{cx} \cdot (\text{cy} \cdot \text{cx})) \cdot \text{cz} \neq \text{cx} \cdot (\text{cy} \cdot (\text{cx} \cdot \text{cz}))$ cnf(prove_left_moufang, negated_conjecture)

RNG028-2.p Left Moufang identity
include('Axioms/RNG004-0.ax')
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$(y \cdot x) \cdot y = y \cdot (x \cdot y)$ cnf(middle_law, axiom)
$\text{associator}(y, x, z) = -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_1$, axiom)
$\text{associator}(z, y, x) = -\text{associator}(x, y, z)$ cnf(associator_skew_symmetry$_2$, axiom)
$(\text{cx} \cdot (\text{cy} \cdot \text{cx})) \cdot \text{cz} \neq \text{cx} \cdot (\text{cy} \cdot (\text{cx} \cdot \text{cz}))$ cnf(prove_left_moufang, negated_conjecture)

RNG028-5.p Left Moufang identity
include('Axioms/RNG003-0.ax')
$(\text{cx} \cdot (\text{cy} \cdot \text{cx})) \cdot \text{cz} \neq \text{cx} \cdot (\text{cy} \cdot (\text{cx} \cdot \text{cz}))$ cnf(prove_left_moufang, negated_conjecture)

RNG028-7.p Left Moufang identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)

$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$(\mathrm{cx} \cdot (\mathrm{cy} \cdot \mathrm{cx})) \cdot \mathrm{cz} \neq \mathrm{cx} \cdot (\mathrm{cy} \cdot (\mathrm{cx} \cdot \mathrm{cz}))$ cnf(prove_left_moufang, negated_conjecture)

RNG028-8.p Left Moufang identity
include('Axioms/RNG003-0.ax')
$\mathrm{associator}(x, y \cdot x, z) \neq x \cdot \mathrm{associator}(x, y, z)$ cnf(prove_left_moufang, negated_conjecture)

RNG028-9.p Left Moufang identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$\mathrm{associator}(x, y \cdot x, z) \neq x \cdot \mathrm{associator}(x, y, z)$ cnf(prove_left_moufang, negated_conjecture)

RNG029-1.p Middle Moufang identity
include('Axioms/RNG004-0.ax')
$(\mathrm{cx} \cdot \mathrm{cy}) \cdot (\mathrm{cz} \cdot \mathrm{cx}) \neq \mathrm{cx} \cdot ((\mathrm{cy} \cdot \mathrm{cz}) \cdot \mathrm{cx})$ cnf(prove_middle_law, negated_conjecture)

RNG029-2.p Middle Moufang identity
include('Axioms/RNG004-0.ax')
$\mathrm{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$(y \cdot x) \cdot y = y \cdot (x \cdot y)$ cnf(middle_law, axiom)
$\mathrm{associator}(y, x, z) = -\mathrm{associator}(x, y, z)$ cnf(associator_skew_symmetry$_1$, axiom)
$\mathrm{associator}(z, y, x) = -\mathrm{associator}(x, y, z)$ cnf(associator_skew_symmetry$_2$, axiom)
$(\mathrm{cx} \cdot \mathrm{cy}) \cdot (\mathrm{cz} \cdot \mathrm{cx}) \neq \mathrm{cx} \cdot ((\mathrm{cy} \cdot \mathrm{cz}) \cdot \mathrm{cx})$ cnf(prove_middle_law, negated_conjecture)

RNG029-3.p Middle Moufang identity
include('Axioms/RNG004-0.ax')
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
$(x \cdot (y \cdot x)) \cdot z = x \cdot (y \cdot (x \cdot z))$ cnf(left_moufang, hypothesis)
$\mathrm{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$(y \cdot x) \cdot y = y \cdot (x \cdot y)$ cnf(middle_law, axiom)
$\mathrm{associator}(y, x, z) = -\mathrm{associator}(x, y, z)$ cnf(associator_skew_symmetry$_1$, axiom)
$\mathrm{associator}(z, y, x) = -\mathrm{associator}(x, y, z)$ cnf(associator_skew_symmetry$_2$, axiom)
$(\mathrm{cx} \cdot \mathrm{cy}) \cdot (\mathrm{cz} \cdot \mathrm{cx}) \neq \mathrm{cx} \cdot ((\mathrm{cy} \cdot \mathrm{cz}) \cdot \mathrm{cx})$ cnf(prove_middle_law, negated_conjecture)

RNG029-5.p Middle Moufang identity
include('Axioms/RNG003-0.ax')
$(\mathrm{cx} \cdot \mathrm{cy}) \cdot (\mathrm{cz} \cdot \mathrm{cx}) \neq \mathrm{cx} \cdot ((\mathrm{cy} \cdot \mathrm{cz}) \cdot \mathrm{cx})$ cnf(prove_middle_law, negated_conjecture)

RNG029-6.p Middle Moufang identity
include('Axioms/RNG003-0.ax')
$(x \cdot y) \cdot (z \cdot x) \neq (x \cdot (y \cdot z)) \cdot x$ cnf(prove_middle_moufang, negated_conjecture)

RNG029-7.p Middle Moufang identity
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$(x \cdot y) \cdot (z \cdot x) \neq (x \cdot (y \cdot z)) \cdot x$ cnf(prove_middle_moufang, negated_conjecture)

RNG030-6.p 2*assr(X,X,Y)∧3 = additive identity
$x + y = y + x$ cnf(commutativity_for_addition, axiom)

$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$\text{associator}(x, x, y) \cdot (\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) + \text{associator}(x, x, y) \cdot (\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) \neq 0$ cnf(prove_conjecture$_1$, negated_conjecture)

RNG030-7.p 2*assr(X,X,Y)∧3 = additive identity
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$\text{associator}(x, x, y) \cdot (\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) + \text{associator}(x, x, y) \cdot (\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) \neq 0$ cnf(prove_conjecture$_1$, negated_conjecture)

RNG031-6.p (W*W)*X*(W*W) = additive identity
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\text{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\text{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$((\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) \cdot x) \cdot (\text{associator}(x, x, y) \cdot \text{associator}(x, x, y)) \neq 0$ cnf(prove_conjecture$_2$, negated_conje

RNG031-7.p (W*W)*X*(W*W) = additive identity
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)

$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\mathrm{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\mathrm{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$((\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y)) \cdot x) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y)) \neq 0$ cnf(prove_conjecture$_2$, negated_conje

RNG032-6.p 6*assr(X,X,Y)∧6 = additive identity
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)
$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
$\mathrm{associator}(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
$\mathrm{commutator}(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
$((((\mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y)) + \mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y))) + \mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y))) + \mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y))) + \mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y))) + \mathrm{associator}(x, x, y) \cdot (\mathrm{associator}(x, x, y) \cdot \mathrm{associator}(x, x, y)) \neq 0$ cnf(prove_conjecture$_3$, negated_conjecture)

RNG032-7.p 6*assr(X,X,Y)∧6 = additive identity
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$x + y = y + x$ cnf(commutativity_for_addition, axiom)
$x + (y + z) = (x + y) + z$ cnf(associativity_for_addition, axiom)
$0 + x = x$ cnf(left_additive_identity, axiom)
$x + 0 = x$ cnf(right_additive_identity, axiom)
$0 \cdot x = 0$ cnf(left_multiplicative_zero, axiom)
$x \cdot 0 = 0$ cnf(right_multiplicative_zero, axiom)
$-x + x = 0$ cnf(left_additive_inverse, axiom)
$x + -x = 0$ cnf(right_additive_inverse, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distribute$_1$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distribute$_2$, axiom)
$-(-x) = x$ cnf(additive_inverse_additive_inverse, axiom)

$(x \cdot y) \cdot y = x \cdot (y \cdot y)$ cnf(right_alternative, axiom)
associator$(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
commutator$(x, y) = y \cdot x + -x \cdot y$ cnf(commutator, axiom)
((((associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y))+associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y)))+ associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y)))+associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y)))+ associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y)))+associator$(x,x,y)\cdot$(associator$(x,x,y)\cdot$associator(x,x,y)) $\neq$ 0 cnf(prove_conjecture$_3$, negated_conjecture)

RNG033-6.p A fairly complex equation with associators
assr(X.Y,Z,W)+assr(X,Y,comm(Z,W)) = X.assr(Y,Z,W)+assr(X,Z,W).Y
include('Axioms/RNG003-0.ax')
associator$(x\cdot y, z, w)$+associator$(x, y,$ commutator$(z, w)) \neq x\cdot$associator(y, z, w)+associator$(x, z, w)\cdot y$ cnf(prove_challenge,

RNG033-7.p A fairly complex equation with associators
assr(X.Y,Z,W)+assr(X,Y,comm(Z,W)) = X.assr(Y,Z,W)+assr(X,Z,W).Y
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
associator$(x\cdot y, z, w)$+associator$(x, y,$ commutator$(z, w)) \neq x\cdot$associator(y, z, w)+associator$(x, z, w)\cdot y$ cnf(prove_challenge,

RNG033-8.p A fairly complex equation with associators
assr(X.Y,Z,W)+assr(X,Y,comm(Z,W)) = X.assr(Y,Z,W)+assr(X,Z,W).Y
include('Axioms/RNG003-0.ax')
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
associator$(x\cdot y, z, w)$+associator$(x, y,$ commutator$(z, w)) \neq x\cdot$associator(y, z, w)+associator$(x, z, w)\cdot y$ cnf(prove_challenge,

RNG033-9.p A fairly complex equation with associators
assr(X.Y,Z,W)+assr(X,Y,comm(Z,W)) = X.assr(Y,Z,W)+assr(X,Z,W).Y
include('Axioms/RNG003-0.ax')
$(-x) \cdot (-y) = x \cdot y$ cnf(product_of_inverses, axiom)
$(-x) \cdot y = -x \cdot y$ cnf(inverse_product$_1$, axiom)
$x \cdot (-y) = -x \cdot y$ cnf(inverse_product$_2$, axiom)
$x \cdot (y + -z) = x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_1$, axiom)
$(x + -y) \cdot z = x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_2$, axiom)
$(-x) \cdot (y + z) = -x \cdot y + -x \cdot z$ cnf(distributivity_of_difference$_3$, axiom)
$(x + y) \cdot (-z) = -x \cdot z + -y \cdot z$ cnf(distributivity_of_difference$_4$, axiom)
$z \cdot (x \cdot (y \cdot x)) = ((z \cdot x) \cdot y) \cdot x$ cnf(right_moufang, hypothesis)
associator$(x\cdot y, z, w)$+associator$(x, y,$ commutator$(z, w)) \neq x\cdot$associator(y, z, w)+associator$(x, z, w)\cdot y$ cnf(prove_challenge,

RNG034-1.p A skew symmetry relation of the associator
include('Axioms/RNG004-0.ax')
associator$(x, y, z) = (x \cdot y) \cdot z + -x \cdot (y \cdot z)$ cnf(associator, axiom)
associator(cy, cx, cz) $\neq -$associator(cx, cy, cz) cnf(prove_skew_symmetry, negated_conjecture)

RNG035-7.p If X*X*X*X = X then the ring is commutative
Given a ring in which for all x, x * x * x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG005-0.ax')
$x \cdot (x \cdot (x \cdot x)) = x$ cnf(x_fourthed_is_x, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG036-7.p If X*X*X*X*X = X then the ring is commutative
Given a ring in which for all x, x * x * x * x * x = x, prove that for all x and y, x * y = y * x.
include('Axioms/RNG005-0.ax')
$x \cdot (x \cdot (x \cdot (x \cdot x))) = x$ cnf(x_fifthed_is_x, hypothesis)
$a \cdot b = c$ cnf(a_times_b_is_c, negated_conjecture)
$b \cdot a \neq c$ cnf(prove_commutativity, negated_conjecture)

RNG037-1.p (X* -Y) + (X*Y) = additive_identity

include('Axioms/RNG001-0.ax')
$a \cdot b = d$ cnf(a_times_b, hypothesis)
$a \cdot (-b) = c$ cnf(a_inverse_times_b, hypothesis)
$\neg\, c + d = 0$ cnf(prove_sum_is_additive_identity, negated_conjecture)

RNG038-1.p Ring property 1
include('Axioms/RNG001-0.ax')
$x = 0 \;\Rightarrow\; x \cdot h(x, y) = y$ cnf(some_property, hypothesis)
$a \cdot b = 0$ cnf(a_times_b, hypothesis)
$a \neq 0$ cnf(a_not_additive_identity, negated_conjecture)
$b \neq 0$ cnf(prove_b_is_additive_identity, negated_conjecture)

RNG039-1.p Ring property 2
include('Axioms/RNG001-0.ax')
$a + (a + b) = b$ cnf(absorbtion_1, axiom)
$(a + b) + b = a$ cnf(absorbtion_2, axiom)
$a + a = 0$ cnf(clause_{32}, axiom)
$a + 0 = a$ cnf(clause_{33}, axiom)
$a + a = 0$ cnf(clause_{34}, axiom)
$a \cdot a = a$ cnf(clause_{35}, axiom)
$a \cdot b = c$ cnf(clause_{36}, axiom)
$b \cdot a = d$ cnf(clause_{37}, axiom)
$a + b = b + a$ cnf(clause_{38}, axiom)
$a \cdot c = c$ cnf(clause_{39}, axiom)
$b \cdot d = d$ cnf(clause_{40}, axiom)
$c \cdot b = c$ cnf(clause_{41}, axiom)
$d \cdot a = d$ cnf(clause_{42}, axiom)
$a \cdot (a \cdot b) = a \cdot b$ cnf(clause_{43}, axiom)
$a \cdot (b \cdot a) = b \cdot a$ cnf(clause_{44}, axiom)
$a \cdot b = c \cdot b$ cnf(clause_{45}, axiom)
$a \cdot (b \cdot c) = c$ cnf(clause_{46}, axiom)
$b \cdot (a \cdot a) = d \cdot a$ cnf(clause_{47}, axiom)
$b \cdot a = d \cdot a$ cnf(clause_{48}, axiom)
$b \cdot (a \cdot d) = d$ cnf(clause_{49}, axiom)
$b \cdot c = d \cdot b$ cnf(clause_{50}, axiom)
$a \cdot d = c \cdot a$ cnf(clause_{51}, axiom)
$(a \cdot b) \cdot b = a \cdot b$ cnf(clause_{52}, axiom)
$(a \cdot a) \cdot b = a \cdot c$ cnf(clause_{53}, axiom)
$a \cdot b = a \cdot c$ cnf(clause_{54}, axiom)
$(c \cdot a) \cdot b = c$ cnf(clause_{55}, axiom)
$d \cdot b = b \cdot c$ cnf(clause_{56}, axiom)
$(a \cdot b) \cdot a = a \cdot d$ cnf(clause_{57}, axiom)
$b \cdot a = b \cdot d$ cnf(clause_{58}, axiom)
$(d \cdot b) \cdot a = d$ cnf(clause_{59}, axiom)
$c \cdot a = a \cdot d$ cnf(clause_{60}, axiom)
$a \cdot (b + a) = c + a$ cnf(clause_{63}, axiom)
$a \cdot (a + b) = a + c$ cnf(clause_{64}, axiom)
$b \cdot (a + b) = d + b$ cnf(clause_{65}, axiom)
$b \cdot (b + a) = b + d$ cnf(clause_{66}, axiom)
$(a + b) \cdot b = c + b$ cnf(clause_{67}, axiom)
$(b + a) \cdot b = b + c$ cnf(clause_{68}, axiom)
$(b + a) \cdot a = d + a$ cnf(clause_{69}, axiom)
$(a + b) \cdot a = a + d$ cnf(clause_{70}, axiom)
$a \cdot a = a$ cnf(clause_{71}, axiom)
$a \cdot b = c$ cnf(a_times_b, negated_conjecture)
$b \cdot a = d$ cnf(b_times_a, negated_conjecture)
$c \neq d$ cnf(prove_c_equals_d, negated_conjecture)

RNG040-1.p Ring property 4
include('Axioms/RNG001-0.ax')
$a \cdot 1 = a$ cnf(right_multiplicative_identity, hypothesis)

$1 \cdot a = a$ cnf(left_multiplicative_identity, hypothesis)
$a \cdot h(a) = 1$ or $a = 0$ cnf(clause$_{30}$, hypothesis)
$h(a) \cdot a = 1$ or $a = 0$ cnf(clause$_{31}$, hypothesis)
$a \cdot b = c \Rightarrow b \cdot a = c$ cnf(product_symmetry, hypothesis)
$b + c = d$ cnf(b_plus_c, negated_conjecture)
$d \cdot a = 0$ cnf(d_plus_a, negated_conjecture)
$b \cdot a = l$ cnf(b_plus_a, negated_conjecture)
$c \cdot a = n$ cnf(c_plus_a, negated_conjecture)
$\neg\, l + n = 0$ cnf(prove_equation, negated_conjecture)

RNG040-2.p Ring property 4
$0 + x = x$ cnf(additive_identity$_1$, axiom)
$x + 0 = x$ cnf(additive_identity$_2$, axiom)
$x \cdot y = x \cdot y$ cnf(closure_of_multiplication, axiom)
$x + y = x + y$ cnf(closure_of_addition, axiom)
$-x + x = 0$ cnf(additive_inverse$_1$, axiom)
$x + -x = 0$ cnf(additive_inverse$_2$, axiom)
$(x + y = u$ and $y + z = v$ and $u + z = w) \Rightarrow x + v = w$ cnf(associativity_of_addition$_1$, axiom)
$(x + y = u$ and $y + z = v$ and $x + v = w) \Rightarrow u + z = w$ cnf(associativity_of_addition$_2$, axiom)
$x + y = z \Rightarrow y + x = z$ cnf(commutativity_of_addition, axiom)
$(x \cdot y = u$ and $y \cdot z = v$ and $u \cdot z = w) \Rightarrow x \cdot v = w$ cnf(associativity_of_multiplication$_1$, axiom)
$(x \cdot y = u$ and $y \cdot z = v$ and $x \cdot v = w) \Rightarrow u \cdot z = w$ cnf(associativity_of_multiplication$_2$, axiom)
$(x \cdot y = v_1$ and $x \cdot z = v_2$ and $y + z = v_3$ and $x \cdot v_3 = v_4) \Rightarrow v_1 + v_2 = v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y = v_1$ and $x \cdot z = v_2$ and $y + z = v_3$ and $v_1 + v_2 = v_4) \Rightarrow x \cdot v_3 = v_4$ cnf(distributivity$_2$, axiom)
$(x + y = u$ and $x + y = v) \Rightarrow u = v$ cnf(addition_is_well_defined, axiom)
$(x \cdot y = u$ and $x \cdot y = v) \Rightarrow u = v$ cnf(multiplication_is_well_defined, axiom)
$a \cdot 1 = a$ cnf(right_multiplicative_identity, hypothesis)
$1 \cdot a = a$ cnf(left_multiplicative_identity, hypothesis)
$a \cdot h(a) = 1$ or $a = 0$ cnf(clause$_{30}$, hypothesis)
$h(a) \cdot a = 1$ or $a = 0$ cnf(clause$_{31}$, hypothesis)
$a \cdot b = c \Rightarrow b \cdot a = c$ cnf(product_symmetry, hypothesis)
$b + c = d$ cnf(b_plus_c, negated_conjecture)
$d \cdot a = 0$ cnf(d_plus_a, negated_conjecture)
$b \cdot a = l$ cnf(b_plus_a, negated_conjecture)
$c \cdot a = n$ cnf(c_plus_a, negated_conjecture)
$\neg\, l + n = 0$ cnf(prove_equation, negated_conjecture)

RNG041-1.p Unknown
include('Axioms/RNG001-0.ax')
$0 \cdot a = 0$ cnf(multiplicative_identity$_1$, hypothesis)
$a \cdot 0 = 0$ cnf(multiplicative_identity$_2$, hypothesis)
$a \cdot 1 = a$ cnf(right_multiplicative_identity, hypothesis)
$1 \cdot a = a$ cnf(left_multiplicative_identity, hypothesis)
$a \cdot h(a) = 1$ or $a = 0$ cnf(clause$_{41}$, hypothesis)
$h(a) \cdot a = 1$ or $a = 0$ cnf(clause$_{42}$, hypothesis)
$a \cdot b = 0$ cnf(a_times_b, negated_conjecture)
$a \neq 0$ cnf(a_not_additive_identity, negated_conjecture)
$b \neq 0$ cnf(prove_b_is_additive_identity, negated_conjecture)

RNG042-1.p Ring theory axioms
include('Axioms/RNG001-0.ax')

RNG042-2.p Ring theory (equality) axioms
include('Axioms/RNG002-0.ax')

RNG042-3.p Ring theory (equality) axioms
include('Axioms/RNG005-0.ax')

RNG043-1.p Alternative ring theory (equality) axioms
include('Axioms/RNG003-0.ax')

RNG043-2.p Alternative ring theory (equality) axioms
include('Axioms/RNG004-0.ax')

RNG044+1.p Cauchy-Bouniakowsky-Schwarz inequality 01, 00 expansion
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mNatSort, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{sz}_{00})$ fof(mZeroNat, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{aNaturalNumber}_0(\mathrm{szszuzczcdt}_0(w_0)) \text{ and } \mathrm{szszuzczcdt}_0(w_0) \neq \mathrm{sz}_{00}))$ fof(mSuccNat, axio
$\forall w_0\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } w_0 \neq \mathrm{sz}_{00}) \Rightarrow \exists w_1\colon (\mathrm{aNaturalNumber}_0(w_1) \text{ and } w_0 = \mathrm{szszuzczcdt}_0(w_1)))$ fof(mNatEx
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{szszuzczcdt}_0(w_0) = \mathrm{szszuzczcdt}_0(w_1) \Rightarrow w_0 = w_1))$ fof(mSuccEqu, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{iLess}_0(w_0, w_1) \Rightarrow \$\mathrm{true}))$ fof(mIHOrd, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \mathrm{iLess}_0(w_0, \mathrm{szszuzczcdt}_0(w_0)))$ fof(mIH, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mScSort, axiom)
$\mathrm{aScalar}_0(\mathrm{sz0z}_{00})$ fof(mSZeroSc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSumSc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mMulSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \mathrm{aScalar}_0(\mathrm{smndt}_0(w_0)))$ fof(mNegSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz0z}_{00}) = w_0 \text{ and } \mathrm{sdtpldt}_0(\mathrm{sz0z}_{00}, w_0) = w_0 \text{ and } \mathrm{sdtasdt}_0(w_0, \mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtasdt}_0(\mathrm{sz0z}_{00}, w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{smndt}_0(\mathrm{smndt}_0(w_0)) = w_0 \text{ and } \mathrm{smndt}_0(\mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00}))$ fof(mScZero, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1), w_2) = \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0($
$\mathrm{sdtpldt}_0(w_1, w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1), w_2) = \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) \text{ and } \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w$
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0,$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistr, axiom)
$\mathrm{aScalar}_0(\mathrm{xx}) \text{ and } \mathrm{aScalar}_0(\mathrm{xy}) \text{ and } \mathrm{aScalar}_0(\mathrm{xu}) \text{ and } \mathrm{aScalar}_0(\mathrm{xv})$ fof(m_674, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{sdtpldt}_0(\mathrm{xx}, \mathrm{xy}), \mathrm{sdtpldt}_0(\mathrm{xu}, \mathrm{xv})) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{xu}), \mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{xv})), \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(\mathrm{xy}, \mathrm{xu}),$

RNG045+1.p Cauchy-Bouniakowsky-Schwarz inequality 01_02, 00 expansion
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mNatSort, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{sz}_{00})$ fof(mZeroNat, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{aNaturalNumber}_0(\mathrm{szszuzczcdt}_0(w_0)) \text{ and } \mathrm{szszuzczcdt}_0(w_0) \neq \mathrm{sz}_{00}))$ fof(mSuccNat, axio
$\forall w_0\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } w_0 \neq \mathrm{sz}_{00}) \Rightarrow \exists w_1\colon (\mathrm{aNaturalNumber}_0(w_1) \text{ and } w_0 = \mathrm{szszuzczcdt}_0(w_1)))$ fof(mNatEx
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{szszuzczcdt}_0(w_0) = \mathrm{szszuzczcdt}_0(w_1) \Rightarrow w_0 = w_1))$ fof(mSuccEqu, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{iLess}_0(w_0, w_1) \Rightarrow \$\mathrm{true}))$ fof(mIHOrd, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \mathrm{iLess}_0(w_0, \mathrm{szszuzczcdt}_0(w_0)))$ fof(mIH, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mScSort, axiom)
$\mathrm{aScalar}_0(\mathrm{sz0z}_{00})$ fof(mSZeroSc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSumSc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mMulSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \mathrm{aScalar}_0(\mathrm{smndt}_0(w_0)))$ fof(mNegSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz0z}_{00}) = w_0 \text{ and } \mathrm{sdtpldt}_0(\mathrm{sz0z}_{00}, w_0) = w_0 \text{ and } \mathrm{sdtasdt}_0(w_0, \mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtasdt}_0(\mathrm{sz0z}_{00}, w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{smndt}_0(\mathrm{smndt}_0(w_0)) = w_0 \text{ and } \mathrm{smndt}_0(\mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00}))$ fof(mScZero, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1), w_2) = \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0($
$\mathrm{sdtpldt}_0(w_1, w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1), w_2) = \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) \text{ and } \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w$
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0,$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistr, axiom)
$\mathrm{aScalar}_0(\mathrm{xx}) \text{ and } \mathrm{aScalar}_0(\mathrm{xy}) \text{ and } \mathrm{aScalar}_0(\mathrm{xu}) \text{ and } \mathrm{aScalar}_0(\mathrm{xv})$ fof(m_674, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{sdtpldt}_0(\mathrm{xx}, \mathrm{xy}), \mathrm{sdtpldt}_0(\mathrm{xu}, \mathrm{xv})) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{sdtpldt}_0(\mathrm{xu}, \mathrm{xv})), \mathrm{sdtasdt}_0(\mathrm{xy}, \mathrm{sdtpldt}_0(\mathrm{xu}, \mathrm{xv})))$ fof(r
$\mathrm{sdtasdt}_0(\mathrm{sdtpldt}_0(\mathrm{xx}, \mathrm{xy}), \mathrm{sdtpldt}_0(\mathrm{xu}, \mathrm{xv})) = \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{xu}), \mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{xv})), \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(\mathrm{xy}, \mathrm{xu}),$

RNG046+1.p Cauchy-Bouniakowsky-Schwarz inequality 02, 00 expansion
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mNatSort, axiom)
$\mathrm{aNaturalNumber}_0(\mathrm{sz}_{00})$ fof(mZeroNat, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow (\mathrm{aNaturalNumber}_0(\mathrm{szszuzczcdt}_0(w_0)) \text{ and } \mathrm{szszuzczcdt}_0(w_0) \neq \mathrm{sz}_{00}))$ fof(mSuccNat, axio
$\forall w_0\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } w_0 \neq \mathrm{sz}_{00}) \Rightarrow \exists w_1\colon (\mathrm{aNaturalNumber}_0(w_1) \text{ and } w_0 = \mathrm{szszuzczcdt}_0(w_1)))$ fof(mNatEx
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{szszuzczcdt}_0(w_0) = \mathrm{szszuzczcdt}_0(w_1) \Rightarrow w_0 = w_1))$ fof(mSuccEqu, axiom)
$\forall w_0, w_1\colon ((\mathrm{aNaturalNumber}_0(w_0) \text{ and } \mathrm{aNaturalNumber}_0(w_1)) \Rightarrow (\mathrm{iLess}_0(w_0, w_1) \Rightarrow \$\mathrm{true}))$ fof(mIHOrd, axiom)
$\forall w_0\colon (\mathrm{aNaturalNumber}_0(w_0) \Rightarrow \mathrm{iLess}_0(w_0, \mathrm{szszuzczcdt}_0(w_0)))$ fof(mIH, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mScSort, axiom)
$\mathrm{aScalar}_0(\mathrm{sz0z}_{00})$ fof(mSZeroSc, axiom)

$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSumSc, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow \mathrm{aScalar}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mMulSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow \mathrm{aScalar}_0(\mathrm{smndt}_0(w_0)))$ fof(mNegSc, axiom)
$\forall w_0\colon (\mathrm{aScalar}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz0z}_{00}) = w_0 \text{ and } \mathrm{sdtpldt}_0(\mathrm{sz0z}_{00}, w_0) = w_0 \text{ and } \mathrm{sdtasdt}_0(w_0, \mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtasdt}_0(\mathrm{sz0z}_{00}, w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz0z}_{00} \text{ and } \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0) = \mathrm{sz0z}_{00} \text{ and } \mathrm{smndt}_0(\mathrm{smndt}_0(w_0)) = w_0 \text{ and } \mathrm{smndt}_0(\mathrm{sz0z}_{00}) = \mathrm{sz0z}_{00}))$ fof(mScZero, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1), w_2) = \mathrm{sdtpldt}_0(w_0, \mathrm{sdtpldt}_0($
$\mathrm{sdtpldt}_0(w_1, w_0) \text{ and } \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1), w_2) = \mathrm{sdtasdt}_0(w_0, \mathrm{sdtasdt}_0(w_1, w_2)) \text{ and } \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w$
$\forall w_0, w_1, w_2\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0,$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_1, w_2))))$ fof(mDistr, axiom)
$\forall w_0, w_1, w_2, w_3\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1) \text{ and } \mathrm{aScalar}_0(w_2) \text{ and } \mathrm{aScalar}_0(w_3)) \Rightarrow \mathrm{sdtasdt}_0(\mathrm{sdtpldt}_0(w_0, w_1), \mathrm{sdtpldt}_0$
$\mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_0, w_2), \mathrm{sdtasdt}_0(w_0, w_3)), \mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_1, w_2), \mathrm{sdtasdt}_0(w_1, w_3))))$ fof(mDistr$_2$, axiom)
$\forall w_0, w_1\colon ((\mathrm{aScalar}_0(w_0) \text{ and } \mathrm{aScalar}_0(w_1)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(w_1)) = \mathrm{smndt}_0(\mathrm{sdtasdt}_0(w_0, w_1)) \text{ and } \mathrm{sdtasdt}_0(\mathrm{smndt}_0($
$\mathrm{smndt}_0(\mathrm{sdtasdt}_0(w_0, w_1))))$ fof(mMNeg, axiom)
$\mathrm{aScalar}_0(\mathrm{xx}) \text{ and } \mathrm{aScalar}_0(\mathrm{xy})$ fof(m$_{_799}$, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{xx}), \mathrm{smndt}_0(\mathrm{xy})) = \mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{xy})$ fof(m$_{__}$, conjecture)

RNG082+1.p Chinese remainder theorem in a ring 01, 00 expansion
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\mathrm{aElement}_0(\mathrm{sz}_{00})$ fof(mSortsC, axiom)
$\mathrm{aElement}_0(\mathrm{sz}_{10})$ fof(mSortsC$_{01}$, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \mathrm{aElement}_0(\mathrm{smndt}_0(w_0)))$ fof(mSortsU, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{aElement}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSortsB, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{aElement}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mSortsB$_{02}$, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1), w_2) = \mathrm{sdtpldt}_0(w_0, \mathrm{sdtp}$
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddInvr, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1), w_2) = \mathrm{sdtasdt}_0(w_0, \mathrm{sdt}$
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulUnit, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasd}$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_1, w_0), \mathrm{sdtasdt}_0(w_2, w_0))))$ fof(mAMDistr, axiom)
$\mathrm{aElement}_0(\mathrm{xx})$ fof(m$_{_444}$, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), \mathrm{xx}) = \mathrm{smndt}_0(\mathrm{xx})$ fof(m$_{__}$, conjecture)

RNG083+1.p Chinese remainder theorem in a ring 02, 00 expansion
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \$\mathrm{true})$ fof(mElmSort, axiom)
$\mathrm{aElement}_0(\mathrm{sz}_{00})$ fof(mSortsC, axiom)
$\mathrm{aElement}_0(\mathrm{sz}_{10})$ fof(mSortsC$_{01}$, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow \mathrm{aElement}_0(\mathrm{smndt}_0(w_0)))$ fof(mSortsU, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{aElement}_0(\mathrm{sdtpldt}_0(w_0, w_1)))$ fof(mSortsB, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{aElement}_0(\mathrm{sdtasdt}_0(w_0, w_1)))$ fof(mSortsB$_{02}$, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{sdtpldt}_0(w_0, w_1) = \mathrm{sdtpldt}_0(w_1, w_0))$ fof(mAddComm, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow \mathrm{sdtpldt}_0(\mathrm{sdtpldt}_0(w_0, w_1), w_2) = \mathrm{sdtpldt}_0(w_0, \mathrm{sdtp}$
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{sz}_{00}) = w_0 \text{ and } w_0 = \mathrm{sdtpldt}_0(\mathrm{sz}_{00}, w_0)))$ fof(mAddZero, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtpldt}_0(w_0, \mathrm{smndt}_0(w_0)) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtpldt}_0(\mathrm{smndt}_0(w_0), w_0)))$ fof(mAddInvr, axiom)
$\forall w_0, w_1\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1)) \Rightarrow \mathrm{sdtasdt}_0(w_0, w_1) = \mathrm{sdtasdt}_0(w_1, w_0))$ fof(mMulComm, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow \mathrm{sdtasdt}_0(\mathrm{sdtasdt}_0(w_0, w_1), w_2) = \mathrm{sdtasdt}_0(w_0, \mathrm{sdt}$
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sz}_{10}) = w_0 \text{ and } w_0 = \mathrm{sdtasdt}_0(\mathrm{sz}_{10}, w_0)))$ fof(mMulUnit, axiom)
$\forall w_0, w_1, w_2\colon ((\mathrm{aElement}_0(w_0) \text{ and } \mathrm{aElement}_0(w_1) \text{ and } \mathrm{aElement}_0(w_2)) \Rightarrow (\mathrm{sdtasdt}_0(w_0, \mathrm{sdtpldt}_0(w_1, w_2)) = \mathrm{sdtpldt}_0(\mathrm{sdtasd}$
$\mathrm{sdtpldt}_0(\mathrm{sdtasdt}_0(w_1, w_0), \mathrm{sdtasdt}_0(w_2, w_0))))$ fof(mAMDistr, axiom)
$\forall w_0\colon (\mathrm{aElement}_0(w_0) \Rightarrow (\mathrm{sdtasdt}_0(\mathrm{smndt}_0(\mathrm{sz}_{10}), w_0) = \mathrm{smndt}_0(w_0) \text{ and } \mathrm{smndt}_0(w_0) = \mathrm{sdtasdt}_0(w_0, \mathrm{smndt}_0(\mathrm{sz}_{10}))))$ fof(
$\mathrm{aElement}_0(\mathrm{xx})$ fof(m$_{_468}$, hypothesis)
$\mathrm{sdtasdt}_0(\mathrm{xx}, \mathrm{sz}_{00}) = \mathrm{sz}_{00} \text{ and } \mathrm{sz}_{00} = \mathrm{sdtasdt}_0(\mathrm{sz}_{00}, \mathrm{xx})$ fof(m$_{__}$, conjecture)

RNG127+1.p Proper integral domains
$\forall a\colon a + m(a) = n$ fof(f_{01}, axiom)
$\forall a\colon a + n = a$ fof(f_{02}, axiom)
$\forall a, b, c\colon a + (b + c) = (a + b) + c$ fof(f_{03}, axiom)
$\forall a, b\colon a + b = b + a$ fof(f_{04}, axiom)

$\forall a$: $a \cdot e = a$ fof(f_{05}, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(f_{06}, axiom)
$\forall a, b$: $a \cdot b = b \cdot a$ fof(f_{07}, axiom)
$\forall a, b, c$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(f_{08}, axiom)
$\forall a, b$: $(a \cdot b = n \Rightarrow (a = n \text{ or } b = n))$ fof(f_{09}, axiom)
$\exists a$: $\forall b$: $(a \neq n \text{ and } a \cdot b \neq e)$ fof(f_{10}, axiom)

RNG128-1.p In commutative semirings with 1+x+x∧2=x, the operations coincide
$a + (b + c) = (a + b) + c$ cnf(sos, axiom)
$a + b = b + a$ cnf(sos_{001}, axiom)
$a \cdot b = b \cdot a$ cnf(sos_{002}, axiom)
$a \cdot (b + c) = a \cdot b + a \cdot c$ cnf(sos_{003}, axiom)
$0 + a = a$ cnf(sos_{004}, axiom)
$a \cdot 1 = a$ cnf(sos_{005}, axiom)
$1 + (a + a \cdot a) = a$ cnf(sos_{006}, axiom)
$x_0 + x_1 \neq x_0 \cdot x_1$ cnf(goals, negated_conjecture)

RNG129-1.p Separativity in rings
$0 + a = a$ cnf(sos, axiom)
$a + 0 = a$ cnf(sos_{001}, axiom)
$-a + a = 0$ cnf(sos_{002}, axiom)
$a + -a = 0$ cnf(sos_{003}, axiom)
$-(-a) = a$ cnf(sos_{004}, axiom)
$(a + b) + c = a + (b + c)$ cnf(sos_{005}, axiom)
$a + b = b + a$ cnf(sos_{006}, axiom)
$a \cdot 1 = a$ cnf(sos_{007}, axiom)
$1 \cdot a = a$ cnf(sos_{008}, axiom)
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ cnf(sos_{009}, axiom)
$0 \cdot a = 0$ cnf(sos_{010}, axiom)
$a \cdot 0 = 0$ cnf(sos_{011}, axiom)
$a \cdot b + a \cdot c = a \cdot (b + c)$ cnf(sos_{012}, axiom)
$a \cdot b + c \cdot b = (a + c) \cdot b$ cnf(sos_{013}, axiom)
$a \cdot (a^{-1} \cdot a) = a$ cnf(sos_{014}, axiom)
$a^{-1} \cdot (a \cdot a^{-1}) = a^{-1}$ cnf(sos_{015}, axiom)
$a_0 + (a_1 + (b_0 + b_1)) = 1$ cnf(sos_{016}, axiom)
$a_0 \cdot a_0 = a_0$ cnf(sos_{017}, axiom)
$a_1 \cdot a_1 = a_1$ cnf(sos_{018}, axiom)
$b_0 \cdot b_0 = b_0$ cnf(sos_{019}, axiom)
$b_1 \cdot b_1 = b_1$ cnf(sos_{020}, axiom)
$a_0 \cdot a_1 = 0$ cnf(sos_{021}, axiom)
$a_1 \cdot a_0 = 0$ cnf(sos_{022}, axiom)
$a_0 \cdot b_0 = 0$ cnf(sos_{023}, axiom)
$b_0 \cdot a_0 = 0$ cnf(sos_{024}, axiom)
$a_0 \cdot b_1 = 0$ cnf(sos_{025}, axiom)
$b_1 \cdot a_0 = 0$ cnf(sos_{026}, axiom)
$a_1 \cdot b_0 = 0$ cnf(sos_{027}, axiom)
$b_0 \cdot a_1 = 0$ cnf(sos_{028}, axiom)
$a_1 \cdot b_1 = 0$ cnf(sos_{029}, axiom)
$b_1 \cdot a_1 = 0$ cnf(sos_{030}, axiom)
$b_0 \cdot b_1 = 0$ cnf(sos_{031}, axiom)
$b_1 \cdot b_0 = 0$ cnf(sos_{032}, axiom)
$u \cdot u = 1$ cnf(sos_{033}, axiom)
$u \cdot (a_0 \cdot u) = a_1$ cnf(sos_{034}, axiom)
$u \cdot (b_0 \cdot u) = b_1$ cnf(sos_{035}, axiom)
$a_0 + a_1 = c \cdot d$ cnf(sos_{036}, axiom)
$a_1 + b_0 = d \cdot c$ cnf(sos_{037}, axiom)
$c = (a_0 + a_1) \cdot (c \cdot (a_1 + b_0))$ cnf(sos_{038}, axiom)
$d = (a_1 + b_0) \cdot (d \cdot (a_0 + a_1))$ cnf(sos_{039}, axiom)
$a_1 + b_0 = e \cdot f$ cnf(sos_{040}, axiom)
$b_0 + b_1 = f \cdot e$ cnf(sos_{041}, axiom)

$e = (a_1 + b_0) \cdot (e \cdot (b_0 + b_1))$ $\quad$ cnf(sos$_{042}$, axiom)
$f = (b_0 + b_1) \cdot (f \cdot (a_1 + b_0))$ $\quad$ cnf(sos$_{043}$, axiom)
$a \cdot b = a_0 \;\Rightarrow\; b \cdot a \neq b_0$ $\quad$ cnf(sos$_{044}$, negated_conjecture)