ROB axioms

ROB001-0.ax Robbins algebra axioms
$x + y = y + x$ cnf(commutativity_of_add, axiom)
$(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
$-(-(x + y) + -(x + -y)) = x$ cnf(robbins_axiom, axiom)

ROB001-1.ax Robbins algebra numbers axioms
$1 \cdot x = x$ cnf(one_times_x, axiom)
positive_integer(x) $\Rightarrow$ successor(v) $\cdot x = x + v \cdot x$ cnf(times_by_adding, axiom)
positive_integer(1) cnf(one, axiom)
positive_integer(x) $\Rightarrow$ positive_integer(successor(x)) cnf(next_integer, axiom)

ROB problems

ROB001-1.p Is every Robbins algebra Boolean?
include('Axioms/ROB001-0.ax')
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB002-1.p –X = X => Boolean
If –X = X then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(-x) = x$ cnf(double_negation, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB003-1.p X + c=c => Boolean
If there exists c such that X+c=c, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$x + c = c$ cnf(there_exists_a_constant, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB004-1.p c = -d, c + d=d, and c + c=c => Boolean
If there exist c, d such that c = -d, c+d=d, and c+c=c, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-d = c$ cnf(negate_d_is_c, hypothesis)
$c + d = d$ cnf(c_plus_d_is_d, hypothesis)
$c + c = c$ cnf(c_plus_c_is_c, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB005-1.p Exists an idempotent element => Boolean
If there is an element c such that c+c=c, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + c = c$ cnf(idempotence, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB006-1.p Exists absorbed element => Boolean
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + d = d$ cnf(absorbtion, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB006-2.p Exists absorbed element => Exists idempotent element
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + d = d$ cnf(absorbtion, hypothesis)
$x + x \neq x$ cnf(prove_idempotence, negated_conjecture)

ROB006-3.p c + d=d => Boolean
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$x + x \neq x$ cnf(idempotence, axiom)
$(-(x + y) = -y$ and positive_integer(v_2)) $\Rightarrow$ $-(y + v_2 \cdot (x + -(x + -y))) = -y$ cnf(corollary_3$_7$, axiom)
$-(x+-y) = -y \Rightarrow y+$successor(successor(1))$\cdot(x+-(x+-y)) = y+$successor(1)$\cdot(x+-(x+-y))$ cnf(corollary_3_9$_1$, axiom)

$-(-y + -(x + -y)) = x \Rightarrow y + \text{successor}(\text{successor}(1)) \cdot (x + -(x + -y)) = y + \text{successor}(1) \cdot (x + -(x + -y))$ cnf(corollary_3_9$_2$, axiom)
$c + d = d$ cnf(absorbtion, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB007-1.p Absorbed within negation element => Boolean
If there exist a, b such that -(a+b) = -b, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(a + b) = -b$ cnf(condition, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB007-2.p Absorbed within negation element => Exists idempotent element
If there exist a, b such that -(a+b) = -b, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(a + b) = -b$ cnf(condition, hypothesis)
$x + x \neq x$ cnf(prove_idempotence, negated_conjecture)

ROB007-3.p Absorbed within negation element => Boolean
If there exist a, b such that -(a+b) = -b, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$x + y \neq y$ cnf(absorbtion, axiom)
$-(x + -y) = -y \Rightarrow y + \text{successor}(\text{successor}(1)) \cdot (x + -(x + -y)) = y + \text{successor}(1) \cdot (x + -(x + -y))$ cnf(corollary_3_9$_1$, axiom)
$-(-y + -(x + -y)) = x \Rightarrow y + \text{successor}(\text{successor}(1)) \cdot (x + -(x + -y)) = y + \text{successor}(1) \cdot (x + -(x + -y))$ cnf(corollary_3_9$_2$, axiom)
$-(a + b) = -b$ cnf(condition, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB007-4.p Absorbed within negation element => Exists idempotent element
If there exist a, b such that -(a+b) = -b, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$x + y \neq y$ cnf(absorbtion, axiom)
$-(x + -y) = -y \Rightarrow y + \text{successor}(\text{successor}(1)) \cdot (x + -(x + -y)) = y + \text{successor}(1) \cdot (x + -(x + -y))$ cnf(corollary_3_9$_1$, axiom)
$-(-y + -(x + -y)) = x \Rightarrow y + \text{successor}(\text{successor}(1)) \cdot (x + -(x + -y)) = y + \text{successor}(1) \cdot (x + -(x + -y))$ cnf(corollary_3_9$_2$, axiom)
$-(a + b) = -b$ cnf(condition, hypothesis)
$x + x \neq x$ cnf(prove_idempotence, negated_conjecture)

ROB008-1.p If -(a + -(b + c)) = -(a + b + -c) then a+b=a
include('Axioms/ROB001-0.ax')
$-(a + -(b + c)) = -(a + (b + -c))$ cnf(condition, hypothesis)
$a + b \neq a$ cnf(prove_result, negated_conjecture)

ROB009-1.p If -(a + -(b + c)) = -(b + -(a + c)) then a = b
include('Axioms/ROB001-0.ax')
$-(a + -(b + c)) = -(b + -(a + c))$ cnf(condition, hypothesis)
$a \neq b$ cnf(prove_result, negated_conjecture)

ROB010-1.p If -(a + -b) = c then -(c + -(b + a)) = a
include('Axioms/ROB001-0.ax')
$-(a + -b) = c$ cnf(condition, hypothesis)
$-(c + -(b + a)) \neq a$ cnf(prove_result, negated_conjecture)

ROB011-1.p If -(a + -b) = c then -(a + -(b + k(a + c))) = c, k=1
This is the base step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(a + -b) = c$ cnf(condition, hypothesis)
$-(a + -(b + 1 \cdot (a + c))) \neq c$ cnf(prove_base_step, negated_conjecture)

ROB012-1.p If -(a + -b) = c then -(a + -(b + k(a + c))) = c, k=k + 1
This is the induction step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')

$-(a+-b)=c$ cnf(condition, hypothesis)
positive_integer(k) cnf(k_an_integer, hypothesis)
$-(a+-(b+k\cdot(a+c)))=c$ cnf(base_step, axiom)
$-(a+-(b+\text{successor}(k)\cdot(a+c)))\neq c$ cnf(prove_induction_step, negated_conjecture)

ROB012-2.p If -(a + -b) = c then -(a + -(b + k(a + c))) = c, k=k + 1
This is the induction step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(x+-y)=z \;\Rightarrow\; -(z+-(y+x))=x$ cnf(lemma_3$_3$, axiom)
$-(a+-b)=c$ cnf(condition, hypothesis)
positive_integer(k) cnf(k_an_integer, hypothesis)
$-(a+-(b+k\cdot(a+c)))=c$ cnf(base_step, axiom)
$-(a+-(b+\text{successor}(k)\cdot(a+c)))\neq c$ cnf(prove_induction_step, negated_conjecture)

ROB013-1.p If -(a + b) = c then -(c + -(-b + a)) = a
include('Axioms/ROB001-0.ax')
$-(a+b)=c$ cnf(condition, hypothesis)
$-(c+-(-b+a))\neq a$ cnf(prove_result, negated_conjecture)

ROB014-1.p If -(-e + -(d + -e)) = d then -(e + k(d + -(d + -e))) = -e, k=1
This is the base step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(-e+-(d+-e))=d$ cnf(condition, hypothesis)
$-(e+1\cdot(d+-(d+-e)))\neq -e$ cnf(prove_base_step, negated_conjecture)

ROB014-2.p If -(-e + -(d + -e)) = d then -(e + k(d + -(d + -e))) = -e, k=1
This is the base step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(x+-(y+z))=-(y+-(x+z)) \;\Rightarrow\; x=y$ cnf(lemma_3$_2$, axiom)
$(-(x+-y)=z \text{ and positive\_integer(vk)}) \;\Rightarrow\; -(x+-(y+\text{vk}\cdot(x+z)))=z$ cnf(lemma_3$_4$, axiom)
$-(-e+-(d+-e))=d$ cnf(condition, hypothesis)
$-(e+1\cdot(d+-(d+-e)))\neq -e$ cnf(prove_base_step, negated_conjecture)

ROB015-1.p If -(-e + -(d + -e)) = d then -(e + k(d + -(d + -e))) = -e
This is the induction step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(-e+-(d+-e))=d$ cnf(condition, hypothesis)
positive_integer(k) cnf(k_positive, axiom)
$-(e+k\cdot(d+-(d+-e)))\neq -e$ cnf(base_step, axiom)
$-(e+\text{successor}(k)\cdot(d+-(d+-e)))\neq -e$ cnf(prove_induction_step, negated_conjecture)

ROB015-2.p If -(-e + -(d + -e)) = d then -(e + k(d + -(d + -e))) = -e
This is the induction step of an induction proof.
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(x+-(y+z))=-(y+-(x+z)) \;\Rightarrow\; x=y$ cnf(lemma_3$_2$, axiom)
$(-(x+-y)=z \text{ and positive\_integer(vk)}) \;\Rightarrow\; -(x+-(y+\text{vk}\cdot(x+z)))=z$ cnf(lemma_3$_4$, axiom)
$-(-e+-(d+-e))=d$ cnf(condition, hypothesis)
positive_integer(k) cnf(k_positive, axiom)
$-(e+k\cdot(d+-(d+-e)))\neq -e$ cnf(base_step, axiom)
$-(e+\text{successor}(k)\cdot(d+-(d+-e)))\neq -e$ cnf(prove_induction_step, negated_conjecture)

ROB016-1.p If -(d + e) = -e then -(e + k(d + -(d + -e))) = -e, for k>0
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(d+e)=-e$ cnf(condition, hypothesis)
positive_integer(k) cnf(k_positive, axiom)
$(-(-y+-(x+-y))=x \text{ and positive\_integer(vk)}) \;\Rightarrow\; -(y+\text{vk}\cdot(x+-(x+-y)))=-y$ cnf(lemma_3$_6$, axiom)
$-(e+k\cdot(d+-(d+-e)))\neq -e$ cnf(prove_result, negated_conjecture)

ROB017-1.p If -(2f + h) = -(3f + h) = -h then 2f + h = 3f + h
That is, 2f+h absorbs f.
include('Axioms/ROB001-0.ax')
$-(f+(f+h)) = -h$ cnf(condition$_1$, hypothesis)
$-(f+(f+(f+h))) = -h$ cnf(condition$_2$, hypothesis)
$-(x+-y) = -y \Rightarrow -(y+(x+-(x+-y))) = -y$ cnf(lemma_3$_7$, axiom)
$f+(f+(f+h)) \neq f+(f+h)$ cnf(prove_result, negated_conjecture)

ROB018-1.p If -(d + e) = -e then e + 2(d + -(d + -e)) absorbs d + -(d + -e)
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')
$-(d+-e) = -e$ cnf(condition, hypothesis)
$e+\text{successor}(\text{successor}(1))\cdot(d+-(d+-e)) \neq e+\text{successor}(1)\cdot(d+-(d+-e))$ cnf(prove_result, negated_conjecture)

ROB020-1.p -(a + -b)=b => Boolean
If there exist a, b such that -(a + -b) = b, the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(a+-b) = b$ cnf(condition$_1$, hypothesis)
$-(a+-b)+-(-a+-b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB020-2.p -(a + -b)=b => Exists idempotent element
If there exist a, b such that -(a + -b) = b, the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(a+-b) = b$ cnf(condition$_1$, hypothesis)
$x+x \neq x$ cnf(prove_idempotence, negated_conjecture)

ROB021-1.p (-X = -Y)=>(X = Y) => Boolean
If (-X = -Y) => (X = Y) then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-x = -y \Rightarrow x = y$ cnf(negative_equality_implies_positive_equality, hypothesis)
$-(a+-b)+-(-a+-b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB022-1.p c + -c=c => Boolean
If there is an element c such that c + -c = c then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c+-c = c$ cnf(condition, hypothesis)
$-(a+-b)+-(-a+-b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB023-1.p X + X=X => Boolean
If for all X X + X = X then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$x+x = x$ cnf(x_plus_x_is_x, hypothesis)
$-(a+-b)+-(-a+-b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB024-1.p -(a + (a + b)) + -(a + -b) = a => Boolean
If there exist a and b so that -(a + (a + b)) + -(a + -b) = a then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(-(a+(a+b))+-(a+-b)) = a$ cnf(the_condition, hypothesis)
$-(a+-b)+-(-a+-b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB025-1.p -(X + Y) = intersection(-X,-Y) => Boolean
If for all X and Y, -(X + Y) = intersection(-X,-Y) then the algebra is Boolean.
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y \Rightarrow y{=}x$ cnf(symmetry, axiom)
$(x{=}y \text{ and } y{=}z) \Rightarrow x{=}z$ cnf(transitivity, axiom)
$a{=}b \Rightarrow a+c{=}b+c$ cnf(add_substitution$_1$, axiom)
$d{=}e \Rightarrow f+d{=}f+e$ cnf(add_substitution$_2$, axiom)
$g{=}h \Rightarrow -g{=}-h$ cnf(inverse_substitution$_1$, axiom)
$x+y{=}y+x$ cnf(commutativity_of_add, axiom)
$(x+y)+z{=}x+(y+z)$ cnf(associativity_of_add, axiom)
$-(-(x+y)+-(x+-y)){=}x$ cnf(robbins_axiom, axiom)
$-(x+y){=}\text{intersect}(-x,-y)$ cnf(the_condition, hypothesis)
$\neg -(a+-b)+-(-a+-b){=}b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB026-1.p c + d = c => Boolean

If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + d = c$ cnf(identity_constant, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB027-1.p -(-c) = c => Boolean
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(-c) = c$ cnf(double_negation, hypothesis)
$-(a + -b) + -(-a + -b) \neq b$ cnf(prove_huntingtons_axiom, negated_conjecture)

ROB028-1.p Robbins algebra axioms
include('Axioms/ROB001-0.ax')

ROB029-1.p Robbins algebra numbers axioms
include('Axioms/ROB001-0.ax')
include('Axioms/ROB001-1.ax')

ROB030-1.p Exists absorbed element => Exists absorbed within negation element
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + d = d$ cnf(absorbtion, hypothesis)
$-(a + b) \neq -b$ cnf(prove_absorption_within_negation, negated_conjecture)

ROB031-1.p Robbins => Exists absorbed within negation element
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$-(a + b) \neq -b$ cnf(prove_absorption_within_negation, negated_conjecture)

ROB031-2.p Robbins => Exists absorbed within negation element
include('Axioms/ROB001-0.ax')
$g(a) = -(a + -a)$ cnf(sos_{04}, axiom)
$h(a) = a + (a + (a + g(a)))$ cnf(sos_{05}, axiom)
$-(x_6 + x_7) \neq -x_6$ cnf(goals, negated_conjecture)

ROB032-1.p Robbins => Exists absorbed element
If there are elements c and d such that c+d=d, then the algebra is Boolean.
include('Axioms/ROB001-0.ax')
$c + d \neq d$ cnf(prove_absorbtion, negated_conjecture)

ROB032-2.p Robbins => Exists absorbed element, with auxilliary definitions
include('Axioms/ROB001-0.ax')
$g(a) = -(a + -a)$ cnf(sos_{04}, axiom)
$h(a) = a + (a + (a + g(a)))$ cnf(sos_{05}, axiom)
$x_4 + x_5 \neq x_5$ cnf(goals, negated_conjecture)

ROB033-1.p Robbins problem with auxilliary definitions
include('Axioms/ROB001-0.ax')
$g(a) = -(a + -a)$ cnf(sos_{04}, axiom)
$h(a) = a + (a + (a + g(a)))$ cnf(sos_{05}, axiom)
$-(x_0 + -x_1) + -(-x_0 + -x_1) \neq x_1$ cnf(goals, negated_conjecture)

ROB034-1.p Robbins => Exists absorbed element, with auxilliary definitions
include('Axioms/ROB001-0.ax')
$g(a) = -(a + -a)$ cnf(sos_{04}, axiom)
$h(a) = a + (a + (a + g(a)))$ cnf(sos_{05}, axiom)
$x_2 + x_3 \neq x_2$ cnf(goals, negated_conjecture)

ROB035-1.p Robbins => Exists absorbed within negation element
include('Axioms/ROB001-0.ax')
$g(a) = -(a + -a)$ cnf(sos_{04}, axiom)
$h(a) = a + (a + (a + g(a)))$ cnf(sos_{05}, axiom)
$-(x_8 + x_9) \neq -x_9$ cnf(goals, negated_conjecture)