# SET axioms

**SET001-0.ax** Membership and subsets
(element $\in$ subset and subset $\subseteq$ superset) $\Rightarrow$ element $\in$ superset  cnf(membership_in_subsets, axiom)
subset $\subseteq$ superset or member_of_1_not_of$_2$(subset, superset) $\in$ subset  cnf(subsets_axiom$_1$, axiom)
member_of_1_not_of$_2$(subset, superset) $\in$ superset $\Rightarrow$ subset $\subseteq$ superset  cnf(subsets_axiom$_2$, axiom)
equal_sets(subset, superset) $\Rightarrow$ subset $\subseteq$ superset  cnf(set_equal_sets_are_subsets$_1$, axiom)
equal_sets(superset, subset) $\Rightarrow$ subset $\subseteq$ superset  cnf(set_equal_sets_are_subsets$_2$, axiom)
(set$_1$ $\subseteq$ set$_2$ and set$_2$ $\subseteq$ set$_1$) $\Rightarrow$ equal_sets(set$_2$, set$_1$)  cnf(subsets_are_set_equal_sets, axiom)

**SET001-1.ax** Membership and union
(union(set$_1$, set$_2$, union) and element $\in$ union) $\Rightarrow$ (element $\in$ set$_1$ or element $\in$ set$_2$)  cnf(member_of_union_is_member_of_
(union(set$_1$, set$_2$, union) and element $\in$ set$_1$) $\Rightarrow$ element $\in$ union  cnf(member_of_set1_is_member_of_union, axiom)
(union(set$_1$, set$_2$, union) and element $\in$ set$_2$) $\Rightarrow$ element $\in$ union  cnf(member_of_set2_is_member_of_union, axiom)
union(set$_1$, set$_2$, union) or $g$(set$_1$, set$_2$, union) $\in$ set$_1$ or $g$(set$_1$, set$_2$, union) $\in$ set$_2$ or $g$(set$_1$, set$_2$, union) $\in$ union  cnf(union_
($g$(set$_1$, set$_2$, union) $\in$ set$_1$ and $g$(set$_1$, set$_2$, union) $\in$ union) $\Rightarrow$ union(set$_1$, set$_2$, union)  cnf(union_axiom$_2$, axiom)
($g$(set$_1$, set$_2$, union) $\in$ set$_2$ and $g$(set$_1$, set$_2$, union) $\in$ union) $\Rightarrow$ union(set$_1$, set$_2$, union)  cnf(union_axiom$_3$, axiom)

**SET001-2.ax** Membership and intersection
(intersection(set$_1$, set$_2$, intersection) and element $\in$ intersection) $\Rightarrow$ element $\in$ set$_1$  cnf(member_of_intersection_is_membe
(intersection(set$_1$, set$_2$, intersection) and element $\in$ intersection) $\Rightarrow$ element $\in$ set$_2$  cnf(member_of_intersection_is_membe
(intersection(set$_1$, set$_2$, intersection) and element $\in$ set$_2$ and element $\in$ set$_1$) $\Rightarrow$ element $\in$ intersection  cnf(member_of_bo
$h$(set$_1$, set$_2$, intersection) $\in$ intersection or intersection(set$_1$, set$_2$, intersection) or $h$(set$_1$, set$_2$, intersection) $\in$ set$_1$  cnf(inter
$h$(set$_1$, set$_2$, intersection) $\in$ intersection or intersection(set$_1$, set$_2$, intersection) or $h$(set$_1$, set$_2$, intersection) $\in$ set$_2$  cnf(inter
($h$(set$_1$, set$_2$, intersection) $\in$ intersection and $h$(set$_1$, set$_2$, intersection) $\in$ set$_2$ and $h$(set$_1$, set$_2$, intersection) $\in$ set$_1$) $\Rightarrow$ intersection(set$_1$, set$_2$, intersection)  cnf(intersection_axiom$_3$, axiom)

**SET001-3.ax** Membership and difference
(set$_1$ $\setminus$ set$_2$=difference and element $\in$ difference) $\Rightarrow$ element $\in$ set$_1$  cnf(member_of_difference, axiom)
(element $\in$ set$_1$ and element $\in$ set$_2$) $\Rightarrow$ $\neg$ a_set $\setminus$ set$_1$=set$_2$  cnf(not_member_of_difference, axiom)
(element $\in$ set$_1$ and set$_1$$\setminus$set$_2$=difference) $\Rightarrow$ (element $\in$ difference or element $\in$ set$_2$)  cnf(member_of_difference_or_set$_2$, a
set$_1$$\setminus$set$_2$=difference or $k$(set$_1$, set$_2$, difference) $\in$ set$_1$ or $k$(set$_1$, set$_2$, difference) $\in$ difference  cnf(difference_axiom$_2$, axiom)
$k$(set$_1$, set$_2$, difference) $\in$ set$_2$ $\Rightarrow$ ($k$(set$_1$, set$_2$, difference) $\in$ difference or set$_1$$\setminus$set$_2$=difference)  cnf(difference_axiom$_1$, axio
($k$(set$_1$, set$_2$, difference) $\in$ difference and $k$(set$_1$, set$_2$, difference) $\in$ set$_1$) $\Rightarrow$ ($k$(set$_1$, set$_2$, difference) $\in$ set$_2$ or set$_1$ $\setminus$ set$_2$=difference)  cnf(difference_axiom$_3$, axiom)

**SET002-0.ax** Set theory axioms
$\neg\, x \in$ empty_set  cnf(empty_set, axiom)
(element $\in$ subset and subset $\subseteq$ superset) $\Rightarrow$ element $\in$ superset  cnf(membership_in_subsets, axiom)
subset $\subseteq$ superset or member_of_1_not_of$_2$(subset, superset) $\in$ subset  cnf(subsets_axiom$_1$, axiom)
member_of_1_not_of$_2$(subset, superset) $\in$ superset $\Rightarrow$ subset $\subseteq$ superset  cnf(subsets_axiom$_2$, axiom)
$x \in$ xs or $x \in$ xs$'$  cnf(member_of_set_or_complement, axiom)
$x \in$ xs $\Rightarrow$ $\neg\, x \in$ xs$'$  cnf(not_member_of_set_and_complement, axiom)
$x \in$ xs $\Rightarrow$ $x \in$ union(xs, ys)  cnf(member_of_set1_is_member_of_union, axiom)
$x \in$ ys $\Rightarrow$ $x \in$ union(xs, ys)  cnf(member_of_set2_is_member_of_union, axiom)
$x \in$ union(xs, ys) $\Rightarrow$ ($x \in$ xs or $x \in$ ys)  cnf(member_of_union_is_member_of_one_set, axiom)
($x \in$ xs and $x \in$ ys) $\Rightarrow$ $x \in$ intersection(xs, ys)  cnf(member_of_both_is_member_of_intersection, axiom)
$x \in$ intersection(xs, ys) $\Rightarrow$ $x \in$ xs  cnf(member_of_intersection_is_member_of_set$_1$, axiom)
$x \in$ intersection(xs, ys) $\Rightarrow$ $x \in$ ys  cnf(member_of_intersection_is_member_of_set$_2$, axiom)
equal_sets(subset, superset) $\Rightarrow$ subset $\subseteq$ superset  cnf(set_equal_sets_are_subsets$_1$, axiom)
equal_sets(superset, subset) $\Rightarrow$ subset $\subseteq$ superset  cnf(set_equal_sets_are_subsets$_2$, axiom)
(set$_1$ $\subseteq$ set$_2$ and set$_2$ $\subseteq$ set$_1$) $\Rightarrow$ equal_sets(set$_2$, set$_1$)  cnf(subsets_are_set_equal_sets, axiom)
equal_sets(xs, xs)  cnf(reflexivity_for_set_equal, axiom)
equal_sets(xs, ys) $\Rightarrow$ equal_sets(ys, xs)  cnf(symmetry_for_set_equal, axiom)
(equal_sets(xs, ys) and equal_sets(ys, zs)) $\Rightarrow$ equal_sets(xs, zs)  cnf(transitivity_for_set_equal, axiom)
equal_elements($x$, $x$)  cnf(reflexivity_for_equal_elements, axiom)
equal_elements($x$, $y$) $\Rightarrow$ equal_elements($y$, $x$)  cnf(symmetry_for_equal_elements, axiom)
(equal_elements($x$, $y$) and equal_elements($y$, $z$)) $\Rightarrow$ equal_elements($x$, $z$)  cnf(transitivity_for_equal_elements, axiom)

**SET004-1.ax** Set theory (Boolean algebra) axioms based on NBG set theory
subclass(compose_class($x$), cross_product(universal_class, universal_class))  cnf(compose_class_definition$_1$, axiom)
ordered_pair($y$, $z$) $\in$ compose_class($x$) $\Rightarrow$ $x \circ y = z$  cnf(compose_class_definition$_2$, axiom)

$(\text{ordered\_pair}(y, z) \in \text{cross\_product}(\text{universal\_class}, \text{universal\_class}) \text{ and } x \circ y = z) \Rightarrow \text{ordered\_pair}(y, z) \in \text{compose\_class}(x)$
$\text{subclass}(\text{composition\_function}, \text{cross\_product}(\text{universal\_class}, \text{cross\_product}(\text{universal\_class}, \text{universal\_class})))$ cnf(definitio
$\text{ordered\_pair}(x, \text{ordered\_pair}(y, z)) \in \text{composition\_function} \Rightarrow x \circ y = z$ cnf(definition_of_composition_function$_2$, axiom)
$\text{ordered\_pair}(x, y) \in \text{cross\_product}(\text{universal\_class}, \text{universal\_class}) \Rightarrow \text{ordered\_pair}(x, \text{ordered\_pair}(y, x \circ y)) \in \text{composition\_function}$ cnf(definition_of_composition_function$_3$, axiom)
$\text{subclass}(\text{domain\_relation}, \text{cross\_product}(\text{universal\_class}, \text{universal\_class}))$ cnf(definition_of_domain_relation$_1$, axiom)
$\text{ordered\_pair}(x, y) \in \text{domain\_relation} \Rightarrow \text{domain\_of}(x) = y$ cnf(definition_of_domain_relation$_2$, axiom)
$x \in \text{universal\_class} \Rightarrow \text{ordered\_pair}(x, \text{domain\_of}(x)) \in \text{domain\_relation}$ cnf(definition_of_domain_relation$_3$, axiom)
$\text{first}(\text{not\_subclass\_element}(x \circ x', \text{identity\_relation})) = \text{single\_valued}_1(x)$ cnf(single_valued_term_defn$_1$, axiom)
$\text{second}(\text{not\_subclass\_element}(x \circ x', \text{identity\_relation})) = \text{single\_valued}_2(x)$ cnf(single_valued_term_defn$_2$, axiom)
$\text{dom}(x) = \text{single\_valued}_3(x)$ cnf(single_valued_term_defn$_3$, axiom)
$\text{intersection}((\text{element\_relation} \circ \text{identity\_relation}')', \text{element\_relation}) = \text{singleton\_relation}$ cnf(compose_can_define_singleto
$\text{subclass}(\text{application\_function}, \text{cross\_product}(\text{universal\_class}, \text{cross\_product}(\text{universal\_class}, \text{universal\_class})))$ cnf(applicatio
$\text{ordered\_pair}(x, \text{ordered\_pair}(y, z)) \in \text{application\_function} \Rightarrow y \in \text{domain\_of}(x)$ cnf(application_function_defn$_2$, axiom)
$\text{ordered\_pair}(x, \text{ordered\_pair}(y, z)) \in \text{application\_function} \Rightarrow \text{apply}(x, y) = z$ cnf(application_function_defn$_3$, axiom)
$(\text{ordered\_pair}(x, \text{ordered\_pair}(y, z)) \in \text{cross\_product}(\text{universal\_class}, \text{cross\_product}(\text{universal\_class}, \text{universal\_class})) \text{ and } y \in \text{domain\_of}(x)) \Rightarrow \text{ordered\_pair}(x, \text{ordered\_pair}(y, \text{apply}(x, y))) \in \text{application\_function}$ cnf(application_function_defn$_4$, axio
$\text{maps}(\text{xf}, x, y) \Rightarrow \text{function}(\text{xf})$ cnf(maps$_1$, axiom)
$\text{maps}(\text{xf}, x, y) \Rightarrow \text{domain\_of}(\text{xf}) = x$ cnf(maps$_2$, axiom)
$\text{maps}(\text{xf}, x, y) \Rightarrow \text{subclass}(\text{range\_of}(\text{xf}), y)$ cnf(maps$_3$, axiom)
$(\text{function}(\text{xf}) \text{ and } \text{subclass}(\text{range\_of}(\text{xf}), y)) \Rightarrow \text{maps}(\text{xf}, \text{domain\_of}(\text{xf}), y)$ cnf(maps$_4$, axiom)

**SET006+0.ax** Naive set theory based on Goedel's set theory
$\forall a, b\colon (a \subseteq b \iff \forall x\colon (x \in a \Rightarrow x \in b))$ fof(subset, axiom)
$\forall a, b\colon (\text{equal\_set}(a, b) \iff (a \subseteq b \text{ and } b \subseteq a))$ fof(equal_set, axiom)
$\forall x, a\colon (x \in \text{power\_set}(a) \iff x \subseteq a)$ fof(power_set, axiom)
$\forall x, a, b\colon (x \in \text{intersection}(a, b) \iff (x \in a \text{ and } x \in b))$ fof(intersection, axiom)
$\forall x, a, b\colon (x \in \text{union}(a, b) \iff (x \in a \text{ or } x \in b))$ fof(union, axiom)
$\forall x\colon \neg\, x \in \text{empty\_set}$ fof(empty_set, axiom)
$\forall b, a, e\colon (b \in (e \setminus a) \iff (b \in e \text{ and } \neg\, b \in a))$ fof(difference, axiom)
$\forall x, a\colon (x \in \text{singleton}(a) \iff x = a)$ fof(singleton, axiom)
$\forall x, a, b\colon (x \in \text{unordered\_pair}(a, b) \iff (x = a \text{ or } x = b))$ fof(unordered_pair, axiom)
$\forall x, a\colon (x \in \text{sum}(a) \iff \exists y\colon (y \in a \text{ and } x \in y))$ fof(sum, axiom)
$\forall x, a\colon (x \in \text{product}(a) \iff \forall y\colon (y \in a \Rightarrow x \in y))$ fof(product, axiom)

**SET006+2.ax** Equivalence relation axioms for the SET006+0 set theory axioms
$\forall a, b\colon (\text{disjoint}(a, b) \iff \neg \exists x\colon (x \in a \text{ and } x \in b))$ fof(disjoint, axiom)
$\forall a, e\colon (\text{partition}(a, e) \iff (\forall x\colon (x \in a \Rightarrow x \subseteq e) \text{ and } \forall x\colon (x \in e \Rightarrow \exists y\colon (y \in a \text{ and } x \in y)) \text{ and } \forall x, y\colon ((x \in a \text{ and } y \in a) \Rightarrow (\exists z\colon (z \in x \text{ and } z \in y) \Rightarrow x = y))))$ fof(partition, axiom)
$\forall a, r\colon (\text{equivalence}(r, a) \iff (\forall x\colon (x \in a \Rightarrow \text{apply}(r, x, x)) \text{ and } \forall x, y\colon ((x \in a \text{ and } y \in a) \Rightarrow (\text{apply}(r, x, y) \Rightarrow \text{apply}(r, y, x))) \text{ and } \forall x, y, z\colon ((x \in a \text{ and } y \in a \text{ and } z \in a) \Rightarrow ((\text{apply}(r, x, y) \text{ and } \text{apply}(r, y, z)) \Rightarrow \text{apply}(r, x, z)))))$ fof(e
$\forall r, e, a, x\colon (x \in \text{equivalence\_class}(a, e, r) \iff (x \in e \text{ and } \text{apply}(r, a, x)))$ fof(equivalence_class, axiom)
$\forall r, e\colon (\text{pre\_order}(r, e) \iff (\forall x\colon (x \in e \Rightarrow \text{apply}(r, x, x)) \text{ and } \forall x, y, z\colon ((x \in e \text{ and } y \in e \text{ and } z \in e) \Rightarrow ((\text{apply}(r, x, y) \text{ and } \text{apply}(r, y, z)) \Rightarrow \text{apply}(r, x, z)))))$ fof(pre_order, axiom)

**SET006+3.ax** Order relation (Naive set theory)
$\forall r, e\colon (\text{order}(r, e) \iff (\forall x\colon (x \in e \Rightarrow \text{apply}(r, x, x)) \text{ and } \forall x, y\colon ((x \in e \text{ and } y \in e) \Rightarrow ((\text{apply}(r, x, y) \text{ and } \text{apply}(r, y, x)) \Rightarrow x = y)) \text{ and } \forall x, y, z\colon ((x \in e \text{ and } y \in e \text{ and } z \in e) \Rightarrow ((\text{apply}(r, x, y) \text{ and } \text{apply}(r, y, z)) \Rightarrow \text{apply}(r, x, z)))))$ fof(order, ax
$\forall r, e\colon (\text{total\_order}(r, e) \iff (\text{order}(r, e) \text{ and } \forall x, y\colon ((x \in e \text{ and } y \in e) \Rightarrow (\text{apply}(r, x, y) \text{ or } \text{apply}(r, y, x)))))$ fof(total_ord
$\forall r, e, m\colon (b \iff \forall x\colon (x \in e \Rightarrow \text{apply}(r, x, m)))$ fof(upper_bound, axiom)
$\forall r, e, m\colon (a \iff \forall x\colon (x \in e \Rightarrow \text{apply}(r, m, x)))$ fof(lower_bound, axiom)
$\forall r, e, m\colon (\text{greatest}(m, r, e) \iff (m \in e \text{ and } \forall x\colon (x \in e \Rightarrow \text{apply}(r, x, m))))$ fof(greatest, axiom)
$\forall r, e, m\colon (\text{least}(m, r, e) \iff (m \in e \text{ and } \forall x\colon (x \in e \Rightarrow \text{apply}(r, m, x))))$ fof(least, axiom)
$\forall r, e, m\colon (\max(m, r, e) \iff (m \in e \text{ and } \forall x\colon ((x \in e \text{ and } \text{apply}(r, m, x)) \Rightarrow m = x)))$ fof(max, axiom)
$\forall r, e, m\colon (\min(m, r, e) \iff (m \in e \text{ and } \forall x\colon ((x \in e \text{ and } \text{apply}(r, x, m)) \Rightarrow m = x)))$ fof(min, axiom)
$\forall a, x, r, e\colon (\text{least\_upper\_bound}(a, x, r, e) \iff (a \in x \text{ and } b \text{ and } \forall m\colon ((m \in e \text{ and } b) \Rightarrow \text{apply}(r, a, m))))$ fof(least_upper_b
$\forall a, x, r, e\colon (\text{greatest\_lower\_bound}(a, x, r, e) \iff (a \in x \text{ and } a \text{ and } \forall m\colon ((m \in e \text{ and } a) \Rightarrow \text{apply}(r, m, a))))$ fof(greatest_lo

**SET006+4.ax** Ordinal numbers for the SET006+0 set theory axioms
$\forall a\colon (a \in \text{on} \iff (\text{set}(a) \text{ and } \text{strict\_well\_order}(\text{member\_predicate}, a) \text{ and } \forall x\colon (x \in a \Rightarrow x \subseteq a)))$ fof(ordinal_number, axio
$\forall r, e\colon (\text{strict\_well\_order}(r, e) \iff (\text{strict\_order}(r, e) \text{ and } \forall a\colon ((a \subseteq e \text{ and } \exists x\colon x \in a) \Rightarrow \exists y\colon \text{least}(y, r, a))))$ fof(strict_well_

$\forall r, e, m\colon (\text{least}(m, r, e) \iff (m \in e \text{ and } \forall x\colon (x \in e \Rightarrow (m = x \text{ or apply}(r, m, x)))))$ fof(least, axiom)
$\forall x, y\colon (\text{apply}(\text{member\_predicate}, x, y) \iff x \in y)$ fof(rel_member, axiom)
$\forall r, e\colon (\text{strict\_order}(r, e) \iff (\forall x, y\colon ((x \in e \text{ and } y \in e) \Rightarrow \neg\,\text{apply}(r, x, y) \text{ and apply}(r, y, x)) \text{ and } \forall x, y, z\colon ((x \in e \text{ and } y \in e \text{ and } z \in e) \Rightarrow ((\text{apply}(r, x, y) \text{ and apply}(r, y, z)) \Rightarrow \text{apply}(r, x, z)))))$ fof(strict_order, axiom)
$\forall x\colon (\text{set}(x) \Rightarrow \forall y\colon (y \in x \Rightarrow \text{set}(y)))$ fof(set_member, axiom)
$\forall x, r, a, y\colon (y \in \text{initial\_segment}(x, r, a) \iff (y \in a \text{ and apply}(r, y, x)))$ fof(initial_segment, axiom)
$\forall a, x\colon (x \in \text{suc}(a) \iff x \in \text{union}(a, \text{singleton}(a)))$ fof(successor, axiom)

**SET008^0.ax** Basic set theory definitions
in: $i → ($i → $o) → $o thf(in_decl, type)
in = ($\lambda x$: $i, $m$: $i → $o: ($m$@$x$)) thf(in, definition)
is_a: $i → ($i → $o) → $o thf(is_a_decl, type)
is_a = ($\lambda x$: $i, $m$: $i → $o: ($m$@$x$)) thf(is_a, definition)
emptyset: $i → $o thf(emptyset_decl, type)
emptyset = ($\lambda x$: $i: $false) thf(emptyset, definition)
unord_pair: $i → $i → $i → $o thf(unord_pair_decl, type)
unord_pair = ($\lambda x$: $i, $y$: $i, $u$: $i: ($u = x$ or $u = y$)) thf(unord_pair, definition)
singleton: $i → $i → $o thf(singleton_decl, type)
singleton = ($\lambda x$: $i, $u$: $i: $u = x$) thf(singleton, definition)
union: ($i → $o) → ($i → $o) → $i → $o thf(union_decl, type)
union = ($\lambda x$: $i → $o, $y$: $i → $o, $u$: $i: ($x$@$u$ or $y$@$u$)) thf(union, definition)
excl_union: ($i → $o) → ($i → $o) → $i → $o thf(excl_union_decl, type)
excl_union = ($\lambda x$: $i → $o, $y$: $i → $o, $u$: $i: (($x$@$u$ and ¬ $y$@$u$) or (¬ $x$@$u$ and $y$@$u$))) thf(excl_union, definition)
intersection: ($i → $o) → ($i → $o) → $i → $o thf(intersection_decl, type)
intersection = ($\lambda x$: $i → $o, $y$: $i → $o, $u$: $i: ($x$@$u$ and $y$@$u$)) thf(intersection, definition)
setminus: ($i → $o) → ($i → $o) → $i → $o thf(setminus_decl, type)
setminus = ($\lambda x$: $i → $o, $y$: $i → $o, $u$: $i: ($x$@$u$ and ¬ $y$@$u$)) thf(setminus, definition)
complement: ($i → $o) → $i → $o thf(complement_decl, type)
complement = ($\lambda x$: $i → $o, $u$: $i: ¬ $x$@$u$) thf(complement, definition)
disjoint: ($i → $o) → ($i → $o) → $o thf(disjoint_decl, type)
disjoint = ($\lambda x$: $i → $o, $y$: $i → $o: (intersection@$x$@$y$) = emptyset) thf(disjoint, definition)
⊆ : ($i → $o) → ($i → $o) → $o thf(subset_decl, type)
⊆= ($\lambda x$: $i → $o, $y$: $i → $o: ∀$u$: $i: (($x$@$u$) ⇒ ($y$@$u$))) thf(subset, definition)
meets: ($i → $o) → ($i → $o) → $o thf(meets_decl, type)
meets = ($\lambda x$: $i → $o, $y$: $i → $o: ∃$u$: $i: ($x$@$u$ and $y$@$u$)) thf(meets, definition)
misses: ($i → $o) → ($i → $o) → $o thf(misses_decl, type)
misses = ($\lambda x$: $i → $o, $y$: $i → $o: ¬ ∃$u$: $i: ($x$@$u$ and $y$@$u$)) thf(misses, definition)

**SET008^1.ax** Definitions for functions
fun_image: ($i → $i) → ($i → $o) → $i → $o thf(fun_image_decl, type)
fun_image = ($\lambda f$: $i → $i, $a$: $i → $o, $y$: $i: ∃$x$: $i: ($a$@$x$ and $y$ = ($f$@$x$))) thf(fun_image, definition)
fun_composition: ($i → $i) → ($i → $i) → $i → $i thf(fun_composition_decl, type)
fun_composition = ($\lambda f$: $i → $i, $g$: $i → $i, $x$: $i: ($g$@($f$@$x$))) thf(fun_composition, definition)
fun_inv_image: ($i → $i) → ($i → $o) → $i → $o thf(fun_inv_image_decl, type)
fun_inv_image = ($\lambda f$: $i → $i, $b$: $i → $o, $x$: $i: ∃$y$: $i: ($b$@$y$ and $y$ = ($f$@$x$))) thf(fun_inv_image, definition)
fun_injective: ($i → $i) → $o thf(fun_injective_decl, type)
fun_injective = ($\lambda f$: $i → $i: ∀$x$: $i, $y$: $i: (($f$@$x$) = ($f$@$y$) ⇒ $x = y$)) thf(fun_injective, definition)
fun_surjective: ($i → $i) → $o thf(fun_surjective_decl, type)
fun_surjective = ($\lambda f$: $i → $i: ∀$y$: $i: ∃$x$: $i: $y$ = ($f$@$x$)) thf(fun_surjective, definition)
fun_bijective: ($i → $i) → $o thf(fun_bijective_decl, type)
fun_bijective = ($\lambda f$: $i → $i: (fun_injective@$f$ and fun_surjective@$f$)) thf(fun_bijective, definition)
fun_decreasing: ($i → $i) → ($i → $i → $o) → $o thf(fun_decreasing_decl, type)
fun_decreasing = ($\lambda f$: $i → $i, sMALLER: $i → $i → $o: ∀$x$: $i, $y$: $i: ((sMALLER@$x$@$y$) ⇒ (sMALLER@($f$@$y$)@($f$@$x$))))
fun_increasing: ($i → $i) → ($i → $i → $o) → $o thf(fun_increasing_decl, type)
fun_increasing = ($\lambda f$: $i → $i, sMALLER: $i → $i → $o: ∀$x$: $i, $y$: $i: ((sMALLER@$x$@$y$) ⇒ (sMALLER@($f$@$x$)@($f$@$y$))))

# SET problems

**SET001-1.p** Set members are superset members
A member of a set is also a member of that set's supersets.

include('Axioms/SET001-0.ax')
equal_sets($b$, bb)     cnf(b_equals_bb, hypothesis)
element_of_b $\in b$     cnf(element_of_b, hypothesis)
$\neg$ element_of_b $\in$ bb     cnf(prove_element_of_bb, negated_conjecture)

**SET002+3.p** Idempotency of union
$\forall b, c\colon (b \subseteq c \Rightarrow \text{union}(b, c) = c)$     fof(subset_union, axiom)
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$     fof(union_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$     fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$     fof(commutativity_of_union, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$     fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$     fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$     fof(equal_member_defn, axiom)
$\forall b\colon \text{union}(b, b) = b$     fof(prove_idempotency_of_union, conjecture)

**SET002+4.p** Idempotency of union
include('Axioms/SET006+0.ax')
$\forall a\colon$ equal_set(union($a, a$), $a$)     fof($\text{thI}_{14}$, conjecture)

**SET002-1.p** Idempotency of union
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')
union($a, a$, aUa)     cnf(a_union_a_is_aUa, hypothesis)
$\neg$ equal_sets(aUa, $a$)     cnf(prove_a_equals_aUa, negated_conjecture)

**SET002-6.p** Idempotency of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, x) \neq x$     cnf($\text{prove\_idempotency\_of\_union}_1$, negated_conjecture)

**SET003-1.p** A set is a subset of the union of itself with itself
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')
union($a, a$, aUa)     cnf(a_union_a_is_aUa, hypothesis)
$\neg\, a \subseteq$ aUa     cnf(prove_a_is_a_subset_of_aUa, negated_conjecture)

**SET004-1.p** A set is a subset of the union of itself and another set
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')
union($a, b$, aUb)     cnf(a_union_b_is_aUb, hypothesis)
$\neg\, a \subseteq$ aUb     cnf(prove_a_is_a_subset_of_aUb, negated_conjecture)

**SET005-1.p** Associativity of set intersection
include('Axioms/SET001-0.ax')
include('Axioms/SET001-2.ax')
intersection($a, b$, aIb)     cnf(a_intersection_b, axiom)
intersection($b, c$, bIc)     cnf(b_intersection_c, axiom)
intersection($a$, bIc, aIbIc)     cnf(a_intersection_bIc, axiom)
$\neg$ intersection(aIb, $c$, aIbIc)     cnf(prove_aIb_intersection_c_is_aIbIc, negated_conjecture)

**SET006-1.p** A = A ∧ B if A (= B
If the intersection of two sets is the first of the two sets, then the first is a subset of the second.
include('Axioms/SET001-0.ax')
include('Axioms/SET001-2.ax')
intersection($d, a, d$)     cnf(d_intersection_a_is_d, hypothesis)
$\neg\, d \subseteq a$     cnf(prove_d_is_a_subset_of_a, negated_conjecture)

**SET007-1.p** Intersection distributes over union
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')
include('Axioms/SET001-2.ax')
union($b, c$, bUc)     cnf(b_union_c, axiom)
intersection($a, b$, aIb)     cnf(a_intersection_b, axiom)
intersection($a, c$, aIc)     cnf(a_intersection_c, axiom)
intersection($a$, bUc, aI_bUc)     cnf(a_intersection_bUc, axiom)

$\neg$ union(aIb, aIc, aI_bUc)    cnf(prove_aIb_union_aIc_is_aI_bUc, negated_conjecture)

**SET008+3.p** (X Y) $\wedge$ Y = the empty set
The intersection of (the difference of X and Y) and Y is the empty set.
$\forall b, c, d$: ($d \in$ intersection$(b, c) \iff (d \in b$ and $d \in c))$    fof(intersection_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$    fof(difference_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set    fof(empty_set_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$    fof(equal_defn, axiom)
$\forall b, c$: intersection$(b, c)$ = intersection$(c, b)$    fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b$: (empty$(b) \iff \forall c$: $\neg\, c \in b)$    fof(empty_defn, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$    fof(equal_member_defn, axiom)
$\forall b, c$: intersection$(b \setminus c, c)$ = empty_set    fof(prove_intersection_difference_empty_set, conjecture)

**SET008-1.p** (X Y) $\wedge$ Y = the empty set
The difference of two sets contains no members of the subtracted set.
include('Axioms/SET001-0.ax')
include('Axioms/SET001-2.ax')
include('Axioms/SET001-3.ax')
$b \setminus a$=bDa    cnf(b_minus_a, hypothesis)
$\neg$ intersection($a$, bDa, aI_bDa)    cnf(a_intersection_bDa, negated_conjecture)
$\neg\, a \in$ aI_bDa    cnf(prove_aI_bDa_is_empty, negated_conjecture)

**SET008^5.p** TPS problem BOOL-PROP-78
Trybulec's 78th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: ($\lambda$xx: $a$: ($x$@xx and $\neg\, y$@xx and $y$@xx)) = ($\lambda$xx: $a$: \$false)    thf(cBOOL_PROP_78_pme, conjecture)

**SET009+3.p** If X is a subset of Y, then Z Y is a subset of Z X
If X is a subset of Y, then the difference of Z and Y is a subset of the difference of Z and X.
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$    fof(difference_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: $(b \subseteq c \Rightarrow (d \setminus c) \subseteq (d \setminus b))$    fof(prove_subset_difference, conjecture)

**SET009-1.p** If X is a subset of Y, then Z Y is a subset of Z X
include('Axioms/SET001-0.ax')
include('Axioms/SET001-3.ax')
$d \subseteq a$    cnf(d_is_a_subset_of_a, hypothesis)
$b \setminus a$=bDa    cnf(b_minus_a, hypothesis)
$b \setminus d$=bDd    cnf(b_minus_d, hypothesis)
$\neg$ bDa $\subseteq$ bDd    cnf(prove_bDa_is_a_subset_of_bDd, negated_conjecture)

**SET009^5.p** TPS problem BOOL-PROP-47
Trybulec's 47th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($y$@xx)) $\Rightarrow$ $\forall$xx: $a$: (($z$@xx and $\neg\, y$@xx) $\Rightarrow$ ($z$@xx and $\neg\, x$@xx)))    thf(cBOOL_PROP_47_pme, conjecture)

**SET010+3.p** X Y $\wedge$ Z = (X Y) U (X Z)
The difference of X and the intersection of Y and Z is the union of (the difference of X and Y) and (the difference of X and Z).
$\forall b, c, d$: ($d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$    fof(union_defn, axiom)
$\forall b, c, d$: ($d \in$ intersection$(b, c) \iff (d \in b$ and $d \in c))$    fof(intersection_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$    fof(difference_defn, axiom)
$\forall b, c, d$: $((b \subseteq c$ and $d \subseteq c) \Rightarrow$ union$(b, d) \subseteq c)$    fof(union_subset, axiom)
$\forall b, c$: intersection$(b, c) \subseteq b$    fof(intersection_is_subset, axiom)
$\forall b, c, d$: $(b \subseteq c \Rightarrow (d \setminus c) \subseteq (d \setminus b))$    fof(subset_difference, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$    fof(equal_defn, axiom)
$\forall b, c$: union$(b, c)$ = union$(c, b)$    fof(commutativity_of_union, axiom)
$\forall b, c$: intersection$(b, c)$ = intersection$(c, b)$    fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$    fof(subset_defn, axiom)

$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d\colon b \setminus \text{intersection}(c, d) = \text{union}(b \setminus c, b \setminus d)$ fof(prove_difference_and_intersection_and_union, conjecture)

**SET010-1.p** X Y ∧ Z = (X Y) U (X Z)
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')
include('Axioms/SET001-2.ax')
include('Axioms/SET001-3.ax')
intersection($a, b$, aIb) cnf(a_intersection_b, hypothesis)
$c \setminus a$=cDa cnf(c_minus_a, hypothesis)
$c \setminus b$=cDb cnf(c_minus_b, hypothesis)
$c \setminus$ aIb=cD_aIb cnf(c_minus_aIb, hypothesis)
¬ union(cDa, cDb, cD_aIb) cnf(prove_cDa_union_cDb_is_cD_aIb, negated_conjecture)

**SET010∧5.p** TPS problem BOOL-PROP-86
Trybulec's 86th Boolean property of sets
$a$: $tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o, z\colon a \to \$o\colon (\lambda xx\colon a\colon (x@xx \text{ and } \neg y@xx \text{ and } z@xx)) = (\lambda xz\colon a\colon ((x@xz \text{ and } \neg y@xz) \text{ or } (x@xz \text{ and } \neg z@$

**SET011+3.p** X (X Y) = X ∧ Y
The difference of X and (the difference of X and Y) is the intersection of X and Y.
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon b \setminus (b \setminus c) = \text{intersection}(b, c)$ fof(prove_difference_difference_intersection, conjecture)

**SET011-1.p** X (X Y) = X ∧ Y
The difference of a first set and the set which is the difference of the first set and a second set, is the intersection of the two sets.
include('Axioms/SET001-0.ax')
include('Axioms/SET001-2.ax')
include('Axioms/SET001-3.ax')
$a \setminus b$=aDb cnf(a_minus_b, hypothesis)
$a \setminus$ aDb=aD_aDb cnf(a_minus_aDb, hypothesis)
¬ intersection($a, b$, aD_aDb) cnf(prove_a_intersection_b_is_aD_aDb, negated_conjecture)

**SET011∧5.p** TPS problem BOOL-PROP-82
Trybulec's 82nd Boolean property of sets
$a$: $tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xx\colon a\colon (x@xx \text{ and } \neg x@xx \text{ and } \neg y@xx)) = (\lambda xx\colon a\colon (x@xx \text{ and } y@xx))$ thf(cBOOL_PROP_82_pr

**SET012+4.p** Complement is an involution
include('Axioms/SET006+0.ax')
$\forall a, e\colon (a \subseteq e \Rightarrow \text{equal\_set}(e \setminus (e \setminus a), a))$ fof($\text{thI}_{23}$, conjecture)

**SET012-1.p** Complement is an involution
include('Axioms/SET002-0.ax')
¬ equal_sets($(a')'$, $a$) cnf(prove_involution, negated_conjecture)

**SET012-2.p** Complement is an involution
include('Axioms/SET002-0.ax')
equal_sets($a'$, $b$) cnf(complement_of_a_is_b, hypothesis)
equal_sets($b'$, $c$) cnf(complement_of_b_is_c, hypothesis)
¬ equal_sets($a$, $c$) cnf(prove_a_equals_c, negated_conjecture)

**SET012-3.p** Complement is an involution
include('Axioms/SET003-0.ax')
as′ = bs cnf(complement_of_a_is_b, hypothesis)
bs′ = cs cnf(complement_of_b_is_c, hypothesis)
as ≠ cs cnf(prove_a_equals_c, negated_conjecture)

**SET012-4.p** Complement is an involution
$x \in y \;\Rightarrow\; \text{little\_set}(x)$ cnf($a_2$, axiom)
little_set($f_1(x, y)$) or $x = y$ cnf(extensionality$_1$, axiom)
$f_1(x, y) \in x$ or $f_1(x, y) \in y$ or $x = y$ cnf(extensionality$_2$, axiom)
$(f_1(x, y) \in x$ and $f_1(x, y) \in y) \;\Rightarrow\; x = y$ cnf(extensionality$_3$, axiom)
$z \in x' \;\Rightarrow\; \neg\, z \in x$ cnf(complement$_1$, axiom)
little_set($z$) $\Rightarrow$ ($z \in x'$ or $z \in x$) cnf(complement$_2$, axiom)
$\neg\, z \in$ empty_set cnf(empty_set, axiom)
little_set($z$) $\Rightarrow$ $z \in$ universal_set cnf(universal_set, axiom)
as$'$ = bs cnf(complement_of_a_is_b, hypothesis)
bs$'$ = cs cnf(complement_of_b_is_c, hypothesis)
as $\neq$ cs cnf(prove_a_equals_c, negated_conjecture)

**SET013+4.p** Commutativity of intersection
include('Axioms/SET006+0.ax')
$\forall a, b$: equal_set(intersection($a, b$), intersection($b, a$)) fof(thI$_{06}$, conjecture)

**SET013-1.p** The intersection of sets is commutative
include('Axioms/SET002-0.ax')
$\neg$ equal_sets(intersection(as, bs), intersection(bs, as)) cnf(prove_commutativity, negated_conjecture)

**SET013-2.p** The intersection of sets is commutative
include('Axioms/SET002-0.ax')
equal_sets(intersection(as, bs), cs) cnf(intersection_of_a_and_b_is_c, hypothesis)
equal_sets(intersection(bs, as), ds) cnf(intersection_of_b_and_a_is_d, hypothesis)
$\neg$ equal_sets(cs, ds) cnf(prove_c_equals_d, negated_conjecture)

**SET013-3.p** The intersection of sets is commutative
include('Axioms/SET003-0.ax')
intersection(as, bs) = cs cnf(intersection_of_a_and_b_is_c, hypothesis)
intersection(bs, as) = ds cnf(intersection_of_b_and_a_is_d, hypothesis)
cs $\neq$ ds cnf(prove_c_equals_d, negated_conjecture)

**SET013-4.p** The intersection of sets is commutative
$x \in y \;\Rightarrow\; \text{little\_set}(x)$ cnf($a_2$, axiom)
little_set($f_1(x, y)$) or $x = y$ cnf(extensionality$_1$, axiom)
$f_1(x, y) \in x$ or $f_1(x, y) \in y$ or $x = y$ cnf(extensionality$_2$, axiom)
$(f_1(x, y) \in x$ and $f_1(x, y) \in y) \;\Rightarrow\; x = y$ cnf(extensionality$_3$, axiom)
$z \in$ intersection($x, y$) $\Rightarrow$ $z \in x$ cnf(intersection$_1$, axiom)
$z \in$ intersection($x, y$) $\Rightarrow$ $z \in y$ cnf(intersection$_2$, axiom)
($z \in x$ and $z \in y$) $\Rightarrow$ $z \in$ intersection($x, y$) cnf(intersection$_3$, axiom)
$\neg\, z \in$ empty_set cnf(empty_set, axiom)
little_set($z$) $\Rightarrow$ $z \in$ universal_set cnf(universal_set, axiom)
intersection(as, bs) = cs cnf(intersection_of_a_and_b_is_c, hypothesis)
intersection(bs, as) = ds cnf(intersection_of_b_and_a_is_d, hypothesis)
cs $\neq$ ds cnf(prove_c_equals_d, negated_conjecture)

**SET014+3.p** If X (= Z and Y (= Z, then X U Y (= Z
If X is a subset of Z and Y is a subset of Z, then the union of X and Y is a subset of Z.
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$)) fof(union_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \;\Rightarrow\; d \in c$)) fof(subset_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$) fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c, d$: (($b \subseteq c$ and $d \subseteq c$) $\Rightarrow$ union($b, d$) $\subseteq c$) fof(prove_union_subset, conjecture)

**SET014+4.p** Union of subsets is a subset
If A and B are contained in C then the union of A and B is also.
include('Axioms/SET006+0.ax')
$\forall a, x, y$: (($x \subseteq a$ and $y \subseteq a$) $\iff$ union($x, y$) $\subseteq a$) fof(thI$_{45}$, conjecture)

**SET014-2.p** Union of subsets is a subset
If A and B are contained in C then the union of A and B is also.
include('Axioms/SET002-0.ax')

$\text{as} \subseteq \text{cs}$ cnf(a_subset_of_c, hypothesis)
$\text{bs} \subseteq \text{cs}$ cnf(b_subset_of_c, hypothesis)
$\neg\, \text{union}(\text{as}, \text{bs}) \subseteq \text{cs}$ cnf(prove_a_union_b_subset_of_c, negated_conjecture)

**SET014-3.p** Union of subsets is a subset
If A and B are contained in C then the union of A and B is also.
include('Axioms/SET003-0.ax')
$\text{as} \subseteq \text{cs}$ cnf(a_subset_of_c, hypothesis)
$\text{bs} \subseteq \text{cs}$ cnf(b_subset_of_c, hypothesis)
$\neg\, \text{union}(\text{as}, \text{bs}) \subseteq \text{cs}$ cnf(prove_a_union_b_subset_of_c, negated_conjecture)

**SET014-4.p** Union of subsets is a subset
If A and B are contained in C then the union of A and B is also.
$x \in y \Rightarrow \text{little\_set}(x)$ cnf($a_2$, axiom)
$\text{little\_set}(f_1(x, y))$ or $x = y$ cnf($\text{extensionality}_1$, axiom)
$f_1(x, y) \in x$ or $f_1(x, y) \in y$ or $x = y$ cnf($\text{extensionality}_2$, axiom)
$(f_1(x, y) \in x$ and $f_1(x, y) \in y) \Rightarrow x = y$ cnf($\text{extensionality}_3$, axiom)
$z \in \text{intersection}(x, y) \Rightarrow z \in x$ cnf($\text{intersection}_1$, axiom)
$z \in \text{intersection}(x, y) \Rightarrow z \in y$ cnf($\text{intersection}_2$, axiom)
$(z \in x$ and $z \in y) \Rightarrow z \in \text{intersection}(x, y)$ cnf($\text{intersection}_3$, axiom)
$z \in x' \Rightarrow \neg\, z \in x$ cnf($\text{complement}_1$, axiom)
$\text{little\_set}(z) \Rightarrow (z \in x'$ or $z \in x)$ cnf($\text{complement}_2$, axiom)
$\text{union}(x, y) = \text{intersection}(x', y')'$ cnf(union, axiom)
$\neg\, z \in \text{empty\_set}$ cnf(empty_set, axiom)
$\text{little\_set}(z) \Rightarrow z \in \text{universal\_set}$ cnf(universal_set, axiom)
$(x \subseteq y$ and $u \in x) \Rightarrow u \in y$ cnf($\text{subset}_1$, axiom)
$x \subseteq y$ or $f_{17}(x, y) \in x$ cnf($\text{subset}_2$, axiom)
$f_{17}(x, y) \in y \Rightarrow x \subseteq y$ cnf($\text{subset}_3$, axiom)
$\text{as} \subseteq \text{cs}$ cnf(a_subset_of_c, hypothesis)
$\text{bs} \subseteq \text{cs}$ cnf(b_subset_of_c, hypothesis)
$\neg\, \text{union}(\text{as}, \text{bs}) \subseteq \text{cs}$ cnf(prove_a_union_b_subset_of_c, negated_conjecture)

**SET014-6.p** If X (= Z and Y (= Z, then X U Y (= Z
If A and B are contained in C then the union of A and B is also.
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{subclass}(x, z)$ cnf($\text{prove\_least\_upper\_bound}_1$, negated_conjecture)
$\text{subclass}(y, z)$ cnf($\text{prove\_least\_upper\_bound}_2$, negated_conjecture)
$\neg\, \text{subclass}(\text{union}(x, y), z)$ cnf($\text{prove\_least\_upper\_bound}_3$, negated_conjecture)

**SET014∧4.p** Union of subsets is a subset
If A and B are contained in C then the union of A and B is also.
include('Axioms/SET008^0.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $a$: \$i → \$o: $((\subseteq @x@a$ and $\subseteq @y@a) \Rightarrow (\subseteq @(\text{union}@x@y)@a))$ thf(thm, conjecture)

**SET014∧5.p** TPS problem BOOL-PROP-32
Trybulec's 32nd Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a$ → \$o, $y$: $a$ → \$o, $z$: $a$ → \$o: $((\forall \text{xx}: a: ((x@\text{xx}) \Rightarrow (z@\text{xx}))$ and $\forall \text{xx}: a: ((y@\text{xx}) \Rightarrow (z@\text{xx}))) \Rightarrow \forall \text{xx}: a: ((x@\text{xx}$ or $y@\text{xx}) \Rightarrow (z@\text{xx})))$ thf(cBOOL_PROP_32_pme, conjecture)

**SET015+4.p** Commutativity of union
include('Axioms/SET006+0.ax')
$\forall a, b$: $\text{equal\_set}(\text{union}(a, b), \text{union}(b, a))$ fof($\text{thI}_{07}$, conjecture)

**SET015-1.p** The union of sets is commutative
include('Axioms/SET002-0.ax')
$\neg\, \text{equal\_sets}(\text{union}(\text{as}, \text{bs}), \text{union}(\text{bs}, \text{as}))$ cnf(prove_commutativity, negated_conjecture)

**SET015-2.p** The union of sets is commutative
include('Axioms/SET002-0.ax')
$\text{equal\_sets}(\text{union}(\text{as}, \text{bs}), \text{cs})$ cnf(a_union_b_is_c, hypothesis)
$\text{equal\_sets}(\text{union}(\text{bs}, \text{as}), \text{ds})$ cnf(b_union_a_is_d, hypothesis)
$\neg\, \text{equal\_sets}(\text{cs}, \text{ds})$ cnf(prove_c_equals_d, negated_conjecture)

**SET015-3.p** The union of sets is commutative
include('Axioms/SET003-0.ax')
$\text{union}(\text{as}, \text{bs}) = \text{cs}$ cnf(a_union_b_is_c, hypothesis)
$\text{union}(\text{bs}, \text{as}) = \text{ds}$ cnf(b_union_a_is_d, hypothesis)
$\text{cs} \neq \text{ds}$ cnf(prove_c_equals_d, negated_conjecture)

**SET015-4.p** The union of sets is commutative
$x \in y \Rightarrow \text{little\_set}(x)$ cnf($a_2$, axiom)
$\text{little\_set}(f_1(x, y))$ or $x = y$ cnf($\text{extensionality}_1$, axiom)
$f_1(x, y) \in x$ or $f_1(x, y) \in y$ or $x = y$ cnf($\text{extensionality}_2$, axiom)
$(f_1(x, y) \in x$ and $f_1(x, y) \in y) \Rightarrow x = y$ cnf($\text{extensionality}_3$, axiom)
$z \in \text{intersection}(x, y) \Rightarrow z \in x$ cnf($\text{intersection}_1$, axiom)
$z \in \text{intersection}(x, y) \Rightarrow z \in y$ cnf($\text{intersection}_2$, axiom)
$(z \in x$ and $z \in y) \Rightarrow z \in \text{intersection}(x, y)$ cnf($\text{intersection}_3$, axiom)
$z \in x' \Rightarrow \neg z \in x$ cnf($\text{complement}_1$, axiom)
$\text{little\_set}(z) \Rightarrow (z \in x'$ or $z \in x)$ cnf($\text{complement}_2$, axiom)
$\text{union}(x, y) = \text{intersection}(x', y')'$ cnf(union, axiom)
$\neg z \in \text{empty\_set}$ cnf(empty_set, axiom)
$\text{little\_set}(z) \Rightarrow z \in \text{universal\_set}$ cnf(universal_set, axiom)
$\text{union}(\text{as}, \text{bs}) = \text{cs}$ cnf(a_union_b_is_c, hypothesis)
$\text{union}(\text{bs}, \text{as}) = \text{ds}$ cnf(b_union_a_is_d, hypothesis)
$\text{cs} \neq \text{ds}$ cnf(prove_c_equals_d, negated_conjecture)

**SET016+1.p** First components of equal ordered pairs are equal
include('Axioms/SET005+0.ax')
$\forall w, x, y, z$: ((ordered_pair$(w, x)$ = ordered_pair$(y, z)$ and $w \in$ universal_class) $\Rightarrow w = y$) fof(ordered_pair_determines_com

**SET016+4.p** First components of equal ordered pairs are equal
If A,A,B = U,U,V then A = U.
include('Axioms/SET006+0.ax')
$\forall a, b, u, v$: (equal_set(unordered_pair(singleton($a$), unordered_pair($a, b$)), unordered_pair(singleton($u$), unordered_pair($u, v$))) =
$a = u$) fof(thI50a, conjecture)

**SET016-1.p** First components of equal ordered pairs are equal
$x \in \text{singleton\_set}(x)$ cnf($\text{singleton}_1$, axiom)
$x \in \text{singleton\_set}(y) \Rightarrow x = y$ cnf($\text{singleton}_2$, axiom)
$x \in \text{unordered\_pair}(x, y)$ cnf($\text{unordered\_pair}_1$, axiom)
$y \in \text{unordered\_pair}(x, y)$ cnf($\text{unordered\_pair}_2$, axiom)
$x \in \text{unordered\_pair}(y, z) \Rightarrow (x = y$ or $x = z)$ cnf($\text{unordered\_pair}_3$, axiom)
$\text{ordered\_pair}(x, y) = \text{unordered\_pair}(\text{singleton\_set}(x), \text{unordered\_pair}(x, y))$ cnf(ordered_pair, axiom)
$\text{ordered\_pair}(m_1, r_1) = \text{ordered\_pair}(m_2, r_2)$ cnf(equal_ordered_pairs, hypothesis)
$m_1 \neq m_2$ cnf(prove_first_components_equal, negated_conjecture)

**SET016-3.p** First components of equal ordered pairs are equal
include('Axioms/SET003-0.ax')
$\text{little\_set}(a)$ cnf(little_set_a, hypothesis)
$\text{little\_set}(b)$ cnf(little_set_b, hypothesis)
$\text{ordered\_pair}(a, c) = \text{ordered\_pair}(b, d)$ cnf(equal_ordered_pairs, hypothesis)
$a \neq b$ cnf(prove_first_components_equal, negated_conjecture)

**SET016-6.p** First components of equal ordered pairs are equal
include('Axioms/SET004-0.ax')
$\text{ordered\_pair}(w, x) = \text{ordered\_pair}(y, z)$ cnf($\text{prove\_ordered\_pair\_determines\_components1}_1$, negated_conjecture)
$w \in \text{universal\_class}$ cnf($\text{prove\_ordered\_pair\_determines\_components1}_2$, negated_conjecture)
$w \neq y$ cnf($\text{prove\_ordered\_pair\_determines\_components1}_3$, negated_conjecture)

**SET017+1.p** Left cancellation for unordered pairs
include('Axioms/SET005+0.ax')
$\forall x, y, z$: (($y \in$ universal_class and $z \in$ universal_class and unordered_pair$(x, y)$ = unordered_pair$(x, z)$) $\Rightarrow y = z$) fof(left_cancellation, conjecture)

**SET017-3.p** Left cancellation for non-ordered pairs
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair$(x, y)$ = ordered_pair$(u, v)$) $\Rightarrow x = u$ cnf(first_components_are_equal, axio

little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
non_ordered_pair($c, a$) = non_ordered_pair($d, b$) cnf(equal_non_ordered_pairs, hypothesis)
$a \neq c$ cnf(prove_left_cancellation, negated_conjecture)

**SET017-4.p** Left cancellation for non-ordered pairs
include('Axioms/SET003-0.ax')
little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
non_ordered_pair($c, a$) = non_ordered_pair($d, b$) cnf(equal_non_ordered_pairs, hypothesis)
$a \neq c$ cnf(prove_left_cancellation, negated_conjecture)

**SET017-6.p** Left cancellation for non-ordered pairs
include('Axioms/SET004-0.ax')
unordered_pair($x, y$) = unordered_pair($x, z$) cnf(prove_left_cancellation$_1$, negated_conjecture)
ordered_pair($y, z$) $\in$ cross_product(universal_class, universal_class) cnf(prove_left_cancellation$_2$, negated_conjecture)
$y \neq z$ cnf(prove_left_cancellation$_3$, negated_conjecture)

**SET017^1.p** Left cancellation for unordered pairs
include('Axioms/SET008^0.ax')
$\forall x$: \$i, $y$: \$i, $z$: \$i: ((unord_pair@$x$@$y$) = (unord_pair@$x$@$z$) $\Rightarrow$ $y = z$) thf(thm, conjecture)

**SET018+1.p** Second components of equal ordered pairs are equal
include('Axioms/SET005+0.ax')
$\forall w, x, y, z$: ((ordered_pair($w, x$) = ordered_pair($y, z$) and $x \in$ universal_class) $\Rightarrow$ $x = z$) fof(ordered_pair_determines_comp

**SET018+4.p** Second components of equal ordered pairs are equal
If A,A,B = U,U,V then B = V.
include('Axioms/SET006+0.ax')
$\forall a, b, u, v$: (equal_set(unordered_pair(singleton($a$), unordered_pair($a, b$)), unordered_pair(singleton($u$), unordered_pair($u, v$))) =
$b = v$) fof(thI50b, conjecture)

**SET018-1.p** Second components of equal ordered pairs are equal
$x \in$ singleton_set($x$) cnf(singleton$_1$, axiom)
$x \in$ singleton_set($y$) $\Rightarrow$ $x = y$ cnf(singleton$_2$, axiom)
$x \in$ unordered_pair($x, y$) cnf(unordered_pair$_1$, axiom)
$y \in$ unordered_pair($x, y$) cnf(unordered_pair$_2$, axiom)
$x \in$ unordered_pair($y, z$) $\Rightarrow$ ($x = y$ or $x = z$) cnf(unordered_pair$_3$, axiom)
ordered_pair($x, y$) = unordered_pair(singleton_set($x$), unordered_pair($x, y$)) cnf(ordered_pair, axiom)
ordered_pair($m_1, r_1$) = ordered_pair($m_2, r_2$) cnf(equal_ordered_pairs, hypothesis)
$r_1 \neq r_2$ cnf(prove_second_components_equal, negated_conjecture)

**SET018-3.p** Second components of equal ordered pairs are equal
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
little_set($c$) cnf(c_little_set, hypothesis)
little_set($d$) cnf(d_little_set, hypothesis)
ordered_pair($a, b$) = ordered_pair($c, d$) cnf(equal_ordered_pair, hypothesis)
$b \neq d$ cnf(prove_second_components_equal, negated_conjecture)

**SET018-4.p** Second components of equal ordered pairs are equal
include('Axioms/SET003-0.ax')
little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
little_set($c$) cnf(c_little_set, hypothesis)
little_set($d$) cnf(d_little_set, hypothesis)
ordered_pair($a, b$) = ordered_pair($c, d$) cnf(equal_ordered_pair, hypothesis)
$b \neq d$ cnf(prove_second_components_equal, negated_conjecture)

**SET018-6.p** Second components of equal ordered pairs are equal
include('Axioms/SET004-0.ax')
ordered_pair($w, x$) = ordered_pair($y, z$) cnf(prove_ordered_pair_determines_components2$_1$, negated_conjecture)

$x \in$ universal_class cnf(prove_ordered_pair_determines_components2$_2$, negated_conjecture)
$x \neq z$ cnf(prove_ordered_pair_determines_components2$_3$, negated_conjecture)

**SET019+4.p** Two sets that contain one another are equal
include('Axioms/SET006+0.ax')
$\forall a, b$: $((a \subseteq b$ and $b \subseteq a) \Rightarrow$ equal_set$(a, b))$ fof(thI$_{02}$, conjecture)

**SET019-3.p** Two sets that contain one another are equal
include('Axioms/SET003-0.ax')
(little_set$(x)$ and little_set$(u)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow x = u$ cnf(first_components_are_equal, axio
(little_set$(x)$ and little_set$(y)$ and non_ordered_pair$(z, x)$ = non_ordered_pair$(z, y)) \Rightarrow x = y$ cnf(left_cancellation, axiom)
(little_set$(x)$ and little_set$(y)$ and little_set$(u)$ and little_set$(v)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow y = v$ cnf(second_components_are_equal, axiom)
$b \subseteq a$ cnf(a_contains_b, hypothesis)
$a \subseteq b$ cnf(b_contains_a, hypothesis)
$a \neq b$ cnf(prove__a_equals_b, negated_conjecture)

**SET019-4.p** Two sets that contain one another are equal
include('Axioms/SET003-0.ax')
$b \subseteq a$ cnf(a_contains_b, hypothesis)
$a \subseteq b$ cnf(b_contains_a, hypothesis)
$a \neq b$ cnf(prove__a_equals_b, negated_conjecture)

**SET020+1.p** Uniqueness of 1st and 2nd when X is an ordered pair of sets
include('Axioms/SET005+0.ax')
$\forall u, v, x$: $((u \in$ universal_class and $v \in$ universal_class and $x$ = ordered_pair$(u, v)) \Rightarrow$ (first$(x) = u$ and second$(x) = v))$ fof(unique_1st_and_2nd_in_pair_of_sets$_1$, conjecture)

**SET020-3.p** 1st is unique when x is an ordered pair of sets
include('Axioms/SET003-0.ax')
(little_set$(x)$ and little_set$(u)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow x = u$ cnf(first_components_are_equal, axio
(little_set$(x)$ and little_set$(y)$ and non_ordered_pair$(z, x)$ = non_ordered_pair$(z, y)) \Rightarrow x = y$ cnf(left_cancellation, axiom)
(little_set$(x)$ and little_set$(y)$ and little_set$(u)$ and little_set$(v)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow y = v$ cnf(second_components_are_equal, axiom)
$(x \subseteq y$ and $y \subseteq x) \Rightarrow x = y$ cnf(two_sets_equal, axiom)
little_set$(a)$ cnf(a_little_set, hypothesis)
little_set$(b)$ cnf(b_little_set, hypothesis)
first(ordered_pair$(a, b)) \neq a$ cnf(prove_first_is_first, negated_conjecture)

**SET020-4.p** 1st is unique when x is an ordered pair of sets
include('Axioms/SET003-0.ax')
little_set$(a)$ cnf(a_little_set, hypothesis)
little_set$(b)$ cnf(b_little_set, hypothesis)
first(ordered_pair$(a, b)) \neq a$ cnf(prove_first_is_first, negated_conjecture)

**SET020-6.p** 1st is unique when x is an ordered pair of sets
include('Axioms/SET004-0.ax')
ordered_pair$(u, v) \in$ cross_product(universal_class, universal_class) cnf(prove_unique_1st_and_2nd_in_pair_of_sets1$_1$, negate
first(ordered_pair$(u, v)) \neq u$ cnf(prove_unique_1st_and_2nd_in_pair_of_sets1$_2$, negated_conjecture)

**SET021-3.p** 2nd is unique when x is an ordered pair of sets
include('Axioms/SET003-0.ax')
(little_set$(x)$ and little_set$(u)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow x = u$ cnf(first_components_are_equal, axio
(little_set$(x)$ and little_set$(y)$ and non_ordered_pair$(z, x)$ = non_ordered_pair$(z, y)) \Rightarrow x = y$ cnf(left_cancellation, axiom)
(little_set$(x)$ and little_set$(y)$ and little_set$(u)$ and little_set$(v)$ and ordered_pair$(x, y)$ = ordered_pair$(u, v)) \Rightarrow y = v$ cnf(second_components_are_equal, axiom)
$(x \subseteq y$ and $y \subseteq x) \Rightarrow x = y$ cnf(two_sets_equal, axiom)
(little_set$(x)$ and little_set$(y)) \Rightarrow$ first(ordered_pair$(x, y)) = x$ cnf(property_of_first, axiom)
little_set$(a)$ cnf(a_little_set, hypothesis)
little_set$(b)$ cnf(b_little_set, hypothesis)
second(ordered_pair$(a, b)) \neq b$ cnf(prove_second_is_second, negated_conjecture)

**SET021-4.p** 2nd is unique when x is an ordered pair of sets
include('Axioms/SET003-0.ax')
little_set$(a)$ cnf(a_little_set, hypothesis)

little_set($b$)  cnf(b_little_set, hypothesis)
second(ordered_pair($a, b$)) $\neq b$  cnf(prove_second_is_second, negated_conjecture)

**SET021-6.p** 2nd is unique when x is an ordered pair of sets
include('Axioms/SET004-0.ax')
ordered_pair($u, v$) $\in$ cross_product(universal_class, universal_class)  cnf(prove_unique_1st_and_2nd_in_pair_of_sets2$_1$, negate
second(ordered_pair($u, v$)) $\neq v$  cnf(prove_unique_1st_and_2nd_in_pair_of_sets2$_2$, negated_conjecture)

**SET022-3.p** The first component of an ordered pair is a little set
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set(first($a$))  cnf(prove_first_component_is_small, negated_conjecture)

**SET022-4.p** The first component of an ordered pair is a little set
include('Axioms/SET003-0.ax')
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set(first($a$))  cnf(prove_first_component_is_small, negated_conjecture)

**SET023-3.p** The second component of an ordered pair is a little set
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set(second($a$))  cnf(prove_second_component_is_small, negated_conjecture)

**SET023-4.p** The second component of an ordered pair is a little set
include('Axioms/SET003-0.ax')
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set(second($a$))  cnf(prove_second_component_is_small, negated_conjecture)

**SET024+1.p** A set belongs to its singleton
include('Axioms/SET005+0.ax')
$\forall x$: ($x \in$ universal_class $\Rightarrow$ $x \in$ singleton($x$))  fof(set_in_its_singleton, conjecture)

**SET024-3.p** A set belongs to its singleton
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($a$)  cnf(a_little_set, hypothesis)
$\neg$ $a \in$ singleton_set($a$)  cnf(prove_membership_of_singleton_set, negated_conjecture)

**SET024-4.p** A set belongs to its singleton
include('Axioms/SET003-0.ax')
little_set($a$)  cnf(a_little_set, hypothesis)

$\neg\, a \in$ singleton_set($a$)  cnf(prove_membership_of_singleton_set, negated_conjecture)

**SET024-6.p** A set belongs to its singleton
include('Axioms/SET004-0.ax')
$x \in$ universal_class  cnf(prove_set_in_its_singleton$_1$, negated_conjecture)
$\neg\, x \in$ singleton($x$)  cnf(prove_set_in_its_singleton$_2$, negated_conjecture)

**SET025+1.p** An ordered pair is a set
include('Axioms/SET005+0.ax')
$\forall x, y$: ordered_pair($x, y$) $\in$ universal_class  fof(ordered_pair_is_set, conjecture)

**SET025-3.p** Ordered pairs are little sets
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) $= x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) $= y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)  cnf(property_of_singleton_sets, axiom)
$\neg$ little_set(ordered_pair($a, b$))  cnf(prove_ordered_pairs_are_small, negated_conjecture)

**SET025-4.p** Ordered pairs are little sets
include('Axioms/SET003-0.ax')
$\neg$ little_set(ordered_pair($a, b$))  cnf(prove_ordered_pairs_are_small, negated_conjecture)

**SET025-6.p** Ordered pairs are little sets
include('Axioms/SET004-0.ax')
$\neg$ ordered_pair($x, y$) $\in$ universal_class  cnf(prove_ordered_pair_is_set$_1$, negated_conjecture)

**SET025-8.p** Ordered pairs are little sets
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) $= x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) $= y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)  cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))  cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set($a$)  cnf(prove_predicate_is_small, negated_conjecture)

**SET025-9.p** Ordered pairs are little sets
include('Axioms/SET003-0.ax')
ordered_pair_predicate($a$)  cnf(an_ordered_pair_predicate, hypothesis)
$\neg$ little_set($a$)  cnf(prove_predicate_is_small, negated_conjecture)

**SET027+1.p** Transitivity of subset
include('Axioms/SET005+0.ax')
$\forall x, y, z$: ((subclass($x, y$) and subclass($y, z$)) $\Rightarrow$ subclass($x, z$))  fof(transitivity, conjecture)

**SET027+3.p** Transitivity of subset
If X is a subset of Y and Y is a subset of Z, then X is a subset of Z.
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$))  fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$  fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: (($b \subseteq c$ and $c \subseteq d$) $\Rightarrow$ $b \subseteq d$)  fof(prove_transitivity_of_subset, conjecture)

**SET027+4.p** Transitivity of subset

include('Axioms/SET006+0.ax')
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$)  fof(thI$_{03}$, conjecture)

**SET027-3.p** Transitivity of subset
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axiom)
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)  cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))  cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$)  cnf(ordered_pairs_are_small$_2$, axiom)
$a \subseteq b$  cnf(a_subset_b, hypothesis)
$b \subseteq c$  cnf(b_subset_c, hypothesis)
$\neg\, a \subseteq c$  cnf(prove_a_subset_c, negated_conjecture)

**SET027-4.p** Transitivity of subset
include('Axioms/SET003-0.ax')
$a \subseteq b$  cnf(a_subset_b, hypothesis)
$b \subseteq c$  cnf(b_subset_c, hypothesis)
$\neg\, a \subseteq c$  cnf(prove_a_subset_c, negated_conjecture)

**SET027-6.p** Transitivity of subset
include('Axioms/SET004-0.ax')
subclass($x, y$)  cnf(prove_transitivity_of_subclass$_1$, negated_conjecture)
subclass($y, z$)  cnf(prove_transitivity_of_subclass$_2$, negated_conjecture)
$\neg$ subclass($x, z$)  cnf(prove_transitivity_of_subclass$_3$, negated_conjecture)

**SET027-7.p** Transitivity of subset
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $x \in$ unordered_pair($x, y$)  cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $y \in$ unordered_pair($x, y$)  cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $u \in$ universal_class  cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $v \in$ universal_class  cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)  cnf(subclass_is_reflexive, axiom)
subclass($x, y$)  cnf(prove_transitivity_of_subclass$_1$, negated_conjecture)
subclass($y, z$)  cnf(prove_transitivity_of_subclass$_2$, negated_conjecture)
$\neg$ subclass($x, z$)  cnf(prove_transitivity_of_subclass$_3$, negated_conjecture)

**SET027^5.p** TPS problem BOOL-PROP-29
Trybulec's 29th Boolean property of sets
$a$: \$tType  thf(a_type, type)
$\forall x$: $a$ $\rightarrow$ \$o, $y$: $a$ $\rightarrow$ \$o, $z$: $a$ $\rightarrow$ \$o: (($\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($y$@xx)) and $\forall$xx: $a$: (($y$@xx) $\Rightarrow$ ($z$@xx))) $\Rightarrow$ $\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($z$@xx)))  thf(cBOOL_PROP_29_pme, conjecture)

**SET027^7.p** Transitivity of subset
include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
$\in$ : mu $\rightarrow$ mu $\rightarrow$ \$i $\rightarrow$ \$o  thf(member_type, type)
$\subseteq$ : mu $\rightarrow$ mu $\rightarrow$ \$i $\rightarrow$ \$o  thf(subset_type, type)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mequiv@($\subseteq$ @$b$@$c$)@(mforall_ind@$\lambda d$: mu: (mimplies@($\in$ @$d$@$b$)@($\in$ @$d$@$c$))))))  thf(subset_defn, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: ($\subseteq$ @$b$@$b$))  thf(reflexivity_of_subset, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mforall_ind@$\lambda d$: mu: (mimplies@(mand@($\subseteq$ @$b$@$c$)@($\subseteq$ @$c$@$d$))@($\subseteq$ @$b$@$d$)))))  thf(prove_transitivity_of_subset, conjecture)

**SET028-3.p** Relationship between apply and image, part 1 of 2

include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
$\neg$ apply(a_function, element) $\subseteq$ sigma(image(singleton_set(element), a_function)) cnf(prove_property_of_image_and_apply$_1$

**SET028-4.p** Relationship between apply and image, part 1 of 2
include('Axioms/SET003-0.ax')
$\neg$ apply(a_function, element) $\subseteq$ sigma(image(singleton_set(element), a_function)) cnf(prove_property_of_image_and_apply$_1$

**SET029-3.p** Relationship between apply and image, part 2 of 2
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
$\neg$ image(singleton_set(element), a_function) $\subseteq$ apply(a_function, element) cnf(prove_image_and_apply$_2$, negated_conjecture

**SET029-4.p** Relationship between apply and image, part 2 of 2
include('Axioms/SET003-0.ax')
$\neg$ image(singleton_set(element), a_function) $\subseteq$ apply(a_function, element) cnf(prove_image_and_apply$_2$, negated_conjecture

**SET030-3.p** Function values are little sets
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function(a_function) cnf(a_function, hypothesis)
$\neg$ little_set(apply(a_function, $b$)) cnf(prove_function_values_are_small, negated_conjecture)

**SET030-4.p** Function values are little sets
include('Axioms/SET003-0.ax')
function($a$)  cnf(a_function, hypothesis)
$\neg$ little_set(apply($a, b$))  cnf(prove_function_values_are_small, negated_conjecture)

**SET030-6.p** Function values are little sets
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)  cnf(prove_application_property8$_1$, negated_conjecture)
$\neg$ apply(xf, $x$) $\in$ universal_class  cnf(prove_application_property8$_2$, negated_conjecture)

**SET031-3.p** The composition of two sets is a relation
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)  cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))  cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$)  cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$  cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf))  cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$)  cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$))  cnf(function_values_are_small, axiom)
$\neg$ relation($a \circ b$)  cnf(prove_composition_is_a_relation, negated_conjecture)

**SET031-4.p** The composition of two sets is a relation
include('Axioms/SET003-0.ax')
$\neg$ relation($a \circ b$)  cnf(prove_composition_is_a_relation, negated_conjecture)

**SET032-3.p** Range of composition
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$  cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$  cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$  cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$  cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$  cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$  cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))  cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))  cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)  cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))  cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$)  cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$  cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf))  cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$)  cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$))  cnf(function_values_are_small, axiom)
relation($y \circ x$)  cnf(composition_is_a_relation, axiom)
$\neg$ range_of($a \circ b$) $\subseteq$ range_of($a$)  cnf(prove_range_of_composition, negated_conjecture)

**SET032-4.p** Range of composition
include('Axioms/SET003-0.ax')
$\neg$ range_of($a \circ b$) $\subseteq$ range_of($a$)  cnf(prove_range_of_composition, negated_conjecture)

**SET032-6.p** Range of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

$\neg$ subclass(xr$\circ$yr, cross_product(domain_of(yr), range_of(xr))) cnf(prove_composition_domain_and_range$_1$, negated_conjectu

**SET033-3.p** Domain of composition
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($a$) $\subseteq$ domain_of($b$) cnf(range_subset_of_domain, hypothesis)
domain_of($a$) $\neq$ domain_of($b \circ a$) cnf(prove_domain_of_composition, negated_conjecture)

**SET033-4.p** Domain of composition
include('Axioms/SET003-0.ax')
range_of($a$) $\subseteq$ domain_of($b$) cnf(range_subset_of_domain, hypothesis)
domain_of($a$) $\neq$ domain_of($b \circ a$) cnf(prove_domain_of_composition, negated_conjecture)

**SET033-6.p** Domain of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(range_of(xr), domain_of(yr)) cnf(prove_boyer_lemma_18$_1$, negated_conjecture)
domain_of(yr $\circ$ xr) $\neq$ domain_of(xr) cnf(prove_boyer_lemma_18$_2$, negated_conjecture)

**SET034-3.p** The composition of functions is a function
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
function(a_function) cnf(a_function, hypothesis)
function(another_function) cnf(another_function, hypothesis)
$\neg$ function(another_function $\circ$ a_function) cnf(prove_their_composition_is_a_function, negated_conjecture)

**SET034-4.p** The composition of functions is a function

include('Axioms/SET003-0.ax')
function($a$)    cnf(a_function, hypothesis)
function($b$)    cnf(b_function, hypothesis)
$\neg$ function($b \circ a$)    cnf(prove_their_composition_is_a_function, negated_conjecture)

**SET034-6.p** The composition of functions is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xf)    cnf(prove_composition_of_functions$_1$, negated_conjecture)
function(yf)    cnf(prove_composition_of_functions$_2$, negated_conjecture)
$\neg$ function(xf $\circ$ yf)    cnf(prove_composition_of_functions$_3$, negated_conjecture)

**SET035-3.p** Maps for composition
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$    cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$    cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$    cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$    cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) $= x$    cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) $= y$    cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))    cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))    cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)    cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))    cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$)    cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$    cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf))    cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$)    cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$))    cnf(function_values_are_small, axiom)
relation($y \circ x$)    cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$)    cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$)    cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$)    cnf(composition_is_a_function, axiom)
maps(function$_1$, $a, b$)    cnf(one_mapping, hypothesis)
maps(function$_2$, $c, d$)    cnf(another_mapping, hypothesis)
$\neg$ maps(function$_2$ $\circ$ function$_1$, $a, d$)    cnf(prove_maps_for_composition, negated_conjecture)

**SET035-4.p** Maps for composition
include('Axioms/SET003-0.ax')
maps(function$_1$, $a, b$)    cnf(one_mapping, hypothesis)
maps(function$_2$, $c, d$)    cnf(another_mapping, hypothesis)
$\neg$ maps(function$_2$ $\circ$ function$_1$, $a, d$)    cnf(prove_maps_for_composition, negated_conjecture)

**SET035-6.p** Maps for composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
maps(xf, $u, v$)    cnf(prove_composition_of_mappings$_1$, negated_conjecture)
maps(xg, $v, w$)    cnf(prove_composition_of_mappings$_2$, negated_conjecture)
$\neg$ maps(xg $\circ$ xf, $u, w$)    cnf(prove_composition_of_mappings$_3$, negated_conjecture)

**SET036-3.p** Properties of apply for functions, part 1 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$    cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$    cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$    cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$    cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) $= x$    cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) $= y$    cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))    cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))    cnf(second_component_is_small, axiom)

little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
function(a_function) cnf(a_function, hypothesis)
ordered_pair($a, b$) $\in$ a_function cnf(ordered_pair_in_function, hypothesis)
apply(a_function, $a$) $\neq b$ cnf(prove_apply_for_functions$_1$, negated_conjecture)

**SET036-4.p** Properties of apply for functions, part 1 of 3
include('Axioms/SET003-0.ax')
little_set($a$) cnf(a_little_set, hypothesis)
little_set($b$) cnf(b_little_set, hypothesis)
function(a_function) cnf(a_function, hypothesis)
ordered_pair($a, b$) $\in$ a_function cnf(ordered_pair_in_function, hypothesis)
apply(a_function, $a$) $\neq b$ cnf(prove_apply_for_functions$_1$, negated_conjecture)

**SET036-6.p** Properties of apply for functions, part 1 of 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf) cnf(prove_application_property9$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ xf cnf(prove_application_property9$_2$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_application_property9$_3$, negated_conjecture)
apply(xf, $x$) $\neq y$ cnf(prove_application_property9$_4$, negated_conjecture)

**SET037-3.p** Properties of apply for functions, part 2 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) = $y$ cnf(apply_for_functions$_1$,
function(a_function) cnf(a_function, hypothesis)
$a \in$ domain_of(a_function) cnf(member_of_function_domain, hypothesis)
apply(a_function, $a$) = $b$ cnf(applying_the_function, hypothesis)

$\neg$ ordered_pair($a, b$) $\in$ a_function    cnf(prove_ordered_pair_in_function, negated_conjecture)

**SET037-4.p** Properties of apply for functions, part 2 of 3
include('Axioms/SET003-0.ax')
function(a_function)    cnf(a_function, hypothesis)
$a \in$ domain_of(a_function)    cnf(member_of_function_domain, hypothesis)
apply(a_function, $a$) $= b$    cnf(applying_the_function, hypothesis)
$\neg$ ordered_pair($a, b$) $\in$ a_function    cnf(prove_ordered_pair_in_function, negated_conjecture)

**SET037-6.p** Properties of apply for functions, part 2 of 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)    cnf(prove_application_property10$_1$, negated_conjecture)
$x \in$ domain_of(xf)    cnf(prove_application_property10$_2$, negated_conjecture)
$\neg$ ordered_pair($x$, apply(xf, $x$)) $\in$ xf    cnf(prove_application_property10$_3$, negated_conjecture)

**SET038-3.p** Properties of apply for functions, part 3 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$    cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$    cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$    cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$    cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) $= x$    cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) $= y$    cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$))    cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$))    cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$)    cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$))    cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$)    cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$    cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf))    cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$)    cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$))    cnf(function_values_are_small, axiom)
relation($y \circ x$)    cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$)    cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$)    cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$)    cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$)    cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) $= y$    cnf(apply_for_functions$_1$,
(function(xf) and $x \in$ domain_of(xf) and apply(xf, $x$) $= y$) $\Rightarrow$ ordered_pair($x, y$) $\in$ xf    cnf(apply_for_functions$_2$, axiom)
maps(a_function, a_domain, a_range)    cnf(a_mapping, hypothesis)
$a \in$ a_domain    cnf(member_of_domain, hypothesis)
$\neg$ apply(a_function, $a$) $\in$ a_range    cnf(prove_mapping_in_range, negated_conjecture)

**SET038-4.p** Properties of apply for functions, part 3 of 3
include('Axioms/SET003-0.ax')
maps(a_function, a_domain, a_range)    cnf(a_mapping, hypothesis)
$a \in$ a_domain    cnf(member_of_domain, hypothesis)
$\neg$ apply(a_function, $a$) $\in$ a_range    cnf(prove_mapping_in_range, negated_conjecture)

**SET038-6.p** Properties of apply for functions, part 3 of 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
maps(xf, xd, xr)    cnf(prove_mapping_property1$_1$, negated_conjecture)
$x \in$ xd    cnf(prove_mapping_property1$_2$, negated_conjecture)
$\neg$ apply(xf, $x$) $\in$ xr    cnf(prove_mapping_property1$_3$, negated_conjecture)

**SET039-3.p** Properties of apply for composition of functions, 1 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$    cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$    cnf(left_cancellation, axiom)

(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) = $y$ cnf(apply_for_functions$_1$,
(function(xf) and $x \in$ domain_of(xf) and apply(xf, $x$) = $y$) $\Rightarrow$ ordered_pair($x, y$) $\in$ xf cnf(apply_for_functions$_2$, axiom)
(maps(xf, xd, xr) and $x \in$ xd) $\Rightarrow$ apply(xf, $x$) $\in$ xr cnf(apply_for_functions$_3$, axiom)
function(a_function) cnf(a_function, hypothesis)
$a \in$ domain_of(a_function) cnf(member_of_domain, hypothesis)
$\neg$ apply(another_function, apply(a_function, $a$)) $\subseteq$ apply(another_function$\circ$a_function, $a$) cnf(prove_apply_for_composition_

**SET039-4.p** Properties of apply for composition of functions, 1 of 3
include('Axioms/SET003-0.ax')
function(a_function) cnf(a_function, hypothesis)
$a \in$ domain_of(a_function) cnf(member_of_domain, hypothesis)
$\neg$ apply(another_function, apply(a_function, $a$)) $\subseteq$ apply(another_function$\circ$a_function, $a$) cnf(prove_apply_for_composition_

**SET040-3.p** Properties of apply for composition of functions, 2 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) = $y$ cnf(apply_for_functions$_1$,
(function(xf) and $x \in$ domain_of(xf) and apply(xf, $x$) = $y$) $\Rightarrow$ ordered_pair($x, y$) $\in$ xf cnf(apply_for_functions$_2$, axiom)
(maps(xf, xd, xr) and $x \in$ xd) $\Rightarrow$ apply(xf, $x$) $\in$ xr cnf(apply_for_functions$_3$, axiom)
(function(xf) and $x \in$ domain_of(xf)) $\Rightarrow$ apply(xg, apply(xf, $x$)) $\subseteq$ apply(xg$\circ$xf, $x$) cnf(apply_for_composition$_1$, axiom)
function(a_function) cnf(a_function, hypothesis)

$\neg$ apply(another_function$\circ$a_function, element) $\subseteq$ apply(another_function, apply(a_function, element)) cnf(prove_apply_for

**SET040-4.p** Properties of apply for composition of functions, 2 of 3
include('Axioms/SET003-0.ax')
function(a_function) cnf(a_function, hypothesis)
$\neg$ apply(another_function$\circ$a_function, element) $\subseteq$ apply(another_function, apply(a_function, element)) cnf(prove_apply_for

**SET040-6.p** Properties of apply for composition of functions, 2 of 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xf) cnf(prove_application_property12$_1$, negated_conjecture)
$\neg$ subclass(apply(yf $\circ$ xf, $x$), apply(yf, apply(xf, $x$))) cnf(prove_application_property12$_2$, negated_conjecture)

**SET041-3.p** Properties of apply for composition of functions, 3 of 3
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)
(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y =$
$v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) = $y$ cnf(apply_for_functions$_1$,
(function(xf) and $x \in$ domain_of(xf) and apply(xf, $x$) = $y$) $\Rightarrow$ ordered_pair($x, y$) $\in$ xf cnf(apply_for_functions$_2$, axiom)
(maps(xf, xd, xr) and $x \in$ xd) $\Rightarrow$ apply(xf, $x$) $\in$ xr cnf(apply_for_functions$_3$, axiom)
(function(xf) and $x \in$ domain_of(xf)) $\Rightarrow$ apply(xg, apply(xf, $x$)) $\subseteq$ apply(xg$\circ$xf, $x$) cnf(apply_for_composition$_1$, axiom)
function(xf) $\Rightarrow$ apply(xg $\circ$ xf, $x$) $\subseteq$ apply(xg, apply(xf, $x$)) cnf(apply_for_composition$_2$, axiom)
function(a_function) cnf(a_function, hypothesis)
$a \in$ domain_of(a_function) cnf(member_of_domain, hypothesis)
apply(another_function, apply(a_function, $a$)) $\neq$ apply(another_function$\circ$a_function, $a$) cnf(prove_apply_for_composition$_3$,

**SET041-4.p** Properties of apply for composition of functions, 3 of 3
include('Axioms/SET003-0.ax')
function(a_function) cnf(a_function, hypothesis)
$a \in$ domain_of(a_function) cnf(member_of_domain, hypothesis)
apply(another_function, apply(a_function, $a$)) $\neq$ apply(another_function$\circ$a_function, $a$) cnf(prove_apply_for_composition$_3$,

**SET041-6.p** Properties of apply for composition of functions, 3 of 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf) cnf(prove_application_property11$_1$, negated_conjecture)
$x \in$ domain_of(xf) cnf(prove_application_property11$_2$, negated_conjecture)
apply(yf $\circ$ xf, $x$) $\neq$ apply(yf, apply(xf, $x$)) cnf(prove_application_property11$_3$, negated_conjecture)

**SET042-3.p** Ordered pairs are in cross products
include('Axioms/SET003-0.ax')
(little_set($x$) and little_set($u$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $x = u$ cnf(first_components_are_equal, axio
(little_set($x$) and little_set($y$) and non_ordered_pair($z, x$) = non_ordered_pair($z, y$)) $\Rightarrow$ $x = y$ cnf(left_cancellation, axiom)

(little_set($x$) and little_set($y$) and little_set($u$) and little_set($v$) and ordered_pair($x, y$) = ordered_pair($u, v$)) $\Rightarrow$ $y = v$ cnf(second_components_are_equal, axiom)
($x \subseteq y$ and $y \subseteq x$) $\Rightarrow$ $x = y$ cnf(two_sets_equal, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ first(ordered_pair($x, y$)) = $x$ cnf(property_of_first, axiom)
(little_set($x$) and little_set($y$)) $\Rightarrow$ second(ordered_pair($x, y$)) = $y$ cnf(property_of_second, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(first($x$)) cnf(first_component_is_small, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set(second($x$)) cnf(second_component_is_small, axiom)
little_set($x$) $\Rightarrow$ $x \in$ singleton_set($x$) cnf(property_of_singleton_sets, axiom)
little_set(ordered_pair($x, y$)) cnf(ordered_pairs_are_small$_1$, axiom)
ordered_pair_predicate($x$) $\Rightarrow$ little_set($x$) cnf(ordered_pairs_are_small$_2$, axiom)
($x \subseteq y$ and $y \subseteq z$) $\Rightarrow$ $x \subseteq z$ cnf(containment_is_transitive, axiom)
apply(xf, $y$) $\subseteq$ sigma(image(singleton_set($y$), xf)) cnf(image_and_apply$_1$, axiom)
image(singleton_set($y$), xf) $\subseteq$ apply(xf, $y$) cnf(image_and_apply$_2$, axiom)
function($y$) $\Rightarrow$ little_set(apply($y, x$)) cnf(function_values_are_small, axiom)
relation($y \circ x$) cnf(composition_is_a_relation, axiom)
range_of($y \circ x$) $\subseteq$ range_of($y$) cnf(range_of_composition, axiom)
range_of($x$) $\subseteq$ domain_of($y$) $\Rightarrow$ domain_of($x$) = domain_of($y \circ x$) cnf(domain_of_composition, axiom)
(function($x$) and function($y$)) $\Rightarrow$ function($y \circ x$) cnf(composition_is_a_function, axiom)
(maps(xf, $u, v$) and maps(xg, $v, w$)) $\Rightarrow$ maps(xg $\circ$ xf, $u, w$) cnf(maps_for_composition, axiom)
(little_set($x$) and little_set($y$) and function(xf) and ordered_pair($x, y$) $\in$ xf) $\Rightarrow$ apply(xf, $x$) = $y$ cnf(apply_for_functions$_1$,
(function(xf) and $x \in$ domain_of(xf) and apply(xf, $x$) = $y$) $\Rightarrow$ ordered_pair($x, y$) $\in$ xf cnf(apply_for_functions$_2$, axiom)
(maps(xf, xd, xr) and $x \in$ xd) $\Rightarrow$ apply(xf, $x$) $\in$ xr cnf(apply_for_functions$_3$, axiom)
(function(xf) and $x \in$ domain_of(xf)) $\Rightarrow$ apply(xg, apply(xf, $x$)) $\subseteq$ apply(xg$\circ$xf, $x$) cnf(apply_for_composition$_1$, axiom)
function(xf) $\Rightarrow$ apply(xg $\circ$ xf, $x$) $\subseteq$ apply(xg, apply(xf, $x$)) cnf(apply_for_composition$_2$, axiom)
(function(xf) and $x \in$ domain_of(xf)) $\Rightarrow$ apply(xg, apply(xf, $x$)) = apply(xg$\circ$xf, $x$) cnf(apply_for_composition$_3$, axiom)
$a \in$ set_a cnf(member_of_set_a, hypothesis)
$b \in$ set_b cnf(member_of_set_b, hypothesis)
$\neg$ ordered_pair($a, b$) $\in$ cross_product(set_a, set_b) cnf(prove_ordered_pair_is_in_cross_product, negated_conjecture)

**SET042-4.p** Ordered pairs are in cross products
include('Axioms/SET003-0.ax')
$a \in$ set_a cnf(member_of_set_a, hypothesis)
$b \in$ set_b cnf(member_of_set_b, hypothesis)
$\neg$ ordered_pair($a, b$) $\in$ cross_product(set_a, set_b) cnf(prove_ordered_pair_is_in_cross_product, negated_conjecture)

**SET043+1.p** Russell's Paradox
Russell's paradox : there is no Russell set (a set which contains exactly those sets which are not members of themselves).
$\neg \exists x$: $\forall y$: (element($y, x$) $\iff$ $\neg$ element($y, y$)) fof(pel$_{39}$, conjecture)

**SET043-5.p** Russell's Paradox
Russell's paradox : there is no Russell set (a set which contains exactly those sets which are not members of themselves).
element($x, a$) $\Rightarrow$ $\neg$ element($x, x$) cnf(clause$_1$, negated_conjecture)
element($x, x$) or element($x, a$) cnf(clause$_2$, negated_conjecture)

**SET043^5.p** TPS problem RUSSELL1
One form of Russell's Paradox.
cE: \$i $\to$ \$i $\to$ \$o thf(cE, type)
$\neg \exists u$: \$i: $\forall v$: \$i: ((cE@$v$@$u$) $\iff$ $\neg$ cE@$v$@$v$) thf(cRUSSELL$_1$, conjecture)

**SET044+1.p** Anti-Russell Sets
If there were an anti-Russell set (a set that contains exactly those sets that are members of themselves), then not every set has a complement.
$\exists y$: $\forall x$: (element($x, y$) $\iff$ element($x, x$)) $\Rightarrow$ $\neg \forall x_1$: $\exists y_1$: $\forall z$: (element($z, y_1$) $\iff$ $\neg$ element($z, x_1$)) fof(pel$_{40}$, conjecture

**SET044-5.p** Anti-Russell Sets
If there were an anti-Russell set (a set that contains exactly those sets that are members of themselves), then not every set has a complement.
element($x, a$) $\Rightarrow$ element($x, x$) cnf(clause$_1$, negated_conjecture)
element($x, x$) $\Rightarrow$ element($x, a$) cnf(clause$_2$, negated_conjecture)
element($y, f(x)$) $\Rightarrow$ $\neg$ element($y, x$) cnf(clause$_3$, negated_conjecture)
element($y, x$) or element($y, f(x)$) cnf(clause$_4$, negated_conjecture)

**SET044^5.p** TPS problem PELL40
If there were an anti-Russell set (a set that contains exactly those sets that are members of themselves), then not every set has a complement.
cF: \$i → \$i → \$o    thf(cF, type)
∃xy: \$i: ∀xx: \$i: ((cF@xx@xy) ⟺ (cF@xx@xx)) ⇒ ¬∀xx: \$i: ∃xy: \$i: ∀xz: \$i: ((cF@xz@xy) ⟺ ¬cF@xz@xx)    thf(cF

**SET045+1.p** No Universal Set
The restricted comprehension axiom says : given a set z, there is a set all of whose members are drawn from z and which satisfy some property. If there were a universal set, then the Russell set could be formed, using this axiom. So given the appropriate instance of this axiom, there is no universal set.
∀$z$: ∃$y$: ∀$x$: (element($x, y$) ⟺ (element($x, z$) and ¬element($x, x$)))    fof(pel41$_1$, axiom)
¬∃$z$: ∀$x$: element($x, z$)    fof(pel$_{41}$, conjecture)

**SET045-5.p** No Universal Set
The restricted comprehension axiom says : given a set z, there is a set all of whose members are drawn from z and which satisfy some property. If there were a universal set, then the Russell set could be formed, using this axiom. So given the appropriate instance of this axiom, there is no universal set.
element($x, f(y)$) ⇒ element($x, y$)    cnf(clause$_1$, axiom)
element($x, f(y)$) ⇒ ¬element($x, x$)    cnf(clause$_2$, axiom)
element($x, y$) ⇒ (element($x, x$) or element($x, f(y)$))    cnf(clause$_3$, axiom)
element($x, a$)    cnf(clause$_4$, negated_conjecture)

**SET045^5.p** TPS problem TTTP5243
Comprehension Theorem.
$b$: \$tType    thf(b_type, type)
$a$: \$tType    thf(a_type, type)
cA: $b$    thf(cA, type)
∃$u$: $a \to b$: ∀$v$: $a$: ($u$@$v$) = cA    thf(cTTTP$_{5243}$, conjecture)

**SET045^7.p** No Universal Set
include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
element: mu → mu → \$i → \$o    thf(element_type, type)
mvalid@(mbox_s$_4$@(mforall_ind@λ$z$: mu: (mexists_ind@λ$y$: mu: (mbox_s$_4$@(mforall_ind@λ$x$: mu: (mand@(mbox_s$_4$@(mimplie
mvalid@(mbox_s$_4$@(mnot@(mexists_ind@λ$z$: mu: (mbox_s$_4$@(mforall_ind@λ$x$: mu: (mbox_s$_4$@(element@$x$@$z$)))))))    thf(pe

**SET046+1.p** No set of non-circular sets
A set is circular if it is a member of another set which in turn is a member of the orginal. Intuitively all sets are non-circular. Prove there is no set of non-circular sets.
¬∃$y$: ∀$x$: (element($x, y$) ⟺ ¬∃$z$: (element($x, z$) and element($z, x$)))    fof(pel$_{42}$, conjecture)

**SET046-5.p** No set of non-circular sets
A set is circular if it is a member of another set which in turn is a member of the orginal. Intuitively all sets are non-circular. Prove there is no set of non-circular sets.
(element($x, a$) and element($x, y$)) ⇒ ¬element($y, x$)    cnf(clause$_1$, negated_conjecture)
element($x, f(x)$) or element($x, a$)    cnf(clause$_2$, negated_conjecture)
element($f(x), x$) or element($x, a$)    cnf(clause$_3$, negated_conjecture)

**SET046^5.p** TPS problem PELL42
There is no set of non-circular sets (where a circular set is a set x s.t. there is a set y, s.t. x belongs to y and reversely).
cF: \$i → \$i → \$o    thf(cF, type)
¬∃xy: \$i: ∀xx: \$i: (((cF@xx@xy) ⇒ ¬∃xz: \$i: (cF@xx@xz and cF@xz@xx)) and (¬∃xz: \$i: (cF@xx@xz and cF@xz@xx) ⇒ (cF@xx@xy)))    thf(cPELL$_{42}$, conjecture)

**SET047+1.p** Set equality is symmetric
Define set equality as having exactly the same members. Prove set equality is symmetric.
∀$x, y$: (set_equal($x, y$) ⟺ ∀$z$: (element($z, x$) ⟺ element($z, y$)))    fof(pel43$_1$, axiom)
∀$x, y$: (set_equal($x, y$) ⟺ set_equal($y, x$))    fof(pel$_{43}$, conjecture)

**SET047-5.p** Set equality is symmetric
Define set equality as having exactly the same members. Prove set equality is symmetric.
(set_equal($x, y$) and element($z, x$)) ⇒ element($z, y$)    cnf(element_substitution$_1$, axiom)
(set_equal($x, y$) and element($z, y$)) ⇒ element($z, x$)    cnf(element_substitution$_2$, axiom)

element$(f(x, y), x)$ or element$(f(x, y), y)$ or set_equal$(x, y)$ cnf(clause$_3$, axiom)
(element$(f(x, y), y)$ and element$(f(x, y), x)$) $\Rightarrow$ set_equal$(x, y)$ cnf(clause$_4$, axiom)
set_equal$(a, b)$ or set_equal$(b, a)$ cnf(prove_symmetry$_1$, negated_conjecture)
set_equal$(b, a)$ $\Rightarrow$ $\neg$ set_equal$(a, b)$ cnf(prove_symmetry$_2$, negated_conjecture)

**SET050-6.p** Corollary to Unordered pair axiom
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v)$ cnf(prove_corollary_1_to_unordered_pair$_1$, negated_conjecture)
$\neg\, x \in$ unordered_pair$(x, y)$ cnf(prove_corollary_1_to_unordered_pair$_2$, negated_conjecture)

**SET051-6.p** Corollary to Unordered pair axiom
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v)$ cnf(prove_corollary_2_to_unordered_pair$_1$, negated_conjecture)
$\neg\, y \in$ unordered_pair$(x, y)$ cnf(prove_corollary_2_to_unordered_pair$_2$, negated_conjecture)

**SET052-6.p** Corollary to Cartesian product axiom
include('Axioms/SET004-0.ax')
ordered_pair$(u, v) \in$ cross_product$(x, y)$ cnf(prove_corollary_1_to_cartesian_product$_1$, negated_conjecture)
$\neg\, u \in$ universal_class cnf(prove_corollary_1_to_cartesian_product$_2$, negated_conjecture)

**SET053-6.p** Corollary to Cartesian product axiom
include('Axioms/SET004-0.ax')
ordered_pair$(u, v) \in$ cross_product$(x, y)$ cnf(prove_corollary_2_to_cartesian_product$_1$, negated_conjecture)
$\neg\, v \in$ universal_class cnf(prove_corollary_2_to_cartesian_product$_2$, negated_conjecture)

**SET054+1.p** Reflexivity of subclass
include('Axioms/SET005+0.ax')
$\forall x$: subclass$(x, x)$ fof(reflexivity_of_subclass, conjecture)

**SET054-6.p** Subclass is reflexive
include('Axioms/SET004-0.ax')
$\neg$ subclass$(x, x)$ cnf(prove_subclass_is_reflexive$_1$, negated_conjecture)

**SET054-7.p** Subclass is reflexive
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v)$ $\Rightarrow$ $x \in$ unordered_pair$(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair$(x, y) \in$ cross_product$(u, v)$ $\Rightarrow$ $y \in$ unordered_pair$(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y)$ $\Rightarrow$ $u \in$ universal_class cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y)$ $\Rightarrow$ $v \in$ universal_class cnf(corollary_2_to_cartesian_product, axiom)
$\neg$ subclass$(x, x)$ cnf(prove_subclass_is_reflexive$_1$, negated_conjecture)

**SET055+1.p** Reflexivity of equality
include('Axioms/SET005+0.ax')
$\forall x$: $x = x$ fof(reflexivity, conjecture)

**SET056+1.p** Expanded equality definition
include('Axioms/SET005+0.ax')
$\forall x, y$: ($x = y$ or $\exists u$: ($u \in x$ and $\neg\, u \in y$) or $\exists w$: ($w \in y$ and $\neg\, w \in x$)) fof(equality$_1$, conjecture)

**SET056-6.p** Expanded equality definition
include('Axioms/SET004-0.ax')
$x \neq y$ cnf(prove_equality1$_1$, negated_conjecture)
$\neg$ not_subclass_element$(x, y) \in x$ cnf(prove_equality1$_2$, negated_conjecture)
$\neg$ not_subclass_element$(y, x) \in y$ cnf(prove_equality1$_3$, negated_conjecture)

**SET056-7.p** Expanded equality definition
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v)$ $\Rightarrow$ $x \in$ unordered_pair$(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair$(x, y) \in$ cross_product$(u, v)$ $\Rightarrow$ $y \in$ unordered_pair$(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y)$ $\Rightarrow$ $u \in$ universal_class cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y)$ $\Rightarrow$ $v \in$ universal_class cnf(corollary_2_to_cartesian_product, axiom)
subclass$(x, x)$ cnf(subclass_is_reflexive, axiom)
(subclass$(x, y)$ and subclass$(y, z)$) $\Rightarrow$ subclass$(x, z)$ cnf(transitivity_of_subclass, axiom)
$x \neq y$ cnf(prove_equality1$_1$, negated_conjecture)
$\neg$ not_subclass_element$(x, y) \in x$ cnf(prove_equality1$_2$, negated_conjecture)
$\neg$ not_subclass_element$(y, x) \in y$ cnf(prove_equality1$_3$, negated_conjecture)

**SET057-6.p** Expanded equality definition
include('Axioms/SET004-0.ax')
not_subclass_element$(x, y) \in y$ cnf(prove_equality2$_1$, negated_conjecture)
$x \neq y$ cnf(prove_equality2$_2$, negated_conjecture)
$\neg$ not_subclass_element$(y, x) \in y$ cnf(prove_equality2$_3$, negated_conjecture)

**SET057-7.p** Expanded equality definition
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow x \in$ unordered_pair$(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow y \in$ unordered_pair$(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow u \in$ universal_class cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow v \in$ universal_class cnf(corollary_2_to_cartesian_product, axiom)
subclass$(x, x)$ cnf(subclass_is_reflexive, axiom)
(subclass$(x, y)$ and subclass$(y, z)$) $\Rightarrow$ subclass$(x, z)$ cnf(transitivity_of_subclass, axiom)
not_subclass_element$(x, y) \in y$ cnf(prove_equality2$_1$, negated_conjecture)
$x \neq y$ cnf(prove_equality2$_2$, negated_conjecture)
$\neg$ not_subclass_element$(y, x) \in y$ cnf(prove_equality2$_3$, negated_conjecture)

**SET058-6.p** Expanded equality definition
include('Axioms/SET004-0.ax')
not_subclass_element$(y, x) \in x$ cnf(prove_equality3$_1$, negated_conjecture)
$x \neq y$ cnf(prove_equality3$_2$, negated_conjecture)
$\neg$ not_subclass_element$(x, y) \in x$ cnf(prove_equality3$_3$, negated_conjecture)

**SET058-7.p** Expanded equality definition
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow x \in$ unordered_pair$(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow y \in$ unordered_pair$(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow u \in$ universal_class cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow v \in$ universal_class cnf(corollary_2_to_cartesian_product, axiom)
subclass$(x, x)$ cnf(subclass_is_reflexive, axiom)
(subclass$(x, y)$ and subclass$(y, z)$) $\Rightarrow$ subclass$(x, z)$ cnf(transitivity_of_subclass, axiom)
not_subclass_element$(y, x) \in x$ cnf(prove_equality3$_1$, negated_conjecture)
$x \neq y$ cnf(prove_equality3$_2$, negated_conjecture)
$\neg$ not_subclass_element$(x, y) \in x$ cnf(prove_equality3$_3$, negated_conjecture)

**SET059-6.p** Expanded equality definition
include('Axioms/SET004-0.ax')
not_subclass_element$(x, y) \in y$ cnf(prove_equality4$_1$, negated_conjecture)
not_subclass_element$(y, x) \in x$ cnf(prove_equality4$_2$, negated_conjecture)
$x \neq y$ cnf(prove_equality4$_3$, negated_conjecture)

**SET059-7.p** Expanded equality definition
include('Axioms/SET004-0.ax')
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow x \in$ unordered_pair$(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair$(x, y) \in$ cross_product$(u, v) \Rightarrow y \in$ unordered_pair$(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow u \in$ universal_class cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair$(u, v) \in$ cross_product$(x, y) \Rightarrow v \in$ universal_class cnf(corollary_2_to_cartesian_product, axiom)
subclass$(x, x)$ cnf(subclass_is_reflexive, axiom)
(subclass$(x, y)$ and subclass$(y, z)$) $\Rightarrow$ subclass$(x, z)$ cnf(transitivity_of_subclass, axiom)
not_subclass_element$(x, y) \in y$ cnf(prove_equality4$_1$, negated_conjecture)
not_subclass_element$(y, x) \in x$ cnf(prove_equality4$_2$, negated_conjecture)
$x \neq y$ cnf(prove_equality4$_3$, negated_conjecture)

**SET060+1.p** Nothing in the intersection of a set and its complement
include('Axioms/SET005+0.ax')
$\forall x, y\colon \neg\, y \in$ intersection$(x', x)$ fof(special_classes_lemma, conjecture)

**SET060-6.p** Nothing in the intersection of a set and its complement
include('Axioms/SET004-0.ax')
$y \in$ intersection$(x', x)$ cnf(prove_special_classes_lemma$_1$, negated_conjecture)

**SET060-7.p** Nothing in the intersection of a set and its complement
include('Axioms/SET004-0.ax')

$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; x \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_1_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; y \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_2_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; u \in \mathrm{universal\_class}$  cnf(corollary_1_to_cartesian_product, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; v \in \mathrm{universal\_class}$  cnf(corollary_2_to_cartesian_product, axiom)
$\mathrm{subclass}(x, x)$  cnf(subclass_is_reflexive, axiom)
$(\mathrm{subclass}(x, y) \text{ and } \mathrm{subclass}(y, z)) \;\Rightarrow\; \mathrm{subclass}(x, z)$  cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \mathrm{not\_subclass\_element}(x, y) \in x \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y$  cnf($\mathrm{equality}_1$, axiom)
$\mathrm{not\_subclass\_element}(x, y) \in y \;\Rightarrow\; (x = y \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y)$  cnf($\mathrm{equality}_2$, axiom)
$\mathrm{not\_subclass\_element}(y, x) \in x \;\Rightarrow\; (x = y \text{ or } \mathrm{not\_subclass\_element}(x, y) \in x)$  cnf($\mathrm{equality}_3$, axiom)
$(\mathrm{not\_subclass\_element}(x, y) \in y \text{ and } \mathrm{not\_subclass\_element}(y, x) \in x) \;\Rightarrow\; x = y$  cnf($\mathrm{equality}_4$, axiom)
$y \in \mathrm{intersection}(x', x)$  cnf($\mathrm{prove\_special\_classes\_lemma}_1$, negated_conjecture)

**SET061+1.p** Existence of a null class
include('Axioms/SET005+0.ax')
$\exists x \colon \forall z \colon \neg\, z \in x$  fof(existence_of_null_class, conjecture)

**SET061-6.p** Existence of the null class
include('Axioms/SET004-0.ax')
$z \in \mathrm{null\_class}$  cnf($\mathrm{prove\_existence\_of\_null\_class}_1$, negated_conjecture)

**SET061-7.p** Existence of the null class
include('Axioms/SET004-0.ax')
$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; x \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_1_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; y \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_2_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; u \in \mathrm{universal\_class}$  cnf(corollary_1_to_cartesian_product, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; v \in \mathrm{universal\_class}$  cnf(corollary_2_to_cartesian_product, axiom)
$\mathrm{subclass}(x, x)$  cnf(subclass_is_reflexive, axiom)
$(\mathrm{subclass}(x, y) \text{ and } \mathrm{subclass}(y, z)) \;\Rightarrow\; \mathrm{subclass}(x, z)$  cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \mathrm{not\_subclass\_element}(x, y) \in x \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y$  cnf($\mathrm{equality}_1$, axiom)
$\mathrm{not\_subclass\_element}(x, y) \in y \;\Rightarrow\; (x = y \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y)$  cnf($\mathrm{equality}_2$, axiom)
$\mathrm{not\_subclass\_element}(y, x) \in x \;\Rightarrow\; (x = y \text{ or } \mathrm{not\_subclass\_element}(x, y) \in x)$  cnf($\mathrm{equality}_3$, axiom)
$(\mathrm{not\_subclass\_element}(x, y) \in y \text{ and } \mathrm{not\_subclass\_element}(y, x) \in x) \;\Rightarrow\; x = y$  cnf($\mathrm{equality}_4$, axiom)
$\neg\, y \in \mathrm{intersection}(x', x)$  cnf(special_classes_lemma, axiom)
$z \in \mathrm{null\_class}$  cnf($\mathrm{prove\_existence\_of\_null\_class}_1$, negated_conjecture)

**SET062+1.p** The empty set is a subset of X
include('Axioms/SET005+0.ax')
$\forall x \colon \mathrm{subclass}(\mathrm{null\_class}, x)$  fof(null_class_is_subclass, conjecture)

**SET062+3.p** The empty set is a subset of X
$\forall b \colon \neg\, b \in \mathrm{empty\_set}$  fof(empty_set_defn, axiom)
$\forall b, c \colon (b \subseteq c \iff \forall d \colon (d \in b \;\Rightarrow\; d \in c))$  fof(subset_defn, axiom)
$\forall b \colon b \subseteq b$  fof(reflexivity_of_subset, axiom)
$\forall b \colon (\mathrm{empty}(b) \iff \forall c \colon \neg\, c \in b)$  fof(empty_defn, axiom)
$\forall b \colon \mathrm{empty\_set} \subseteq b$  fof(prove_empty_set_subset, conjecture)

**SET062+4.p** The empty set is a subset of all sets
include('Axioms/SET006+0.ax')
$\forall a \colon \mathrm{empty\_set} \subseteq a$  fof($\mathrm{thI}_{15}$, conjecture)

**SET062-6.p** The empty set is a subset of X
include('Axioms/SET004-0.ax')
$\neg\, \mathrm{subclass}(\mathrm{null\_class}, x)$  cnf($\mathrm{prove\_null\_class\_is\_subclass}_1$, negated_conjecture)

**SET062-7.p** The empty set is a subset of X
include('Axioms/SET004-0.ax')
$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; x \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_1_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(x, y) \in \mathrm{cross\_product}(u, v) \;\Rightarrow\; y \in \mathrm{unordered\_pair}(x, y)$  cnf(corollary_2_to_unordered_pair, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; u \in \mathrm{universal\_class}$  cnf(corollary_1_to_cartesian_product, axiom)
$\mathrm{ordered\_pair}(u, v) \in \mathrm{cross\_product}(x, y) \;\Rightarrow\; v \in \mathrm{universal\_class}$  cnf(corollary_2_to_cartesian_product, axiom)
$\mathrm{subclass}(x, x)$  cnf(subclass_is_reflexive, axiom)
$(\mathrm{subclass}(x, y) \text{ and } \mathrm{subclass}(y, z)) \;\Rightarrow\; \mathrm{subclass}(x, z)$  cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \mathrm{not\_subclass\_element}(x, y) \in x \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y$  cnf($\mathrm{equality}_1$, axiom)
$\mathrm{not\_subclass\_element}(x, y) \in y \;\Rightarrow\; (x = y \text{ or } \mathrm{not\_subclass\_element}(y, x) \in y)$  cnf($\mathrm{equality}_2$, axiom)

$\text{not\_subclass\_element}(y, x) \in x \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(x, y) \in x)$ cnf(equality$_3$, axiom)
$(\text{not\_subclass\_element}(x, y) \in y \text{ and } \text{not\_subclass\_element}(y, x) \in x) \Rightarrow x = y$ cnf(equality$_4$, axiom)
$\neg\, y \in \text{intersection}(x', x)$ cnf(special_classes_lemma, axiom)
$\neg\, z \in \text{null\_class}$ cnf(existence_of_null_class, axiom)
$\neg\, \text{subclass}(\text{null\_class}, x)$ cnf(prove_null_class_is_subclass$_1$, negated_conjecture)

**SET062^5.p** TPS problem BOOL-PROP-27
Trybulec's 27th Boolean property of sets.
$a$: \$tType thf(a_type, type)
$\forall x: a \rightarrow \$o, xx: a: (\$false \Rightarrow (x@xx))$ thf(cBOOL_PROP_27_pme, conjecture)

**SET062^6.p** TPS problem from BASIC-FO-THMS
Trybulec's 27th Boolean property of sets.
cA: \$i → \$o thf(cA, type)
$\forall z_3: \$i: (\$false \Rightarrow (cA@z_3))$ thf(cSET76_pme, conjecture)

**SET063+1.p** If X is a subset of the empty set, then X is the empty set
include('Axioms/SET005+0.ax')
$\forall x: (\text{subclass}(x, \text{null\_class}) \Rightarrow x = \text{null\_class})$ fof(corollary_of_null_class_is_subclass, conjecture)

**SET063+3.p** If X is a subset of the empty set, then X is the empty set
$\forall b: \text{empty\_set} \subseteq b$ fof(empty_set_subset, axiom)
$\forall b: \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c: (b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c: (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b: b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b: (\text{empty}(b) \iff \forall c: \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b: (b \subseteq \text{empty\_set} \Rightarrow b = \text{empty\_set})$ fof(prove_subset_of_empty_set_is_empty_set, conjecture)

**SET063+4.p** The intersection of a set and empty set is empty
include('Axioms/SET006+0.ax')
$\forall a: \text{equal\_set}(\text{intersection}(a, \text{empty\_set}), \text{empty\_set})$ fof(thI$_{17}$, conjecture)

**SET063-6.p** If X is a subset of the empty set, then X is the empty set
include('Axioms/SET004-0.ax')
$\text{subclass}(x, \text{null\_class})$ cnf(prove_corollary_of_null_class_is_subclass$_1$, negated_conjecture)
$x \neq \text{null\_class}$ cnf(prove_corollary_of_null_class_is_subclass$_2$, negated_conjecture)

**SET063-7.p** If X is a subset of the empty set, then X is the empty set
include('Axioms/SET004-0.ax')
$\text{ordered\_pair}(x, y) \in \text{cross\_product}(u, v) \Rightarrow x \in \text{unordered\_pair}(x, y)$ cnf(corollary_1_to_unordered_pair, axiom)
$\text{ordered\_pair}(x, y) \in \text{cross\_product}(u, v) \Rightarrow y \in \text{unordered\_pair}(x, y)$ cnf(corollary_2_to_unordered_pair, axiom)
$\text{ordered\_pair}(u, v) \in \text{cross\_product}(x, y) \Rightarrow u \in \text{universal\_class}$ cnf(corollary_1_to_cartesian_product, axiom)
$\text{ordered\_pair}(u, v) \in \text{cross\_product}(x, y) \Rightarrow v \in \text{universal\_class}$ cnf(corollary_2_to_cartesian_product, axiom)
$\text{subclass}(x, x)$ cnf(subclass_is_reflexive, axiom)
$(\text{subclass}(x, y) \text{ and } \text{subclass}(y, z)) \Rightarrow \text{subclass}(x, z)$ cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \text{not\_subclass\_element}(x, y) \in x \text{ or } \text{not\_subclass\_element}(y, x) \in y$ cnf(equality$_1$, axiom)
$\text{not\_subclass\_element}(x, y) \in y \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(y, x) \in y)$ cnf(equality$_2$, axiom)
$\text{not\_subclass\_element}(y, x) \in x \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(x, y) \in x)$ cnf(equality$_3$, axiom)
$(\text{not\_subclass\_element}(x, y) \in y \text{ and } \text{not\_subclass\_element}(y, x) \in x) \Rightarrow x = y$ cnf(equality$_4$, axiom)
$\neg\, y \in \text{intersection}(x', x)$ cnf(special_classes_lemma, axiom)
$\neg\, z \in \text{null\_class}$ cnf(existence_of_null_class, axiom)
$\text{subclass}(\text{null\_class}, x)$ cnf(null_class_is_subclass, axiom)
$\text{subclass}(x, \text{null\_class})$ cnf(prove_corollary_of_null_class_is_subclass$_1$, negated_conjecture)
$x \neq \text{null\_class}$ cnf(prove_corollary_of_null_class_is_subclass$_2$, negated_conjecture)

**SET063^5.p** TPS problem BOOL-PROP-30
Trybulec's 30th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow \$false) \Rightarrow x = (\lambda xx: a: \$false))$ thf(cBOOL_PROP_30_pme, conjecture)

**SET064+1.p** Uniqueness of null class
include('Axioms/SET005+0.ax')
$\forall z: (z = \text{null\_class} \text{ or } \exists y: y \in z)$ fof(null_class_is_unique, conjecture)

**SET064-6.p** The null class is unique
include('Axioms/SET004-0.ax')
$z \neq$ null_class  cnf(prove_null_class_is_unique$_1$, negated_conjecture)
$\neg$ not_subclass_element($z$, null_class) $\in z$  cnf(prove_null_class_is_unique$_2$, negated_conjecture)

**SET064-7.p** The null class is unique
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $x \in$ unordered_pair($x, y$)  cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $y \in$ unordered_pair($x, y$)  cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $u \in$ universal_class  cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $v \in$ universal_class  cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)  cnf(subclass_is_reflexive, axiom)
(subclass($x, y$) and subclass($y, z$)) $\Rightarrow$ subclass($x, z$)  cnf(transitivity_of_subclass, axiom)
$x = y$ or not_subclass_element($x, y$) $\in x$ or not_subclass_element($y, x$) $\in y$  cnf(equality$_1$, axiom)
not_subclass_element($x, y$) $\in y$ $\Rightarrow$ ($x = y$ or not_subclass_element($y, x$) $\in y$)  cnf(equality$_2$, axiom)
not_subclass_element($y, x$) $\in x$ $\Rightarrow$ ($x = y$ or not_subclass_element($x, y$) $\in x$)  cnf(equality$_3$, axiom)
(not_subclass_element($x, y$) $\in y$ and not_subclass_element($y, x$) $\in x$) $\Rightarrow$ $x = y$  cnf(equality$_4$, axiom)
$\neg\, y \in$ intersection($x'$, $x$)  cnf(special_classes_lemma, axiom)
$\neg\, z \in$ null_class  cnf(existence_of_null_class, axiom)
subclass(null_class, $x$)  cnf(null_class_is_subclass, axiom)
subclass($x$, null_class) $\Rightarrow$ $x =$ null_class  cnf(corollary_of_null_class_is_subclass, axiom)
$z \neq$ null_class  cnf(prove_null_class_is_unique$_1$, negated_conjecture)
$\neg$ not_subclass_element($z$, null_class) $\in z$  cnf(prove_null_class_is_unique$_2$, negated_conjecture)

**SET065+1.p** Null class is a set (follows from axiom of infinity)
include('Axioms/SET005+0.ax')
null_class $\in$ universal_class  fof(null_class_is_a_set, conjecture)

**SET065-6.p** The null class is a set
include('Axioms/SET004-0.ax')
$\neg$ null_class $\in$ universal_class  cnf(prove_null_class_is_a_set$_1$, negated_conjecture)

**SET065-7.p** The null class is a set
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $x \in$ unordered_pair($x, y$)  cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $y \in$ unordered_pair($x, y$)  cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $u \in$ universal_class  cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) $\in$ cross_product($x, y$) $\Rightarrow$ $v \in$ universal_class  cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)  cnf(subclass_is_reflexive, axiom)
(subclass($x, y$) and subclass($y, z$)) $\Rightarrow$ subclass($x, z$)  cnf(transitivity_of_subclass, axiom)
$x = y$ or not_subclass_element($x, y$) $\in x$ or not_subclass_element($y, x$) $\in y$  cnf(equality$_1$, axiom)
not_subclass_element($x, y$) $\in y$ $\Rightarrow$ ($x = y$ or not_subclass_element($y, x$) $\in y$)  cnf(equality$_2$, axiom)
not_subclass_element($y, x$) $\in x$ $\Rightarrow$ ($x = y$ or not_subclass_element($x, y$) $\in x$)  cnf(equality$_3$, axiom)
(not_subclass_element($x, y$) $\in y$ and not_subclass_element($y, x$) $\in x$) $\Rightarrow$ $x = y$  cnf(equality$_4$, axiom)
$\neg\, y \in$ intersection($x'$, $x$)  cnf(special_classes_lemma, axiom)
$\neg\, z \in$ null_class  cnf(existence_of_null_class, axiom)
subclass(null_class, $x$)  cnf(null_class_is_subclass, axiom)
subclass($x$, null_class) $\Rightarrow$ $x =$ null_class  cnf(corollary_of_null_class_is_subclass, axiom)
$z =$ null_class or not_subclass_element($z$, null_class) $\in z$  cnf(null_class_is_unique, axiom)
$\neg$ null_class $\in$ universal_class  cnf(prove_null_class_is_a_set$_1$, negated_conjecture)

**SET066+1.p** Unordered pair is commutative
include('Axioms/SET005+0.ax')
$\forall x, y$: unordered_pair($x, y$) = unordered_pair($y, x$)  fof(commutativity_of_unordered_pair, conjecture)

**SET066-6.p** Unordered pair is commutative
include('Axioms/SET004-0.ax')
unordered_pair($x, y$) $\neq$ unordered_pair($y, x$)  cnf(prove_commutativity_of_unordered_pair$_1$, negated_conjecture)

**SET066-7.p** Unordered pair is commutative
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $x \in$ unordered_pair($x, y$)  cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair($x, y$) $\in$ cross_product($u, v$) $\Rightarrow$ $y \in$ unordered_pair($x, y$)  cnf(corollary_2_to_unordered_pair, axiom)

ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $u$ ∈ universal_class    cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $v$ ∈ universal_class    cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)    cnf(subclass_is_reflexive, axiom)
(subclass($x, y$) and subclass($y, z$)) ⇒ subclass($x, z$)    cnf(transitivity_of_subclass, axiom)
$x = y$ or not_subclass_element($x, y$) ∈ $x$ or not_subclass_element($y, x$) ∈ $y$    cnf(equality$_1$, axiom)
not_subclass_element($x, y$) ∈ $y$ ⇒ ($x = y$ or not_subclass_element($y, x$) ∈ $y$)    cnf(equality$_2$, axiom)
not_subclass_element($y, x$) ∈ $x$ ⇒ ($x = y$ or not_subclass_element($x, y$) ∈ $x$)    cnf(equality$_3$, axiom)
(not_subclass_element($x, y$) ∈ $y$ and not_subclass_element($y, x$) ∈ $x$) ⇒ $x = y$    cnf(equality$_4$, axiom)
¬ $y$ ∈ intersection($x', x$)    cnf(special_classes_lemma, axiom)
¬ $z$ ∈ null_class    cnf(existence_of_null_class, axiom)
subclass(null_class, $x$)    cnf(null_class_is_subclass, axiom)
subclass($x$, null_class) ⇒ $x$ = null_class    cnf(corollary_of_null_class_is_subclass, axiom)
$z$ = null_class or not_subclass_element($z$, null_class) ∈ $z$    cnf(null_class_is_unique, axiom)
null_class ∈ universal_class    cnf(null_class_is_a_set, axiom)
unordered_pair($x, y$) ≠ unordered_pair($y, x$)    cnf(prove_commutativity_of_unordered_pair$_1$, negated_conjecture)

**SET066^1.p** Unordered pair is commutative
include('Axioms/SET008^0.ax')
∀$x$: \$i, $y$: \$i: (unord_pair@$x$@$y$) = (unord_pair@$y$@$x$)    thf(thm, conjecture)

**SET067+1.p** If one argument is a proper class, pair contains only the other
include('Axioms/SET005+0.ax')
∀$x, y$: subclass(unordered_pair($x, x$), unordered_pair($x, y$))    fof(pair_contains_other, conjecture)

**SET067-6.p** Proper class in an unordered pair, part 1
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
¬ subclass(singleton($x$), unordered_pair($x, y$))    cnf(prove_singleton_in_unordered_pair1$_1$, negated_conjecture)

**SET067-7.p** Proper class in an unordered pair, part 1
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) ∈ cross_product($u, v$) ⇒ $x$ ∈ unordered_pair($x, y$)    cnf(corollary_1_to_unordered_pair, axiom)
ordered_pair($x, y$) ∈ cross_product($u, v$) ⇒ $y$ ∈ unordered_pair($x, y$)    cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $u$ ∈ universal_class    cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $v$ ∈ universal_class    cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)    cnf(subclass_is_reflexive, axiom)
(subclass($x, y$) and subclass($y, z$)) ⇒ subclass($x, z$)    cnf(transitivity_of_subclass, axiom)
$x = y$ or not_subclass_element($x, y$) ∈ $x$ or not_subclass_element($y, x$) ∈ $y$    cnf(equality$_1$, axiom)
not_subclass_element($x, y$) ∈ $y$ ⇒ ($x = y$ or not_subclass_element($y, x$) ∈ $y$)    cnf(equality$_2$, axiom)
not_subclass_element($y, x$) ∈ $x$ ⇒ ($x = y$ or not_subclass_element($x, y$) ∈ $x$)    cnf(equality$_3$, axiom)
(not_subclass_element($x, y$) ∈ $y$ and not_subclass_element($y, x$) ∈ $x$) ⇒ $x = y$    cnf(equality$_4$, axiom)
¬ $y$ ∈ intersection($x', x$)    cnf(special_classes_lemma, axiom)
¬ $z$ ∈ null_class    cnf(existence_of_null_class, axiom)
subclass(null_class, $x$)    cnf(null_class_is_subclass, axiom)
subclass($x$, null_class) ⇒ $x$ = null_class    cnf(corollary_of_null_class_is_subclass, axiom)
$z$ = null_class or not_subclass_element($z$, null_class) ∈ $z$    cnf(null_class_is_unique, axiom)
null_class ∈ universal_class    cnf(null_class_is_a_set, axiom)
unordered_pair($x, y$) = unordered_pair($y, x$)    cnf(commutativity_of_unordered_pair, axiom)
¬ subclass(singleton($x$), unordered_pair($x, y$))    cnf(prove_singleton_in_unordered_pair1$_1$, negated_conjecture)

**SET067^1.p** If one argument is a proper class, pair contains only the other
include('Axioms/SET008^0.ax')
∀$x$: \$i, $y$: \$i: (⊆ @(unord_pair@$x$@$y$)@(unord_pair@$y$@$x$))    thf(thm, conjecture)

**SET068-6.p** Proper class in an unordered pair, part 2
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
¬ subclass(singleton($y$), unordered_pair($x, y$))    cnf(prove_singleton_in_unordered_pair2$_1$, negated_conjecture)

**SET068-7.p** Proper class in an unordered pair, part 2
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')

$\text{ordered\_pair}(x,y) \in \text{cross\_product}(u,v) \Rightarrow x \in \text{unordered\_pair}(x,y)$ cnf(corollary_1_to_unordered_pair, axiom)
$\text{ordered\_pair}(x,y) \in \text{cross\_product}(u,v) \Rightarrow y \in \text{unordered\_pair}(x,y)$ cnf(corollary_2_to_unordered_pair, axiom)
$\text{ordered\_pair}(u,v) \in \text{cross\_product}(x,y) \Rightarrow u \in \text{universal\_class}$ cnf(corollary_1_to_cartesian_product, axiom)
$\text{ordered\_pair}(u,v) \in \text{cross\_product}(x,y) \Rightarrow v \in \text{universal\_class}$ cnf(corollary_2_to_cartesian_product, axiom)
$\text{subclass}(x,x)$ cnf(subclass_is_reflexive, axiom)
$(\text{subclass}(x,y) \text{ and } \text{subclass}(y,z)) \Rightarrow \text{subclass}(x,z)$ cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \text{not\_subclass\_element}(x,y) \in x \text{ or } \text{not\_subclass\_element}(y,x) \in y$ cnf($\text{equality}_1$, axiom)
$\text{not\_subclass\_element}(x,y) \in y \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(y,x) \in y)$ cnf($\text{equality}_2$, axiom)
$\text{not\_subclass\_element}(y,x) \in x \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(x,y) \in x)$ cnf($\text{equality}_3$, axiom)
$(\text{not\_subclass\_element}(x,y) \in y \text{ and } \text{not\_subclass\_element}(y,x) \in x) \Rightarrow x = y$ cnf($\text{equality}_4$, axiom)
$\neg\, y \in \text{intersection}(x',x)$ cnf(special_classes_lemma, axiom)
$\neg\, z \in \text{null\_class}$ cnf(existence_of_null_class, axiom)
$\text{subclass}(\text{null\_class},x)$ cnf(null_class_is_subclass, axiom)
$\text{subclass}(x,\text{null\_class}) \Rightarrow x = \text{null\_class}$ cnf(corollary_of_null_class_is_subclass, axiom)
$z = \text{null\_class} \text{ or } \text{not\_subclass\_element}(z,\text{null\_class}) \in z$ cnf(null_class_is_unique, axiom)
$\text{null\_class} \in \text{universal\_class}$ cnf(null_class_is_a_set, axiom)
$\text{unordered\_pair}(x,y) = \text{unordered\_pair}(y,x)$ cnf(commutativity_of_unordered_pair, axiom)
$\neg\, \text{subclass}(\text{singleton}(y),\text{unordered\_pair}(x,y))$ cnf($\text{prove\_singleton\_in\_unordered\_pair2}_1$, negated_conjecture)

**SET069+1.p** If one argument is a proper class, pair contains only the other
include('Axioms/SET005+0.ax')
$\forall x,y\colon (\neg\, y \in \text{universal\_class} \Rightarrow \text{unordered\_pair}(x,y) = \text{singleton}(x))$ fof($\text{pair\_contains\_only\_other}_2$, conjecture)

**SET069-6.p** Proper class in an unordered pair, part 3
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
$\neg\, y \in \text{universal\_class}$ cnf($\text{prove\_unordered\_pair\_equals\_singleton1}_1$, negated_conjecture)
$\text{unordered\_pair}(x,y) \neq \text{singleton}(x)$ cnf($\text{prove\_unordered\_pair\_equals\_singleton1}_2$, negated_conjecture)

**SET069-7.p** Proper class in an unordered pair, part 3
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
$\text{ordered\_pair}(x,y) \in \text{cross\_product}(u,v) \Rightarrow x \in \text{unordered\_pair}(x,y)$ cnf(corollary_1_to_unordered_pair, axiom)
$\text{ordered\_pair}(x,y) \in \text{cross\_product}(u,v) \Rightarrow y \in \text{unordered\_pair}(x,y)$ cnf(corollary_2_to_unordered_pair, axiom)
$\text{ordered\_pair}(u,v) \in \text{cross\_product}(x,y) \Rightarrow u \in \text{universal\_class}$ cnf(corollary_1_to_cartesian_product, axiom)
$\text{ordered\_pair}(u,v) \in \text{cross\_product}(x,y) \Rightarrow v \in \text{universal\_class}$ cnf(corollary_2_to_cartesian_product, axiom)
$\text{subclass}(x,x)$ cnf(subclass_is_reflexive, axiom)
$(\text{subclass}(x,y) \text{ and } \text{subclass}(y,z)) \Rightarrow \text{subclass}(x,z)$ cnf(transitivity_of_subclass, axiom)
$x = y \text{ or } \text{not\_subclass\_element}(x,y) \in x \text{ or } \text{not\_subclass\_element}(y,x) \in y$ cnf($\text{equality}_1$, axiom)
$\text{not\_subclass\_element}(x,y) \in y \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(y,x) \in y)$ cnf($\text{equality}_2$, axiom)
$\text{not\_subclass\_element}(y,x) \in x \Rightarrow (x = y \text{ or } \text{not\_subclass\_element}(x,y) \in x)$ cnf($\text{equality}_3$, axiom)
$(\text{not\_subclass\_element}(x,y) \in y \text{ and } \text{not\_subclass\_element}(y,x) \in x) \Rightarrow x = y$ cnf($\text{equality}_4$, axiom)
$\neg\, y \in \text{intersection}(x',x)$ cnf(special_classes_lemma, axiom)
$\neg\, z \in \text{null\_class}$ cnf(existence_of_null_class, axiom)
$\text{subclass}(\text{null\_class},x)$ cnf(null_class_is_subclass, axiom)
$\text{subclass}(x,\text{null\_class}) \Rightarrow x = \text{null\_class}$ cnf(corollary_of_null_class_is_subclass, axiom)
$z = \text{null\_class} \text{ or } \text{not\_subclass\_element}(z,\text{null\_class}) \in z$ cnf(null_class_is_unique, axiom)
$\text{null\_class} \in \text{universal\_class}$ cnf(null_class_is_a_set, axiom)
$\text{unordered\_pair}(x,y) = \text{unordered\_pair}(y,x)$ cnf(commutativity_of_unordered_pair, axiom)
$\neg\, y \in \text{universal\_class}$ cnf($\text{prove\_unordered\_pair\_equals\_singleton1}_1$, negated_conjecture)
$\text{unordered\_pair}(x,y) \neq \text{singleton}(x)$ cnf($\text{prove\_unordered\_pair\_equals\_singleton1}_2$, negated_conjecture)

**SET070-6.p** Proper class in an unordered pair, part 4
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
$\neg\, x \in \text{universal\_class}$ cnf($\text{prove\_unordered\_pair\_equals\_singleton2}_1$, negated_conjecture)
$\text{unordered\_pair}(x,y) \neq \text{singleton}(y)$ cnf($\text{prove\_unordered\_pair\_equals\_singleton2}_2$, negated_conjecture)

**SET070-7.p** Proper class in an unordered pair, part 4
If one argument of an unordered pair is a proper class, the pair contains only the other.
include('Axioms/SET004-0.ax')
$\text{ordered\_pair}(x,y) \in \text{cross\_product}(u,v) \Rightarrow x \in \text{unordered\_pair}(x,y)$ cnf(corollary_1_to_unordered_pair, axiom)

ordered_pair($x, y$) ∈ cross_product($u, v$) ⇒ $y$ ∈ unordered_pair($x, y$)    cnf(corollary_2_to_unordered_pair, axiom)
ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $u$ ∈ universal_class    cnf(corollary_1_to_cartesian_product, axiom)
ordered_pair($u, v$) ∈ cross_product($x, y$) ⇒ $v$ ∈ universal_class    cnf(corollary_2_to_cartesian_product, axiom)
subclass($x, x$)    cnf(subclass_is_reflexive, axiom)
(subclass($x, y$) and subclass($y, z$)) ⇒ subclass($x, z$)    cnf(transitivity_of_subclass, axiom)
$x = y$ or not_subclass_element($x, y$) ∈ $x$ or not_subclass_element($y, x$) ∈ $y$    cnf(equality$_1$, axiom)
not_subclass_element($x, y$) ∈ $y$ ⇒ ($x = y$ or not_subclass_element($y, x$) ∈ $y$)    cnf(equality$_2$, axiom)
not_subclass_element($y, x$) ∈ $x$ ⇒ ($x = y$ or not_subclass_element($x, y$) ∈ $x$)    cnf(equality$_3$, axiom)
(not_subclass_element($x, y$) ∈ $y$ and not_subclass_element($y, x$) ∈ $x$) ⇒ $x = y$    cnf(equality$_4$, axiom)
¬ $y$ ∈ intersection($x', x$)    cnf(special_classes_lemma, axiom)
¬ $z$ ∈ null_class    cnf(existence_of_null_class, axiom)
subclass(null_class, $x$)    cnf(null_class_is_subclass, axiom)
subclass($x$, null_class) ⇒ $x$ = null_class    cnf(corollary_of_null_class_is_subclass, axiom)
$z$ = null_class or not_subclass_element($z$, null_class) ∈ $z$    cnf(null_class_is_unique, axiom)
null_class ∈ universal_class    cnf(null_class_is_a_set, axiom)
unordered_pair($x, y$) = unordered_pair($y, x$)    cnf(commutativity_of_unordered_pair, axiom)
¬ $x$ ∈ universal_class    cnf(prove_unordered_pair_equals_singleton2$_1$, negated_conjecture)
unordered_pair($x, y$) ≠ singleton($y$)    cnf(prove_unordered_pair_equals_singleton2$_2$, negated_conjecture)

**SET071+1.p** If both arguments are proper classes, pair is null
include('Axioms/SET005+0.ax')
∀$x, y$: ((¬ $x$ ∈ universal_class and ¬ $y$ ∈ universal_class) ⇒ unordered_pair($x, y$) = null_class)    fof(null_unordered_pair, con

**SET071-6.p** Null unordered pair
If both arguments of an unordered pair are proper classes, the pair is null.
include('Axioms/SET004-0.ax')
unordered_pair($x, y$) ≠ null_class    cnf(prove_null_unordered_pair$_1$, negated_conjecture)
¬ $x$ ∈ universal_class    cnf(prove_null_unordered_pair$_2$, negated_conjecture)
¬ $y$ ∈ universal_class    cnf(prove_null_unordered_pair$_3$, negated_conjecture)

**SET072+1.p** Right cancellation for unordered pairs
include('Axioms/SET005+0.ax')
∀$x, y, z$: (($x$ ∈ universal_class and $y$ ∈ universal_class and unordered_pair($x, z$) = unordered_pair($y, z$)) ⇒ $x = y$)    fof(right_cancellation, conjecture)

**SET072-6.p** Right cancellation for unordered pairs
include('Axioms/SET004-0.ax')
unordered_pair($x, z$) = unordered_pair($y, z$)    cnf(prove_right_cancellation$_1$, negated_conjecture)
ordered_pair($x, y$) ∈ cross_product(universal_class, universal_class)    cnf(prove_right_cancellation$_2$, negated_conjecture)
$x \neq y$    cnf(prove_right_cancellation$_3$, negated_conjecture)

**SET073+1.p** Corollary to unordered pair axiom
include('Axioms/SET005+0.ax')
∀$x, y$: ($x$ ∈ universal_class ⇒ unordered_pair($x, y$) ≠ null_class)    fof(corollary1$_1$, conjecture)

**SET073-6.p** Corollary to unordered pair axiom
include('Axioms/SET004-0.ax')
$x$ ∈ universal_class    cnf(prove_corollary_to_unordered_pair_axiom1$_1$, negated_conjecture)
unordered_pair($x, y$) = null_class    cnf(prove_corollary_to_unordered_pair_axiom1$_2$, negated_conjecture)

**SET074+1.p** Corollary to unordered pair axiom
include('Axioms/SET005+0.ax')
∀$x, y$: ($y$ ∈ universal_class ⇒ unordered_pair($x, y$) ≠ null_class)    fof(corollary1$_2$, conjecture)

**SET074-6.p** Corollary to unordered pair axiom
include('Axioms/SET004-0.ax')
$y$ ∈ universal_class    cnf(prove_corollary_to_unordered_pair_axiom2$_1$, negated_conjecture)
unordered_pair($x, y$) = null_class    cnf(prove_corollary_to_unordered_pair_axiom2$_2$, negated_conjecture)

**SET075-6.p** Corollary to unordered pair axiom
include('Axioms/SET004-0.ax')
ordered_pair($x, y$) ∈ cross_product($u, v$)    cnf(prove_corollary_to_unordered_pair_axiom3$_1$, negated_conjecture)
unordered_pair($x, y$) = null_class    cnf(prove_corollary_to_unordered_pair_axiom3$_2$, negated_conjecture)

**SET076+1.p** If both members of a pair belong to a set, the pair is a subset

include('Axioms/SET005+0.ax')
$\forall x, y, z$: $((x \in z \text{ and } y \in z) \Rightarrow \text{subclass}(\text{unordered\_pair}(x, y), z))$ fof(unordered_pair_is_subset, conjecture)

**SET076-6.p** Unorderd pair is a subset
If both members of an unordered pair belong to a set, the pair is a subset.
include('Axioms/SET004-0.ax')
$x \in z$ cnf(prove_unordered_pair_is_subset$_1$, negated_conjecture)
$y \in z$ cnf(prove_unordered_pair_is_subset$_2$, negated_conjecture)
$\neg\, \text{subclass}(\text{unordered\_pair}(x, y), z)$ cnf(prove_unordered_pair_is_subset$_3$, negated_conjecture)

**SET076^1.p** If both members of a pair belong to a set, the pair is a subset
include('Axioms/SET008^0.ax')
$\forall x$: \$i, $y$: \$i, $z$: \$i → \$o: $((z@x \text{ and } z@y) \Rightarrow (\subseteq @(\text{unord\_pair}@x@y)@z))$ thf(thm, conjecture)

**SET077+1.p** Every singleton is a set
include('Axioms/SET005+0.ax')
$\forall x$: $\text{singleton}(x) \in \text{universal\_class}$ fof(singletons_are_sets, conjecture)

**SET077-6.p** Every singleton is a set
include('Axioms/SET004-0.ax')
$\neg\, \text{singleton}(x) \in \text{universal\_class}$ cnf(prove_singletons_are_sets$_1$, negated_conjecture)

**SET078-6.p** Corollary to every singleton is a set
include('Axioms/SET004-0.ax')
$\neg\, \text{singleton}(y) \in \text{unordered\_pair}(x, \text{singleton}(y))$ cnf(prove_corollary_1_to_singletons_are_sets$_1$, negated_conjecture)

**SET079+1.p** A set belongs to its singleton
include('Axioms/SET005+0.ax')
$\forall x$: $(x \in \text{universal\_class} \Rightarrow \text{singleton}(x) \neq \text{null\_class})$ fof(corollary_to_set_in_its_singleton, conjecture)

**SET079-6.p** Corollary to a set belongs to its singleton
include('Axioms/SET004-0.ax')
$x \in \text{universal\_class}$ cnf(prove_corollary_to_set_in_its_singleton$_1$, negated_conjecture)
$\text{singleton}(x) = \text{null\_class}$ cnf(prove_corollary_to_set_in_its_singleton$_2$, negated_conjecture)

**SET080-6.p** Corollary to a set belongs to its singleton
include('Axioms/SET004-0.ax')
$\neg\, \text{null\_class} \in \text{singleton}(\text{null\_class})$ cnf(prove_null_class_in_its_singleton$_1$, negated_conjecture)

**SET081+1.p** Only X can belong to X
include('Axioms/SET005+0.ax')
$\forall x, y$: $(y \in \text{singleton}(x) \Rightarrow y = x)$ fof(only_member_in_singleton, conjecture)

**SET081-6.p** Only the element can belong to its singleton
include('Axioms/SET004-0.ax')
$y \in \text{singleton}(x)$ cnf(prove_only_member_in_singleton$_1$, negated_conjecture)
$y \neq x$ cnf(prove_only_member_in_singleton$_2$, negated_conjecture)

**SET082+1.p** If X is not a set, X = null class
include('Axioms/SET005+0.ax')
$\forall x$: $(\neg\, x \in \text{universal\_class} \Rightarrow \text{singleton}(x) = \text{null\_class})$ fof(singleton_is_null_class, conjecture)

**SET082-6.p** The singleton of a non-set is the null class
include('Axioms/SET004-0.ax')
$\neg\, x \in \text{universal\_class}$ cnf(prove_singleton_is_null_class$_1$, negated_conjecture)
$\text{singleton}(x) \neq \text{null\_class}$ cnf(prove_singleton_is_null_class$_2$, negated_conjecture)

**SET083+1.p** A singleton set is determined by its element
include('Axioms/SET005+0.ax')
$\forall x, y$: $((\text{singleton}(x) = \text{singleton}(y) \text{ and } x \in \text{universal\_class}) \Rightarrow x = y)$ fof(singleton_identified_by_element$_1$, conjecture)

**SET083-6.p** A singleton set depends on its element, part 1
include('Axioms/SET004-0.ax')
$\text{singleton}(x) = \text{singleton}(y)$ cnf(prove_singleton_identified_by_element1$_1$, negated_conjecture)
$x \in \text{universal\_class}$ cnf(prove_singleton_identified_by_element1$_2$, negated_conjecture)
$x \neq y$ cnf(prove_singleton_identified_by_element1$_3$, negated_conjecture)

**SET084+1.p** A singleton set is determined by its element
include('Axioms/SET005+0.ax')

$\forall x, y$: ((singleton($x$) = singleton($y$) and $y \in$ universal_class) $\Rightarrow$ $x = y$) fof(singleton_identified_by_element$_2$, conjecture)

**SET084-6.p** A singleton set depends on its element, part 2
include('Axioms/SET004-0.ax')
singleton($x$) = singleton($y$) cnf(prove_singleton_identified_by_element2$_1$, negated_conjecture)
$y \in$ universal_class cnf(prove_singleton_identified_by_element2$_2$, negated_conjecture)
$x \neq y$ cnf(prove_singleton_identified_by_element2$_3$, negated_conjecture)

**SET085-6.p** Unordered pair that is a singleton
include('Axioms/SET004-0.ax')
unordered_pair($y, z$) = singleton($x$) cnf(prove_singleton_in_unordered_pair3$_1$, negated_conjecture)
$x \in$ universal_class cnf(prove_singleton_in_unordered_pair3$_2$, negated_conjecture)
$x \neq y$ cnf(prove_singleton_in_unordered_pair3$_3$, negated_conjecture)
$x \neq z$ cnf(prove_singleton_in_unordered_pair3$_4$, negated_conjecture)

**SET086+1.p** A singleton set has a member
include('Axioms/SET005+0.ax')
$\forall x$: $\exists u$: (($u \in$ universal_class and $x$ = singleton($u$)) or ($\neg \exists y$: ($y \in$ universal_class and $x$ = singleton($y$)) and $u = x$)) fof(member_of_substitution, conjecture)

**SET086-6.p** A singleton set has a member, part 1
include('Axioms/SET004-0.ax')
$y \in$ universal_class cnf(prove_member_exists1$_1$, negated_conjecture)
$\neg$ member_of(singleton($y$)) $\in$ universal_class cnf(prove_member_exists1$_2$, negated_conjecture)

**SET086^1.p** A singleton set has a member
include('Axioms/SET008^0.ax')
$\forall x$: \$i: $\exists y$: \$i: (singleton@$x$@$y$) thf(thm, conjecture)

**SET087-6.p** A singleton set has a member, part 2
include('Axioms/SET004-0.ax')
$y \in$ universal_class cnf(prove_member_exists2$_1$, negated_conjecture)
singleton(member_of(singleton($y$))) $\neq$ singleton($y$) cnf(prove_member_exists2$_2$, negated_conjecture)

**SET088-6.p** A singleton set has a member, part 3
include('Axioms/SET004-0.ax')
$\neg$ member_of($x$) $\in$ universal_class cnf(prove_member_exists3$_1$, negated_conjecture)
member_of($x$) $\neq x$ cnf(prove_member_exists3$_2$, negated_conjecture)

**SET089-6.p** A singleton set has a member, part 4
include('Axioms/SET004-0.ax')
singleton(member_of($x$)) $\neq x$ cnf(prove_member_exists4$_1$, negated_conjecture)
member_of($x$) $\neq x$ cnf(prove_member_exists4$_2$, negated_conjecture)

**SET090+1.p** Uniqueness of member_of of a singleton set
include('Axioms/SET005+0.ax')
$\forall x, u$: (($u \in$ universal_class and $x$ = singleton($u$)) $\Rightarrow$ member_of($x$) = $u$) fof(member_of_singleton, conjecture)

**SET090-6.p** The member of a singleton set is unique
include('Axioms/SET004-0.ax')
$u \in$ universal_class cnf(prove_member_of_singleton_is_unique$_1$, negated_conjecture)
member_of(singleton($u$)) $\neq u$ cnf(prove_member_of_singleton_is_unique$_2$, negated_conjecture)

**SET091+1.p** Uniqueness of member_of when X is not a singleton of a set
include('Axioms/SET005+0.ax')
$\forall x, u$: (($\neg \exists y$: ($y \in$ universal_class and $x$ = singleton($y$)) and $x = u$) $\Rightarrow$ member_of($x$) = $u$) fof(member_when_not_a_singl

**SET091-6.p** Member_of(X) is unique if X is not a singleton, part 1
include('Axioms/SET004-0.ax')
$\neg$ member_of$_1$($x$) $\in$ universal_class cnf(prove_member_of_non_singleton_unique1$_1$, negated_conjecture)
member_of($x$) $\neq x$ cnf(prove_member_of_non_singleton_unique1$_2$, negated_conjecture)

**SET092-6.p** Member_of(X) is unique if X is not a singleton, part 2
include('Axioms/SET004-0.ax')
singleton(member_of$_1$($x$)) $\neq x$ cnf(prove_member_of_non_singleton_unique2$_1$, negated_conjecture)
member_of($x$) $\neq x$ cnf(prove_member_of_non_singleton_unique2$_2$, negated_conjecture)

**SET093+1.p** Corollary to every singleton is a set

include('Axioms/SET005+0.ax')
$\forall x$: (singleton(member_of($x$)) $= x \Rightarrow x \in$ universal_class) fof(corollary_2_to_singletons_are_sets, conjecture)

**SET093-6.p** Corollary to every singleton is a set
include('Axioms/SET004-0.ax')
singleton(member_of($x$)) $= x$ cnf(prove_corollary_2_to_singletons_are_sets$_1$, negated_conjecture)
$\neg\, x \in$ universal_class cnf(prove_corollary_2_to_singletons_are_sets$_2$, negated_conjecture)

**SET094+1.p** Property 1 of singletons
include('Axioms/SET005+0.ax')
$\forall x, y$: ((singleton(member_of($x$)) $= x$ and $y \in x$) $\Rightarrow$ member_of($x$) $= y$) fof(property_of_singletons1$_1$, conjecture)

**SET094-6.p** Property 1 of singleton sets
include('Axioms/SET004-0.ax')
singleton(member_of($x$)) $= x$ cnf(prove_property_of_singletons1$_1$, negated_conjecture)
$y \in x$ cnf(prove_property_of_singletons1$_2$, negated_conjecture)
member_of($x$) $\neq y$ cnf(prove_property_of_singletons1$_3$, negated_conjecture)

**SET095+1.p** Property 2 of singletons
include('Axioms/SET005+0.ax')
$\forall x, y$: ($x \in y \Rightarrow$ subclass(singleton($x$), $y$)) fof(property_of_singletons$_2$, conjecture)

**SET095+4.p** If X is in Y, then the singleton containing X is a subset of Y
include('Axioms/SET006+0.ax')
$\forall a, x$: ($x \in a \Rightarrow$ singleton($x$) $\subseteq a$) fof(thI$_{44}$, conjecture)

**SET095-6.p** If X is in Y, then the singleton containing X is a subset of Y
include('Axioms/SET004-0.ax')
$x \in y$ cnf(prove_property_of_singletons2$_1$, negated_conjecture)
$\neg$ subclass(singleton($x$), $y$) cnf(prove_property_of_singletons2$_2$, negated_conjecture)

**SET096+1.p** There are at most two subsets of a singleton set
include('Axioms/SET005+0.ax')
$\forall x, y$: (subclass($x$, singleton($y$)) $\Rightarrow$ ($x =$ null_class or singleton($y$) $= x$)) fof(two_subsets_of_singleton, conjecture)

**SET096-6.p** There are at most two subsets of a singleton set
include('Axioms/SET004-0.ax')
subclass($x$, singleton($y$)) cnf(prove_two_subsets_of_singleton$_1$, negated_conjecture)
$x \neq$ null_class cnf(prove_two_subsets_of_singleton$_2$, negated_conjecture)
singleton($y$) $\neq x$ cnf(prove_two_subsets_of_singleton$_3$, negated_conjecture)

**SET096^1.p** There are at most two subsets of a singleton set
include('Axioms/SET008^0.ax')
$\forall x$: \$i $\rightarrow$ \$o, $y$: \$i: (($\subseteq$ @$x$@(singleton@$y$)) $\Rightarrow$ ($x =$ emptyset or $x =$ (singleton@$y$))) thf(thm, conjecture)

**SET097+1.p** A class contains 0, 1 or at least 2 members.
include('Axioms/SET005+0.ax')
$\forall x$: ($x =$ null_class or $\exists y$: singleton($y$) $= x$ or $\exists v$: ($v \in x$ and $\exists w$: $w \in$ intersection(singleton($v$)$'$, $x$))) fof(number_of_eleme

**SET097-6.p** A class contains 0, 1, or at least 2 members
include('Axioms/SET004-0.ax')
$\neg$ not_subclass_element(intersection(singleton(not_subclass_element($x$, null_class))$'$, $x$), null_class) $\in$ intersection(singleton(not
singleton(not_subclass_element($x$, null_class)) $\neq x$ cnf(prove_number_of_elements_in_class$_2$, negated_conjecture)
$x \neq$ null_class cnf(prove_number_of_elements_in_class$_3$, negated_conjecture)

**SET098+1.p** Corollary 1 to a class contains 0, 1, or at least 2 members
include('Axioms/SET005+0.ax')
$\forall x$: ($x =$ null_class or $\exists y$: singleton($y$) $= x$ or $\exists v$: ($v \in x$ and $\exists w$: ($w \in$ intersection(singleton($v$)$'$, $x$) and $w \in x$))) fof(corollary_1_to_number_of_elements_in_class, conjecture)

**SET098-6.p** Corollary 1 to a class contains 0, 1, or at least 2 members
include('Axioms/SET004-0.ax')
$\neg$ not_subclass_element(intersection(singleton(not_subclass_element($x$, null_class))$'$, $x$), null_class) $\in x$ cnf(prove_corollary_
singleton(not_subclass_element($x$, null_class)) $\neq x$ cnf(prove_corollary_1_to_number_of_elements_in_class$_2$, negated_conjectu
$x \neq$ null_class cnf(prove_corollary_1_to_number_of_elements_in_class$_3$, negated_conjecture)

**SET099+1.p** Corollary 2 to a class contains 0, 1, or at least 2 members
include('Axioms/SET005+0.ax')

∀x: (∀u, v: ((u ∈ x and v ∈ intersection(singleton(u)′, x)) ⇒ u = v) ⇒ (x = null_class or ∃y: singleton(y) = x)) fof(corollary_2_to_number_of_elements_in_class, conjecture)

**SET099-6.p** Corollary 2 to a class contains 0, 1, or at least 2 members
include('Axioms/SET004-0.ax')
not_subclass_element(intersection(singleton(not_subclass_element(x, null_class))′, x), null_class) = not_subclass_element(x, null
singleton(not_subclass_element(x, null_class)) ≠ x cnf(prove_corollary_2_to_number_of_elements_in_class$_2$, negated_conjectu
x ≠ null_class cnf(prove_corollary_2_to_number_of_elements_in_class$_3$, negated_conjecture)

**SET100-6.p** The relationship of singleton sets to ordered pairs
include('Axioms/SET004-0.ax')
unordered_pair(x, y) ≠ union(singleton(x), singleton(y)) cnf(prove_unordered_pairs_and_singletons$_1$, negated_conjecture)

**SET101+1.p** Singleton of the first is a member of an ordered pair
include('Axioms/SET005+0.ax')
∀x, y: singleton(x) ∈ ordered_pair(x, y) fof(singleton_member_of_ordered_pair, conjecture)

**SET101-6.p** Singleton of the first is a member of an ordered pair
include('Axioms/SET004-0.ax')
¬ singleton(x) ∈ ordered_pair(x, y) cnf(prove_singleton_member_of_ordered_pair$_1$, negated_conjecture)

**SET102+1.p** Ordered pair member of ordered pair
include('Axioms/SET005+0.ax')
∀x, y: unordered_pair(x, singleton(y)) ∈ ordered_pair(x, y) fof(unordered_pair_member_of_ordered_pair, conjecture)

**SET102-6.p** Ordered pair member of ordered pair
include('Axioms/SET004-0.ax')
¬ unordered_pair(x, singleton(y)) ∈ ordered_pair(x, y) cnf(prove_unordered_pair_member_of_ordered_pair$_1$, negated_conjec

**SET103+1.p** Special member 1 of an ordered pair
include('Axioms/SET005+0.ax')
∀x, y: (unordered_pair(singleton(x), unordered_pair(x, null_class)) = ordered_pair(x, y) or y ∈ universal_class) fof(property

**SET103-6.p** Special member 1 of an ordered pair
include('Axioms/SET004-0.ax')
unordered_pair(singleton(x), unordered_pair(x, null_class)) ≠ ordered_pair(x, y) cnf(prove_property_1_of_ordered_pair$_1$, neg
¬ y ∈ universal_class cnf(prove_property_1_of_ordered_pair$_2$, negated_conjecture)

**SET104+1.p** Special member 2 of an ordered pair
include('Axioms/SET005+0.ax')
∀x, y: (unordered_pair(null_class, singleton(singleton(y))) = ordered_pair(x, y) or x ∈ universal_class) fof(property_2_of_or

**SET104-6.p** Special member 2 of an ordered pair
include('Axioms/SET004-0.ax')
unordered_pair(null_class, singleton(singleton(y))) ≠ ordered_pair(x, y) cnf(prove_property_2_of_ordered_pair$_2$, negated_co
¬ x ∈ universal_class cnf(prove_property_2_of_ordered_pair$_3$, negated_conjecture)

**SET105+1.p** Special member 3 of an ordered pair
include('Axioms/SET005+0.ax')
∀x, y: (unordered_pair(null_class, singleton(null_class)) = ordered_pair(x, y) or x ∈ universal_class or y ∈ universal_class) f

**SET105-6.p** Special member 3 of an ordered pair
include('Axioms/SET004-0.ax')
unordered_pair(null_class, singleton(null_class)) ≠ ordered_pair(x, y) cnf(prove_property_3_of_ordered_pair$_1$, negated_conje
¬ x ∈ universal_class cnf(prove_property_3_of_ordered_pair$_2$, negated_conjecture)
¬ y ∈ universal_class cnf(prove_property_3_of_ordered_pair$_3$, negated_conjecture)

**SET108+1.p** 1st and 2nd make the ordered pair
include('Axioms/SET005+0.ax')
∀x: ∃u, v: ((u ∈ universal_class and v ∈ universal_class and x = ordered_pair(u, v)) or (¬ ∃y, z: (y ∈ universal_class and z ∈ universal_class and x = ordered_pair(y, z)) and u = x and v = x)) fof(existence_of_first_and_second, conjecture)

**SET108-6.p** 1st and 2nd make the ordered pair
include('Axioms/SET004-0.ax')
ordered_pair(y, z) ∈ cross_product(universal_class, universal_class) cnf(prove_existence_of_1st_and_2nd_1$_1$, negated_conject
¬ ordered_pair(first(ordered_pair(y, z)), second(ordered_pair(y, z))) ∈ cross_product(universal_class, universal_class) cnf(pr

**SET109-6.p** 1st is the ordered pair, first condition
include('Axioms/SET004-0.ax')

$\neg$ ordered_pair(first($x$), second($x$)) $\in$ cross_product(universal_class, universal_class) cnf(prove_existence_of_1st_and_2nd_2$_1$,
first($x$) $\neq x$ cnf(prove_existence_of_1st_and_2nd_2$_2$, negated_conjecture)

**SET110-6.p** 2nd is the ordered pair, first condition
include('Axioms/SET004-0.ax')
$\neg$ ordered_pair(first($x$), second($x$)) $\in$ cross_product(universal_class, universal_class) cnf(prove_existence_of_1st_and_2nd_3$_1$,
second($x$) $\neq x$ cnf(prove_existence_of_1st_and_2nd_3$_2$, negated_conjecture)

**SET111-6.p** 1st is the ordered pair, second condition
include('Axioms/SET004-0.ax')
ordered_pair(first($x$), second($x$)) $\neq x$ cnf(prove_existence_of_1st_and_2nd_4$_1$, negated_conjecture)
first($x$) $\neq x$ cnf(prove_existence_of_1st_and_2nd_4$_2$, negated_conjecture)

**SET112-6.p** 2nd is the ordered pair, second condition
include('Axioms/SET004-0.ax')
ordered_pair(first($x$), second($x$)) $\neq x$ cnf(prove_existence_of_1st_and_2nd_5$_1$, negated_conjecture)
second($x$) $\neq x$ cnf(prove_existence_of_1st_and_2nd_5$_2$, negated_conjecture)

**SET113+1.p** Uniqueness of 1st and 2nd when X is not an ordered pair of sets
include('Axioms/SET005+0.ax')
$\forall u, v, x$: (($\exists y, z$: ($y \in$ universal_class and $z \in$ universal_class and $x =$ ordered_pair($y, z$)) and $x = u$ and $v = x$) or (first($x$) = $u$ and second($x$) = $v$)) fof(unique_1st_and_2nd_in_pair_of_non_sets$_1$, conjecture)

**SET113-6.p** 1st is unique if x is not an ordered pair of sets, part 1
include('Axioms/SET004-0.ax')
$\neg$ ordered_pair(first$_1$($x$), second$_1$($x$)) $\in$ cross_product(universal_class, universal_class) cnf(prove_unique_1st_and_2nd_in_pai
first($x$) $\neq x$ cnf(prove_unique_1st_in_pair_of_non_sets, negated_conjecture)

**SET114-6.p** 2nd is unique if x is not an ordered pair of sets, part 1
include('Axioms/SET004-0.ax')
$\neg$ ordered_pair(first$_1$($x$), second$_1$($x$)) $\in$ cross_product(universal_class, universal_class) cnf(prove_unique_1st_and_2nd_in_pai
second($x$) $\neq x$ cnf(prove_unique_2nd_in_pair_of_non_sets, negated_conjecture)

**SET115-6.p** 1st is unique if x is not an ordered pair of sets, part 2
include('Axioms/SET004-0.ax')
ordered_pair(first$_1$($x$), second$_1$($x$)) $\neq x$ cnf(prove_unique_1st_and_2nd_in_pair_of_non_sets3$_1$, negated_conjecture)
first($x$) $\neq x$ cnf(prove_unique_1st_in_pair_of_non_sets, negated_conjecture)

**SET116-6.p** 2nd is unique if x is not an ordered pair of sets, part 2
include('Axioms/SET004-0.ax')
ordered_pair(first$_1$($x$), second$_1$($x$)) $\neq x$ cnf(prove_unique_1st_and_2nd_in_pair_of_non_sets4$_1$, negated_conjecture)
second($x$) $\neq x$ cnf(prove_unique_2nd_in_pair_of_non_sets, negated_conjecture)

**SET117+1.p** Corollary 1 to every ordered pair being a set
include('Axioms/SET005+0.ax')
$\forall x$: (ordered_pair(first($x$), second($x$)) = $x \Rightarrow x \in$ universal_class) fof(corollary_1_to_ordered_pairs_are_sets, conjecture)

**SET117-6.p** Corollary 1 to every ordered pair being a set
include('Axioms/SET004-0.ax')
ordered_pair(first($x$), second($x$)) = $x$ cnf(prove_corollary_1_to_ordered_pairs_are_sets$_1$, negated_conjecture)
$\neg x \in$ universal_class cnf(prove_corollary_1_to_ordered_pairs_are_sets$_2$, negated_conjecture)

**SET118-6.p** Corollary 2 to every ordered pair being a set
include('Axioms/SET004-0.ax')
$x \in$ cross_product(universal_class, universal_class) cnf(prove_corollary_2_to_ordered_pairs_are_sets$_1$, negated_conjecture)
$\neg x \in$ universal_class cnf(prove_corollary_2_to_ordered_pairs_are_sets$_2$, negated_conjecture)

**SET119+1.p** Corollary 1 to components of equal ordered pairs being equal
include('Axioms/SET005+0.ax')
$\forall x, y$: ($x \in$ universal_class or first(ordered_pair($x, y$)) = ordered_pair($x, y$)) fof(corollary_1_to_OP_determines_components

**SET119-6.p** Corollary 1 to components of equal ordered pairs being equal
include('Axioms/SET004-0.ax')
$\neg x \in$ universal_class cnf(prove_corollary_1_to_OP_determines_components1$_1$, negated_conjecture)
first(ordered_pair($x, y$)) $\neq$ ordered_pair($x, y$) cnf(prove_corollary_1_to_OP_determines_components1$_2$, negated_conjecture)

**SET120+1.p** Corollary 2 to components of equal ordered pairs being equal
include('Axioms/SET005+0.ax')

$\forall x, y$: ($x \in$ universal_class or second(ordered_pair($x, y$)) = ordered_pair($x, y$)) fof(corollary_2_to_OP_determines_componen

**SET120-6.p** Corollary 2 to components of equal ordered pairs being equal
include('Axioms/SET004-0.ax')
$\neg\, x \in$ universal_class cnf(prove_corollary_2_to_OP_determines_components1$_1$, negated_conjecture)
second(ordered_pair($x, y$)) $\neq$ ordered_pair($x, y$) cnf(prove_corollary_2_to_OP_determines_components1$_2$, negated_conjectur

**SET121+1.p** Corollary 3 to components of equal ordered pairs being equal
include('Axioms/SET005+0.ax')
$\forall x, y$: ($y \in$ universal_class or first(ordered_pair($x, y$)) = ordered_pair($x, y$)) fof(corollary_1_to_OP_determines_components

**SET121-6.p** Corollary 3 to components of equal ordered pairs being equal
include('Axioms/SET004-0.ax')
$\neg\, y \in$ universal_class cnf(prove_corollary_1_to_OP_determines_components2$_1$, negated_conjecture)
first(ordered_pair($x, y$)) $\neq$ ordered_pair($x, y$) cnf(prove_corollary_1_to_OP_determines_components2$_2$, negated_conjecture)

**SET122+1.p** Corollary 4 to components of equal ordered pairs being equal
include('Axioms/SET005+0.ax')
$\forall x, y$: ($y \in$ universal_class or second(ordered_pair($x, y$)) = ordered_pair($x, y$)) fof(corollary_2_to_OP_determines_componen

**SET122-6.p** Corollary 4 to components of equal ordered pairs being equal
include('Axioms/SET004-0.ax')
$\neg\, y \in$ universal_class cnf(prove_corollary_2_to_OP_determines_components2$_1$, negated_conjecture)
second(ordered_pair($x, y$)) $\neq$ ordered_pair($x, y$) cnf(prove_corollary_2_to_OP_determines_components2$_2$, negated_conjectur

**SET123-6.p** Alternative definition of set builder, part 1
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
$x \in$ set_builder($y, z$) cnf(prove_set_builder_alternate_defn1$_1$, negated_conjecture)
$x \neq y$ cnf(prove_set_builder_alternate_defn1$_2$, negated_conjecture)
$\neg\, x \in z$ cnf(prove_set_builder_alternate_defn1$_3$, negated_conjecture)

**SET124-6.p** Alternative definition of set builder, part 2
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
$x \in$ universal_class cnf(prove_set_builder_alternate_defn2$_1$, negated_conjecture)
$\neg\, x \in$ set_builder($x, z$) cnf(prove_set_builder_alternate_defn2$_2$, negated_conjecture)

**SET125-6.p** Alternative definition of set builder, part 3
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
$x \in z$ cnf(prove_set_builder_alternate_defn3$_1$, negated_conjecture)
$\neg\, x \in$ set_builder($y, z$) cnf(prove_set_builder_alternate_defn3$_2$, negated_conjecture)

**SET126-6.p** Relation to singleton
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
set_builder($x$, null_class) $\neq$ singleton($x$) cnf(prove_set_builder_and_singleton$_1$, negated_conjecture)

**SET127-6.p** Relation to unordered pair
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
set_builder($x$, singleton($y$)) $\neq$ unordered_pair($x, y$) cnf(prove_set_builder_and_unordered_pair$_1$, negated_conjecture)

**SET128-6.p** Building a triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
union(singleton($x$), union(singleton($y$), singleton($z$))) $\neq$ set_builder($x$, set_builder($y$, set_builder($z$, null_class))) cnf(prove_b

**SET129-6.p** Membership in a built unordered triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x, y$) cnf(definition_of_set_builder, axiom)
$u \in$ set_builder($x$, set_builder($y$, set_builder($z$, null_class))) cnf(prove_members_of_built_triple$_1$, negated_conjecture)
$u \neq x$ cnf(prove_members_of_built_triple$_2$, negated_conjecture)
$u \neq y$ cnf(prove_members_of_built_triple$_3$, negated_conjecture)
$u \neq z$ cnf(prove_members_of_built_triple$_4$, negated_conjecture)

**SET130-6.p** Membership in unordered triple, part 1
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_member_of_triple1$_1$, negated_conjecture)
$\neg\, u \in$ set_builder($u$, set_builder($y$, set_builder($z$, null_class)))    cnf(prove_member_of_triple1$_2$, negated_conjecture)

**SET131-6.p** Membership in unordered triple, part 2
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_member_of_triple2$_1$, negated_conjecture)
$\neg\, u \in$ set_builder($x$, set_builder($u$, set_builder($z$, null_class)))    cnf(prove_member_of_triple2$_2$, negated_conjecture)

**SET132-6.p** Membership in unordered triple, part 3
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_member_of_triple3$_1$, negated_conjecture)
$\neg\, u \in$ set_builder($x$, set_builder($y$, set_builder($u$, null_class)))    cnf(prove_member_of_triple3$_2$, negated_conjecture)

**SET133-6.p** Corollary 1 to membership in unordered triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in x$    cnf(prove_corollary_1_to_member_of_triple$_1$, negated_conjecture)
$v \in x$    cnf(prove_corollary_1_to_member_of_triple$_2$, negated_conjecture)
$w \in x$    cnf(prove_corollary_1_to_member_of_triple$_3$, negated_conjecture)
$\neg$ subclass(set_builder($u$, set_builder($v$, set_builder($w$, null_class))), $x$)    cnf(prove_corollary_1_to_member_of_triple$_4$, negated_

**SET134-6.p** Corollary 2 to membership in unordered triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_corollary_2_to_member_of_triple$_1$, negated_conjecture)
set_builder($u$, set_builder($y$, set_builder($z$, null_class))) = null_class    cnf(prove_corollary_2_to_member_of_triple$_2$, negated_co

**SET135-6.p** Corollary 3 to membership in unordered triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_corollary_3_to_member_of_triple$_1$, negated_conjecture)
set_builder($x$, set_builder($u$, set_builder($z$, null_class))) = null_class    cnf(prove_corollary_3_to_member_of_triple$_2$, negated_co

**SET136-6.p** Corollary 4 to membership in unordered triple
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_corollary_4_to_member_of_triple$_1$, negated_conjecture)
set_builder($x$, set_builder($y$, set_builder($u$, null_class))) = null_class    cnf(prove_corollary_4_to_member_of_triple$_2$, negated_co

**SET137-6.p** Kludge 1 to instantiate variables in unordered triples
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in x$    cnf(prove_member_of_triple_kludge1$_1$, negated_conjecture)
$v \in x$    cnf(prove_member_of_triple_kludge1$_2$, negated_conjecture)
$w \in x$    cnf(prove_member_of_triple_kludge1$_3$, negated_conjecture)
set_builder($u$, set_builder($v$, set_builder($w$, null_class))) = null_class    cnf(prove_member_of_triple_kludge1$_4$, negated_conjectu

**SET138-6.p** Kludge 2 to instantiate variables in unordered triples
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
$u \in$ universal_class    cnf(prove_member_of_triple_kludge2$_1$, negated_conjecture)
$v \in$ universal_class    cnf(prove_member_of_triple_kludge2$_2$, negated_conjecture)
$w \in$ universal_class    cnf(prove_member_of_triple_kludge2$_3$, negated_conjecture)
$\neg\, u \in$ set_builder($u$, set_builder($v$, set_builder($w$, null_class)))    cnf(prove_member_of_triple_kludge2$_4$, negated_conjecture)

**SET139-6.p** Triple reduction 1
include('Axioms/SET004-0.ax')
union(singleton($x$), $y$) = set_builder($x$, $y$)    cnf(definition_of_set_builder, axiom)
set_builder($x$, set_builder($x$, set_builder($y$, null_class))) $\neq$ unordered_pair($x$, $y$)    cnf(prove_triple_reduction1$_1$, negated_conjec

**SET140-6.p** Triple reduction 2
include('Axioms/SET004-0.ax')
$\text{union}(\text{singleton}(x), y) = \text{set\_builder}(x, y)$ cnf(definition_of_set_builder, axiom)
$\text{set\_builder}(x, \text{set\_builder}(y, \text{set\_builder}(x, \text{null\_class}))) \neq \text{unordered\_pair}(x, y)$ cnf(prove_triple_reduction2$_1$, negated_conjec

**SET141-6.p** Triple reduction 3
include('Axioms/SET004-0.ax')
$\text{union}(\text{singleton}(x), y) = \text{set\_builder}(x, y)$ cnf(definition_of_set_builder, axiom)
$\text{set\_builder}(x, \text{set\_builder}(y, \text{set\_builder}(y, \text{null\_class}))) \neq \text{unordered\_pair}(x, y)$ cnf(prove_triple_reduction3$_1$, negated_conjec

**SET142-6.p** Lexical ordering in unordered triples is irrelevant
include('Axioms/SET004-0.ax')
$\text{union}(\text{singleton}(x), y) = \text{set\_builder}(x, y)$ cnf(definition_of_set_builder, axiom)
$\text{set\_builder}(x, \text{set\_builder}(y, \text{set\_builder}(z, \text{null\_class}))) \neq \text{set\_builder}(y, \text{set\_builder}(x, \text{set\_builder}(z, \text{null\_class})))$ cnf(prove_

**SET143+3.p** Associativity of intersection
The intersection of (the intersection of X and Y) and Z is the intersection of X and (the intersection of Y and Z).
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{intersection}(\text{intersection}(b, c), d) = \text{intersection}(b, \text{intersection}(c, d))$ fof(prove_associativity_of_intersection, conject

**SET143+4.p** Associativity of intersection
The intersection of (the intersection of X and Y) and Z is the intersection of X and (the intersection of Y and Z).
include('Axioms/SET006+0.ax')
$\forall a, b, c$: $\text{equal\_set}(\text{intersection}(\text{intersection}(a, b), c), \text{intersection}(a, \text{intersection}(b, c)))$ fof(thI$_{08}$, conjecture)

**SET143-6.p** Associativity of intersection
The intersection of (the intersection of X and Y) and Z is the intersection of X and (the intersection of Y and Z).
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(\text{intersection}(x, y), z) \neq \text{intersection}(x, \text{intersection}(y, z))$ cnf(prove_associativity_of_intersection$_1$, negated_conj

**SET143^3.p** Associativity of intersection
The intersection of (the intersection of X and Y) and Z is the intersection of X and (the intersection of Y and Z).
include('Axioms/SET008^0.ax')
$\forall x$: $\$i \to \$o, y$: $\$i \to \$o, z$: $\$i \to \$o$: $(\text{intersection}@(\text{intersection}@x@y)@z) = (\text{intersection}@x@(\text{intersection}@y@z))$ thf(thm

**SET143^5.p** TPS problem BOOL-PROP-67
Trybulec's 67th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to \$o, y$: $a \to \$o, z$: $a \to \$o$: $(\lambda \text{xx}$: $a$: $(x@\text{xx} \text{ and } y@\text{xx} \text{ and } z@\text{xx})) = (\lambda \text{xx}$: $a$: $(x@\text{xx} \text{ and } y@\text{xx} \text{ and } z@\text{xx}))$ thf(cBO

**SET144+3.p** If X is a subset of Z, then X U Y ∧ Z = (X U Y) ∧ Z
If X is a subset of Z, then the union of X and the intersection of Y and Z is the intersection of (the union of X and Y) and Z.
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(b \subseteq c \Rightarrow \text{union}(b, \text{intersection}(d, c)) = \text{intersection}(\text{union}(b, d), c))$ fof(prove_th$_{44}$, conjecture)

**SET144-6.p** Commutativity of intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x, y) \neq \text{intersection}(y, x)$ cnf(prove_commutativity_of_intersection$_1$, negated_conjecture)

**SET144^5.p** TPS problem BOOL-PROP-44

Trybulec's 44th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to \$o, y$: $a \to \$o, z$: $a \to \$o$: ($\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($z$@xx)) $\Rightarrow$ ($\lambda$xz: $a$: ($x$@xz or ($y$@xz and $z$@xz))) = ($\lambda$xx: $a$: (($x$@xx or $y$@xx) and $z$@xx)))    thf(cBOOL_PROP_44_pme, conjecture)

**SET145-6.p** Commutativity outside intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection($x$, intersection($y, z$)) $\neq$ intersection($y$, intersection($x, z$))    cnf(prove_commutativity_outside_intersection$_1$, nega

**SET146+3.p** The intersection of X and the empty set is the empty set
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set    fof(empty_set_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$))    fof(equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$))    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff \forall c$: $\neg\, c \in b$)    fof(empty_defn, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$))    fof(equal_member_defn, axiom)
$\forall b$: intersection($b$, empty_set) = empty_set    fof(prove_th$_{61}$, conjecture)

**SET146-6.p** The intersection of X and the empty set is the empty set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(null_class, $x$) $\neq$ null_class    cnf(prove_intersection_with_null_class$_1$, negated_conjecture)

**SET147-6.p** Universal class is identity for intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(universal_class, $x$) $\neq x$    cnf(prove_identity_for_intersection$_1$, negated_conjecture)

**SET148+3.p** Idempotency of intersection
$\forall b, c$: ($b \subseteq c \Rightarrow$ intersection($b, c$) = $b$)    fof(subset_intersection, axiom)
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$))    fof(equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$))    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$))    fof(equal_member_defn, axiom)
$\forall b$: intersection($b, b$) = $b$    fof(prove_idempotency_of_intersection, conjecture)

**SET148+4.p** A set intersection itself is itself
include('Axioms/SET006+0.ax')
$\forall a$: equal_set(intersection($a, a$), $a$)    fof(thI$_{13}$, conjecture)

**SET148-6.p** Idempotency of intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection($x, x$) $\neq x$    cnf(prove_idempotency_of_intersection$_1$, negated_conjecture)

**SET149-6.p** Corollary to idempotency of intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection($x$, intersection($x, y$)) $\neq$ intersection($x, y$)    cnf(prove_corollary_to_idempotency_of_intersection$_1$, negated_conjec

**SET150-6.p** Complement is an involution
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$(x')' \neq x$    cnf(prove_complement_is_self_cancelling$_1$, negated_conjecture)

**SET151-6.p** Complement of null class is universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
null_class$'$ $\neq$ universal_class    cnf(prove_complement_of_null_class$_1$, negated_conjecture)

**SET152-6.p** Complement of universal class is null class

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{universal\_class}' \neq \text{null\_class}$ cnf($\text{prove\_complement\_of\_universal\_class}_1$, negated_conjecture)

**SET153-6.p** Intersection with complement is null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x', x) \neq \text{null\_class}$ cnf($\text{prove\_intersection\_with\_complement}_1$, negated_conjecture)

**SET154-6.p** Union with complement is universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x', x) \neq \text{universal\_class}$ cnf($\text{prove\_union\_with\_complement}_1$, negated_conjecture)

**SET155+4.p** De Morgans law 1
include('Axioms/SET006+0.ax')
$\forall a, b, e$: $((a \subseteq e \text{ and } b \subseteq e) \Rightarrow \text{equal\_set}(e \setminus \text{union}(a, b), \text{intersection}(e \setminus a, e \setminus b)))$ fof($\text{thI}_{26}$, conjecture)

**SET155-6.p** De Morgans law 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, y)' \neq \text{intersection}(x', y')$ cnf($\text{prove\_demorgans\_law1}_1$, negated_conjecture)

**SET156+4.p** De Morgans law 2
include('Axioms/SET006+0.ax')
$\forall a, b, e$: $((a \subseteq e \text{ and } b \subseteq e) \Rightarrow \text{equal\_set}(e \setminus \text{intersection}(a, b), \text{union}(e \setminus a, e \setminus b)))$ fof($\text{thI}_{25}$, conjecture)

**SET156-6.p** De Morgans law 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x, y)' \neq \text{union}(x', y')$ cnf($\text{prove\_demorgans\_law2}_1$, negated_conjecture)

**SET157-6.p** Complement is unique
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, y) = \text{universal\_class}$ cnf($\text{prove\_complement\_is\_unique}_1$, negated_conjecture)
$\text{intersection}(x, y) = \text{null\_class}$ cnf($\text{prove\_complement\_is\_unique}_2$, negated_conjecture)
$x' \neq y$ cnf($\text{prove\_complement\_is\_unique}_3$, negated_conjecture)

**SET158-6.p** Corollary to complement axiom
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in x$ cnf($\text{prove\_corollary\_to\_complement\_axiom}_1$, negated_conjecture)
$z \in x'$ cnf($\text{prove\_corollary\_to\_complement\_axiom}_2$, negated_conjecture)
$y = z$ cnf($\text{prove\_corollary\_to\_complement\_axiom}_3$, negated_conjecture)

**SET159+3.p** Associativity of union
The union of (the union of X and Y) and Z is the union of X and (the union of Y and Z).
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{union}(\text{union}(b, c), d) = \text{union}(b, \text{union}(c, d))$ fof(prove_associativity_of_union, conjecture)

**SET159+4.p** Associativity of union
include('Axioms/SET006+0.ax')
$\forall a, b, c$: $\text{equal\_set}(\text{union}(\text{union}(a, b), c), \text{union}(a, \text{union}(b, c)))$ fof($\text{thI}_{09}$, conjecture)

**SET159-6.p** Associativity of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{union}(x, y), z) \neq \text{union}(x, \text{union}(y, z))$ cnf($\text{prove\_associativity\_of\_union}_1$, negated_conjecture)

**SET159^5.p** TPS problem BOOL-PROP-64
Trybulec's 64th Boolean property of sets

$a$: \$tType    thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o, z\colon a \to \$o\colon (\lambda xz\colon a\colon (x@xz \text{ or } y@xz \text{ or } z@xz)) = (\lambda xz\colon a\colon (x@xz \text{ or } y@xz \text{ or } z@xz))$    thf(cBOOL_PRO

**SET160-6.p** Commutativity of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, y) \neq \text{union}(y, x)$    cnf(prove_commutativity_of_union$_1$, negated_conjecture)

**SET161-6.p** Commutativity outside union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, \text{union}(y, z)) \neq \text{union}(y, \text{union}(x, z))$    cnf(prove_commutativity_outside_union$_1$, negated_conjecture)

**SET162+3.p** The union of X and the empty set is X
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$    fof(union_defn, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$    fof(empty_set_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$    fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$    fof(commutativity_of_union, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$    fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$    fof(empty_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$    fof(equal_member_defn, axiom)
$\forall b\colon \text{union}(b, \text{empty\_set}) = b$    fof(prove_union_empty_set, conjecture)

**SET162+4.p** The union of a set and empty set is equal to the set
include('Axioms/SET006+0.ax')
$\forall a\colon \text{equal\_set}(\text{union}(a, \text{empty\_set}), a)$    fof(thI$_{18}$, conjecture)

**SET162-6.p** Null class is identity for union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{null\_class}, x) \neq x$    cnf(prove_union_with_null_class$_1$, negated_conjecture)

**SET162^5.p** TPS problem BOOL-PROP-60
Trybulec's 60th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x\colon a \to \$o\colon (\lambda xz\colon a\colon (x@xz \text{ or } \$false)) = x$    thf(cBOOL_PROP_60_pme, conjecture)

**SET163-6.p** Union with universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{universal\_class}, x) \neq \text{universal\_class}$    cnf(prove_union_with_universal_class$_1$, negated_conjecture)

**SET165-6.p** Corollary to idempotency of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, \text{union}(x, y)) \neq \text{union}(x, y)$    cnf(prove_corollary_to_idempotency_of_union$_1$, negated_conjecture)

**SET166-6.p** Members of union 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in \text{union}(y, z)$    cnf(prove_members_of_union1$_1$, negated_conjecture)
$\neg\, x \in y$    cnf(prove_members_of_union1$_2$, negated_conjecture)
$\neg\, x \in z$    cnf(prove_members_of_union1$_3$, negated_conjecture)

**SET167-6.p** Members of union 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in y$    cnf(prove_members_of_union2$_1$, negated_conjecture)
$\neg\, x \in \text{union}(y, z)$    cnf(prove_members_of_union2$_2$, negated_conjecture)

**SET168-6.p** Members of union 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in z$    cnf(prove_members_of_union3$_1$, negated_conjecture)
$\neg\, x \in \text{union}(y, z)$    cnf(prove_members_of_union3$_2$, negated_conjecture)

**SET169+3.p** Intersection distributes over union
The intersection of X and (the union of Y and Z) is the union of (the intersection of X and Y) and (the intersection of X and Z).
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{intersection}(b, \text{union}(c, d)) = \text{union}(\text{intersection}(b, c), \text{intersection}(b, d))$ fof(prove_intersection_distributes_over_uni

**SET169+4.p** Distribution of intersection over union
include('Axioms/SET006+0.ax')
$\forall a, b, c$: $\text{equal\_set}(\text{intersection}(a, \text{union}(b, c)), \text{union}(\text{intersection}(a, b), \text{intersection}(a, c)))$ fof($\text{thI}_{10}$, conjecture)

**SET169-6.p** Intersection distributes over union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{intersection}(x, y), \text{intersection}(x, z)) \neq \text{intersection}(x, \text{union}(y, z))$ cnf(prove_intersection_over_union$1_1$, negated_con

**SET169^5.p** TPS problem BOOL-PROP-70
Trybulec's 70th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x: a \to \$o, y: a \to \$o, z: a \to \$o: (\lambda \text{xx}: a: (x@\text{xx} \text{ and } (y@\text{xx} \text{ or } z@\text{xx}))) = (\lambda \text{xz}: a: ((x@\text{xz} \text{ and } y@\text{xz}) \text{ or } (x@\text{xz} \text{ and } z@\text{xz})))$

**SET170-6.p** Distribution of intersection over union 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{intersection}(x, z), \text{intersection}(y, z)) \neq \text{intersection}(\text{union}(x, y), z)$ cnf(prove_intersection_over_union$2_1$, negated_con

**SET171+3.p** Union distributes over intersection
The union of X and (the intersection of Y and Z) is the intersection of (the union of X and Y) and (the union of X and Z).
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{union}(b, \text{intersection}(c, d)) = \text{intersection}(\text{union}(b, c), \text{union}(b, d))$ fof(prove_union_distributes_over_intersection, co

**SET171+4.p** Distribution of union over intersection 1
The union of X and (the intersection of Y and Z) is the intersection of (the union of X and Y) and (the union of X and Z).
include('Axioms/SET006+0.ax')
$\forall a, b, c$: $\text{equal\_set}(\text{union}(a, \text{intersection}(b, c)), \text{intersection}(\text{union}(a, b), \text{union}(a, c)))$ fof($\text{thI}_{11}$, conjecture)

**SET171-6.p** Union distributes over intersection
The union of X and (the intersection of Y and Z) is the intersection of (the union of X and Y) and (the union of X and Z).
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(\text{union}(x, y), \text{union}(x, z)) \neq \text{union}(x, \text{intersection}(y, z))$ cnf(prove_union_over_intersection$1_1$, negated_conjectur

**SET171^3.p** Union distributes over intersection
The union of X and (the intersection of Y and Z) is the intersection of (the union of X and Y) and (the union of X and Z).
include('Axioms/SET008^0.ax')
$\forall a: \$i \to \$o, b: \$i \to \$o, c: \$i \to \$o: (\text{union}@a@(\text{intersection}@b@c)) = (\text{intersection}@(\text{union}@a@b)@(\text{union}@a@c))$ thf(unio

**SET171^5.p** TPS problem BOOL-PROP-71

Trybulec's 71st Boolean property of sets
$a$: \$tType  thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o, z\colon a \to \$o\colon (\lambda xz\colon a\colon (x@xz \text{ or } (y@xz \text{ and } z@xz))) = (\lambda xx\colon a\colon ((x@xx \text{ or } y@xx) \text{ and } (x@xx \text{ or } z@xx)))$

**SET172-6.p** Distribution of union over intersection 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(\text{union}(x, z), \text{union}(y, z)) \neq \text{union}(\text{intersection}(x, y), z)$  cnf(prove_union_over_intersection2$_1$, negated_conjectur

**SET173+3.p** Absorbtion for intersection
The intersection of X and the union of X and Y is X.
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$  fof(union_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$  fof(intersection_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$  fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$  fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$  fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$  fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$  fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$  fof(equal_member_defn, axiom)
$\forall b, c\colon \text{intersection}(b, \text{union}(b, c)) = b$  fof(prove_absorbtion_for_intersection, conjecture)

**SET173-6.p** Absorbtion for intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x, \text{union}(x, y)) \neq x$  cnf(prove_absorbtion_for_intersection$_1$, negated_conjecture)

**SET173^5.p** TPS problem BOOL-PROP-68
Trybulec's 68th Boolean property of sets
$a$: \$tType  thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xx\colon a\colon (x@xx \text{ and } (x@xx \text{ or } y@xx))) = x$  thf(cBOOL_PROP_68_pme, conjecture)

**SET174-6.p** Corollary to absorbtion for intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x, \text{intersection}(y, \text{union}(x, z))) \neq \text{intersection}(x, y)$  cnf(prove_corollary_to_absorbtion_for_intersection$_1$, negate

**SET175+3.p** Absorbtion for union
The union of X and the intersection of X and Y is X.
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$  fof(union_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$  fof(intersection_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$  fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$  fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$  fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$  fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$  fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$  fof(equal_member_defn, axiom)
$\forall b, c\colon \text{union}(b, \text{intersection}(b, c)) = b$  fof(prove_absorbtion_for_union, conjecture)

**SET175-6.p** Absorbtion for union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, \text{intersection}(x, y)) \neq x$  cnf(prove_absorbtion_for_union$_1$, negated_conjecture)

**SET175^5.p** TPS problem BOOL-PROP-69
Trybulec's 69th Boolean property of sets
$a$: \$tType  thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xz\colon a\colon (x@xz \text{ or } (x@xz \text{ and } y@xz))) = x$  thf(cBOOL_PROP_69_pme, conjecture)

**SET176-6.p** Corollary to absorbtion for union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, \text{union}(y, \text{intersection}(x, z))) \neq \text{union}(x, y)$  cnf(prove_corollary_to_absorbtion_for_union$_1$, negated_conjecture)

**SET177-6.p** Distribution property 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
$\text{union}(x, \text{intersection}(x', z)) \neq \text{union}(x, z)$ cnf(prove_distribution_property1$_1$, negated_conjecture)

**SET178-6.p** Corollary 1 to distribution property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x, \text{union}(y, \text{intersection}(x', z))) \neq \text{union}(x, \text{union}(y, z))$ cnf(prove_corollary_1_to_distribution_property1$_1$, negated_co

**SET179-6.p** Corollary 2 to distribution property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{intersection}(x, z), \text{intersection}(x', \text{intersection}(y, z))) \neq \text{union}(\text{intersection}(x, z), \text{intersection}(y, z))$ cnf(prove_corolla

**SET180-6.p** Distribution property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x', \text{intersection}(x, z)) \neq \text{union}(x', z)$ cnf(prove_distribution_property2$_1$, negated_conjecture)

**SET181-6.p** Corollary to distribution property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(x', \text{union}(y, \text{intersection}(x, z))) \neq \text{union}(x', \text{union}(y, z))$ cnf(prove_corollary_to_distribution_property2$_1$, negated_con

**SET182-6.p** Distribution property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{union}(\text{intersection}(x', y), \text{intersection}(x, y)) \neq y$ cnf(prove_distribution_property3$_1$, negated_conjecture)

**SET183+3.p** If X is a subset of Y, then the intersection of X and Y is X
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) \subseteq b$ fof(intersection_is_subset, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \Rightarrow \text{intersection}(b, c) = b)$ fof(prove_subset_intersection, conjecture)

**SET183-6.p** If X is a subset of Y, then the intersection of X and Y is X
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{subclass}(x, y)$ cnf(prove_subclass_property1$_1$, negated_conjecture)
$\text{intersection}(x, y) \neq x$ cnf(prove_subclass_property1$_2$, negated_conjecture)

**SET183^5.p** TPS problem BOOL-PROP-42
Trybulec's 42nd Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\forall xx\colon a\colon ((x@xx) \Rightarrow (y@xx)) \Rightarrow (\lambda xx\colon a\colon (x@xx \text{ and } y@xx)) = x)$ thf(cBOOL_PROP_42_pme, con

**SET184-6.p** Subclass property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(x, y) = x$ cnf(prove_subclass_property2$_1$, negated_conjecture)
$\neg\,\text{subclass}(x, y)$ cnf(prove_subclass_property2$_2$, negated_conjecture)

**SET185+3.p** If X is a subset of Y, then the union of X and Y is Y
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \Rightarrow \text{union}(b, c) = c)$ fof(prove_subset_union, conjecture)

**SET185-6.p** If X is a subset of Y, then the union of X and Y is Y
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_subclass_property3$_1$, negated_conjecture)
union$(x, y) \neq y$ cnf(prove_subclass_property3$_2$, negated_conjecture)

**SET185^5.p** TPS problem BOOL-PROP-35
Trybulec's 35th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x: a \to \$o, y: a \to \$o$: $(\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: a: ((x@xx \text{ or } y@xx) \Rightarrow (y@xx)))$ thf(cBOOL_PROP_35_pm

**SET186-6.p** Subclass property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union$(x, y) = y$ cnf(prove_subclass_property4$_1$, negated_conjecture)
$\neg$ subclass$(x, y)$ cnf(prove_subclass_property4$_2$, negated_conjecture)

**SET187-6.p** Subclass property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_subclass_property5$_1$, negated_conjecture)
intersection$(y', x) \neq$ null_class cnf(prove_subclass_property5$_2$, negated_conjecture)

**SET188-6.p** Subclass property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection$(y', x) =$ null_class cnf(prove_subclass_property6$_1$, negated_conjecture)
$\neg$ subclass$(x, y)$ cnf(prove_subclass_property6$_2$, negated_conjecture)

**SET189-6.p** Corollary to subclass property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection$(y', x) =$ null_class cnf(prove_corollary_to_subclass_property6$_1$, negated_conjecture)
subclass$(y, x)$ cnf(prove_corollary_to_subclass_property6$_2$, negated_conjecture)
$x \neq y$ cnf(prove_corollary_to_subclass_property6$_3$, negated_conjecture)

**SET190-6.p** Subclass property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_subclass_property7$_1$, negated_conjecture)
union$(x', y) \neq$ universal_class cnf(prove_subclass_property7$_2$, negated_conjecture)

**SET191-6.p** Subclass property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union$(x', y) =$ universal_class cnf(prove_subclass_property8$_1$, negated_conjecture)
$\neg$ subclass$(x, y)$ cnf(prove_subclass_property8$_2$, negated_conjecture)

**SET192-6.p** Subclass property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_subclass_property9$_1$, negated_conjecture)
$\neg$ subclass$(y', x')$ cnf(prove_subclass_property9$_2$, negated_conjecture)

**SET193-6.p** Subclass property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(y', x')$ cnf(prove_subclass_property10$_1$, negated_conjecture)
$\neg$ subclass$(x, y)$ cnf(prove_subclass_property10$_2$, negated_conjecture)

**SET194+3.p** X is a subset of the union of X and Y
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $b \subseteq$ union$(b, c)$ fof(prove_subset_of_union, conjecture)

**SET194-6.p** X is a subset of the union of X and Y
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass($x$, union($x$, $y$))    cnf(prove_lattice_upper_bound1$_1$, negated_conjecture)

**SET194^5.p** TPS problem BOOL-PROP-31
Trybulec's 31st Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, xx: $a$: (($x$@xx) $\Rightarrow$ ($x$@xx or $y$@xx))    thf(cBOOL_PROP_31_pme, conjecture)

**SET195-6.p** Lattice upper bound 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass($y$, union($x$, $y$))    cnf(prove_lattice_upper_bound2$_1$, negated_conjecture)

**SET196+3.p** The intersection of X and Y is a subset of X
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$))    fof(subset_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$))    fof(equal_member_defn, axiom)
$\forall b, c$: intersection($b, c$) $\subseteq b$    fof(prove_intersection_is_subset, conjecture)

**SET196-6.p** The intersection of X and Y is a subset of X
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection($x$, $y$), $x$)    cnf(prove_lattice_lower_bound1$_1$, negated_conjecture)

**SET196^5.p** TPS problem BOOL-PROP-37
Trybulec's 37th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, xx: $a$: (($x$@xx and $y$@xx) $\Rightarrow$ ($x$@xx))    thf(cBOOL_PROP_37_pme, conjecture)

**SET197-6.p** Lattice lower bound 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection($x$, $y$), $y$)    cnf(prove_lattice_lower_bound2$_1$, negated_conjecture)

**SET199+3.p** If Z (= X and Z (= Y, then Z (= X $\wedge$ Y
If Z is a subset of X and Z is a subset of Y, then Z is a subset of the intersection of X and Y.
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$))    fof(subset_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$))    fof(equal_member_defn, axiom)
$\forall b, c, d$: (($b \subseteq c$ and $b \subseteq d$) $\Rightarrow$ $b \subseteq$ intersection($c, d$))    fof(prove_intersection_of_subsets, conjecture)

**SET199+4.p** If Z (= X and Z (= Y, then Z (= X $\wedge$ Y
If Z is a subset of X and Y, then Z is a subset the intersection.
include('Axioms/SET006+0.ax')
$\forall a, x, y$: (($a \subseteq x$ and $a \subseteq y$) $\iff$ $a \subseteq$ intersection($x, y$))    fof(thI$_{46}$, conjecture)

**SET199-6.p** If Z (= X and Z (= Y, then Z (= X $\wedge$ Y
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($z$, $x$)    cnf(prove_greatest_lower_bound$_1$, negated_conjecture)
subclass($z$, $y$)    cnf(prove_greatest_lower_bound$_2$, negated_conjecture)
$\neg$ subclass($z$, intersection($x$, $y$))    cnf(prove_greatest_lower_bound$_3$, negated_conjecture)

**SET199^5.p** TPS problem BOOL-PROP-39
Trybulec's 39th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: (($\forall$xx: $a$: (($z$@xx) $\Rightarrow$ ($x$@xx)) and $\forall$xx: $a$: (($z$@xx) $\Rightarrow$ ($y$@xx))) $\Rightarrow$ $\forall$xx: $a$: (($z$@xx) $\Rightarrow$ ($x$@xx or $y$@xx)))    thf(cBOOL_PROP_39_pme, conjecture)

**SET200+3.p** Union is monotonic

If X is a subset of Y and Z is a subset of V, then the union of X and Z is a subset of the union of Y and V.
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d, e$: $((b \subseteq c \text{ and } d \subseteq e) \Rightarrow \text{union}(b, d) \subseteq \text{union}(c, e))$ fof(prove_$\text{th}_{34}$, conjecture)

**SET200-6.p** Union is monotonic
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_union_is_$\text{monotonic}_1$, negated_conjecture)
subclass$(z, w)$ cnf(prove_union_is_$\text{monotonic}_2$, negated_conjecture)
$\neg$ subclass(union$(x, z)$, union$(y, w)$) cnf(prove_union_is_$\text{monotonic}_3$, negated_conjecture)

**SET200^5.p** TPS problem BOOL-PROP-34
Trybulec's 34th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to \$o, y$: $a \to \$o, z$: $a \to \$o, v$: $a \to \$o$: $((\forall xx$: $a$: $((x@xx) \Rightarrow (y@xx))$ and $\forall xx$: $a$: $((z@xx) \Rightarrow (v@xx))) \Rightarrow \forall xx$: $a$: $((x@xx \text{ or } z@xx) \Rightarrow (y@xx \text{ or } v@xx)))$ thf(cBOOL_PROP_34_pme, conjecture)

**SET201+3.p** Intersection is monotonic
If X is a subset of Y and Z is a subset of V, then the intersection of X and Z is a subset of the intersection of Y and V.
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d, e$: $((b \subseteq c \text{ and } d \subseteq e) \Rightarrow \text{intersection}(b, d) \subseteq \text{intersection}(c, e))$ fof(prove_$\text{th}_{41}$, conjecture)

**SET201-6.p** Intersection is monotonic
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass$(x, y)$ cnf(prove_intersection_is_$\text{monotonic}_1$, negated_conjecture)
subclass$(z, w)$ cnf(prove_intersection_is_$\text{monotonic}_2$, negated_conjecture)
$\neg$ subclass(intersection$(x, z)$, intersection$(y, w)$) cnf(prove_intersection_is_$\text{monotonic}_3$, negated_conjecture)

**SET201^5.p** TPS problem BOOL-PROP-41
Trybulec's 41st Boolean property of sets
$a$: \$tType thf(a_type, type)
cV: $a \to \$o$ thf(cV, type)
$\forall x$: $a \to \$o, y$: $a \to \$o, z$: $a \to \$o$: $((\forall xx$: $a$: $((x@xx) \Rightarrow (y@xx))$ and $\forall xx$: $a$: $((z@xx) \Rightarrow (cV@xx))) \Rightarrow \forall xx$: $a$: $((x@xx \text{ and } z@xx) \Rightarrow (y@xx \text{ and } cV@xx)))$ thf(cBOOL_PROP_41_pme, conjecture)

**SET202-6.p** Cross product property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product$(x, y)$, cross_product(universal_class, universal_class)) cnf(prove_cross_product_$\text{property1}_1$, negate

**SET203-6.p** Corollary to cross product product property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$u \in x$ cnf(prove_corollary_to_X_product_$\text{property1}_1$, negated_conjecture)
$v \in y$ cnf(prove_corollary_to_X_product_$\text{property1}_2$, negated_conjecture)
$\neg$ ordered_pair$(u, v) \in$ cross_product(universal_class, universal_class) cnf(prove_corollary_to_X_product_$\text{property1}_3$, negate

**SET204-6.p** Cross product property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair$(u, v) \in$ cross_product$(x, y)$ cnf(prove_cross_product_$\text{property2}_1$, negated_conjecture)
$\neg$ ordered_pair$(v, u) \in$ cross_product$(y, x)$ cnf(prove_cross_product_$\text{property2}_2$, negated_conjecture)

**SET205-6.p** Cross product with null class 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
$\text{cross\_product}(x, \text{null\_class}) \neq \text{null\_class}$ cnf(prove_cross_product_with_null_class1$_1$, negated_conjecture)

**SET206-6.p** Cross product with null class 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{cross\_product}(\text{null\_class}, y) \neq \text{null\_class}$ cnf(prove_cross_product_with_null_class2$_1$, negated_conjecture)

**SET207-6.p** Cross product property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{intersection}(x, \text{cross\_product}(\text{universal\_class}, \text{universal\_class})), \text{cross\_product}(\text{domain\_of}(x), \text{universal\_class}))$ cn

**SET208-6.p** Cross product is monotonic 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{subclass}(x, y)$ cnf(prove_cross_product_is_monotonic1$_1$, negated_conjecture)
$\neg\,\text{subclass}(\text{cross\_product}(x, z), \text{cross\_product}(y, z))$ cnf(prove_cross_product_is_monotonic1$_2$, negated_conjecture)

**SET209-6.p** Cross product is monotonic 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{subclass}(y, z)$ cnf(prove_cross_product_is_monotonic2$_1$, negated_conjecture)
$\neg\,\text{subclass}(\text{cross\_product}(x, y), \text{cross\_product}(x, z))$ cnf(prove_cross_product_is_monotonic2$_2$, negated_conjecture)

**SET210-6.p** Corollary 1 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, z), \text{cross\_product}(\text{union}(x, y), z))$ cnf(prove_corollary_1_to_X_product_monotonicity$_1$, negated_c

**SET211-6.p** Corollary 2 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(y, z), \text{cross\_product}(\text{union}(x, y), z))$ cnf(prove_corollary_2_to_X_product_monotonicity$_1$, negated_c

**SET212-6.p** Corollary 3 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, y), \text{cross\_product}(x, \text{union}(y, z)))$ cnf(prove_corollary_3_to_X_product_monotonicity$_1$, negated_c

**SET213-6.p** Corollary 4 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, z), \text{cross\_product}(x, \text{union}(y, z)))$ cnf(prove_corollary_4_to_X_product_monotonicity$_1$, negated_c

**SET214-6.p** Corollary 5 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(\text{intersection}(x, y), z), \text{cross\_product}(x, z))$ cnf(prove_corollary_5_to_X_product_monotonicity$_1$, neg

**SET215-6.p** Corollary 6 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(\text{intersection}(x, y), z), \text{cross\_product}(y, z))$ cnf(prove_corollary_6_to_X_product_monotonicity$_1$, neg

**SET216-6.p** Corollary 7 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, \text{intersection}(y, z)), \text{cross\_product}(x, y))$ cnf(prove_corollary_7_to_X_product_monotonicity$_1$, neg

**SET217-6.p** Corollary 8 to cross product product monotonicity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, \text{intersection}(y, z)), \text{cross\_product}(x, z))$ cnf(prove_corollary_8_to_X_product_monotonicity$_1$, neg

**SET218-6.p** Cross product distributes over union 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
union(cross_product($x, z$), cross_product($y, z$)) $\neq$ cross_product(union($x, y$), $z$)     cnf(prove_cross_product_over_union$1_1$, nega

**SET219-6.p** Cross product distributes over union 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union(cross_product($x, y$), cross_product($x, z$)) $\neq$ cross_product($x$, union($y, z$))     cnf(prove_cross_product_over_union$2_1$, nega

**SET220-6.p** Cross product distributes over intersection 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(cross_product($x, z$), cross_product($y, z$)) $\neq$ cross_product(intersection($x, y$), $z$)     cnf(prove_cross_product_over_i

**SET221-6.p** Cross product distributes over intersection 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(cross_product($x, y$), cross_product($x, z$)) $\neq$ cross_product($x$, intersection($y, z$))     cnf(prove_cross_product_over_i

**SET222-6.p** Cross product property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection(cross_product($w, x$), cross_product($y, z$)), cross_product($w, z$))     cnf(prove_cross_product_property$4_1$

**SET223-6.p** Cross product property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product(intersection($w, y$), intersection($x, z$)), cross_product($w, z$))     cnf(prove_cross_product_property$5_1$, n

**SET224-6.p** Cross product double distribution for intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(cross_product($w, x$), cross_product($y, z$)) $\neq$ cross_product(intersection($w, y$), intersection($x, z$))     cnf(prove_cros

**SET225-6.p** Inverse of cross product squared
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($x, x$)$'$ $\neq$ cross_product($x, x$)     cnf(prove_inverse_of_square$_1$, negated_conjecture)

**SET226-6.p** Cross product left cancellation 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($u, v$) = cross_product($w, x$)     cnf(prove_cross_product_left_cancellation$1_1$, negated_conjecture)
$u \neq$ null_class     cnf(prove_cross_product_left_cancellation$1_2$, negated_conjecture)
$v \neq x$     cnf(prove_cross_product_left_cancellation$1_3$, negated_conjecture)

**SET227-6.p** Cross product left cancellation 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($u, v$) = cross_product($w, x$)     cnf(prove_cross_product_left_cancellation$2_1$, negated_conjecture)
$w \neq$ null_class     cnf(prove_cross_product_left_cancellation$2_2$, negated_conjecture)
$v \neq x$     cnf(prove_cross_product_left_cancellation$2_3$, negated_conjecture)

**SET228-6.p** Cross product right cancellation 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($u, v$) = cross_product($w, x$)     cnf(prove_cross_product_right_cancellation$1_1$, negated_conjecture)
$v \neq$ null_class     cnf(prove_cross_product_right_cancellation$1_2$, negated_conjecture)
$u \neq w$     cnf(prove_cross_product_right_cancellation$1_3$, negated_conjecture)

**SET229-6.p** Cross product right cancellation 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($u, v$) = cross_product($w, x$)     cnf(prove_cross_product_right_cancellation$2_1$, negated_conjecture)
$x \neq$ null_class     cnf(prove_cross_product_right_cancellation$2_2$, negated_conjecture)
$u \neq w$     cnf(prove_cross_product_right_cancellation$2_3$, negated_conjecture)

**SET230-6.p** Corollary to cross product cancellation

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($u,u$) = cross_product($w,w$)    cnf(prove_corollary_to_cross_product_cancellation$_1$, negated_conjecture)
$u \neq w$    cnf(prove_corollary_to_cross_product_cancellation$_2$, negated_conjecture)

**SET231-6.p** Cross product property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product($x,y$), $z$)    cnf(prove_cross_product_property6$_1$, negated_conjecture)
ordered_pair(first(not_subclass_element(cross_product($x,y$), $z$)), second(not_subclass_element(cross_product($x,y$), $z$))) $\neq$ not_subclass_element(cross_product($x,y$), $z$)    cnf(prove_cross_product_property6$_2$, negated_conjecture)

**SET232-6.p** Cross product property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product($x,y$), $z$)    cnf(prove_cross_product_property7$_1$, negated_conjecture)
$\neg$ first(not_subclass_element(cross_product($x,y$), $z$)) $\in x$    cnf(prove_cross_product_property7$_2$, negated_conjecture)

**SET233-6.p** Cross product property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product($x,y$), $z$)    cnf(prove_cross_product_property8$_1$, negated_conjecture)
$\neg$ second(not_subclass_element(cross_product($x,y$), $z$)) $\in y$    cnf(prove_cross_product_property8$_2$, negated_conjecture)

**SET234-6.p** Cross product property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first(not_subclass_element(cross_product($x,y$), $z$)), second(not_subclass_element(cross_product($x,y$), $z$))) $\in$ $z$    cnf(prove_cross_product_property9$_1$, negated_conjecture)
$\neg$ subclass(cross_product($x,y$), $z$)    cnf(prove_cross_product_property9$_2$, negated_conjecture)

**SET235-6.p** Cross product property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))    cnf(prove_cross_product_property10$_1$, negated_conjecture)
$\neg$ ordered_pair(first(not_subclass_element($x,y$)), second(not_subclass_element($x,y$))) $\in x$    cnf(prove_cross_product_property
$\neg$ subclass($x,y$)    cnf(prove_cross_product_property10$_3$, negated_conjecture)

**SET236-6.p** Cross product property 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))    cnf(prove_cross_product_property11$_1$, negated_conjecture)
ordered_pair(first(not_subclass_element($x,y$)), second(not_subclass_element($x,y$))) $\in y$    cnf(prove_cross_product_property1
$\neg$ subclass($x,y$)    cnf(prove_cross_product_property11$_3$, negated_conjecture)

**SET237-6.p** Restriction alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(restrict(xr, $x,y$), cross_product(universal_class, universal_class))    cnf(prove_restriction_alternate_defn1$_1$, negated

**SET238-6.p** Corollary to restriction alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ restrict(xr, $x,y$)    cnf(prove_corollary_to_restriction_alternate_defn1$_1$, negated_conjecture)
$\neg$ ordered_pair(first($z$), second($z$)) $\in$ restrict(xr, $x,y$)    cnf(prove_corollary_to_restriction_alternate_defn1$_2$, negated_conjectu

**SET239-6.p** Restriction alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ restrict(xr, $x,y$)    cnf(prove_restriction_alternate_defn2$_1$, negated_conjecture)
$\neg z \in$ xr    cnf(prove_restriction_alternate_defn2$_2$, negated_conjecture)

**SET240-6.p** Restriction alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

ordered_pair($u, v$) $\in$ restrict(xr, $x, y$)  cnf(prove_restriction_alternate_defn3$_1$, negated_conjecture)
$\neg\, u \in x$  cnf(prove_restriction_alternate_defn3$_2$, negated_conjecture)

**SET241-6.p** Restriction alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($u, v$) $\in$ restrict(xr, $x, y$)  cnf(prove_restriction_alternate_defn4$_1$, negated_conjecture)
$\neg\, v \in y$  cnf(prove_restriction_alternate_defn4$_2$, negated_conjecture)

**SET242-6.p** Restriction alternate definition 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ xr  cnf(prove_restriction_alternate_defn5$_1$, negated_conjecture)
$z \in$ cross_product($x, y$)  cnf(prove_restriction_alternate_defn5$_2$, negated_conjecture)
$\neg\, z \in$ restrict(xr, $x, y$)  cnf(prove_restriction_alternate_defn5$_3$, negated_conjecture)

**SET243-6.p** Restriction property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(restrict(xf, $x_1, y_1$), $x_2, y_2$) $\neq$ restrict(xf, intersection($x_1, x_2$), intersection($y_1, y_2$))  cnf(prove_restriction_property1$_1$, n

**SET244-6.p** Restriction with universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(universal_class, $x, y$) $\neq$ cross_product($x, y$)  cnf(prove_restriction_with_universal_class$_1$, negated_conjecture)

**SET245-6.p** Restriction with null class 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(null_class, $x, y$) $\neq$ null_class  cnf(prove_restriction_with_null_class1$_1$, negated_conjecture)

**SET246-6.p** Restriction with null class 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, null_class, $y$) $\neq$ null_class  cnf(prove_restriction_with_null_class2$_1$, negated_conjecture)

**SET247-6.p** Restriction with null class 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, $x$, null_class) $\neq$ null_class  cnf(prove_restriction_with_null_class3$_1$, negated_conjecture)

**SET248-6.p** Restriction preserves intersections
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection(restrict(xr$_1$, $x_1, y_1$), restrict(xr$_2$, $x_2, y_2$)) $\neq$ restrict(intersection(xr$_1$, xr$_2$), intersection($x_1, x_2$), intersection($y_1, y_2$))

**SET249-6.p** Restriction property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union(restrict(xr$_1$, $x, y$), restrict(xr$_2$, $x, y$)) $\neq$ restrict(union(xr$_1$, xr$_2$), $x, y$)  cnf(prove_restriction_property2$_1$, negated_conje

**SET250-6.p** Corollary to restriction property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union(restrict($x, y, y$), restrict($x, y, y)'$) $\neq$ restrict(union($x, x'$), $y, y$)  cnf(prove_corollary_to_restriction_property2$_1$, negated

**SET251-6.p** Restriction of element relation, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ universal_class  cnf(prove_restriction_of_element_relation$_1$, negated_conjecture)
domain_of(restrict(element_relation, $x$, singleton($y$))) $\neq$ intersection($x, y$)  cnf(prove_restriction_of_element_relation$_2$, negat

**SET252-6.p** Restriction property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(restrict($x, y, z$), $x$)  cnf(prove_restriction_property3$_1$, negated_conjecture)

**SET253-6.p** Restriction property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(restrict($x, y, z$), cross_product($y, z$))    cnf(prove_restriction_property4$_1$, negated_conjecture)

**SET254-6.p** Monotonicity of restriction 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x_1, x_2$)    cnf(prove_monotonicity_of_restriction1$_1$, negated_conjecture)
¬ subclass(restrict($x_1, y, z$), restrict($x_2, y, z$))    cnf(prove_monotonicity_of_restriction1$_2$, negated_conjecture)

**SET255-6.p** Monotonicity of restriction 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($y_1, y_2$)    cnf(prove_monotonicity_of_restriction2$_1$, negated_conjecture)
¬ subclass(restrict($x, y_1, z$), restrict($x, y_2, z$))    cnf(prove_monotonicity_of_restriction2$_2$, negated_conjecture)

**SET256-6.p** Monotonicity of restriction 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($z_1, z_2$)    cnf(prove_monotonicity_of_restriction3$_1$, negated_conjecture)
¬ subclass(restrict($x, y, z_1$), restrict($x, y, z_2$))    cnf(prove_monotonicity_of_restriction3$_2$, negated_conjecture)

**SET257-6.p** Restriction property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(cross_product($x, y$), $x, y$) ≠ cross_product($x, y$)    cnf(prove_restriction_property5$_1$, negated_conjecture)

**SET258-6.p** Domain alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∈ domain_of(xr)    cnf(prove_domain_alternate_defn1$_1$, negated_conjecture)
¬ ordered_pair($x$, range(xr, $x$, universal_class)) ∈ cross_product(universal_class, universal_class)    cnf(prove_domain_alternat

**SET259-6.p** Domain alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∈ domain_of(xr)    cnf(prove_domain_alternate_defn2$_1$, negated_conjecture)
¬ ordered_pair($x$, range(xr, $x$, universal_class)) ∈ xr    cnf(prove_domain_alternate_defn2$_2$, negated_conjecture)

**SET260-6.p** Domain alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ xr    cnf(prove_domain_alternate_defn3$_1$, negated_conjecture)
ordered_pair($x, y$) ∈ cross_product(universal_class, universal_class)    cnf(prove_domain_alternate_defn3$_2$, negated_conjecture
¬ $x$ ∈ domain_of(xr)    cnf(prove_domain_alternate_defn3$_3$, negated_conjecture)

**SET261-6.p** Domain of null class is the null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(null_class) ≠ null_class    cnf(prove_domain_of_null_class$_1$, negated_conjecture)

**SET262-6.p** Domain of universal class is the universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(universal_class) ≠ universal_class    cnf(prove_domain_of_universal_class$_1$, negated_conjecture)

**SET263-6.p** Domain preserves union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union(domain_of($x$), domain_of($y$)) ≠ domain_of(union($x, y$))    cnf(prove_domain_preserves_union$_1$, negated_conjecture)

**SET264-6.p** Domain is monotonic 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$)    cnf(prove_domain_is_monotonic1$_1$, negated_conjecture)

$\neg$ subclass(domain_of($x$), domain_of($y$))  cnf(prove_domain_is_monotonic1$_2$, negated_conjecture)

**SET265-6.p** Domain is monotonic 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(domain_of(intersection($x, y$)), domain_of($x$))  cnf(prove_domain_is_monotonic2$_1$, negated_conjecture)

**SET266-6.p** Domain is monotonic 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(domain_of(intersection($x, y$)), domain_of($y$))  cnf(prove_domain_is_monotonic3$_1$, negated_conjecture)

**SET267-6.p** Domain is monotonic 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(domain_of($x$)$'$, domain_of($x'$))  cnf(prove_domain_is_monotonic4$_1$, negated_conjecture)

**SET268-6.p** Domain property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection($x$, cross_product(universal_class, universal_class)), cross_product(domain_of($x$), universal_class))  cn

**SET269-6.p** Domain only considers ordered pairs
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(intersection($x$, cross_product(universal_class, universal_class))) $\neq$ domain_of($x$)  cnf(prove_domain_does_ordered

**SET270-6.p** Domain property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(cross_product($x, y$)) $\neq x$  cnf(prove_domain_property2$_1$, negated_conjecture)
$y \neq$ null_class  cnf(prove_domain_property2$_2$, negated_conjecture)

**SET271-6.p** Corollary to domain property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(cross_product($x, x$)) $\neq x$  cnf(prove_corollary_to_domain_property2$_1$, negated_conjecture)

**SET272-6.p** Domain property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, intersection(domain_of(xr), $x$), $y$) $\neq$ restrict(xr, $x, y$)  cnf(prove_domain_property3$_1$, negated_conjecture)

**SET273-6.p** Corollary to domain property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, domain_of(xr), $y$) $\neq$ restrict(xr, universal_class, $y$)  cnf(prove_corollary_to_domain_property3$_1$, negated_conjectu

**SET274-6.p** Domain property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(restrict($x, y$, universal_class)) $\neq$ intersection(domain_of($x$), $y$)  cnf(prove_domain_property4$_1$, negated_conjecture

**SET275-6.p** Corollary 1 to domain property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(domain_of(restrict($x, y, z$)), intersection(domain_of($x$), $y$))  cnf(prove_corollary_1_to_domain_property4$_1$, negated

**SET276-6.p** Corollary 2 to domain property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product(domain_of(restrict($x, y, z$)), $u$), cross_product(intersection(domain_of($x$), $y$), $u$))  cnf(prove_corolla

**SET277-6.p** Corollary 3 to domain property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product($u$, domain_of(restrict($x, y, z$))), cross_product($u$, intersection(domain_of($x$), $y$)))  cnf(prove_corolla

**SET278-6.p** Corollary 4 to domain property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(cross_product(domain_of(restrict($x_1, y_1, z_1$)), domain_of(restrict($x_2, y_2, z_2$))), cross_product($y_1, y_2$)) cnf(prove_c

**SET279-6.p** Domain property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$u \in$ domain_of(restrict(xr, $y$, singleton($z$))) cnf(prove_domain_property5$_1$, negated_conjecture)
$z \in$ universal_class cnf(prove_domain_property5$_2$, negated_conjecture)
¬ ordered_pair($u, z$) $\in$ xr cnf(prove_domain_property5$_3$, negated_conjecture)

**SET280-6.p** Domain property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of($x$) = null_class cnf(prove_domain_property6$_1$, negated_conjecture)
subclass($x$, cross_product(universal_class, universal_class)) cnf(prove_domain_property6$_2$, negated_conjecture)
$x \neq$ null_class cnf(prove_domain_property6$_3$, negated_conjecture)

**SET281-6.p** Domain relation is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ function(domain_relation) cnf(prove_domain_relation_is_a_function$_1$, negated_conjecture)

**SET282-6.p** Domain of domain relation
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(domain_relation) $\neq$ universal_class cnf(prove_domain_of_domain_relation$_1$, negated_conjecture)

**SET283-6.p** Apply domain relation
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ universal_class cnf(prove_apply_domain_relation$_1$, negated_conjecture)
apply(domain_relation, $x$) $\neq$ domain_of($x$) cnf(prove_apply_domain_relation$_2$, negated_conjecture)

**SET284-6.p** Image of domain relation
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(image(domain_relation, $x$)) $\neq$ domain_of(sum_class($x$)) cnf(prove_image_of_domain_relation$_1$, negated_conjectur

**SET285-6.p** Domain property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_domain_property7$_1$, negated_conjecture)
domain_of(singleton(ordered_pair($x, y$))) $\neq$ singleton($x$) cnf(prove_domain_property7$_2$, negated_conjecture)

**SET286-6.p** Corollary to domain property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_corollary_to_domain_property7$_1$, negated_con
domain_of(union($x$, singleton(ordered_pair(domain_of($x$), $y$)))) $\neq$ successor(domain_of($x$)) cnf(prove_corollary_to_domain_

**SET287-6.p** Domain property 8
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$) cnf(prove_domain_property8$_1$, negated_conjecture)
domain_of(intersection($y', x$)) $\neq$ null_class cnf(prove_domain_property8$_2$, negated_conjecture)

**SET288-6.p** Domain property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class)) cnf(prove_domain_property9$_1$, negated_conjecture)
domain_of(intersection($y', x$)) = null_class cnf(prove_domain_property9$_2$, negated_conjecture)
¬ subclass($x, y$) cnf(prove_domain_property9$_3$, negated_conjecture)

**SET289-6.p** Proof of Goedel's axiom B6, part 1

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass($x'$, cross_product(universal_class, universal_class)) cnf(prove_inverse_property1$_1$, negated_conjecture)

**SET290-6.p** Proof of Goedel's axiom B6, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($u, v$) $\in x'$ cnf(prove_inverse_property2$_1$, negated_conjecture)
$\neg$ ordered_pair($v, u$) $\in x$ cnf(prove_inverse_property2$_2$, negated_conjecture)

**SET291-6.p** Proof of Goedel's axiom B6, part 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($v, u$) $\in x$ cnf(prove_inverse_property3$_1$, negated_conjecture)
ordered_pair($u, v$) $\in$ cross_product(universal_class, universal_class) cnf(prove_inverse_property3$_2$, negated_conjecture)
$\neg$ ordered_pair($u, v$) $\in x'$ cnf(prove_inverse_property3$_3$, negated_conjecture)

**SET292-6.p** Inverse of null class is the null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
null_class$'$ $\neq$ null_class cnf(prove_inverse_of_null_class$_1$, negated_conjecture)

**SET293-6.p** Inverse of universal class is the universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
universal_class$'$ $\neq$ cross_product(universal_class, universal_class) cnf(prove_inverse_of_universal_class$_1$, negated_conjecture)

**SET294-6.p** Inverse distributes over union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union($x', y'$) $\neq$ union($x, y$)$'$ cnf(prove_inverse_over_union$_1$, negated_conjecture)

**SET295-6.p** Inverse distributes over intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection($x', y'$) $\neq$ intersection($x, y$)$'$ cnf(prove_inverse_over_intersection$_1$, negated_conjecture)

**SET296-6.p** Domain of inverse
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of($x'$) $\neq$ range_of($x$) cnf(prove_domain_of_inverse$_1$, negated_conjecture)

**SET297-6.p** Range of inverse
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of($x'$) $\neq$ domain_of($x$) cnf(prove_range_of_inverse$_1$, negated_conjecture)

**SET298-6.p** Inverse of complement
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x'' \neq$ intersection(($x'$)$'$, cross_product(universal_class, universal_class)) cnf(prove_inverse_of_complement$_1$, negated_conject

**SET299-6.p** Inverse of product
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($x, y$)$'$ $\neq$ cross_product($y, x$) cnf(prove_inverse_of_product$_1$, negated_conjecture)

**SET300-6.p** Inverse of inverse
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
($x'$)$'$ $\neq$ restrict($x$, universal_class, universal_class) cnf(prove_inverse_of_inverse$_1$, negated_conjecture)

**SET301-6.p** Inverse commutes with restriction
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, $y, x$)$'$ $\neq$ restrict(xr$'$, $x, y$) cnf(prove_inverse_commutes_restriction$_1$, negated_conjecture)

**SET302-6.p** Range alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ range_of(xr)  cnf(prove_range_alternate_defn1$_1$, negated_conjecture)
$\neg$ ordered_pair(dom(xr), $y$) $\in$ cross_product(universal_class, universal_class)  cnf(prove_range_alternate_defn1$_2$, negated_con

**SET303-6.p** Range alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ range_of(xr)  cnf(prove_range_alternate_defn2$_1$, negated_conjecture)
$\neg$ ordered_pair(dom(xr), $y$) $\in$ xr  cnf(prove_range_alternate_defn2$_2$, negated_conjecture)

**SET304-6.p** Range alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ xr  cnf(prove_range_alternate_defn3$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)  cnf(prove_range_alternate_defn3$_2$, negated_conjecture)
$\neg\, y \in$ range_of(xr)  cnf(prove_range_alternate_defn3$_3$, negated_conjecture)

**SET305-6.p** Range of null class is the null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(null_class) $\neq$ null_class  cnf(prove_range_of_null_class$_1$, negated_conjecture)

**SET306-6.p** Range of universal class is the universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(universal_class) $\neq$ universal_class  cnf(prove_range_of_universal_class$_1$, negated_conjecture)

**SET307-6.p** Range preserves union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union(range_of($x$), range_of($y$)) $\neq$ range_of(union($x, y$))  cnf(prove_range_preserves_union$_1$, negated_conjecture)

**SET308-6.p** Monotonicity of range 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$)  cnf(prove_monotonicity_of_range1$_1$, negated_conjecture)
$\neg$ subclass(range_of($x$), range_of($y$))  cnf(prove_monotonicity_of_range1$_2$, negated_conjecture)

**SET309-6.p** Monotonicity of range 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(range_of(intersection($x, y$)), range_of($y$))  cnf(prove_monotonicity_of_range2$_1$, negated_conjecture)

**SET310-6.p** Monotonicity of range 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(range_of(intersection($x, y$)), range_of($x$))  cnf(prove_monotonicity_of_range3$_1$, negated_conjecture)

**SET311-6.p** Range property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection($x$, cross_product(universal_class, universal_class)), cross_product(universal_class, range_of($x$)))  cnf(p

**SET312-6.p** Range only considers ordered pairs
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(intersection($x$, cross_product(universal_class, universal_class))) $\neq$ range_of($x$)  cnf(prove_range_does_ordered_pairs

**SET313-6.p** Range property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(cross_product($x, y$)) $\neq y$  cnf(prove_range_property2$_1$, negated_conjecture)
$x \neq$ null_class  cnf(prove_range_property2$_2$, negated_conjecture)

**SET314-6.p** Range property 3

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, $x$, intersection($y$, range_of(xr))) $\neq$ restrict(xr, $x$, $y$)    cnf(prove_range_property3$_1$, negated_conjecture)

**SET315-6.p** Corollary to range property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict(xr, $x$, range_of(xr)) $\neq$ restrict(xr, $x$, universal_class)    cnf(prove_corollary_to_range_property3$_1$, negated_conjecture)

**SET316-6.p** Range property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(restrict($x$, universal_class, $z$)) $\neq$ intersection(range_of($x$), $z$)    cnf(prove_range_property4$_1$, negated_conjecture)

**SET317-6.p** Corollary 1 to range property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(range_of(restrict($x, y, z$)), intersection($z$, range_of($x$)))    cnf(prove_corollary_1_to_range_property4$_1$, negated_conj

**SET318-6.p** Corollary 2 to range property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product(range_of(restrict($x, y, z$)), $u$), cross_product(intersection($z$, range_of($x$)), $u$))    cnf(prove_corollary_2

**SET319-6.p** Corollary 3 to range property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product($u$, range_of(restrict($x, y, z$))), cross_product($u$, intersection($z$, range_of($x$))))    cnf(prove_corollary_3

**SET320-6.p** Corollary 4 to range property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cross_product(range_of(restrict($x_1, y_1, z_1$)), range_of(restrict($x_2, y_2, z_2$))), cross_product($z_1, z_2$))    cnf(prove_coroll

**SET321-6.p** Range property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ range_of($z$)    cnf(prove_range_property5$_1$, negated_conjecture)
$\neg$ dom($z$) $\in$ domain_of($z$)    cnf(prove_range_property5$_2$, negated_conjecture)

**SET322-6.p** Range property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ domain_of($z$)    cnf(prove_range_property6$_1$, negated_conjecture)
$\neg$ range($z, x$, universal_class) $\in$ range_of($z$)    cnf(prove_range_property6$_2$, negated_conjecture)

**SET323-6.p** Range property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of($x$) = null_class    cnf(prove_range_property7$_1$, negated_conjecture)
subclass($x$, cross_product(universal_class, universal_class))    cnf(prove_range_property7$_2$, negated_conjecture)
$x \neq$ null_class    cnf(prove_range_property7$_3$, negated_conjecture)

**SET324-6.p** Image alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ image(xr, $x$)    cnf(prove_image_alternate_defn1$_1$, negated_conjecture)
$\neg$ ordered_pair(dom(xr), $y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_image_alternate_defn1$_2$, negated_con

**SET325-6.p** Image alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ image(xr, $x$)    cnf(prove_image_alternate_defn2$_1$, negated_conjecture)
$\neg$ ordered_pair(dom(xr), $y$) $\in$ xr    cnf(prove_image_alternate_defn2$_2$, negated_conjecture)

**SET326-6.p** Corollary to image alternate definition 2
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
$y \in$ image(xr, $x$) cnf(prove_corollary_to_image_alternate_defn2$_1$, negated_conjecture)
$\neg$ dom(xr) $\in$ domain_of(xr) cnf(prove_corollary_to_image_alternate_defn2$_2$, negated_conjecture)

**SET327-6.p** Image alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ image(xr, $x$) cnf(prove_image_alternate_defn3$_1$, negated_conjecture)
$\neg$ dom(xr) $\in x$ cnf(prove_image_alternate_defn3$_2$, negated_conjecture)

**SET328-6.p** Corollary to image alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ image(xr, singleton($x$)) cnf(prove_corollary_to_image_alternate_defn3$_1$, negated_conjecture)
$x \in$ universal_class cnf(prove_corollary_to_image_alternate_defn3$_2$, negated_conjecture)
$\neg$ ordered_pair($x, y$) $\in$ xr cnf(prove_corollary_to_image_alternate_defn3$_3$, negated_conjecture)

**SET329-6.p** Image alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ xr cnf(prove_image_alternate_defn4$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_image_alternate_defn4$_2$, negated_conjecture)
$x \in z$ cnf(prove_image_alternate_defn4$_3$, negated_conjecture)
$\neg y \in$ image(xr, $z$) cnf(prove_image_alternate_defn4$_4$, negated_conjecture)

**SET330-6.p** Corollary to image alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ xr cnf(prove_corollary_to_image_alternate_defn4$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_corollary_to_image_alternate_defn4$_2$, negated
$\neg y \in$ image(xr, singleton($x$)) cnf(prove_corollary_to_image_alternate_defn4$_3$, negated_conjecture)

**SET331-6.p** Range is image of the domain
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(xr, domain_of(xr)) $\neq$ range_of(xr) cnf(prove_image_of_domain$_1$, negated_conjecture)

**SET332-6.p** Corollary to range is image of domain
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(xr, universal_class) $\neq$ range_of(xr) cnf(prove_corollary_to_image_of_domain$_1$, negated_conjecture)

**SET333-6.p** Monotonicity of image 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($y, z$) cnf(prove_monotonicity_of_image1$_1$, negated_conjecture)
$\neg$ subclass(image(xr, $y$), image(xr, $z$)) cnf(prove_monotonicity_of_image1$_2$, negated_conjecture)

**SET334-6.p** Monotonicity of image 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(xr, yr) cnf(prove_monotonicity_of_image2$_1$, negated_conjecture)
$\neg$ subclass(image(xr, $z$), image(yr, $z$)) cnf(prove_monotonicity_of_image2$_2$, negated_conjecture)

**SET335-6.p** Image property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
intersection($x$, domain_of($z$)) = null_class cnf(prove_image_property1$_1$, negated_conjecture)
image($z, x$) $\neq$ null_class cnf(prove_image_property1$_2$, negated_conjecture)

**SET336-6.p** Corollary 1 image property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image($z$, null_class) $\neq$ null_class cnf(prove_corollary_1_image_property1$_1$, negated_conjecture)

**SET337-6.p** Corollary 2 image property 1

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(null_class, $x$) $\neq$ null_class cnf(prove_corollary_2_image_property1$_1$, negated_conjecture)

**SET338-6.p** Corollary 3 image property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg x \in$ domain_of($z$) cnf(prove_corollary_3_image_property1$_1$, negated_conjecture)
image($z$, singleton($x$)) $\neq$ null_class cnf(prove_corollary_3_image_property1$_2$, negated_conjecture)

**SET339-6.p** Subclass alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ image(element_relation, $y'$)$'$ cnf(prove_subclass_alternate_defn1$_1$, negated_conjecture)
$\neg$ subclass($x, y$) cnf(prove_subclass_alternate_defn1$_2$, negated_conjecture)

**SET340-6.p** Subclass alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$) cnf(prove_subclass_alternate_defn2$_1$, negated_conjecture)
$x \in$ universal_class cnf(prove_subclass_alternate_defn2$_2$, negated_conjecture)
$\neg x \in$ image(element_relation, $y'$)$'$ cnf(prove_subclass_alternate_defn2$_3$, negated_conjecture)

**SET341-6.p** Image under universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(universal_class, $x$) $\neq$ universal_class cnf(prove_image_under_universal_class$_1$, negated_conjecture)
$x \neq$ null_class cnf(prove_image_under_universal_class$_2$, negated_conjecture)

**SET342-6.p** Image of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(image(xr, union($y, z$)), union(image(xr, $y$), image(xr, $z$))) cnf(prove_image_of_union$_1$, negated_conjecture)

**SET343-6.p** Image of intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(xr, intersection($y, z$)) $\neq$ intersection(image(xr, $y$), image(xr, $z$)) cnf(prove_image_of_intersection$_1$, negated_conjectur

**SET344-6.p** Sum class alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ sum_class($y$) cnf(prove_sum_class_alternate_defn1$_1$, negated_conjecture)
$\neg x \in$ range(element_relation, $x, y$) cnf(prove_sum_class_alternate_defn1$_2$, negated_conjecture)

**SET345-6.p** Sum class alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ sum_class($y$) cnf(prove_sum_class_alternate_defn2$_1$, negated_conjecture)
$\neg$ range(element_relation, $x, y$) $\in y$ cnf(prove_sum_class_alternate_defn2$_2$, negated_conjecture)

**SET346-6.p** Sum class alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in y$ cnf(prove_sum_class_alternate_defn3$_1$, negated_conjecture)
$y \in x$ cnf(prove_sum_class_alternate_defn3$_2$, negated_conjecture)
$\neg z \in$ sum_class($x$) cnf(prove_sum_class_alternate_defn3$_3$, negated_conjecture)

**SET347+4.p** Sum of the empty set is the empty set
include('Axioms/SET006+0.ax')
equal_set(sum(empty_set), empty_set) fof(thI$_{38}$, conjecture)

**SET347-6.p** Sum class of the empty set is the empty set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(null_class) $\neq$ null_class cnf(prove_sum_class_of_null_class$_1$, negated_conjecture)

**SET348-6.p** Sum class of universal class is universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(universal_class) $\neq$ universal_class    cnf(prove_sum_class_of_universal_class$_1$, negated_conjecture)

**SET349-6.p** Sum class of singleton null is null class 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(singleton(null_class)) $\neq$ null_class    cnf(prove_sum_class_of_singleton_null$_1$, negated_conjecture)

**SET350-6.p** Sum class of singleton null is null class 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class($x$) = null_class    cnf(prove_sum_class_is_null_class$_1$, negated_conjecture)
$x \neq$ null_class    cnf(prove_sum_class_is_null_class$_2$, negated_conjecture)
$x \neq$ singleton(null_class)    cnf(prove_sum_class_is_null_class$_3$, negated_conjecture)

**SET351+4.p** Sum of a singleton is the singleton member
include('Axioms/SET006+0.ax')
$\forall a$: equal_set(sum(singleton($a$)), $a$)    fof(thI$_{39}$, conjecture)

**SET351-6.p** Sum class of a singleton is the singleton member
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ universal_class    cnf(prove_sum_of_singleton$_1$, negated_conjecture)
sum_class(singleton($x$)) $\neq x$    cnf(prove_sum_of_singleton$_2$, negated_conjecture)

**SET352+4.p** The sum of an unordered pair is the union of the pair
include('Axioms/SET006+0.ax')
$\forall a, b$: equal_set(sum(unordered_pair($a, b$)), union($a, b$))    fof(thI$_{40}$, conjecture)

**SET352-6.p** The sum class of an unordered pair is the union of the pair
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_sum_of_pair$_1$, negated_conjecture)
sum_class(unordered_pair($x, y$)) $\neq$ union($x, y$)    cnf(prove_sum_of_pair$_2$, negated_conjecture)

**SET353-6.p** Corollary to sum of pair
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_corollary_to_sum_of_pair$_1$, negated_conjecture
$\neg$ union($x, y$) $\in$ universal_class    cnf(prove_corollary_to_sum_of_pair$_2$, negated_conjecture)

**SET354-6.p** Sum of ordered pair
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_sum_of_ordered_pair$_1$, negated_conjecture)
sum_class(ordered_pair($x, y$)) $\neq$ unordered_pair($x$, singleton($y$))    cnf(prove_sum_of_ordered_pair$_2$, negated_conjecture)

**SET355+4.p** If X is in Y, then X is a subset of the sum of Y
include('Axioms/SET006+0.ax')
$\forall a, x$: ($x \in a \Rightarrow x \subseteq$ sum($a$))    fof(thI$_{43}$, conjecture)

**SET355-6.p** If X is in Y, then X is a subset of the sum class of Y
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in y$    cnf(prove_subclass_of_union$_1$, negated_conjecture)
$\neg$ subclass($x$, sum_class($y$))    cnf(prove_subclass_of_union$_2$, negated_conjecture)

**SET356-6.p** Corollary to subclass of union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass($y$, power_class(sum_class($y$)))    cnf(prove_corollary_to_subclass_of_union$_1$, negated_conjecture)

**SET357-6.p** Sum class alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

image(element_relation$'$, $x$) $\neq$ sum_class($x$)    cnf(prove_sum_class_alternate_defn4$_1$, negated_conjecture)

**SET358+4.p** Sum distributes over union
The union of sum(A) and sum(B) is equal to the sum of the union of A and B.
include('Axioms/SET006+0.ax')
$\forall a, b$: equal_set(union(sum($a$), sum($b$)), sum(union($a, b$)))    fof(thI$_{37}$, conjecture)

**SET358-6.p** Sum class distributes over union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(union($x, y$)) $\neq$ union(sum_class($x$), sum_class($y$))    cnf(prove_sum_over_union$_1$, negated_conjecture)

**SET359-6.p** Sum class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(sum_class(intersection($x, y$)), intersection(sum_class($x$), sum_class($y$)))    cnf(prove_sum_class_property1$_1$, negated

**SET360-6.p** Domain is sum squared
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(domain_of($x$), sum_class(sum_class($x$)))    cnf(prove_domain_is_sum_squared$_1$, negated_conjecture)

**SET361-6.p** Range is sum squared
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(range_of($x$), sum_class(sum_class(sum_class($x$))))    cnf(prove_range_is_sum_squared$_1$, negated_conjecture)

**SET362-6.p** Monotonicity of sum
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$)    cnf(prove_monotonicity_of_sum$_1$, negated_conjecture)
$\neg$ subclass(sum_class($x$), sum_class($y$))    cnf(prove_monotonicity_of_sum$_2$, negated_conjecture)

**SET363-6.p** Power class alternative definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ power_class($x$)    cnf(prove_power_class_alternative_defn1$_1$, negated_conjecture)
$\neg$ subclass($z, x$)    cnf(prove_power_class_alternative_defn1$_2$, negated_conjecture)

**SET364-6.p** Power class alternative definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($z, x$)    cnf(prove_power_class_alternative_defn2$_1$, negated_conjecture)
$z \in$ universal_class    cnf(prove_power_class_alternative_defn2$_2$, negated_conjecture)
$\neg z \in$ power_class($x$)    cnf(prove_power_class_alternative_defn2$_3$, negated_conjecture)

**SET365-6.p** Monotonicity of power
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$)    cnf(prove_monotonicity_of_power$_1$, negated_conjecture)
$\neg$ subclass(power_class($x$), power_class($y$))    cnf(prove_monotonicity_of_power$_2$, negated_conjecture)

**SET366+4.p** The empty set is in every power set
include('Axioms/SET006+0.ax')
$\forall a$: empty_set $\in$ power_set($a$)    fof(thI$_{47}$, conjecture)

**SET366-6.p** The empty set is in every power set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ null_class $\in$ power_class($x$)    cnf(prove_null_class_in_power_class$_1$, negated_conjecture)

**SET367-6.p** Power class not in null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
power_class($x$) $=$ null_class    cnf(prove_power_class_not_null_class$_1$, negated_conjecture)

**SET368-6.p** Power class of universal class is universal class

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{power\_class}(\text{universal\_class}) \neq \text{universal\_class}$ cnf($\text{prove\_power\_class\_of\_universal\_class}_1$, negated_conjecture)

**SET369-6.p** Power class of set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in \text{universal\_class}$ cnf($\text{prove\_power\_class\_of\_set}_1$, negated_conjecture)
$\text{image}(\text{subset\_relation}', \text{singleton}(x)) \neq \text{power\_class}(x)$ cnf($\text{prove\_power\_class\_of\_set}_2$, negated_conjecture)

**SET370-6.p** Power class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(\text{cross\_product}(x, y), \text{power\_class}(\text{power\_class}(\text{union}(x, \text{power\_class}(y)))))$ cnf($\text{prove\_power\_class\_property1}_1$, neg

**SET371-6.p** Power class property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg\,\text{subclass}(x, \text{sum\_class}(\text{power\_class}(x)))$ cnf($\text{prove\_power\_class\_property2}_1$, negated_conjecture)

**SET372+4.p** Power set distributes over intersection
The power_set of the intersection of A and B is equal to the intersection of the power_set of A and the power_set of B.
include('Axioms/SET006+0.ax')
$\forall a, b$: $\text{equal\_set}(\text{power\_set}(\text{intersection}(a, b)), \text{intersection}(\text{power\_set}(a), \text{power\_set}(b)))$ fof($\text{thI}_{21}$, conjecture)

**SET372-6.p** Power set distributes over intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(\text{power\_class}(x), \text{power\_class}(y)) \neq \text{power\_class}(\text{intersection}(x, y))$ cnf($\text{prove\_power\_class\_property3}_1$, negated_c

**SET373-6.p** Power class property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\text{intersection}(\text{power\_class}(x), \text{power\_class}(y)) \neq \text{power\_class}(\text{intersection}(x, y))$ cnf($\text{prove\_power\_class\_property4}_1$, negated_c

**SET374-6.p** Power class is closed under union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_union}_1$, negated_conjecture)
$y \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_union}_2$, negated_conjecture)
$\neg\,\text{union}(x, y) \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_union}_3$, negated_conjecture)

**SET375-6.p** Power class is closed under intersection
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_intersection}_1$, negated_conjecture)
$y \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_intersection}_2$, negated_conjecture)
$\neg\,\text{intersection}(x, y) \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_under\_intersection}_3$, negated_conjecture)

**SET376-6.p** Power class set builder
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in z$ cnf($\text{prove\_power\_class\_set\_builder}_1$, negated_conjecture)
$y \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_set\_builder}_2$, negated_conjecture)
$\neg\,\text{union}(\text{singleton}(x), y) \in \text{power\_class}(z)$ cnf($\text{prove\_power\_class\_set\_builder}_3$, negated_conjecture)

**SET377-6.p** Corollary 1 to power class set builder
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in \text{intersection}(\text{power\_class}(z), z)$ cnf($\text{prove\_corollary\_1\_to\_power\_class\_set\_builder}_1$, negated_conjecture)
$\neg\,\text{successor}(x) \in \text{power\_class}(z)$ cnf($\text{prove\_corollary\_1\_to\_power\_class\_set\_builder}_2$, negated_conjecture)

**SET378-6.p** Corollary 2 to power class set builder
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

$\neg$ subclass(image(successor_relation, intersection(power_class($z$), $z$)), power_class($z$))  cnf(prove_corollary_1_to_power_class_

**SET379-6.p** Corollary 3 to power class set builder
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(image(successor_relation, $z$), $z$)  cnf(prove_corollary_1_to_power_class_set_builder$_1$, negated_conjecture)
$\neg$ subclass(image(successor_relation, intersection(power_class($z$), $z$)), intersection(power_class($z$), $z$))  cnf(prove_corollary_1

**SET380-6.p** Relation property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(restrict(xf, $x$, $y$), cross_product(universal_class, universal_class))  cnf(prove_relation_property1$_1$, negated_conjectu

**SET381-6.p** Relation property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))  cnf(prove_relation_property2$_1$, negated_conjecture)
$\neg$ subclass($x$, cross_product(domain_of($x$), range_of($x$)))  cnf(prove_relation_property2$_2$, negated_conjecture)

**SET382-6.p** Corollary 1 to relation property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))  cnf(prove_corollary_1_to_relation_property2$_1$, negated_conjectur
restrict($x$, universal_class, universal_class) $\neq x$  cnf(prove_corollary_1_to_relation_property2$_2$, negated_conjecture)

**SET383-6.p** Corollary 2 to relation property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(restrict($x$, universal_class, universal_class), cross_product(domain_of($x$), range_of($x$)))  cnf(prove_corollary_2_to_r

**SET384-6.p** Corollary 1 to relation property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$((x')')' \neq x'$  cnf(prove_relation_property3$_1$, negated_conjecture)

**SET385-6.p** Corollary 2 to relation property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
(element_relation$'$)$'$ $\neq$ element_relation  cnf(prove_corollary_to_relation_property3$_1$, negated_conjecture)

**SET386-6.p** Relation property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
restrict($x$, universal_class, universal_class) $= x$  cnf(prove_relation_property4$_1$, negated_conjecture)
$\neg$ subclass($x$, cross_product(universal_class, universal_class))  cnf(prove_relation_property4$_2$, negated_conjecture)

**SET387-6.p** Composition alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($u, v$) $\in$ xf $\circ$ yf  cnf(prove_composition_alternate_defn1$_1$, negated_conjecture)
$\neg$ ordered_pair($u$, dom(xf)) $\in$ yf  cnf(prove_composition_alternate_defn1$_2$, negated_conjecture)

**SET388-6.p** Composition alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($u, v$) $\in$ xf $\circ$ yf  cnf(prove_composition_alternate_defn2$_1$, negated_conjecture)
$\neg$ ordered_pair($u$, dom(xf)) $\in$ cross_product(universal_class, universal_class)  cnf(prove_composition_alternate_defn2$_2$, negat

**SET389-6.p** Composition alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($u, v$) $\in$ xf $\circ$ yf  cnf(prove_composition_alternate_defn3$_1$, negated_conjecture)
$\neg$ ordered_pair(dom(xf), $v$) $\in$ xf  cnf(prove_composition_alternate_defn3$_2$, negated_conjecture)

**SET390-6.p** Composition alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

ordered_pair($u, v$) ∈ xf ∘ yf    cnf(prove_composition_alternate_defn4$_1$, negated_conjecture)
¬ ordered_pair(dom(xf), $v$) ∈ cross_product(universal_class, universal_class)    cnf(prove_composition_alternate_defn4$_2$, negat

**SET391-6.p** Composition property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ xr    cnf(prove_composition_property1$_1$, negated_conjecture)
ordered_pair($y, z$) ∈ yr    cnf(prove_composition_property1$_2$, negated_conjecture)
ordered_pair($x, y$) ∈ cross_product(universal_class, universal_class)    cnf(prove_composition_property1$_3$, negated_conjecture)
ordered_pair($y, z$) ∈ cross_product(universal_class, universal_class)    cnf(prove_composition_property1$_4$, negated_conjecture)
¬ ordered_pair($x, z$) ∈ yr ∘ xr    cnf(prove_composition_property1$_5$, negated_conjecture)

**SET392-6.p** Right identity for composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$∘identity_relation ≠ restrict($x$, universal_class, universal_class)    cnf(prove_right_identity_for_composition$_1$, negated_conject

**SET393-6.p** Left identity for composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
identity_relation∘$x$ ≠ restrict($x$, universal_class, universal_class)    cnf(prove_left_identity_for_composition$_1$, negated_conjectu

**SET394-6.p** Composition property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(restrict(identity_relation, domain_of($x$), universal_class), $x'$∘$x$)    cnf(prove_composition_property2$_1$, negated_conj

**SET395-6.p** Composition relates to image
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(xr ∘ yr, $x$) ≠ image(xr, image(yr, $x$))    cnf(prove_compositions_relates_to_image$_1$, negated_conjecture)

**SET396-6.p** Domain of composition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(domain_of(xr ∘ yr), domain_of(yr))    cnf(prove_domain_of_composition$_1$, negated_conjecture)

**SET397-6.p** Range of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(range_of(xr ∘ yr), range_of(xr))    cnf(prove_range_of_composition$_1$, negated_conjecture)

**SET398-6.p** Associativity of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
(xr ∘ yr) ∘ zr ≠ xr ∘ (yr ∘ zr)    cnf(prove_associativity_of_composition$_1$, negated_conjecture)

**SET399-6.p** Left compose with null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
null_class ∘ $x$ ≠ null_class    cnf(prove_left_compose_with_null_class$_1$, negated_conjecture)

**SET400-6.p** Right compose with null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∘ null_class ≠ null_class    cnf(prove_right_compose_with_null_class$_1$, negated_conjecture)

**SET401-6.p** Left compose with universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
universal_class∘$x$ ≠ cross_product(domain_of($x$), universal_class)    cnf(prove_left_compose_with_universal_class$_1$, negated_co

**SET402-6.p** Right compose with universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$∘universal_class ≠ cross_product(universal_class, range_of($x$))    cnf(prove_right_compose_with_universal_class$_1$, negated_co

**SET403-6.p** Domain of composition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(domain_of(yr), range_of(xr)) cnf(prove_dual_of_boyer_lemma_18$_1$, negated_conjecture)
range_of(yr ∘ xr) ≠ range_of(yr) cnf(prove_dual_of_boyer_lemma_18$_2$, negated_conjecture)

**SET404-6.p** Monotonicity of composition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(xr, yr) cnf(prove_monotonicity_of_composition1$_1$, negated_conjecture)
¬ subclass(zr ∘ xr, zr ∘ yr) cnf(prove_monotonicity_of_composition1$_2$, negated_conjecture)

**SET405-6.p** Monotonicity of composition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(xr, yr) cnf(prove_monotonicity_of_composition2$_1$, negated_conjecture)
¬ subclass(xr ∘ zr, yr ∘ zr) cnf(prove_monotonicity_of_composition2$_2$, negated_conjecture)

**SET406-6.p** Corollary 1 monotonicity of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(xr, yr) cnf(prove_corollary_1_monotonicity_of_composition$_1$, negated_conjecture)
¬ subclass(zr ∘ (xr ∘ ur), zr ∘ (yr ∘ ur)) cnf(prove_corollary_1_monotonicity_of_composition$_2$, negated_conjecture)

**SET407-6.p** Corollary 2 monotonicity of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(xr, identity_relation) cnf(prove_corollary_2_monotonicity_of_composition$_1$, negated_conjecture)
¬ subclass(zr ∘ (xr ∘ ur), zr ∘ ur) cnf(prove_corollary_2_monotonicity_of_composition$_2$, negated_conjecture)

**SET408-6.p** Inverse of composition
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
(xr ∘ yr)′ ≠ yr′ ∘ xr′ cnf(prove_inverse_of_composition$_1$, negated_conjecture)

**SET409-6.p** Composition of element relation 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ element_relation∘element_relation cnf(prove_composition_of_element_relation1$_1$, negated_conjecture)
¬ $x$ ∈ sum_class($y$) cnf(prove_composition_of_element_relation1$_2$, negated_conjecture)

**SET410-6.p** Composition of element relation 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∈ sum_class($y$) cnf(prove_composition_of_element_relation2$_1$, negated_conjecture)
$y$ ∈ universal_class cnf(prove_composition_of_element_relation2$_2$, negated_conjecture)
¬ ordered_pair($x, y$) ∈ element_relation∘element_relation cnf(prove_composition_of_element_relation2$_3$, negated_conjecture

**SET411-6.p** Compose condition for singleton membership 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ singleton_relation cnf(prove_compose_condition_for_singleton_membership1$_1$, negated_conjecture)
¬ $x$ ∈ universal_class cnf(prove_compose_condition_for_singleton_membership1$_2$, negated_conjecture)

**SET412-6.p** Compose condition for singleton membership 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ singleton_relation cnf(prove_compose_condition_for_singleton_membership2$_1$, negated_conjecture)
singleton($x$) ≠ $y$ cnf(prove_compose_condition_for_singleton_membership2$_2$, negated_conjecture)

**SET413-6.p** Compose condition for singleton membership 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
singleton($x$) = $y$ cnf(prove_compose_condition_for_singleton_membership3$_1$, negated_conjecture)
$x$ ∈ universal_class cnf(prove_compose_condition_for_singleton_membership3$_2$, negated_conjecture)

¬ ordered_pair($x, y$) ∈ singleton_relation    cnf(prove_compose_condition_for_singleton_membership3$_3$, negated_conjecture)

**SET414-6.p** Composition distributes over union
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∘ union($y, z$) ≠ union($x$ ∘ $y$, $x$ ∘ $z$)    cnf(prove_composition_over_union$_1$, negated_conjecture)

**SET415-6.p** Composition with singleton function 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y$ ∈ domain_of($z$)    cnf(prove_composition_with_singleton_function1$_1$, negated_conjecture)
$z$∘singleton(ordered_pair($x, y$)) ≠ singleton(ordered_pair($x$, apply($z, y$)))    cnf(prove_composition_with_singleton_function1$_2$

**SET416-6.p** Composition with singleton function 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ $y$ ∈ domain_of($z$)    cnf(prove_composition_with_singleton_function2$_1$, negated_conjecture)
$z$∘singleton(ordered_pair($x, y$)) ≠ null_class    cnf(prove_composition_with_singleton_function2$_2$, negated_conjecture)

**SET417-6.p** Composition property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ subclass(image(composition_function, singleton($x$)), cross_product(universal_class, universal_class))    cnf(prove_compositio

**SET418-6.p** Composition property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ function(image(composition_function, singleton($x$)))    cnf(prove_composition_property2$_1$, negated_conjecture)

**SET419-6.p** Composition property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) ∈ cross_product(universal_class, universal_class)    cnf(prove_composition_property3$_1$, negated_conjecture)
apply(image(composition_function, singleton($x$)), $y$) ≠ $x$∘$y$    cnf(prove_composition_property3$_2$, negated_conjecture)

**SET420-6.p** Composition property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∈ universal_class    cnf(prove_composition_property4$_1$, negated_conjecture)
sum_class(image(image(composition_function, singleton($x$)), $y$)) ≠ $x$∘sum_class($y$)    cnf(prove_composition_property4$_2$, nega

**SET421-6.p** Compose class is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ function(compose_class($x$))    cnf(prove_compose_class_is_a_function$_1$, negated_conjecture)

**SET422-6.p** Compose class and apply
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y$ ∈ universal_class    cnf(prove_compose_class_and_apply$_1$, negated_conjecture)
apply(compose_class($x$), $y$) ≠ $x$ ∘ $y$    cnf(prove_compose_class_and_apply$_2$, negated_conjecture)

**SET423-6.p** Sum compose class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
sum_class(image(compose_class($x$), $y$)) ≠ $x$ ∘ sum_class($y$)    cnf(prove_sum_compose_class$_1$, negated_conjecture)

**SET424-6.p** Compose class and composition function are related
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x$ ∈ $v$    cnf(prove_compose_class_and_composition_function$_1$, negated_conjecture)
image(composition_function, singleton($x$)) ≠ compose_class($x$)    cnf(prove_compose_class_and_composition_function$_2$, negat

**SET425-6.p** Single valued class alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($z$)    cnf(prove_single_valued_class_alternate_defn1$_1$, negated_conjecture)

ordered_pair($u, v$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_single_valued_class_alternate_defn1$_2$, negated
ordered_pair($u, w$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_single_valued_class_alternate_defn1$_3$, negated
ordered_pair($u, v$) $\in z$    cnf(prove_single_valued_class_alternate_defn1$_4$, negated_conjecture)
ordered_pair($u, w$) $\in z$    cnf(prove_single_valued_class_alternate_defn1$_5$, negated_conjecture)
$v \neq w$    cnf(prove_single_valued_class_alternate_defn1$_6$, negated_conjecture)

**SET426-6.p** Single valued class alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ ordered_pair(single_valued$_3$($x$), single_valued$_1$($x$)) $\in x$    cnf(prove_single_valued_class_alternate_defn2$_1$, negated_conjecture
$\neg$ single_valued_class($x$)    cnf(prove_single_valued_class_alternate_defn2$_2$, negated_conjecture)

**SET427-6.p** Single valued class alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ ordered_pair(single_valued$_3$($x$), single_valued$_2$($x$)) $\in x$    cnf(prove_single_valued_class_alternate_defn3$_1$, negated_conjecture
$\neg$ single_valued_class($x$)    cnf(prove_single_valued_class_alternate_defn3$_2$, negated_conjecture)

**SET428-6.p** Single valued class alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued$_1$($x$) = single_valued$_2$($x$)    cnf(prove_single_valued_class_alternate_defn4$_1$, negated_conjecture)
$\neg$ single_valued_class($x$)    cnf(prove_single_valued_class_alternate_defn4$_2$, negated_conjecture)

**SET429-6.p** A subclass of a single-valued class is single-valued
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($x$)    cnf(prove_single_valued_subclass$_1$, negated_conjecture)
$\neg$ single_valued_class(intersection($x, y$))    cnf(prove_single_valued_subclass$_2$, negated_conjecture)

**SET430-6.p** In a single-valued class, each image is a singleton
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($x$)    cnf(prove_image_of_single_valued_class_domain$_1$, negated_conjecture)
$z \in$ domain_of($x$)    cnf(prove_image_of_single_valued_class_domain$_2$, negated_conjecture)
singleton(member_of(image($x$, singleton($z$)))) $\neq$ image($x$, singleton($z$))    cnf(prove_image_of_single_valued_class_domain$_3$, ne

**SET431-6.p** The composition of single-valued classes is single-valued
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xr)    cnf(prove_compose_single_valued_classes$_1$, negated_conjecture)
single_valued_class(yr)    cnf(prove_compose_single_valued_classes$_2$, negated_conjecture)
$\neg$ single_valued_class(xr $\circ$ yr)    cnf(prove_compose_single_valued_classes$_3$, negated_conjecture)

**SET432-6.p** Function alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($z$)    cnf(prove_function_alternate_defn1$_1$, negated_conjecture)
ordered_pair($u, v$) $\in z$    cnf(prove_function_alternate_defn1$_2$, negated_conjecture)
ordered_pair($u, w$) $\in z$    cnf(prove_function_alternate_defn1$_3$, negated_conjecture)
$v \neq w$    cnf(prove_function_alternate_defn1$_4$, negated_conjecture)

**SET433-6.p** Function alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))    cnf(prove_function_alternate_defn2$_1$, negated_conjecture)
$\neg$ ordered_pair(single_valued$_3$($x$), single_valued$_1$($x$)) $\in x$    cnf(prove_function_alternate_defn2$_2$, negated_conjecture)
$\neg$ function($x$)    cnf(prove_function_alternate_defn2$_3$, negated_conjecture)

**SET434-6.p** Function alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))    cnf(prove_function_alternate_defn3$_1$, negated_conjecture)
$\neg$ ordered_pair(single_valued$_3$($x$), single_valued$_2$($x$)) $\in x$    cnf(prove_function_alternate_defn3$_2$, negated_conjecture)
$\neg$ function($x$)    cnf(prove_function_alternate_defn3$_3$, negated_conjecture)

**SET435-6.p** Function alternate definition 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x$, cross_product(universal_class, universal_class))   cnf(prove_function_alternate_defn4$_1$, negated_conjecture)
single_valued$_1$($x$) = single_valued$_2$($x$)   cnf(prove_function_alternate_defn4$_2$, negated_conjecture)
$\neg$ function($x$)   cnf(prove_function_alternate_defn4$_3$, negated_conjecture)

**SET436-6.p** Subclass of function is a function, part 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_subclass_of_function1$_1$, negated_conjecture)
$\neg$ function(intersection($x, y$))   cnf(prove_subclass_of_function1$_2$, negated_conjecture)

**SET437-6.p** Subclass of function is a function, part 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_subclass_of_function2$_1$, negated_conjecture)
$\neg$ function(intersection($y, x$))   cnf(prove_subclass_of_function2$_2$, negated_conjecture)

**SET438-6.p** In a function, the image of each domain element is a singleton
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_images_of_functions_are_singletons$_1$, negated_conjecture)
$z \in$ domain_of($x$)   cnf(prove_images_of_functions_are_singletons$_2$, negated_conjecture)
singleton(member_of(image($x$, singleton($z$)))) $\neq$ image($x$, singleton($z$))   cnf(prove_images_of_functions_are_singletons$_3$, nega

**SET439-6.p** Null class is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ function(null_class)   cnf(prove_null_class_is_a_function$_1$, negated_conjecture)

**SET440-6.p** The restriction of function is function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xf)   cnf(prove_restriction_of_function_is_function$_1$, negated_conjecture)
$\neg$ function(restrict(xf, $x, y$))   cnf(prove_restriction_of_function_is_function$_2$, negated_conjecture)

**SET441-6.p** The intersection of functions is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_intersection_of_functions$_1$, negated_conjecture)
function($y$)   cnf(prove_intersection_of_functions$_2$, negated_conjecture)
$\neg$ function(intersection($x, y$))   cnf(prove_intersection_of_functions$_3$, negated_conjecture)

**SET442-6.p** Restriction of function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xf)   cnf(prove_restrict_a_function$_1$, negated_conjecture)
restrict(xf, universal_class, universal_class) $\neq$ xf   cnf(prove_restrict_a_function$_2$, negated_conjecture)

**SET443-6.p** Difference of functions is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_symmetric_difference_of_functions$_1$, negated_conjecture)
function($y$)   cnf(prove_symmetric_difference_of_functions$_2$, negated_conjecture)
$\neg$ function(intersection($x', y$))   cnf(prove_symmetric_difference_of_functions$_3$, negated_conjecture)

**SET444-6.p** Function property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)   cnf(prove_function_property1$_1$, negated_conjecture)
function($y$)   cnf(prove_function_property1$_2$, negated_conjecture)
intersection(domain_of($x$), domain_of($y$)) = null_class   cnf(prove_function_property1$_3$, negated_conjecture)
$\neg$ function(union($x, y$))   cnf(prove_function_property1$_4$, negated_conjecture)

**SET445-6.p** Corollary to function property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)    cnf(prove_corollary_to_function_property1$_1$, negated_conjecture)
$\neg$ function(union($x$, singleton(ordered_pair($y, z$))))    cnf(prove_corollary_to_function_property1$_2$, negated_conjecture)
$\neg\, y \in$ domain_of($x$)    cnf(prove_corollary_to_function_property1$_3$, negated_conjecture)

**SET446-6.p** Function property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ universal_class    cnf(prove_function_property2$_1$, negated_conjecture)
$\neg$ function(cross_product($x$, singleton($y$)))    cnf(prove_function_property2$_2$, negated_conjecture)

**SET447-6.p** Function property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($y$)    cnf(prove_function_property3$_1$, negated_conjecture)
subclass($x, y$)    cnf(prove_function_property3$_2$, negated_conjecture)
restrict($y$, domain_of($x$), universal_class) $\neq x$    cnf(prove_function_property3$_3$, negated_conjecture)

**SET448-6.p** Function property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)    cnf(prove_function_property4$_1$, negated_conjecture)
subclass($y, x$)    cnf(prove_function_property4$_2$, negated_conjecture)
subclass($z$, domain_of($y$))    cnf(prove_function_property4$_3$, negated_conjecture)
restrict($x, z$, universal_class) $\neq$ restrict($y, z$, universal_class)    cnf(prove_function_property4$_4$, negated_conjecture)

**SET449-6.p** Condition 1 for one function to be a subset of another
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)    cnf(prove_function_subset_function1$_1$, negated_conjecture)
subclass(domain_of($x$), domain_of($y$))    cnf(prove_function_subset_function1$_2$, negated_conjecture)
$\neg$ subclass($x, y$)    cnf(prove_function_subset_function1$_3$, negated_conjecture)
$\neg$ not_subfunction($x, y$) $\in$ domain_of($x$)    cnf(prove_function_subset_function1$_4$, negated_conjecture)

**SET450-6.p** Condition 2 for one function to be a subset of another
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)    cnf(prove_function_subset_function2$_1$, negated_conjecture)
subclass(domain_of($x$), domain_of($y$))    cnf(prove_function_subset_function2$_2$, negated_conjecture)
apply($x$, not_subfunction($x, y$)) = apply($y$, not_subfunction($x, y$))    cnf(prove_function_subset_function2$_3$, negated_conjectur
$\neg$ subclass($x, y$)    cnf(prove_function_subset_function2$_4$, negated_conjecture)

**SET451-6.p** Subset relation alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(subset_relation, cross_product(universal_class, universal_class))    cnf(prove_subset_relation_alternate_defn1$_1$, nega

**SET452-6.p** Subset relation alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ subset_relation    cnf(prove_subset_relation_alternate_defn2$_1$, negated_conjecture)
$\neg$ subclass($x, y$)    cnf(prove_subset_relation_alternate_defn2$_2$, negated_conjecture)

**SET453-6.p** Subset relation alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass($x, y$)    cnf(prove_subset_relation_alternate_defn3$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_subset_relation_alternate_defn3$_2$, negated_con
$\neg$ ordered_pair($x, y$) $\in$ subset_relation    cnf(prove_subset_relation_alternate_defn3$_3$, negated_conjecture)

**SET454-6.p** Identity alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

$\neg$ subclass(identity_relation, cross_product(universal_class, universal_class)) cnf(prove_identity_alternate_defn1$_1$, negated_c

**SET455-6.p** Identity alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ identity_relation cnf(prove_identity_alternate_defn2$_1$, negated_conjecture)
$x \neq y$ cnf(prove_identity_alternate_defn2$_2$, negated_conjecture)

**SET456-6.p** Identity alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_identity_alternate_defn3$_1$, negated_conjecture
$x = y$ cnf(prove_identity_alternate_defn3$_2$, negated_conjecture)
$\neg$ ordered_pair($x, y$) $\in$ identity_relation cnf(prove_identity_alternate_defn3$_3$, negated_conjecture)

**SET457-6.p** Identity is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ function(identity_relation) cnf(prove_identity_is_a_function$_1$, negated_conjecture)

**SET458-6.p** Corollary to identity is a function
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ function(restrict(identity_relation, $x, y$)) cnf(prove_corollary_to_identity_is_a_function$_1$, negated_conjecture)

**SET459-6.p** Domain of identity is the universal class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(identity_relation) $\neq$ universal_class cnf(prove_domain_of_identity$_1$, negated_conjecture)

**SET460-6.p** Range of identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(identity_relation) $\neq$ universal_class cnf(prove_range_of_identity$_1$, negated_conjecture)

**SET461-6.p** Domain of restricted identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(restrict(identity_relation, $x, y$)) $\neq$ intersection($x, y$) cnf(prove_domain_of_restricted_identity$_1$, negated_conjectu

**SET462-6.p** Range of restricted identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(restrict(identity_relation, $x, y$)) $\neq$ intersection($x, y$) cnf(prove_range_of_restricted_identity$_1$, negated_conjecture)

**SET463-6.p** Corollary to domain and range of identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(identity_relation, $x$) $\neq x$ cnf(prove_corollary_to_domain_and_range_of_identity$_1$, negated_conjecture)

**SET464-6.p** Class image under identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(restrict(identity_relation, $x, x$), $y$) $\neq$ intersection($x, y$) cnf(prove_class_image_under_identity$_1$, negated_conjecture)

**SET465-6.p** Identity is one-to-one
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ one_to_one(identity_relation) cnf(prove_identity_is_1_to_1$_1$, negated_conjecture)

**SET466-6.p** Inverse of identity is identity
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
identity_relation$'$ $\neq$ identity_relation cnf(prove_inverse_of_identity$_1$, negated_conjecture)

**SET467-6.p** Sets with at most one member 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')
subclass(cross_product($x, x$), identity_relation)      cnf(prove_sets_with_one_member1$_1$, negated_conjecture)
singleton(not_subclass_element($x$, null_class)) $\neq x$      cnf(prove_sets_with_one_member1$_2$, negated_conjecture)
$x \neq$ null_class      cnf(prove_sets_with_one_member1$_3$, negated_conjecture)

**SET468-6.p** Sets with at most one member 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x =$ null_class      cnf(prove_sets_with_one_member2$_1$, negated_conjecture)
$\neg$ subclass(cross_product($x, x$), identity_relation)      cnf(prove_sets_with_one_member2$_2$, negated_conjecture)

**SET469-6.p** Sets with at most one member 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
singleton(member_of($x$)) $= x$      cnf(prove_sets_with_one_member3$_1$, negated_conjecture)
$\neg$ subclass(cross_product($x, x$), identity_relation)      cnf(prove_sets_with_one_member3$_2$, negated_conjecture)

**SET470-6.p** Corollary to sets with one member
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(cross_product($x, x$), identity_relation)      cnf(prove_corollary_to_sets_with_one_member$_1$, negated_conjecture)
singleton(member_of($x$)) $\neq x$      cnf(prove_corollary_to_sets_with_one_member$_2$, negated_conjecture)
$x \neq$ null_class      cnf(prove_corollary_to_sets_with_one_member$_3$, negated_conjecture)

**SET471-6.p** Sets with more than one member 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$u \in x$      cnf(prove_sets_with_many_members1$_1$, negated_conjecture)
$\neg$ not_subclass_element(intersection($x$, singleton($u$)$'$), null_class) $\in x$      cnf(prove_sets_with_many_members1$_2$, negated_conjec
$\neg$ subclass(cross_product($x, x$), identity_relation)      cnf(prove_sets_with_many_members1$_3$, negated_conjecture)

**SET472-6.p** Sets with more than one member 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$u \in x$      cnf(prove_sets_with_many_members2$_1$, negated_conjecture)
not_subclass_element(intersection($x$, singleton($u$)$'$), null_class) $= u$      cnf(prove_sets_with_many_members2$_2$, negated_conjectu
$\neg$ subclass(cross_product($x, x$), identity_relation)      cnf(prove_sets_with_many_members2$_3$, negated_conjecture)

**SET473-6.p** Lemma 1 to restricted domain
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y =$ null_class      cnf(prove_lemma_1_to_restricted_domain$_1$, negated_conjecture)
domain_of(restrict($x, y, y$)) $\neq y$      cnf(prove_lemma_1_to_restricted_domain$_2$, negated_conjecture)

**SET474-6.p** Lemma 2 to restricted domain
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(cross_product($y, y$), union($x$, identity_relation))      cnf(prove_lemma_2_to_restricted_domain$_1$, negated_conjecture)
domain_of(restrict($x, y, y$)) $\neq y$      cnf(prove_lemma_2_to_restricted_domain$_2$, negated_conjecture)
$\neg$ subclass(cross_product($y, y$), identity_relation)      cnf(prove_lemma_2_to_restricted_domain$_3$, negated_conjecture)

**SET475-6.p** Restricted domain
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(cross_product($y, y$), union($x$, identity_relation))      cnf(prove_restricted_domain$_1$, negated_conjecture)
domain_of(restrict($x, y, y$)) $\neq y$      cnf(prove_restricted_domain$_2$, negated_conjecture)
singleton(member_of($y$)) $\neq y$      cnf(prove_restricted_domain$_3$, negated_conjecture)

**SET476-6.p** Intersection subclass
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(intersection($x$, identity_relation), intersection($x, x'$))      cnf(prove_intersection_subclass$_1$, negated_conjecture)

**SET477-6.p** Axiom of subsets 1
include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$y \in$ universal_class cnf(prove_axiom_of_subsets1$_1$, negated_conjecture)

subclass($x, y$) cnf(prove_axiom_of_subsets1$_2$, negated_conjecture)

$\neg\, x \in$ universal_class cnf(prove_axiom_of_subsets1$_3$, negated_conjecture)

**SET478-6.p** Axiom of subsets 2

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$y \in$ universal_class cnf(prove_axiom_of_subsets2$_1$, negated_conjecture)

$\neg$ intersection($x, y$) $\in$ universal_class cnf(prove_axiom_of_subsets2$_2$, negated_conjecture)

**SET479-6.p** Replacement property 1

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$x \in$ universal_class cnf(prove_replacement_property1$_1$, negated_conjecture)

$\neg$ domain_of($x$) $\in$ universal_class cnf(prove_replacement_property1$_2$, negated_conjecture)

**SET480-6.p** Replacement property 2

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$x \in$ universal_class cnf(prove_replacement_property2$_1$, negated_conjecture)

$\neg$ range_of($x$) $\in$ universal_class cnf(prove_replacement_property2$_2$, negated_conjecture)

**SET481-6.p** Replacement property 3

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_replacement_property3$_1$, negated_conjecture)

$\neg$ cross_product($x, y$) $\in$ universal_class cnf(prove_replacement_property3$_2$, negated_conjecture)

**SET482-6.p** Replacement property 4

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

ordered_pair(domain_of($x$), range_of($x$)) $\in$ cross_product(universal_class, universal_class) cnf(prove_replacement_property4

subclass($x$, cross_product(universal_class, universal_class)) cnf(prove_replacement_property4$_2$, negated_conjecture)

$\neg\, x \in$ universal_class cnf(prove_replacement_property4$_3$, negated_conjecture)

**SET483-6.p** Replacement property 5

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$x \in$ universal_class cnf(prove_replacement_property5$_1$, negated_conjecture)

$\neg\, x' \in$ universal_class cnf(prove_replacement_property5$_2$, negated_conjecture)

**SET484-6.p** Replacement property 6

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

function(xf) cnf(prove_replacement_property6$_1$, negated_conjecture)

domain_of(xf) $\in$ universal_class cnf(prove_replacement_property6$_2$, negated_conjecture)

$\neg$ xf $\in$ universal_class cnf(prove_replacement_property6$_3$, negated_conjecture)

**SET485-6.p** Replacement property 7

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

single_valued_class($x$) cnf(prove_replacement_property7$_1$, negated_conjecture)

$y \in$ universal_class cnf(prove_replacement_property7$_2$, negated_conjecture)

$\neg$ restrict($x, y$, universal_class) $\in$ universal_class cnf(prove_replacement_property7$_3$, negated_conjecture)

**SET486-6.p** Replacement property 8

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

sum_class($x$) $\in$ universal_class cnf(prove_replacement_property8$_1$, negated_conjecture)

$\neg\, x \in$ universal_class cnf(prove_replacement_property8$_2$, negated_conjecture)

**SET487-6.p** Replacement property 9

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

power_class($x$) $\in$ universal_class    cnf(prove_replacement_property9$_1$, negated_conjecture)
$\neg\, x \in$ universal_class    cnf(prove_replacement_property9$_2$, negated_conjecture)

**SET488-6.p** Replacement property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
union($x, y$) $\in$ universal_class    cnf(prove_replacement_property10$_1$, negated_conjecture)
$\neg\, x \in$ universal_class    cnf(prove_replacement_property10$_2$, negated_conjecture)

**SET489-6.p** Replacement property 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($x, y$) $\in$ universal_class    cnf(prove_replacement_property11$_1$, negated_conjecture)
$\neg\, x \in$ universal_class    cnf(prove_replacement_property11$_2$, negated_conjecture)
$y \neq$ null_class    cnf(prove_replacement_property11$_3$, negated_conjecture)

**SET490-6.p** Replacement property 12
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product($x, y$) $\in$ universal_class    cnf(prove_replacement_property12$_1$, negated_conjecture)
$\neg\, y \in$ universal_class    cnf(prove_replacement_property12$_2$, negated_conjecture)
$x \neq$ null_class    cnf(prove_replacement_property12$_3$, negated_conjecture)

**SET491-6.p** Diagonalization lemma 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ universal_class    cnf(prove_diagonalization_lemma1$_0$, negated_conjecture)
ordered_pair($x, x$) $\in y$    cnf(prove_diagonalization_lemma1$_1$, negated_conjecture)
$\neg\, x \in$ domain_of(intersection($y$, identity_relation))    cnf(prove_diagonalization_lemma1$_2$, negated_conjecture)

**SET492-6.p** Diagonalization lemma 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ domain_of(intersection($y$, identity_relation))    cnf(prove_diagonalization_lemma2$_1$, negated_conjecture)
$\neg$ ordered_pair($x, x$) $\in y$    cnf(prove_diagonalization_lemma2$_2$, negated_conjecture)

**SET493-6.p** Diagonalization corollary
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ domain_of(intersection($y$, identity_relation))    cnf(prove_diagonalization_corollary$_1$, negated_conjecture)
function($y$)    cnf(prove_diagonalization_corollary$_2$, negated_conjecture)
apply($y, x$) $\neq x$    cnf(prove_diagonalization_corollary$_3$, negated_conjecture)

**SET494-6.p** Diagonalization alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
domain_of(intersection($x'$, identity_relation)) $\neq$ diagonalise($x$)    cnf(prove_diagonalization_alternate_defn1$_1$, negated_conject

**SET495-6.p** Diagonalization alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ diagonalise(xr)    cnf(prove_diagonalization_alternate_defn2$_1$, negated_conjecture)
ordered_pair($z, z$) $\in$ xr    cnf(prove_diagonalization_alternate_defn2$_2$, negated_conjecture)

**SET496-6.p** Diagonalization alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ universal_class    cnf(prove_diagonalization_alternate_defn3$_1$, negated_conjecture)
$\neg\, z \in$ diagonalise(xr)    cnf(prove_diagonalization_alternate_defn3$_2$, negated_conjecture)
$\neg$ ordered_pair($z, z$) $\in$ xr    cnf(prove_diagonalization_alternate_defn3$_3$, negated_conjecture)

**SET497-6.p** Special case of the Russell class, without the regularity axiom
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ diagonalise(element_relation)    cnf(prove_russell_class1$_1$, negated_conjecture)

$z \in z$ cnf(prove_russell_class1$_2$, negated_conjecture)

**SET498-6.p** Special case of the Russell class, without the regularity axiom
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ universal_class cnf(prove_russell_class2$_1$, negated_conjecture)
$\neg z \in$ diagonalise(element_relation) cnf(prove_russell_class2$_2$, negated_conjecture)
$\neg z \in z$ cnf(prove_russell_class2$_3$, negated_conjecture)

**SET499-6.p** The Russell class not a set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
diagonalise(element_relation) $\in$ universal_class cnf(prove_russell_class_not_a_set$_1$, negated_conjecture)

**SET500-6.p** Diagonalization property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
range_of(intersection(xr, identity_relation))$'$ $\neq$ diagonalise(xr) cnf(prove_diagonalization_property1$_1$, negated_conjecture)

**SET501-6.p** Diagonalization property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
diagonalise(xr$'$ $\circ$ xs) $\neq$ domain_of(intersection(xr, xs))$'$ cnf(prove_diagonalization_property2$_1$, negated_conjecture)

**SET502-6.p** Diagonalization property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
diagonalise(xr $\circ$ xs$'$) $\neq$ range_of(intersection(xr, xs))$'$ cnf(prove_diagonalization_property3$_1$, negated_conjecture)

**SET503-6.p** The universal class not set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
universal_class $\in x$ cnf(prove_universal_class_not_set$_1$, negated_conjecture)

**SET504-6.p** Corollary 1 to universal class not set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x$, universal_class) $\in$ cross_product(universal_class, universal_class) cnf(prove_corollary_1_to_universal_class_n

**SET505-6.p** Corollary 2 to universal class not set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(universal_class, $y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_corollary_2_to_universal_class_n

**SET506-6.p** Universal class not null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
universal_class = null_class cnf(prove_universal_class_not_null_class$_1$, negated_conjecture)

**SET507-6.p** Universal class not subclass of null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
subclass(universal_class, null_class) cnf(prove_universal_class_not_subclass_of_null_class$_1$, negated_conjecture)

**SET508-6.p** Corollary 1 to singleton in unordered pair axiom
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
unordered_pair($x$, universal_class) $\neq$ singleton($x$) cnf(prove_corollary_1_to_singleton_in_unordered_pair$_1$, negated_conjectur

**SET509-6.p** Corollary 2 to singleton in unordered pair axiom
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
unordered_pair(universal_class, $x$) $\neq$ singleton($x$) cnf(prove_corollary_2_to_singleton_in_unordered_pair$_1$, negated_conjectur

**SET510-6.p** Corollary to singleton is null class
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

singleton(universal_class) $\neq$ null_class cnf(prove_corollary_to_singleton_is_null_class$_1$, negated_conjecture)

**SET511-6.p** Corollary 1 to special members of ordered pairs
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
unordered_pair(singleton($x$), unordered_pair($x$, null_class)) $\neq$ ordered_pair($x$, universal_class) cnf(prove_corollary_1_to_prop

**SET512-6.p** Corollary 2 to special members of ordered pairs
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
unordered_pair(null_class, singleton(singleton($y$))) $\neq$ ordered_pair(universal_class, $y$) cnf(prove_corollary_2_to_property_1_o

**SET513-6.p** Corollary 3 to special members of ordered pairs
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(universal_class, universal_class) $\neq$ unordered_pair(null_class, singleton(null_class)) cnf(prove_corollary_3_to_pr

**SET514-6.p** Class of ordered pairs is not a set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
cross_product(universal_class, universal_class) $\in x$ cnf(prove_class_of_ordered_pairs_not_set$_1$, negated_conjecture)

**SET515-6.p** No class belongs to itself
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in x$ cnf(prove_no_class_belongs_to_itself$_1$, negated_conjecture)

**SET516-6.p** Corollary to no class belongs to itself
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
singleton($x$) $= x$ cnf(prove_corollary_to_no_class_belongs_to_itself$_1$, negated_conjecture)

**SET517-6.p** If member of X is X then X is not a singleton of a set
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
singleton(member_of($x$)) $= x$ cnf(prove_not_singleton_of_set$_1$, negated_conjecture)
member_of($x$) $= x$ cnf(prove_not_singleton_of_set$_2$, negated_conjecture)

**SET518-6.p** There are no cycles of length 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in y$ cnf(prove_no_cycles_length_2$_1$, negated_conjecture)
$y \in x$ cnf(prove_no_cycles_length_2$_2$, negated_conjecture)

**SET519-6.p** Corollary 1 to no cycles of length 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x$, $y$) $= x$ cnf(prove_corollary_1_no_cycles_length_2$_1$, negated_conjecture)

**SET520-6.p** Corollary 2 to no cycles of length 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x$, $y$) $= y$ cnf(prove_corollary_2_no_cycles_length_2$_1$, negated_conjecture)

**SET521-6.p** Ordered pair determines components 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
first(ordered_pair($x$, $y$)) = ordered_pair($x$, $y$) cnf(prove_ordered_pair_determines_components1$_1$, negated_conjecture)
ordered_pair($x$, $y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_ordered_pair_determines_components1$_2$, nega

**SET522-6.p** Ordered pair determines components 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
second(ordered_pair($x$, $y$)) = ordered_pair($x$, $y$) cnf(prove_ordered_pair_determines_components2$_1$, negated_conjecture)
ordered_pair($x$, $y$) $\in$ cross_product(universal_class, universal_class) cnf(prove_ordered_pair_determines_components2$_2$, nega

**SET523-6.p** Element and complement can't both be sets

include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in y$ cnf(prove_element_and_complement_not_both_sets$_1$, negated_conjecture)
$x' \in z$ cnf(prove_element_and_complement_not_both_sets$_2$, negated_conjecture)

**SET524-6.p** Equivalent condition 1 for x not to be an ordered pair
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
first($x$) $= x$ cnf(prove_condition_for_x_not_to_be_an_ordered_pair1$_1$, negated_conjecture)
second($x$) $\neq x$ cnf(prove_condition_for_x_not_to_be_an_ordered_pair1$_2$, negated_conjecture)

**SET525-6.p** Equivalent condition 2 for x not to be an ordered pair
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
second($x$) $= x$ cnf(prove_condition_for_x_not_to_be_an_ordered_pair2$_1$, negated_conjecture)
first($x$) $\neq x$ cnf(prove_condition_for_x_not_to_be_an_ordered_pair2$_2$, negated_conjecture)

**SET526-6.p** Ordered pair components are sets 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first($x$), second($x$)) $= x$ cnf(prove_ordered_pair_components_are_sets1$_1$, negated_conjecture)
$\neg$ first($x$) $\in$ universal_class cnf(prove_ordered_pair_components_are_sets1$_2$, negated_conjecture)

**SET527-6.p** Ordered pair components are sets 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first($x$), second($x$)) $= x$ cnf(prove_ordered_pair_components_are_sets2$_1$, negated_conjecture)
$\neg$ second($x$) $\in$ universal_class cnf(prove_ordered_pair_components_are_sets2$_2$, negated_conjecture)

**SET528-6.p** Corollary 1 to ordered pair components are sets
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first($x$), second($x$)) $= x$ cnf(prove_corollary$_1$, negated_conjecture)
$\neg x \in$ cross_product(universal_class, universal_class) cnf(prove_corollary$_2$, negated_conjecture)

**SET529-6.p** Corollary 2 to ordered pair components are sets
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first(ordered_pair($x, y$)), second(ordered_pair($x, y$))) = ordered_pair($x, y$) cnf(prove_corollary$_1$, negated_conje
$\neg x \in$ universal_class cnf(prove_corollary$_2$, negated_conjecture)

**SET530-6.p** Corollary 3 to ordered pair components are sets
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair(first(ordered_pair($x, y$)), second(ordered_pair($x, y$))) = ordered_pair($x, y$) cnf(prove_corollary$_1$, negated_conje
$\neg y \in$ universal_class cnf(prove_corollary$_2$, negated_conjecture)

**SET531-6.p** Application property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($z$) cnf(prove_application_property1$_1$, negated_conjecture)
$x \in$ domain_of($z$) cnf(prove_application_property1$_2$, negated_conjecture)
member_of(image($z$, singleton($x$))) $\neq$ apply($z, x$) cnf(prove_application_property1$_3$, negated_conjecture)

**SET532-6.p** Application property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($z$) cnf(prove_application_property2$_1$, negated_conjecture)
$x \in$ domain_of($z$) cnf(prove_application_property2$_2$, negated_conjecture)
image($z$, singleton($x$)) $\neq$ singleton(apply($z, x$)) cnf(prove_application_property2$_3$, negated_conjecture)

**SET533-6.p** The range of Z is the class of applications of Z to Z's domain 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($z$) cnf(prove_class_of_applications_to_domain1$_1$, negated_conjecture)

$x \in$ domain_of($z$)    cnf(prove_class_of_applications_to_domain1$_2$, negated_conjecture)
$\neg$ apply($z, x$) $\in$ range_of($z$)    cnf(prove_class_of_applications_to_domain1$_3$, negated_conjecture)

**SET534-6.p** The range of Z is the class of applications of Z to Z's domain 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class($z$)    cnf(prove_class_of_applications_to_domain2$_1$, negated_conjecture)
$y \in$ range_of($z$)    cnf(prove_class_of_applications_to_domain2$_2$, negated_conjecture)
apply($z$, dom($z$)) $\neq y$    cnf(prove_class_of_applications_to_domain2$_3$, negated_conjecture)

**SET535-6.p** Application property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$y \in$ image(xf, singleton($x$))    cnf(prove_application_property3$_1$, negated_conjecture)
$\neg$ subclass($y$, apply(xf, $x$))    cnf(prove_application_property3$_2$, negated_conjecture)

**SET536-6.p** Corollary 1 to application property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(image(xf, singleton($x$)), power_class(apply(xf, $x$)))    cnf(prove_corollary_1_to_application_property3$_1$, negated_co

**SET537-6.p** Corollary 2 to application property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
ordered_pair($x, y$) $\in$ xf    cnf(prove_corollary_2_to_application_property3$_1$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(universal_class, universal_class)    cnf(prove_corollary_2_to_application_property3$_2$, negate
$\neg$ subclass($y$, apply(xf, $x$))    cnf(prove_corollary_2_to_application_property3$_3$, negated_conjecture)

**SET538-6.p** Application property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
image(sum_class(xf), image(xf, singleton($x$))) $\neq$ apply(xf, $x$)    cnf(prove_application_property4$_1$, negated_conjecture)

**SET539-6.p** Application property 5
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ apply(xf, $z$)    cnf(prove_application_property5$_1$, negated_conjecture)
$z \in$ diagonalise(element_relation$'$ $\circ$ xf)    cnf(prove_application_property5$_2$, negated_conjecture)

**SET540-6.p** Application property 6
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ universal_class    cnf(prove_application_property6$_1$, negated_conjecture)
$\neg z \in$ apply(xf, $z$)    cnf(prove_application_property6$_2$, negated_conjecture)
$\neg z \in$ diagonalise(element_relation$'$ $\circ$ xf)    cnf(prove_application_property6$_3$, negated_conjecture)

**SET541-6.p** Application property 7
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
apply($z, x$) $\neq$ null_class    cnf(prove_application_property7$_1$, negated_conjecture)
$\neg x \in$ domain_of($z$)    cnf(prove_application_property7$_2$, negated_conjecture)

**SET542-6.p** Corollary to application property 9
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)    cnf(prove_corollary_to_application_property9$_1$, negated_conjecture)
$y \in$ image(xf, $x$)    cnf(prove_corollary_to_application_property9$_2$, negated_conjecture)
apply(xf, dom(xf)) $\neq y$    cnf(prove_corollary_to_application_property9$_3$, negated_conjecture)

**SET543-6.p** Corollary to application property 10
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)    cnf(prove_corollary_to_application_property10$_1$, negated_conjecture)
$x \in y$    cnf(prove_corollary_to_application_property10$_2$, negated_conjecture)
$x \in$ domain_of(xf)    cnf(prove_corollary_to_application_property10$_3$, negated_conjecture)

$\neg$ apply(xf, $x$) $\in$ image(xf, $y$)     cnf(prove_corollary_to_application_property10$_4$, negated_conjecture)

**SET544-6.p** Corollary to application property 11
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)     cnf(prove_corollary_to_application_property11$_1$, negated_conjecture)
$x \in$ domain_of(yf $\circ$ xf)     cnf(prove_corollary_to_application_property11$_2$, negated_conjecture)
apply(yf $\circ$ xf, $x$) $\neq$ apply(yf, apply(xf, $x$))     cnf(prove_corollary_to_application_property11$_3$, negated_conjecture)

**SET545-6.p** Application special case 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$x \in$ universal_class     cnf(prove_application_special_case1$_1$, negated_conjecture)
apply(universal_class, $x$) $\neq$ universal_class     cnf(prove_application_special_case1$_2$, negated_conjecture)

**SET546-6.p** Application special case 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
apply(null_class, $x$) $\neq$ null_class     cnf(prove_application_special_case2$_1$, negated_conjecture)
$\neg\, x \in$ universal_class     cnf(prove_application_special_case2$_2$, negated_conjecture)

**SET547-6.p** Application special case 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
apply($x$, universal_class) $\neq$ null_class     cnf(prove_application_special_case3$_1$, negated_conjecture)

**SET548-6.p** Application property 16
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)     cnf(prove_application_property13$_1$, negated_conjecture)
$u \in$ domain_of(intersection($y, x$))     cnf(prove_application_property13$_2$, negated_conjecture)
apply(intersection($y, x$), $u$) $\neq$ apply($x, u$)     cnf(prove_application_property13$_3$, negated_conjecture)

**SET549-6.p** Application property 17
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(image(application_function, singleton($x$)), cross_product(universal_class, universal_class))     cnf(prove_application

**SET550-6.p** Application property 18
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ function(image(application_function, singleton($x$)))     cnf(prove_application_property15$_1$, negated_conjecture)

**SET551-6.p** Application property 19
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function($x$)     cnf(prove_application_property16$_1$, negated_conjecture)
$x \in$ universal_class     cnf(prove_application_property16$_2$, negated_conjecture)
image(application_function, singleton($x$)) $\neq x$     cnf(prove_application_property16$_3$, negated_conjecture)

**SET552-6.p** Application property 20
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xf)     cnf(prove_application_property17$_1$, negated_conjecture)
subclass($x$, domain_of(intersection(xf, identity_relation)))     cnf(prove_application_property17$_2$, negated_conjecture)
image(xf, $x$) $\neq x$     cnf(prove_application_property17$_3$, negated_conjecture)

**SET553-6.p** Cantor class alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$\neg$ subclass(cantor($x$), domain_of($x$))     cnf(prove_cantor_class_alternate_defn$_1$, negated_conjecture)

**SET554-6.p** Cantor class alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ cantor(xr)     cnf(prove_cantor_class_property1$_1$, negated_conjecture)

$z \in$ apply(xr, $z$)    cnf(prove_cantor_class_property$1_2$, negated_conjecture)

**SET555-6.p** Cantor class alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ domain_of(xr)    cnf(prove_cantor_class_property$2_1$, negated_conjecture)
$\neg z \in$ apply(xr, $z$)    cnf(prove_cantor_class_property$2_2$, negated_conjecture)
$\neg z \in$ cantor(xr)    cnf(prove_cantor_class_property$2_3$, negated_conjecture)

**SET556-6.p** Cantor class property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
$z \in$ domain_of(xr)    cnf(prove_cantor_class_property$3_1$, negated_conjecture)
apply(xr, $z$) = cantor(xr)    cnf(prove_cantor_class_property$3_2$, negated_conjecture)

**SET557-6.p** Cantor's theorem
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
single_valued_class(xf)    cnf(prove_cantors_theorem$_1$, negated_conjecture)
domain_of(xf) $\in$ universal_class    cnf(prove_cantors_theorem$_2$, negated_conjecture)
subclass(power_class(domain_of(xf)), range_of(xf))    cnf(prove_cantors_theorem$_3$, negated_conjecture)

**SET557^1.p** Cantor's theorem
$\neg \exists g$: \$i $\rightarrow$ \$i $\rightarrow$ \$o: $\forall f$: \$i $\rightarrow$ \$o: $\exists x$: \$i: $(g@x) = f$    thf(surjectiveCantorThm, conjecture)

**SET557^6.p** TPS problem THM43
Restatement of Cantor's theorem.
$\forall s$: \$i $\rightarrow$ \$o: $\neg \exists g$: \$i $\rightarrow$ \$i $\rightarrow$ \$o: $\forall f$: \$i $\rightarrow$ \$o: ($\forall$xx: \$i: (($f$@xx) $\Rightarrow$ ($s$@xx)) $\Rightarrow$ $\exists j$: \$i: ($s@j$ and ($g@j$) = $f$))    thf(cTHM43_pme, conjecture)

**SET558-6.p** Compatible functions alternate definition 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
operation(xf$_1$)    cnf(prove_compatible_functions_alternate_defn$1_1$, negated_conjecture)
compatible(xh, xf$_1$, xf$_2$)    cnf(prove_compatible_functions_alternate_defn$1_2$, negated_conjecture)
cross_product(domain_of(xh), domain_of(xh)) $\neq$ domain_of(xf$_1$)    cnf(prove_compatible_functions_alternate_defn$1_3$, negated

**SET559-6.p** Compatible functions alternate definition 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
operation(xf$_2$)    cnf(prove_compatible_functions_alternate_defn$2_1$, negated_conjecture)
compatible(xh, xf$_1$, xf$_2$)    cnf(prove_compatible_functions_alternate_defn$2_2$, negated_conjecture)
$\neg$ subclass(cross_product(range_of(xh), range_of(xh)), domain_of(xf$_2$))    cnf(prove_compatible_functions_alternate_defn$2_3$, ne

**SET560-6.p** Compatible functions alternate definition 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
function(xh)    cnf(prove_compatible_functions_alternate_defn$3_1$, negated_conjecture)
cross_product(domain_of(xh), domain_of(xh)) = domain_of(xf$_1$)    cnf(prove_compatible_functions_alternate_defn$3_2$, negated
subclass(cross_product(range_of(xh), range_of(xh)), domain_of(xf$_2$))    cnf(prove_compatible_functions_alternate_defn$3_3$, nega
$\neg$ compatible(xh, xf$_1$, xf$_2$)    cnf(prove_compatible_functions_alternate_defn$3_4$, negated_conjecture)

**SET561-6.p** Compatible function property 1
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
operation(xf$_1$)    cnf(prove_compatible_function_property$1_1$, negated_conjecture)
compatible(xh, xf$_1$, xf$_2$)    cnf(prove_compatible_function_property$1_2$, negated_conjecture)
ordered_pair($x, y$) $\in$ cross_product(domain_of(xh), domain_of(xh))    cnf(prove_compatible_function_property$1_3$, negated_con
$\neg$ apply(xf$_1$, ordered_pair($x, y$)) $\in$ domain_of(xh)    cnf(prove_compatible_function_property$1_4$, negated_conjecture)

**SET562-6.p** Compatible function property 2
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
operation(xf$_2$)    cnf(prove_compatible_function_property$2_0$, negated_conjecture)
compatible(xh, xf$_1$, xf$_2$)    cnf(prove_compatible_function_property$2_1$, negated_conjecture)

ordered_pair$(x, y)$ ∈ cross_product(domain_of(xh), domain_of(xh)) cnf(prove_compatible_function_property$2_2$, negated_con
¬ ordered_pair(apply(xh, $x$), apply(xh, $y$)) ∈ domain_of($xf_2$) cnf(prove_compatible_function_property$2_3$, negated_conjecture

**SET563-6.p** Compatible function property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
compatible($xh_1, xf_1, xf_2$) cnf(prove_compatible_function_property$3_1$, negated_conjecture)
compatible($xh_2, xf_2, xf_3$) cnf(prove_compatible_function_property$3_2$, negated_conjecture)
¬ subclass(range_of($xh_1$), domain_of($xh_2$)) cnf(prove_compatible_function_property$3_3$, negated_conjecture)

**SET564-6.p** Corollary 1 to compatible function property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
compatible($xh_1, xf_1, xf_2$) cnf(prove_corollary_1_to_compatible_function_property$3_1$, negated_conjecture)
compatible($xh_2, xf_2, xf_3$) cnf(prove_corollary_1_to_compatible_function_property$3_2$, negated_conjecture)
domain_of($xh_2$∘$xh_1$) ≠ domain_of($xh_1$) cnf(prove_corollary_1_to_compatible_function_property$3_3$, negated_conjecture)

**SET565-6.p** Corollary 2 to compatible function property 3
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
compatible($xh_1, xf_1, xf_2$) cnf(prove_corollary_2_to_compatible_function_property$3_1$, negated_conjecture)
compatible($xh_2, xf_2, xf_3$) cnf(prove_corollary_2_to_compatible_function_property$3_2$, negated_conjecture)
cross_product(domain_of($xh_2$∘$xh_1$), domain_of($xh_2$∘$xh_1$)) ≠ cross_product(domain_of($xh_1$), domain_of($xh_1$)) cnf(prove_cor

**SET566-6.p** Compatible function property 4
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
compatible($xh_1, xf_1, xf_2$) cnf(prove_compatible_function_property$4_1$, negated_conjecture)
compatible($xh_2, xf_2, xf_3$) cnf(prove_compatible_function_property$4_2$, negated_conjecture)
¬ compatible($xh_2$ ∘ $xh_1, xf_1, xf_3$) cnf(prove_compatible_function_property$4_3$, negated_conjecture)

**SET567-6.p** Compatible function special case
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')
¬ operation(null_class) cnf(prove_compatible_function_special_case$_1$, negated_conjecture)

**SET573+3.p** Trybulec's 12th Boolean property of sets
$\forall b, c$: (intersect$(b, c)$ ⟺ $\exists d$: $(d \in b$ and $d \in c)$) fof(intersect_defn, axiom)
$\forall b, c$: (disjoint$(b, c)$ ⟺ ¬ intersect$(b, c)$) fof(disjoint_defn, axiom)
$\forall b, c$: (intersect$(b, c)$ ⇒ intersect$(c, b)$) fof(symmetry_of_intersect, axiom)
$\forall b, c, d$: (($b \in c$ and disjoint$(c, d)$) ⇒ $\neg\, b \in d$) fof(prove_th$_{12}$, conjecture)

**SET574+3.p** Trybulec's 13th Boolean property of sets
$\forall b, c$: (intersect$(b, c)$ ⟺ $\exists d$: $(d \in b$ and $d \in c)$) fof(intersect_defn, axiom)
$\forall b, c$: (intersect$(b, c)$ ⇒ intersect$(c, b)$) fof(symmetry_of_intersect, axiom)
$\forall b, c, d$: (($b \in c$ and $b \in d$) ⇒ intersect$(c, d)$) fof(prove_th$_{13}$, conjecture)

**SET575+3.p** Trybulec's 15th Boolean property of sets
$\forall b, c$: (intersect$(b, c)$ ⟺ $\exists d$: $(d \in b$ and $d \in c)$) fof(intersect_defn, axiom)
$\forall b, c$: (intersect$(b, c)$ ⇒ intersect$(c, b)$) fof(symmetry_of_intersect, axiom)
$\forall b, c$: (intersect$(b, c)$ ⇒ $\exists d$: $(d \in b$ and $d \in c)$) fof(prove_th$_{15}$, conjecture)

**SET575^7.p** Trybulec's 15th Boolean property of sets
include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
∈ : mu → mu → $i → $o thf(member_type, type)
intersect: mu → mu → $i → $o thf(intersect_type, type)
mvalid@(mbox_$s_4$@(mforall_ind@$\lambda b$: mu: (mbox_$s_4$@(mforall_ind@$\lambda c$: mu: (mand@(mbox_$s_4$@(mimplies@(mbox_$s_4$@(intersect
@$d$@$b$))@(mbox_$s_4$@(∈ @$d$@$c$))))))@(mbox_$s_4$@(mimplies@(mexists_ind@$\lambda d$: mu: (mand@(mbox_$s_4$@(∈ @$d$@$b$))@(mbox_$s_4$@(
@$d$@$c$))))@(mbox_$s_4$@(intersect@$b$@$c$))))))))) thf(intersect_defn, axiom)
mvalid@(mbox_$s_4$@(mforall_ind@$\lambda b$: mu: (mbox_$s_4$@(mforall_ind@$\lambda c$: mu: (mbox_$s_4$@(mimplies@(mbox_$s_4$@(intersect@$b$@$c$))@
mvalid@(mbox_$s_4$@(mforall_ind@$\lambda b$: mu: (mbox_$s_4$@(mforall_ind@$\lambda c$: mu: (mbox_$s_4$@(mimplies@(mbox_$s_4$@(intersect@$b$@$c$))@
@$d$@$b$))@(mbox_$s_4$@(∈ @$d$@$c$)))))))))) thf(prove_th$_{15}$, conjecture)

**SET576+3.p** Trybulec's 17th Boolean property of sets
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \neg\, \text{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c\colon (\forall d\colon (d \in b \Rightarrow \neg\, d \in c) \Rightarrow \text{disjoint}(b, c))$ fof(prove_th$_{17}$, conjecture)

**SET576^7.p** Trybulec's 17th Boolean property of sets
include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
intersect: mu → mu → \$i → \$o thf(intersect_type, type)
disjoint: mu → mu → \$i → \$o thf(disjoint_type, type)
∈ : mu → mu → \$i → \$o thf(member_type, type)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mequiv@(intersect@$b$@$c$)@(mexists_ind@$\lambda d$: mu: (mand@(∈ @$d$@$b$)@(∈ @$d$@$c$)))))) thf(intersect_defn, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mequiv@(disjoint@$b$@$c$)@(mnot@(intersect@$b$@$c$))))) thf(disjoint_de
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(intersect@$b$@$c$)@(intersect@$c$@$b$)))) thf(symmetry_of_inte
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mforall_ind@$\lambda d$: mu: (mimplies@(∈ @$d$@$b$)@(mnot@(∈ @$d$@$c$))))@(disjoint@$b$@$c$)))) thf(prove_th$_{17}$, conjecture)

**SET577+3.p** Trybulec's 18th Boolean property of sets
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d\colon (\forall e\colon (e \in b \iff (e \in c \text{ or } e \in d)) \Rightarrow b = \text{union}(c, d))$ fof(prove_th$_{18}$, conjecture)

**SET578+3.p** Trybulec's 19th Boolean property of sets
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d\colon (\forall e\colon (e \in b \iff (e \in c \text{ and } e \in d)) \Rightarrow b = \text{intersection}(c, d))$ fof(prove_th$_{19}$, conjecture)

**SET579+3.p** Trybulec's 20th Boolean property of sets
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d\colon (\forall e\colon (e \in b \iff (e \in c \text{ and } \neg\, e \in d)) \Rightarrow b = c \setminus d)$ fof(prove_th$_{20}$, conjecture)

**SET580+3.p** x is in X sym Y iff x is in X iff x is not in Y
x is in the symmetric difference of X and Y iff it is not the case x is in X iff x is in Y.
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d\colon (b \in \text{symmetric\_difference}(c, d) \iff \neg\, b \in c \iff b \in d)$ fof(prove_th$_{23}$, conjecture)

**SET580^3.p** x is in X sym Y iff x is in X iff x is not in Y
x is in the symmetric difference of X and Y iff it is not the case x is in X iff x is in Y.
include('Axioms/SET008^0.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $u$: \$i: ((excl_union@$x$@$y$@$u$) $\iff$ (($x$@$u$) $\iff$ ¬ $y$@$u$)) thf(thm, conjecture)

**SET580^5.p** TPS problem BOOL-PROP-23
Trybulec's 23rd Boolean property of sets
$a$: \$tType thf(a_type, type)

$\forall$xx: $a, x$: $a \to$ \$o, $y$: $a \to$ \$o: (((($x$@xx and $\neg y$@xx) or ($y$@xx and $\neg x$@xx)) $\iff$ ($x$@xx)) $\iff$ $\neg y$@xx) thf(cBOOL_PI

**SET581+3.p** Trybulec's 24th Boolean property of sets
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$)) fof(intersection_defn, axiom)
$\forall b$: $\neg b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c$: (not_equal($b, c$) $\iff b \neq c$) fof(not_equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$) fof(commutativity_of_intersection, axiom)
$\forall b$: (empty($b$) $\iff \forall c$: $\neg c \in b$) fof(empty_defn, axiom)
$\forall b, c, d$: (($b \in c$ and $b \in d$) $\Rightarrow$ not_equal(intersection($c, d$), empty_set)) fof(prove_th$_{24}$, conjecture)

**SET582+3.p** If x not in X iff x in Y iff x in Z, then X = Y sym Z
If for every x : x is not in X iff x is in Y iff x is in Z, then X is the symmetric difference of Y and Z.
$\forall b, c$: ($\forall d$: ($d \in b \iff d \in c$) $\Rightarrow b = c$) fof(member_equal, axiom)
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$)) fof(union_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c) \iff$ ($d \in b$ and $\neg d \in c$)) fof(difference_defn, axiom)
$\forall b, c$: symmetric_difference($b, c$) = union($b \setminus c, c \setminus b$) fof(symmetric_difference_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$) fof(commutativity_of_union, axiom)
$\forall b, c$: symmetric_difference($b, c$) = symmetric_difference($c, b$) fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: ($\forall e$: ($\neg e \in b \iff$ ($e \in c \iff e \in d$)) $\Rightarrow b =$ symmetric_difference($c, d$)) fof(prove_th$_{25}$, conjecture)

**SET582^5.p** TPS problem BOOL-PROP-25
Trybulec's 25th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\forall$xx: $a$: (($\neg x$@xx $\iff$ ($y$@xx)) $\iff$ ($z$@xx)) $\Rightarrow x = (\lambda$xz: $a$: (($y$@xz and $\neg z$@xz) or ($z$@

**SET583+3.p** Extensionality
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: (($b \subseteq c$ and $c \subseteq b$) $\Rightarrow b = c$) fof(prove_extensionality, conjecture)

**SET583^7.p** Extensionality
include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
$\in$ : mu $\to$ mu $\to$ \$i $\to$ \$o thf(member_type, type)
$\subseteq$ : mu $\to$ mu $\to$ \$i $\to$ \$o thf(subset_type, type)
mvalid@(mforall_ind@$\lambda x$: mu: (qmltpeq@$x$@$x$)) thf(reflexivity, axiom)
mvalid@(mforall_ind@$\lambda x$: mu: (mforall_ind@$\lambda y$: mu: (mimplies@(qmltpeq@$x$@$y$)@(qmltpeq@$y$@$x$)))) thf(symmetry, axio
mvalid@(mforall_ind@$\lambda x$: mu: (mforall_ind@$\lambda y$: mu: (mforall_ind@$\lambda z$: mu: (mimplies@(mand@(qmltpeq@$x$@$y$)@(qmltpeq@$y$
mvalid@(mforall_ind@$\lambda a$: mu: (mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mand@(qmltpeq@$a$@$b$)@($\in$ @$a$@$c$))@($\in$ @$b$@$c$))))) thf(member_substitution$_1$, axiom)
mvalid@(mforall_ind@$\lambda a$: mu: (mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mand@(qmltpeq@$a$@$b$)@($\in$ @$c$@$a$))@($\in$ @$c$@$b$))))) thf(member_substitution$_2$, axiom)
mvalid@(mforall_ind@$\lambda a$: mu: (mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mand@(qmltpeq@$a$@$b$)@($\subseteq$ @$a$@$c$))@($\subseteq$ @$b$@$c$))))) thf(subset_substitution$_1$, axiom)
mvalid@(mforall_ind@$\lambda a$: mu: (mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mand@(qmltpeq@$a$@$b$)@($\subseteq$ @$c$@$a$))@($\subseteq$ @$c$@$b$))))) thf(subset_substitution$_2$, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mequiv@(qmltpeq@$b$@$c$)@(mand@($\subseteq$ @$b$@$c$)@($\subseteq$ @$c$@$b$))))) thf(equa
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mequiv@($\subseteq$ @$b$@$c$)@(mforall_ind@$\lambda d$: mu: (mimplies@($\in$ @$d$@$b$)@($\in$ @$d$@$c$)))))) thf(subset_defn, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: ($\subseteq$ @$b$@$b$)) thf(reflexivity_of_subset, axiom)
mvalid@(mforall_ind@$\lambda b$: mu: (mforall_ind@$\lambda c$: mu: (mimplies@(mand@($\subseteq$ @$b$@$c$)@($\subseteq$ @$c$@$b$))@(qmltpeq@$b$@$c$)))) thf(pr

**SET584+3.p** If X (= Y, then X U Z (= Y U Z
If X is a subset of Y, then the union of X and Z is a subset of the union of Y and Z.
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$)) fof(union_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)

$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(b \subseteq c \Rightarrow \text{union}(b, d) \subseteq \text{union}(c, d))$ fof(prove_th$_{33}$, conjecture)

**SET584^5.p** TPS problem BOOL-PROP-33
Trybulec's 33rd Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: $(\forall \text{xx}$: $a$: $((x@\text{xx}) \Rightarrow (y@\text{xx})) \Rightarrow \forall \text{xx}$: $a$: $((x@\text{xx}$ or $z@\text{xx}) \Rightarrow (y@\text{xx}$ or $z@\text{xx})))$ thf(cB

**SET585+3.p** The intersection of X and Y is a subset of the union of X and Z
$\forall b, c, d$: $((b \subseteq c$ and $c \subseteq d) \Rightarrow b \subseteq d)$ fof(transitivity_of_subset, axiom)
$\forall b, c$: $b \subseteq \text{union}(b, c)$ fof(subset_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) \subseteq b$ fof(intersection_is_subset, axiom)
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b$ and $d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{intersection}(b, c) \subseteq \text{union}(b, d)$ fof(prove_intersection_subset_of_union, conjecture)

**SET585^5.p** TPS problem BOOL-PROP-38
Trybulec's 38th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o, xx: $a$: $((x@\text{xx}$ and $y@\text{xx}) \Rightarrow (x@\text{xx}$ or $z@\text{xx}))$ thf(cBOOL_PROP_38_pme, conjecture)

**SET586+3.p** If X (= Y, then X ∧ Z (= Y ∧ Z
If X is a subset of Y, then the intersection of X and Z is a subset of the intersection of Y and Z.
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b$ and $d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(b \subseteq c \Rightarrow \text{intersection}(b, d) \subseteq \text{intersection}(c, d))$ fof(prove_intersection_of_subset, conjecture)

**SET586^5.p** TPS problem BOOL-PROP-40
Trybulec's 40th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: $(\forall \text{xx}$: $a$: $((x@\text{xx}) \Rightarrow (y@\text{xx})) \Rightarrow \forall \text{xx}$: $a$: $((x@\text{xx}$ and $z@\text{xx}) \Rightarrow (y@\text{xx}$ and $z@\text{xx})))$ thf(cBOOL_PROP_40_pme, conjecture)

**SET587+3.p** X Y = the empty set iff X (= Y
The difference of X and Y is the empty set iff X is a subset of Y.
$\forall b, c$: $(\forall d$: $(d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg d \in c))$ fof(difference_defn, axiom)
$\forall b$: $\neg b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: $(\text{empty}(b) \iff \forall c$: $\neg c \in b)$ fof(empty_defn, axiom)
$\forall b, c$: $(b \setminus c = \text{empty\_set} \iff b \subseteq c)$ fof(prove_difference_empty_set, conjecture)

**SET587^5.p** TPS problem BOOL-PROP-45
Trybulec's 45th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: $((\lambda \text{xx}$: $a$: $(x@\text{xx}$ and $\neg y@\text{xx})) = (\lambda \text{xx}$: $a$: \$false$) \iff \forall \text{xx}$: $a$: $((x@\text{xx}) \Rightarrow (y@\text{xx})))$ thf(cBOOL_P

**SET588+3.p** If X (= Y, then X Z (= Y Z
If X is a subset of Y, then the difference of X and Z is a subset of the difference of Y and Z.
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)

$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d\colon (b \subseteq c \Rightarrow (b \setminus d) \subseteq (c \setminus d))$ fof(prove_difference_subset$_1$, conjecture)

**SET588^5.p** TPS problem BOOL-PROP-46
Trybulec's 46th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o, z\colon a \rightarrow \$o\colon (\forall xx\colon a\colon ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx\colon a\colon ((x@xx \text{ and } \neg z@xx) \Rightarrow (y@xx \text{ and } \neg z@xx)))$ thf(cBOOL_PROP_46_pme, conjecture)

**SET589+3.p** If X (= Y and Z (= V, then X V (= Y Z
If X is a subset of Y and Z is a subset of V, then the difference of X and V is a subset of the difference of Y and Z.
$\forall b, c, d\colon ((b \subseteq c \text{ and } c \subseteq d) \Rightarrow b \subseteq d)$ fof(transitivity_of_subset, axiom)
$\forall b, c, d\colon (b \subseteq c \Rightarrow (b \setminus d) \subseteq (c \setminus d))$ fof(difference_subset$_1$, axiom)
$\forall b, c, d\colon (b \subseteq c \Rightarrow (d \setminus c) \subseteq (d \setminus b))$ fof(difference_subset$_2$, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d, e\colon ((b \subseteq c \text{ and } d \subseteq e) \Rightarrow (b \setminus e) \subseteq (c \setminus d))$ fof(prove_th$_{48}$, conjecture)

**SET589^5.p** TPS problem BOOL-PROP-48
Trybulec's 48th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o, z\colon a \rightarrow \$o, v\colon a \rightarrow \$o\colon ((\forall xx\colon a\colon ((x@xx) \Rightarrow (y@xx)) \text{ and } \forall xx\colon a\colon ((z@xx) \Rightarrow (v@xx))) \Rightarrow \forall xx\colon a\colon ((x@xx \text{ and } \neg v@xx) \Rightarrow (y@xx \text{ and } \neg z@xx)))$ thf(cBOOL_PROP_48_pme, conjecture)

**SET590+3.p** The difference of X and Y is a subset of X
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b \setminus c) \subseteq b$ fof(prove_th$_{49}$, conjecture)

**SET590^5.p** TPS problem BOOL-PROP-49
Trybulec's 49th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o, xx\colon a\colon ((x@xx \text{ and } \neg y@xx) \Rightarrow (x@xx))$ thf(cBOOL_PROP_49_pme, conjecture)

**SET591+3.p** If X (= Y X, then X = the empty set
If X is a subset of the difference of Y and X, then X is the empty set.
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg d \in c))$ fof(difference_defn, axiom)
$\forall b\colon \neg b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (b \subseteq (c \setminus b) \Rightarrow b = \text{empty\_set})$ fof(prove_th$_{50}$, conjecture)

**SET591^5.p** TPS problem BOOL-PROP-50
Trybulec's 50th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o\colon (\forall xx\colon a\colon ((x@xx) \Rightarrow (y@xx \text{ and } \neg x@xx)) \Rightarrow x = (\lambda xx\colon a\colon \$\text{false}))$ thf(cBOOL_PROP_50_pme, co

**SET592+3.p** If X (= Y and X (= Z and Y ∧ Z = empty set, then X = empty set
If X is a subset of Y and X is a subset of Z and the intersection of Y and Z is the empty set, then X is the empty set.
$\forall b\colon (b \subseteq \text{empty\_set} \Rightarrow b = \text{empty\_set})$ fof(subset_of_empty_set_is_empty_set, axiom)
$\forall b, c, d\colon ((b \subseteq c \text{ and } b \subseteq d) \Rightarrow b \subseteq \text{intersection}(c, d))$ fof(intersection_of_subsets, axiom)
$\forall b\colon \neg b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)

$\forall b, c, d$: $((b \subseteq c$ and $b \subseteq d$ and intersection$(c, d) =$ empty_set$) \Rightarrow b =$ empty_set$)$ fof(prove_th$_{51}$, conjecture)

**SET592^5.p** TPS problem BOOL-PROP-51
Trybulec's 51st Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: $((\forall$xx: $a$: $((x$@xx$) \Rightarrow (y$@xx$))$ and $\forall$xx: $a$: $((x$@xx$) \Rightarrow (z$@xx$))$ and $(\lambda$xx: $a$: $(y$@xx and $z$@
$(\lambda$xx: $a$: \$false$)) \Rightarrow x = (\lambda$xx: $a$: \$false$))$ thf(cBOOL_PROP_51_pme, conjecture)

**SET593+3.p** If X (= Y U Z, then X Y (= Z and X Z (= Y
If X is a subset of the union of Y and Z, then the difference of X and Y is a subset of Z and the difference of X and Z is a subset of Y.
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(b \subseteq$ union$(c, d) \Rightarrow ((b \setminus c) \subseteq d$ and $(b \setminus d) \subseteq c))$ fof(prove_th$_{52}$, conjecture)

**SET593^5.p** TPS problem BOOL-PROP-52
Trybulec's 52nd Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: $(\forall$xx: $a$: $((x$@xx$) \Rightarrow (y$@xx or $z$@xx$)) \Rightarrow (\forall$xx: $a$: $((x$@xx and $\neg y$@xx$) \Rightarrow$ $(z$@xx$))$ and $\forall$xx: $a$: $((x$@xx and $\neg z$@xx$) \Rightarrow (y$@xx$))))$ thf(cBOOL_PROP_52_pme, conjecture)

**SET594+3.p** If X ∧ Y U X ∧ Z = X, then X (= Y U Z
If the intersection of X and the union of Y and the intersection of X and Z is X, then X is a subset of the union of Y and Z.
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in$ intersection$(b, c) \iff (d \in b$ and $d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: intersection$(b, c) =$ intersection$(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: (union(intersection$(b, c)$, intersection$(b, d)) = b \Rightarrow b \subseteq$ union$(c, d))$ fof(prove_th$_{53}$, conjecture)

**SET594^5.p** TPS problem BOOL-PROP-53
Trybulec's 53rd Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: $((\lambda$xz: $a$: $((x$@xz and $y$@xz$)$ or $(x$@xz and $z$@xz$))) = x \Rightarrow \forall$xx: $a$: $((x$@xx$) \Rightarrow$ $(y$@xx or $z$@xx$)))$ thf(cBOOL_PROP_53_pme, conjecture)

**SET595+3.p** If X (= Y, then Y = X U (Y X)
If X is a subset of Y, then Y is the union of X and (the difference of Y and X).
$\forall b, c$: $(\forall d$: $(d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b \subseteq c \Rightarrow c =$ union$(b, c \setminus b))$ fof(prove_th$_{54}$, conjecture)

**SET595+4.p** If X (= Y, then Y = X U (Y X)
include('Axioms/SET006+0.ax')
$\forall a, e$: $(a \subseteq e \Rightarrow$ equal_set(union$(e \setminus a, a), e))$ fof(thI$_{27}$, conjecture)

**SET595^5.p** TPS problem BOOL-PROP-54
Trybulec's 54th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: $(\forall$xx: $a$: $((x$@xx$) \Rightarrow (y$@xx$)) \Rightarrow y = (\lambda$xz: $a$: $(x$@xz or $(y$@xz and $\neg x$@xz$))))$ thf(cBOOL_PROP

**SET596+3.p** If X (= Y and Y ∧ Z = the empty set, then X ∧ Z = the empty set
If X is a subset of Y and the intersection of Y and Z is the empty set, then the intersection of X and Z is the empty set.
$\forall b$: ($b \subseteq$ empty_set $\Rightarrow$ $b =$ empty_set) fof(subset_of_empty_set_is_empty_set, axiom)
$\forall b, c, d$: ($b \subseteq c$ $\Rightarrow$ intersection($b, d$) $\subseteq$ intersection($c, d$)) fof(intersection_of_subset, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$)) fof(intersection_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$) fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff \forall c$: $\neg\, c \in b$) fof(empty_defn, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c, d$: (($b \subseteq c$ and intersection($c, d$) = empty_set) $\Rightarrow$ intersection($b, d$) = empty_set) fof(prove_th$_{55}$, conjecture)

**SET596^5.p** TPS problem BOOL-PROP-55
Trybulec's 55th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \rightarrow$ \$o, $y$: $a \rightarrow$ \$o, $z$: $a \rightarrow$ \$o: (($\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($y$@xx)) and ($\lambda$xx: $a$: ($y$@xx and $z$@xx)) = ($\lambda$xx: $a$: \$false)) $\Rightarrow$ ($\lambda$xx: $a$: ($x$@xx and $z$@xx)) = ($\lambda$xx: $a$: \$false)) thf(cBOOL_PROP_55_pme, conjecture)

**SET597+3.p** X = Y U Z iff Y (= X, Z (= X, !V: Y (= V & Z (= V, X (= V
X is the union of Y and Z if and only if the following conditions are satisfied: 1. Y is a subset of X, 2. Z is a subset of X, and 3. for every V such that Y is a subset of V and Z is a subset of V : X is a subset of V.
$\forall b, c$: $b \subseteq$ union($b, c$) fof(subset_of_union, axiom)
$\forall b, c, d$: (($b \subseteq c$ and $d \subseteq c$) $\Rightarrow$ union($b, d$) $\subseteq c$) fof(union_subset, axiom)
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$)) fof(union_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$) fof(commutativity_of_union, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c, d$: ($b =$ union($c, d$) $\iff$ ($c \subseteq b$ and $d \subseteq b$ and $\forall e$: (($c \subseteq e$ and $d \subseteq e$) $\Rightarrow b \subseteq e$))) fof(prove_th$_{56}$, conjecture)

**SET597^5.p** TPS problem BOOL-PROP-56
Trybulec's 56th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \rightarrow$ \$o, $y$: $a \rightarrow$ \$o, $z$: $a \rightarrow$ \$o: ($x$ = ($\lambda$xz: $a$: ($y$@xz or $z$@xz)) $\iff$ ($\forall$xx: $a$: (($y$@xx) $\Rightarrow$ ($x$@xx)) and $\forall$xx: $a$: (($z$@xx) $\Rightarrow$ ($x$@xx)) and $\forall v$: $a \rightarrow$ \$o: (($\forall$xx: $a$: (($y$@xx) $\Rightarrow$ ($v$@xx)) and $\forall$xx: $a$: (($z$@xx) $\Rightarrow$ ($v$@xx))) $\Rightarrow$ $\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($v$@xx))))) thf(cBOOL_PROP_56_pme, conjecture)

**SET598+3.p** X = Y ∧ Z iff X (= Y, X (= Z, !V: V (= Y & V (= Z, V (= X
X is the intersection of Y and Z if and only if the following conditions are satisfied: 1. X is a subset of Y, 2. X is a subset of Z, and 3. for every V such that V is a subset of Y and V is a subset of Z : V is a subset of X.
$\forall b, c$: intersection($b, c$) $\subseteq b$ fof(intersection_is_subset, axiom)
$\forall b, c, d$: (($b \subseteq c$ and $b \subseteq d$) $\Rightarrow$ $b \subseteq$ intersection($c, d$)) fof(intersection_of_subsets, axiom)
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$)) fof(intersection_defn, axiom)
$\forall b, c$: ($b \subseteq c \iff \forall d$: ($d \in b \Rightarrow d \in c$)) fof(subset_defn, axiom)
$\forall b, c$: ($b = c \iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$) fof(commutativity_of_intersection, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c \iff \forall d$: ($d \in b \iff d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c, d$: ($b =$ intersection($c, d$) $\iff$ ($b \subseteq c$ and $b \subseteq d$ and $\forall e$: (($e \subseteq c$ and $e \subseteq d$) $\Rightarrow e \subseteq b$))) fof(prove_th$_{57}$, conjecture)

**SET598^5.p** TPS problem BOOL-PROP-57
Trybulec's 57th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \rightarrow$ \$o, $y$: $a \rightarrow$ \$o, $z$: $a \rightarrow$ \$o: ($x$ = ($\lambda$xx: $a$: ($y$@xx and $z$@xx)) $\iff$ ($\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($y$@xx)) and $\forall$xx: $a$: (($x$@xx) $\Rightarrow$ ($z$@xx)) and $\forall v$: $a \rightarrow$ \$o: (($\forall$xx: $a$: (($v$@xx) $\Rightarrow$ ($y$@xx)) and $\forall$xx: $a$: (($v$@xx) $\Rightarrow$ ($z$@xx))) $\Rightarrow$ $\forall$xx: $a$: (($v$@xx) $\Rightarrow$ ($x$@xx))))) thf(cBOOL_PROP_57_pme, conjecture)

**SET599+3.p** X  Y (= X sym Y
The difference of X and Y is a subset of the symmetric difference of X and Y.

$\forall b, c$: symmetric_difference$(b, c) =$ union$(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c$: $b \subseteq$ union$(b, c)$ fof(subset_of_union, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: symmetric_difference$(b, c) =$ symmetric_difference$(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b \setminus c) \subseteq$ symmetric_difference$(b, c)$ fof(prove_th$_{58}$, conjecture)

**SET599^5.p** TPS problem BOOL-PROP-58
Trybulec's 58th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, xx: $a$: $((x$@xx and $\neg\, y$@xx$) \Rightarrow ((x$@xx and $\neg\, y$@xx$)$ or $(y$@xx and $\neg\, x$@xx$)))$ thf(cBOOL_PROP_

**SET600+3.p** X U Y = empty set iff X = empty set & Y = empty set
The union of X and Y is the empty set iff X is the empty set and Y is the empty set.
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty$(b) \iff \forall c$: $\neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: (union$(b, c) =$ empty_set $\iff (b =$ empty_set and $c =$ empty_set$))$ fof(prove_th$_{59}$, conjecture)

**SET600^5.p** TPS problem BOOL-PROP-59
Trybulec's 59th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: $((\lambda$xz: $a$: $(x$@xz or $y$@xz$)) = (\lambda$xx: $a$: \$false$) \iff (x = (\lambda$xx: $a$: \$false$)$ and $y = (\lambda$xx: $a$: \$false$)))$ thf(cBOOL_PROP_59_pme, conjecture)

**SET601+3.p** X ∧ Y U Y ∧ Z U Z ∧ X = (X U Y) ∧ (Y U Z) ∧ (Z U X)
The intersection of X and the union of Y and the intersection of Y and the union of Z and the intersection of Z and X is the intersection of (the union of X and Y) and the intersection of (the union of Y and Z) and (the union of Z and X).
$\forall b, c, d$: union(union$(b, c), d) =$ union$(b,$ union$(c, d))$ fof(associativity_of_union, axiom)
$\forall b$: intersection$(b, b) = b$ fof(idempotency_of_intersection, axiom)
$\forall b, c, d$: intersection(intersection$(b, c), d) =$ intersection$(b,$ intersection$(c, d))$ fof(associativity_of_intersection, axiom)
$\forall b, c$: union$(b,$ intersection$(b, c)) = b$ fof(union_intersection, axiom)
$\forall b, c, d$: union$(b,$ intersection$(c, d)) =$ intersection(union$(b, c),$ union$(b, d))$ fof(union_distributes_over_intersection, axiom)
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in$ intersection$(b, c) \iff (d \in b$ and $d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c) =$ union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: intersection$(b, c) =$ intersection$(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: union(union(intersection$(b, c),$ intersection$(c, d)),$ intersection$(d, b)) =$ intersection(intersection(union$(b, c),$ union$(c, d)$

**SET601^3.p** X ∧ Y U Y ∧ Z U Z ∧ X = (X U Y) ∧ (Y U Z) ∧ (Z U X)
The intersection of X and the union of Y and the intersection of Y and the union of Z and the intersection of Z and X is the intersection of (the union of X and Y) and the intersection of (the union of Y and Z) and (the union of Z and X).
include('Axioms/SET008^0.ax')
$\forall x$: \$i $\to$ \$o, $y$: \$i $\to$ \$o, $z$: \$i $\to$ \$o: (union@(intersection@$x$@$y$)@(union@(intersection@$y$@$z$)@(intersection@$z$@$x$))) = (intersection@(union@$x$@$y$)@(intersection@(union@$y$@$z$)@(union@$z$@$x$))) thf(thm, conjecture)

**SET601^5.p** TPS problem BOOL-PROP-72
Trybulec's 72nd Boolean property of sets
$a$: \$tType thf(a_type, type)

$\forall x$: $a \rightarrow$ \$o, $y$: $a \rightarrow$ \$o, $z$: $a \rightarrow$ \$o: ($\lambda$xz: $a$: (($x$@xz and $y$@xz) or ($y$@xz and $z$@xz) or ($z$@xz and $x$@xz))) = ($\lambda$xx: $a$: (($x$@xx or $y$@xx) and ($y$@xx or $z$@xx) and ($z$@xx or $x$@xx))) thf(cBOOL_PROP_72_pme, conjecture)

**SET602+3.p** The difference of X and X is the empty set
$\forall b, c$: ($b \setminus c$ = empty_set $\iff$ $b \subseteq c$) fof(difference_empty_set, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$)) fof(difference_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff$ $\forall c$: $\neg\, c \in b$) fof(empty_defn, axiom)
$\forall b$: $b \setminus b$ = empty_set fof(prove_self_difference_is_empty_set, conjecture)

**SET602+4.p** The difference of X and X is the empty set
include('Axioms/SET006+0.ax')
$\forall e$: equal_set($e \setminus e$, empty_set) fof(thI$_{29}$, conjecture)

**SET603+3.p** The difference of X and the empty set is X
$\forall b, c$: ($\forall d$: ($d \in b$ $\iff$ $d \in c$) $\Rightarrow$ $b = c$) fof(member_equal, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$)) fof(difference_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$)) fof(equal_member_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff$ $\forall c$: $\neg\, c \in b$) fof(empty_defn, axiom)
$\forall b$: $b \setminus$ empty_set $= b$ fof(prove_th$_{74}$, conjecture)

**SET603+4.p** The difference of X and the empty set is X
include('Axioms/SET006+0.ax')
$\forall e$: equal_set($e \setminus$ empty_set, $e$) fof(thI$_{30}$, conjecture)

**SET603^5.p** TPS problem BOOL-PROP-74
Trybulec's 74th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \rightarrow$ \$o: ($\lambda$xx: $a$: ($x$@xx and $\neg$ \$false)) $= x$ thf(cBOOL_PROP_74_pme, conjecture)

**SET604+3.p** The difference of the empty set and X is the empty set
$\forall b$: empty_set $\subseteq b$ fof(empty_set_subset, axiom)
$\forall b, c$: ($b \setminus c$ = empty_set $\iff$ $b \subseteq c$) fof(difference_empty_set, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$)) fof(difference_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff$ $\forall c$: $\neg\, c \in b$) fof(empty_defn, axiom)
$\forall b$: empty_set $\setminus b$ = empty_set fof(prove_no_difference_with_empty_set, conjecture)

**SET604^5.p** TPS problem BOOL-PROP-75
Trybulec's 75th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \rightarrow$ \$o: ($\lambda$xx: $a$: (\$false and $\neg\, x$@xx)) = ($\lambda$xx: $a$: \$false) thf(cBOOL_PROP_75_pme, conjecture)

**SET605+3.p** The difference of X and the union of X and Y is the empty set
$\forall b, c$: $b \subseteq$ union($b, c$) fof(subset_of_union, axiom)
$\forall b, c$: ($b \setminus c$ = empty_set $\iff$ $b \subseteq c$) fof(difference_empty_set, axiom)
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$)) fof(union_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set fof(empty_set_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$)) fof(difference_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$)) fof(equal_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$) fof(commutativity_of_union, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$)) fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff$ $\forall c$: $\neg\, c \in b$) fof(empty_defn, axiom)

$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon b \setminus \mathrm{union}(b, c) = \mathrm{empty\_set}$ fof(prove_th$_{76}$, conjecture)

**SET605^5.p** TPS problem BOOL-PROP-76
Trybulec's 76th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xx\colon a\colon (x@xx \text{ and } \neg\, x@xx \text{ or } y@xx)) = (\lambda xx\colon a\colon \$false)$ thf(cBOOL_PROP_76_pme, conjecture)

**SET606+3.p** X X ∧ Y = X Y
The difference of X and the intersection of X and Y is the difference of X and Y.
$\forall b, c\colon (\forall d\colon (d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d\colon (d \in \mathrm{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \mathrm{intersection}(b, c) = \mathrm{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon b \setminus \mathrm{intersection}(b, c) = b \setminus c$ fof(prove_difference_into_intersection, conjecture)

**SET606^3.p** X X ∧ Y = X Y
The difference of X and the intersection of X and Y is the difference of X and Y.
include('Axioms/SET008^0.ax')
$\forall x\colon \$i \to \$o, y\colon \$i \to \$o\colon (\mathrm{setminus}@x@(\mathrm{intersection}@x@y)) = (\mathrm{setminus}@x@y)$ thf(thm, conjecture)

**SET606^5.p** TPS problem BOOL-PROP-77
Trybulec's 77th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xx\colon a\colon (x@xx \text{ and } \neg\, x@xx \text{ and } y@xx)) = (\lambda xx\colon a\colon (x@xx \text{ and } \neg\, y@xx))$ thf(cBOOL_PROP_77_pn

**SET607+3.p** X U (Y X) = X U Y
The union of X and the difference of Y and X is the union of X and Y.
$\forall b, c, d\colon (d \in \mathrm{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon \mathrm{union}(b, c \setminus b) = \mathrm{union}(b, c)$ fof(prove_th$_{79}$, conjecture)

**SET607^3.p** X U (Y X) = X U Y
The union of X and the difference of Y and X is the union of X and Y.
include('Axioms/SET008^0.ax')
$\forall x\colon \$i \to \$o, y\colon \$i \to \$o\colon (\mathrm{union}@x@(\mathrm{setminus}@y@x)) = (\mathrm{union}@x@y)$ thf(thm, conjecture)

**SET607^5.p** TPS problem BOOL-PROP-79
Trybulec's 79th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \to \$o, y\colon a \to \$o\colon (\lambda xz\colon a\colon (x@xz \text{ or } (y@xz \text{ and } \neg\, x@xz))) = (\lambda xz\colon a\colon (x@xz \text{ or } y@xz))$ thf(cBOOL_PROP_79_pme, co

**SET608+3.p** X ∧ Y U (X Y) = X
The intersection of X and the union of Y and (the difference of X and Y) is X.
$\forall b, c, d\colon (d \in \mathrm{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d\colon (d \in \mathrm{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \mathrm{intersection}(b, c) = \mathrm{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon \mathrm{union}(\mathrm{intersection}(b, c), b \setminus c) = b$ fof(prove_union_intersection_difference, conjecture)

**SET608^5.p** TPS problem BOOL-PROP-80

Trybulec's 80th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: ($\lambda$xz: $a$: (($x$@xz and $y$@xz) or ($x$@xz and $\neg\, y$@xz))) = $x$    thf(cBOOL_PROP_80_pme, conjecture)

**SET609+3.p** X  (Y  Z) = (X  Y) U X ∧ Z
The difference of X and (the difference of Y and Z) is the union of (the difference of X and Y) and the intersection of X and Z.
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$))    fof(union_defn, axiom)
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$))    fof(difference_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$))    fof(equal_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$)    fof(commutativity_of_union, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$))    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$))    fof(equal_member_defn, axiom)
$\forall b, c, d$: $b \setminus (c \setminus d)$ = union($b \setminus c$, intersection($b, d$))    fof(prove_th$_{81}$, conjecture)

**SET609^3.p** X  (Y  Z) = (X  Y) U X ∧ Z
The difference of X and (the difference of Y and Z) is the union of (the difference of X and Y) and the intersection of X and Z.
include('Axioms/SET008^0.ax')
$\forall x$: \$i $\to$ \$o, $y$: \$i $\to$ \$o, $z$: \$i $\to$ \$o: (setminus@$x$@(setminus@$y$@$z$)) = (union@(setminus@$x$@$y$)@(intersection@$x$@$z$))    thf

**SET609^5.p** TPS problem BOOL-PROP-81
Trybulec's 81st Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\lambda$xx: $a$: ($x$@xx and $\neg\, y$@xx and $\neg\, z$@xx)) = ($\lambda$xz: $a$: (($x$@xz and $\neg\, y$@xz) or ($x$@xz and $z$@

**SET610+3.p** (X U Y)  Y = X  Y
The difference of (the union of X and Y) and Y is the difference of X and Y.
$\forall b, c$: ($\forall d$: ($d \in b$ $\iff$ $d \in c$) $\Rightarrow$ $b = c$)    fof(member_equal, axiom)
$\forall b, c, d$: ($d \in$ union($b, c$) $\iff$ ($d \in b$ or $d \in c$))    fof(union_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$))    fof(difference_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$))    fof(equal_defn, axiom)
$\forall b, c$: union($b, c$) = union($c, b$)    fof(commutativity_of_union, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$))    fof(equal_member_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$))    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b, c$: union($b, c$) $\setminus c = b \setminus c$    fof(prove_th$_{83}$, conjecture)

**SET610^5.p** TPS problem BOOL-PROP-83
Trybulec's 83th Boolean property of sets
$a$: \$tType    thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: ($\lambda$xx: $a$: (($x$@xx or $y$@xx) and $\neg\, y$@xx)) = ($\lambda$xx: $a$: ($x$@xx and $\neg\, y$@xx))    thf(cBOOL_PROP_83_pn

**SET611+3.p** X ∧ Y = the empty set iff X  Y = X
The intersection of X and Y is the empty set iff the difference of X and Y is X.
$\forall b, c$: ($\forall d$: ($d \in b$ $\iff$ $d \in c$) $\Rightarrow$ $b = c$)    fof(member_equal, axiom)
$\forall b, c, d$: ($d \in$ intersection($b, c$) $\iff$ ($d \in b$ and $d \in c$))    fof(intersection_defn, axiom)
$\forall b, c, d$: ($d \in (b \setminus c)$ $\iff$ ($d \in b$ and $\neg\, d \in c$))    fof(difference_defn, axiom)
$\forall b$: $\neg\, b \in$ empty_set    fof(empty_set_defn, axiom)
$\forall b, c$: ($b = c$ $\iff$ ($b \subseteq c$ and $c \subseteq b$))    fof(equal_defn, axiom)
$\forall b, c$: intersection($b, c$) = intersection($c, b$)    fof(commutativity_of_intersection, axiom)
$\forall b, c$: ($b = c$ $\iff$ $\forall d$: ($d \in b$ $\iff$ $d \in c$))    fof(equal_member_defn, axiom)
$\forall b, c$: ($b \subseteq c$ $\iff$ $\forall d$: ($d \in b$ $\Rightarrow$ $d \in c$))    fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$    fof(reflexivity_of_subset, axiom)
$\forall b$: (empty($b$) $\iff$ $\forall c$: $\neg\, c \in b$)    fof(empty_defn, axiom)
$\forall b, c$: (intersection($b, c$) = empty_set $\iff$ $b \setminus c = b$)    fof(prove_th$_{84}$, conjecture)

**SET611^3.p** X ∧ Y = the empty set iff X  Y = X
The intersection of X and Y is the empty set iff the difference of X and Y is X.
include('Axioms/SET008^0.ax')

$\forall a \colon \$i \to \$o, b \colon \$i \to \$o \colon ((\mathrm{intersection}@a@b) = \mathrm{emptyset} \iff (\mathrm{setminus}@a@b) = a)$ thf(thm, conjecture)

**SET611^5.p** TPS problem BOOL-PROP-84
Trybulec's 84th Boolean property of sets
$a \colon \$tType$ thf(a_type, type)
$\forall x \colon a \to \$o, y \colon a \to \$o \colon ((\lambda xx \colon a \colon (x@xx \text{ and } y@xx)) = (\lambda xx \colon a \colon \$false) \iff (\lambda xx \colon a \colon (x@xx \text{ and } \neg\, y@xx)) = x)$ thf(cBOOL_PROP_84_pme, conjecture)

**SET612+3.p** X (Y U Z) = (X Y) ∧ (X Z)
The difference of X and (the union of Y and Z) is the intersection of (the difference of X and Y) and (the difference of X and Z).
$\forall b, c, d \colon (d \in \mathrm{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d \colon (d \in \mathrm{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d \colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c \colon b \subseteq \mathrm{union}(b, c)$ fof(subset_of_union, axiom)
$\forall b, c, d \colon ((b \subseteq c \text{ and } b \subseteq d) \Rightarrow b \subseteq \mathrm{intersection}(c, d))$ fof(intersection_of_subsets, axiom)
$\forall b, c, d \colon (b \subseteq c \Rightarrow (d \setminus c) \subseteq (d \setminus b))$ fof(subset_difference, axiom)
$\forall b, c \colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c \colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c \colon \mathrm{intersection}(b, c) = \mathrm{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c \colon (b \subseteq c \iff \forall d \colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b \colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c \colon (b = c \iff \forall d \colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d \colon b \setminus \mathrm{union}(c, d) = \mathrm{intersection}(b \setminus c, b \setminus d)$ fof(prove_th$_{85}$, conjecture)

**SET612^3.p** X (Y U Z) = (X Y) ∧ (X Z)
The difference of X and (the union of Y and Z) is the intersection of (the difference of X and Y) and (the difference of X and Z).
include('Axioms/SET008^0.ax')
$\forall x \colon \$i \to \$o, y \colon \$i \to \$o, z \colon \$i \to \$o \colon (\mathrm{setminus}@x@(\mathrm{union}@y@z)) = (\mathrm{intersection}@(\mathrm{setminus}@x@y)@(\mathrm{setminus}@x@z))$ thf

**SET612^5.p** TPS problem BOOL-PROP-85
Trybulec's 85th Boolean property of sets
$a \colon \$tType$ thf(a_type, type)
$\forall x \colon a \to \$o, y \colon a \to \$o, z \colon a \to \$o \colon (\lambda xx \colon a \colon (x@xx \text{ and } \neg\, y@xx \text{ or } z@xx)) = (\lambda xx \colon a \colon (x@xx \text{ and } \neg\, y@xx \text{ and } x@xx \text{ and } \neg\, z@xx$

**SET613+3.p** (X U Y) X ∧ Y = (X Y) U (Y X)
The difference of (the union of X and Y) and the intersection of X and Y is the union of (the difference of X and Y) and (the difference of Y and X).
$\forall b, c \colon (\forall d \colon (d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d \colon (d \in \mathrm{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d \colon (d \in \mathrm{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d \colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c \colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c \colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c \colon \mathrm{intersection}(b, c) = \mathrm{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c \colon (b = c \iff \forall d \colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c \colon (b \subseteq c \iff \forall d \colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b \colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c \colon \mathrm{union}(b, c) \setminus \mathrm{intersection}(b, c) = \mathrm{union}(b \setminus c, c \setminus b)$ fof(prove_difference_union_intersection, conjecture)

**SET613^5.p** TPS problem BOOL-PROP-87
Trybulec's 87th Boolean property of sets
$a \colon \$tType$ thf(a_type, type)
$\forall x \colon a \to \$o, y \colon a \to \$o \colon (\lambda xx \colon a \colon ((x@xx \text{ or } y@xx) \text{ and } \neg\, x@xx \text{ and } y@xx)) = (\lambda xz \colon a \colon ((x@xz \text{ and } \neg\, y@xz) \text{ or } (y@xz \text{ and } \neg\, x@$

**SET614+3.p** X Y Z = X (Y U Z)
The difference of X and the difference of Y and Z is the difference of X and (the union of Y and Z).
$\forall b, c, d \colon (d \in \mathrm{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d \colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c \colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c \colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c \colon (b \subseteq c \iff \forall d \colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)

$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(b \setminus c) \setminus d = b \setminus \text{union}(c, d)$ fof(prove_difference_difference_union, conjecture)

**SET614^3.p** X  Y  Z = X  (Y U Z)
The difference of X and the difference of Y and Z is the difference of X and (the union of Y and Z).
include('Axioms/SET008^0.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $z$: \$i → \$o: (setminus@(setminus@$x$@$y$)@$z$) = (setminus@$x$@(union@$y$@$z$)) thf(thm, conjecture)

**SET614^5.p** TPS problem BOOL-PROP-88
Trybulec's 88th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a$ → \$o, $y$: $a$ → \$o, $z$: $a$ → \$o: (λxx: $a$: ($x$@xx and ¬ $y$@xx and ¬ $z$@xx)) = (λxx: $a$: ($x$@xx and ¬ $y$@xx or $z$@xx)) thf(

**SET615+3.p** (X U Y)  Z = (X  Z) U (Y  Z)
The difference of (the union of X and Y) and Z is the union of (the difference of X and Z) and (the difference of Y and Z).
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $\text{union}(b, c) \setminus d = \text{union}(b \setminus d, c \setminus d)$ fof(prove_difference_distributes_over_union, conjecture)

**SET615^3.p** (X U Y)  Z = (X  Z) U (Y  Z)
The difference of (the union of X and Y) and Z is the union of (the difference of X and Z) and (the difference of Y and Z).
include('Axioms/SET008^0.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $z$: \$i → \$o: (setminus@(union@$x$@$y$)@$z$) = (union@(setminus@$x$@$z$)@(setminus@$y$@$z$)) thf(thm,

**SET615^5.p** TPS problem BOOL-PROP-89
Trybulec's 89th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a$ → \$o, $y$: $a$ → \$o, $z$: $a$ → \$o: (λxx: $a$: (($x$@xx or $y$@xx) and ¬ $z$@xx)) = (λxz: $a$: (($x$@xz and ¬ $z$@xz) or ($y$@xz and ¬ $z$@

**SET616+3.p** If X  Y = Y  X, then X = Y
If the difference of X and Y is the difference of Y and X, then X is Y.
$\forall b, c$: $(\forall d$: $(d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: $(b \setminus c = c \setminus b \Rightarrow b = c)$ fof(prove_th$_{90}$, conjecture)

**SET616^5.p** TPS problem BOOL-PROP-90
Trybulec's 90th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a$ → \$o, $y$: $a$ → \$o: ((λxx: $a$: ($x$@xx and ¬ $y$@xx)) = (λxx: $a$: ($y$@xx and ¬ $x$@xx)) ⇒ $x = y$) thf(cBOOL_PROP_90_p

**SET617+3.p** X sym the empty set = X and the empty set sym X = X
The symmetric difference of X and the empty set is X and the symmetric difference of the empty set and X is X.
$\forall b, c$: $\text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b$: $\text{union}(b, \text{empty\_set}) = b$ fof(union_empty_set, axiom)
$\forall b$: $b \setminus \text{empty\_set} = b$ fof(no_difference_with_empty_set$_1$, axiom)
$\forall b$: $\text{empty\_set} \setminus b = \text{empty\_set}$ fof(no_difference_with_empty_set$_2$, axiom)
$\forall b$: $\neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)

$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b\colon (\text{symmetric\_difference}(b, \text{empty\_set}) = b \text{ and } \text{symmetric\_difference}(\text{empty\_set}, b) = b)$ fof(prove_th$_{92}$, conjecture)

**SET617^5.p** TPS problem BOOL-PROP-92
Trybulec's 92nd Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}\colon ((\lambda \text{xz}\colon a\colon ((x@\text{xz} \text{ and } \neg\,\$\text{false}) \text{ or } (\$\text{false} \text{ and } \neg\, x@\text{xz}))) = x \text{ and } (\lambda \text{xz}\colon a\colon ((\$\text{false} \text{ and } \neg\, x@\text{xz}) \text{ or } (x@\text{xz} \text{ and } \neg\,\$\text{f}$
$x)$ thf(cBOOL_PROP_92_pme, conjecture)

**SET618+3.p** The symmetric difference of X and X is the empty set
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b\colon \text{union}(b, b) = b$ fof(idempotency_of_union, axiom)
$\forall b\colon b \setminus b = \text{empty\_set}$ fof(self_difference_is_empty_set, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b\colon \text{symmetric\_difference}(b, b) = \text{empty\_set}$ fof(prove_th$_{93}$, conjecture)

**SET618^5.p** TPS problem BOOL-PROP-93
Trybulec's 93th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}\colon (\lambda \text{xz}\colon a\colon ((x@\text{xz} \text{ and } \neg\, x@\text{xz}) \text{ or } (x@\text{xz} \text{ and } \neg\, x@\text{xz}))) = (\lambda \text{xx}\colon a\colon \$\text{false})$ thf(cBOOL_PROP_93_pme, conjectu

**SET619+3.p** X U Y = (X sym Y) U X ∧ Y
The union of X and Y is the union of (the symmetric difference of X and Y) and the intersection of X and Y.
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c, d\colon \text{union}(\text{union}(b, c), d) = \text{union}(b, \text{union}(c, d))$ fof(associativity_of_union, axiom)
$\forall b, c\colon \text{union}(b, \text{intersection}(b, c)) = b$ fof(union_intersection, axiom)
$\forall b, c\colon \text{union}(\text{intersection}(b, c), b \setminus c) = b$ fof(union_intersection_difference, axiom)
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(\text{symmetric\_difference}(b, c), \text{intersection}(b, c))$ fof(prove_th$_{95}$, conjecture)

**SET619^5.p** TPS problem BOOL-PROP-95
Trybulec's 95th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}, y\colon a \to \$\text{o}\colon (\lambda \text{xz}\colon a\colon (x@\text{xz} \text{ or } y@\text{xz})) = (\lambda \text{xz}\colon a\colon ((x@\text{xz} \text{ and } \neg\, y@\text{xz}) \text{ or } (y@\text{xz} \text{ and } \neg\, x@\text{xz}) \text{ or } (x@\text{xz} \text{ and } y@\text{xz}))$

**SET620+3.p** X sym Y = (X U Y)  X ∧ Y
The symmetric difference of X and Y is the difference of (the union of X and Y) and the intersection of X and Y.
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c\colon b \setminus \text{intersection}(b, c) = b \setminus c$ fof(difference_into_intersection, axiom)
$\forall b, c, d\colon \text{union}(b, c) \setminus d = \text{union}(b \setminus d, c \setminus d)$ fof(difference_distributes_over_union, axiom)
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)

$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c$: symmetric_difference$(b, c)$ = union$(b, c) \setminus$ intersection$(b, c)$ fof(prove_th$_{96}$, conjecture)

**SET620^5.p** TPS problem BOOL-PROP-96
Trybulec's 96th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o: ($\lambda$xz: $a$: (($x$@xz and $\neg\, y$@xz) or ($y$@xz and $\neg\, x$@xz))) = ($\lambda$xx: $a$: (($x$@xx or $y$@xx) and $\neg\, x$@xx and $y$@

**SET621+3.p** (X sym Y)  Z = (X  (Y U Z)) U (Y  (X U Z))
The difference of (the symmetric difference of X and Y) and Z is the union of (the difference of X and (the union of Y and Z)) and (the difference of Y and (the union of X and Z)).
$\forall b, c$: symmetric_difference$(b, c)$ = union$(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c, d$: $(b \setminus c) \setminus d = b \setminus$ union$(c, d)$ fof(difference_difference_union, axiom)
$\forall b, c, d$: union$(b, c) \setminus d$ = union$(b \setminus d, c \setminus d)$ fof(difference_distributes_over_union, axiom)
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c)$ = union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: symmetric_difference$(b, c)$ = symmetric_difference$(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: symmetric_difference$(b, c) \setminus d$ = union$(b \setminus$ union$(c, d), c \setminus$ union$(b, d))$ fof(prove_th$_{97}$, conjecture)

**SET621^5.p** TPS problem BOOL-PROP-97
Trybulec's 97th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\lambda$xx: $a$: ((($x$@xx and $\neg\, y$@xx) or ($y$@xx and $\neg\, x$@xx)) and $\neg\, z$@xx)) = ($\lambda$xz: $a$: (($x$@xz and

**SET622+3.p** X  (Y sym Z) = (X  (Y U Z)) U X ∧ Y ∧ Z
The difference of X and (the symmetric difference of Y and Z) is the union of (the difference of X and (the union of Y and Z)) and the intersection of X and the intersection of Y and Z.
$\forall b, c, d$: intersection(intersection$(b, c), d)$ = intersection$(b,$ intersection$(c, d))$ fof(associativity_of_intersection, axiom)
$\forall b, c, d$: $b \setminus (c \setminus d)$ = union$(b \setminus c,$ intersection$(b, d))$ fof(difference_difference_union$_2$, axiom)
$\forall b, c$: symmetric_difference$(b, c)$ = union$(b, c) \setminus$ intersection$(b, c)$ fof(symmetric_difference_and_difference, axiom)
$\forall b, c, d$: $(d \in$ union$(b, c) \iff (d \in b$ or $d \in c))$ fof(union_defn, axiom)
$\forall b, c, d$: $(d \in$ intersection$(b, c) \iff (d \in b$ and $d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d$: $(d \in (b \setminus c) \iff (d \in b$ and $\neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c$: symmetric_difference$(b, c)$ = union$(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c$ and $c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: union$(b, c)$ = union$(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: intersection$(b, c)$ = intersection$(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: symmetric_difference$(b, c)$ = symmetric_difference$(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: $(b = c \iff \forall d$: $(d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d$: $(d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: $b \setminus$symmetric_difference$(c, d)$ = union$(b \setminus$union$(c, d),$ intersection(intersection$(b, c), d))$ fof(prove_th$_{98}$, conjecture)

**SET622^5.p** TPS problem BOOL-PROP-98
Trybulec's 98th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\lambda$xx: $a$: ($x$@xx and $\neg$ ($y$@xx and $\neg\, z$@xx) or ($z$@xx and $\neg\, y$@xx))) = ($\lambda$xz: $a$: (($x$@xz and $\neg$

**SET623+3.p** (X sym Y) sym Z = X sym (Y sym Z)
The symmetric difference of (the symmetric difference of X and Y) and Z is the symmetric difference of X and (the symmetric difference of Y and Z).
$\forall b, c$: symmetric_difference$(b, c)$ = union$(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c, d$: union(union$(b, c), d)$ = union$(b,$ union$(c, d))$ fof(associativity_of_union, axiom)
$\forall b, c, d$: intersection(intersection$(b, c), d)$ = intersection$(b,$ intersection$(c, d))$ fof(associativity_of_intersection, axiom)
$\forall b, c, d$: $b \setminus (c \setminus d)$ = union$(b \setminus c,$ intersection$(b, d))$ fof(difference_difference_union$_1$, axiom)
$\forall b, c$: union$(b, c) \setminus$ intersection$(b, c)$ = union$(b \setminus c, c \setminus b)$ fof(difference_union_intersection, axiom)

$\forall b, c, d$: $(b \setminus c) \setminus d = b \setminus \text{union}(c, d)$ fof(difference_difference_union$_2$, axiom)
$\forall b, c, d$: $\text{union}(b, c) \setminus d = \text{union}(b \setminus d, c \setminus d)$ fof(difference_distributes_over_union, axiom)
$\forall b, c$: $(b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $\text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: $\text{symmetric\_difference}(\text{symmetric\_difference}(b, c), d) = \text{symmetric\_difference}(b, \text{symmetric\_difference}(c, d))$ fof(prove

**SET623^3.p** (X sym Y) sym Z = X sym (Y sym Z)
The symmetric difference of (the symmetric difference of X and Y) and Z is the symmetric difference of X and (the symmetric difference of Y and Z).
include('Axioms/SET008^0.ax')
$\forall x$: \$i $\to$ \$o, $y$: \$i $\to$ \$o, $z$: \$i $\to$ \$o: $(\text{excl\_union}@(\text{excl\_union}@x@y)@z) = (\text{excl\_union}@x@(\text{excl\_union}@y@z))$ thf(thm, con

**SET623^5.p** TPS problem BOOL-PROP-99
Trybulec's 99th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\lambda$xz: $a$: (($x$@xz and $\neg$ ($y$@xz and $\neg$ $z$@xz) or ($z$@xz and $\neg$ $y$@xz)) or ((($y$@xz and $\neg$ $z$@xz)
($\lambda$xz: $a$: (((($x$@xz and $\neg$ $y$@xz) or ($y$@xz and $\neg$ $x$@xz)) and $\neg$ $z$@xz) or ($z$@xz and $\neg$ ($x$@xz and $\neg$ $y$@xz) or ($y$@xz and $\neg$ $x$@x

**SET624+3.p** X intersects Y U Z iff X intersects Y or X intersects Z
X intersects the union of Y and Z iff X intersects Y or X intersects Z.
$\forall b, c, d$: $(d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b, c$: $(\text{intersect}(b, c) \iff \exists d: (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c$: $\text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c$: $(\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(\text{intersect}(b, \text{union}(c, d)) \iff (\text{intersect}(b, c) \text{ or } \text{intersect}(b, d)))$ fof(prove_intersect_with_union, conjecture)

**SET624^3.p** X intersects Y U Z iff X intersects Y or X intersects Z
X intersects the union of Y and Z iff X intersects Y or X intersects Z.
include('Axioms/SET008^0.ax')
$\forall x$: \$i $\to$ \$o, $y$: \$i $\to$ \$o, $z$: \$i $\to$ \$o: $((\text{meets}@x@(\text{union}@y@z)) \iff (\text{meets}@x@y \text{ or } \text{meets}@x@z))$ thf(thm, conjecture)

**SET624^5.p** TPS problem BOOL-PROP-100
Trybulec's 100th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: ($\exists$xx: $a$: ($x$@xx and ($y$@xx or $z$@xx)) $\iff$ ($\exists$xx: $a$: ($x$@xx and $y$@xx) or $\exists$xx: $a$: ($x$@xx and

**SET625+3.p** If X intersects Y and Y is a subset of Z, then X intersects Z
$\forall b, c$: $(\text{intersect}(b, c) \iff \exists d: (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c$: $(b \subseteq c \iff \forall d: (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c$: $(\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b$: $b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d$: $((\text{intersect}(b, c) \text{ and } c \subseteq d) \Rightarrow \text{intersect}(b, d))$ fof(prove_th$_{101}$, conjecture)

**SET625^5.p** TPS problem BOOL-PROP-101
Trybulec's 101th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x$: $a \to$ \$o, $y$: $a \to$ \$o, $z$: $a \to$ \$o: (($\exists$xx: $a$: ($x$@xx and $y$@xx) and $\forall$xx: $a$: (($y$@xx) $\Rightarrow$ ($z$@xx))) $\Rightarrow$ $\exists$xx: $a$: ($x$@xx and $z$@xx)

**SET626+3.p** If X intersects the intersection of Y and Z, then X intersects Y
$\forall b, c, d$: $(d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c$: $(\text{intersect}(b, c) \iff \exists d: (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c$: $\text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c$: $(\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c$: $(b = c \iff \forall d: (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d$: $(\text{intersect}(b, \text{intersection}(c, d)) \Rightarrow \text{intersect}(b, c))$ fof(prove_th$_{102}$, conjecture)

**SET626^5.p** TPS problem BOOL-PROP-102
Trybulec's 102th Boolean property of sets
$a$: \$tType thf(a_type, type)

cZ: $a \rightarrow \$o$ thf(cZ, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o\colon (\exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx} \text{ and } \mathrm{cZ}@\mathrm{xx}) \Rightarrow \exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx}))$ thf(cBOOL_PROP_102_pme

**SET627+3.p** X is disjoint from the empty set
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \neg\, \text{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b\colon \text{disjoint}(b, \text{empty\_set})$ fof(prove_th$_{104}$, conjecture)

**SET627^5.p** TPS problem BOOL-PROP-104
Trybulec's 104th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o\colon \neg\, \exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } \$\text{false})$ thf(cBOOL_PROP_104_pme, conjecture)

**SET628+3.p** X intersects X iff X is not the empty set
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (\text{not\_equal}(b, c) \iff b \neq c)$ fof(not_equal_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b\colon (\text{intersect}(b, b) \iff \text{not\_equal}(b, \text{empty\_set}))$ fof(prove_th$_{110}$, conjecture)

**SET628^5.p** TPS problem BOOL-PROP-110
Trybulec's 110th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o\colon (\exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } x@\mathrm{xx}) \iff x \neq (\lambda \mathrm{xx}\colon a\colon \$\text{false}))$ thf(cBOOL_PROP_110_pme, conjecture)

**SET629+3.p** X ∧ Y is disjoint from X  Y
The intersection of X and Y is disjoint from the difference of X and Y.
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \neg\, \text{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon \text{disjoint}(\text{intersection}(b, c), b \setminus c)$ fof(prove_intersection_and_difference_disjoint, conjecture)

**SET629^5.p** TPS problem BOOL-PROP-111
Trybulec's 111th Boolean property of sets
$a$: \$tType thf(a_type, type)
$\forall x\colon a \rightarrow \$o, y\colon a \rightarrow \$o\colon \neg\, \exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx} \text{ and } x@\mathrm{xx} \text{ and } \neg\, y@\mathrm{xx})$ thf(cBOOL_PROP_111_pme, conjecture)

**SET630+3.p** X ∧ Y is disjoint from X sym Y
The intersection of X and Y is disjoint from the symmetric difference of X and Y.
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c, d\colon (\text{intersect}(b, \text{union}(c, d)) \iff (\text{intersect}(b, c) \text{ or } \text{intersect}(b, d)))$ fof(intersect_with_union, axiom)
$\forall b, c\colon \text{disjoint}(\text{intersection}(b, c), b \setminus c)$ fof(intersection_and_union_disjoint, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \neg\, \text{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon \text{symmetric\_difference}(b, c) = \text{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon \text{disjoint}(\text{intersection}(b, c), \text{symmetric\_difference}(b, c))$ fof(prove_intersection_and_symmetric_difference_disjoint, conjec

**SET630^3.p** X ∧ Y is disjoint from X sym Y
The intersection of X and Y is disjoint from the symmetric difference of X and Y.
include('Axioms/SET008^0.ax')

$\forall x\colon \$\mathrm{i} \to \$\mathrm{o}, y\colon \$\mathrm{i} \to \$\mathrm{o}\colon (\mathrm{misses}@(\mathrm{intersection}@x@y)@(\mathrm{excl\_union}@x@y))$ thf(thm, conjecture)

**SET630^5.p** TPS problem BOOL-PROP-112
Trybulec's 112th Boolean property of sets.
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\mathrm{o}, y\colon a \to \$\mathrm{o}\colon \neg\, \exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx} \text{ and } ((x@\mathrm{xx} \text{ and } \neg\, y@\mathrm{xx}) \text{ or } (y@\mathrm{xx} \text{ and } \neg\, x@\mathrm{xx})))$ thf(cBOOL_PROP_

**SET631+3.p** If X intersects the difference of Y and Z, then X intersects Y
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (\mathrm{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b, c\colon (\mathrm{intersect}(b, c) \Rightarrow \mathrm{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c, d\colon (\mathrm{intersect}(b, c \setminus d) \Rightarrow \mathrm{intersect}(b, c))$ fof(prove_th$_{113}$, conjecture)

**SET631^5.p** TPS problem BOOL-PROP-113
Trybulec's 113th Boolean property of sets
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\mathrm{o}, y\colon a \to \$\mathrm{o}, z\colon a \to \$\mathrm{o}\colon (\exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx} \text{ and } \neg\, z@\mathrm{xx}) \Rightarrow \exists \mathrm{xx}\colon a\colon (x@\mathrm{xx} \text{ and } y@\mathrm{xx}))$ thf(cBOOL_PRO

**SET632+3.p** If X (= Y & X (= Z & Y disjoint from Z, then X = empty set
If X is a subset of Y and X is a subset of Z and Y is disjoint from Z, then X is the empty set.
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (\mathrm{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b\colon \neg\, b \in \mathrm{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (\mathrm{disjoint}(b, c) \iff \neg\, \mathrm{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon (\mathrm{intersect}(b, c) \Rightarrow \mathrm{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\mathrm{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c, d\colon ((b \subseteq c \text{ and } b \subseteq d \text{ and } \mathrm{disjoint}(c, d)) \Rightarrow b = \mathrm{empty\_set})$ fof(prove_th$_{114}$, conjecture)

**SET632^5.p** TPS problem BOOL-PROP-114
Trybulec's 114th Boolean property of sets
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\mathrm{o}, y\colon a \to \$\mathrm{o}, z\colon a \to \$\mathrm{o}\colon ((\forall \mathrm{xx}\colon a\colon ((x@\mathrm{xx}) \Rightarrow (y@\mathrm{xx})) \text{ and } \forall \mathrm{xx}\colon a\colon ((x@\mathrm{xx}) \Rightarrow (z@\mathrm{xx})) \text{ and } \neg\, \exists \mathrm{xx}\colon a\colon (y@\mathrm{xx} \text{ and } z$
$x = (\lambda \mathrm{xx}\colon a\colon \$\mathrm{false}))$ thf(cBOOL_PROP_114_pme, conjecture)

**SET633+3.p** If X  Y (= Z and Y  X (= Z, then X sym Y (= Z
If the difference of X and Y is a subset of Z and the difference of Y and X is a subset of Z, then the symmetric difference of X and Y is a subset of Z.
$\forall b, c\colon \mathrm{symmetric\_difference}(b, c) = \mathrm{union}(b \setminus c, c \setminus b)$ fof(symmetric_difference_defn, axiom)
$\forall b, c, d\colon ((b \subseteq c \text{ and } d \subseteq c) \Rightarrow \mathrm{union}(b, d) \subseteq c)$ fof(union_subset, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon \mathrm{union}(b, c) = \mathrm{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \mathrm{symmetric\_difference}(b, c) = \mathrm{symmetric\_difference}(c, b)$ fof(commutativity_of_symmetric_difference, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d\colon (((b \setminus c) \subseteq d \text{ and } (c \setminus b) \subseteq d) \Rightarrow \mathrm{symmetric\_difference}(b, c) \subseteq d)$ fof(prove_th$_{115}$, conjecture)

**SET633^5.p** TPS problem BOOL-PROP-115
Trybulec's 115th Boolean property of sets
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\mathrm{o}, y\colon a \to \$\mathrm{o}, z\colon a \to \$\mathrm{o}\colon ((\forall \mathrm{xx}\colon a\colon ((x@\mathrm{xx} \text{ and } \neg\, y@\mathrm{xx}) \Rightarrow (z@\mathrm{xx})) \text{ and } \forall \mathrm{xx}\colon a\colon ((y@\mathrm{xx} \text{ and } \neg\, x@\mathrm{xx}) \Rightarrow$
$(z@\mathrm{xx}))) \Rightarrow \forall \mathrm{xx}\colon a\colon (((x@\mathrm{xx} \text{ and } \neg\, y@\mathrm{xx}) \text{ or } (y@\mathrm{xx} \text{ and } \neg\, x@\mathrm{xx})) \Rightarrow (z@\mathrm{xx})))$ thf(cBOOL_PROP_115_pme, conjecture

**SET634+3.p** X ∧ (Y  Z) = X ∧ Y  Z
The intersection of X and the difference of Y and Z is the intersection of X and the difference of Y and Z.
$\forall b, c\colon (\forall d\colon (d \in b \iff d \in c) \Rightarrow b = c)$ fof(member_equal, axiom)
$\forall b, c, d\colon (d \in \mathrm{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \mathrm{intersection}(b, c) = \mathrm{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)

$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c, d\colon \text{intersection}(b, c \setminus d) = \text{intersection}(b, c) \setminus d$ fof(prove_difference_and_intersection, conjecture)

**SET634∧5.p** TPS problem BOOL-PROP-116
Trybulec's 116th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}, y\colon a \to \$\text{o}, z\colon a \to \$\text{o}\colon (\lambda \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx and } \neg\, z@\text{xx})) = (\lambda \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx and } \neg\, z@\text{xx}))$ thf(

**SET635+3.p** X ∧ (Y Z) = X ∧ Y X ∧ Z
The intersection of X and the difference of Y and Z is the intersection of X and the difference of Y and the intersection of X and Z.
$\forall b, c\colon \text{intersection}(b, c) \subseteq b$ fof(intersection_is_subset, axiom)
$\forall b, c\colon (b \setminus c = \text{empty\_set} \iff b \subseteq c)$ fof(difference_empty_set, axiom)
$\forall b\colon \text{union}(b, \text{empty\_set}) = b$ fof(union_empty_set, axiom)
$\forall b, c, d\colon b \setminus \text{intersection}(c, d) = \text{union}(b \setminus c, b \setminus d)$ fof(difference_and_intersection_and_union, axiom)
$\forall b, c, d\colon \text{intersection}(b, c \setminus d) = \text{intersection}(b, c) \setminus d$ fof(difference_and_intersection, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c, d\colon (d \in (b \setminus c) \iff (d \in b \text{ and } \neg\, d \in c))$ fof(difference_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c, d\colon \text{intersection}(b, c \setminus d) = \text{intersection}(b, c) \setminus \text{intersection}(b, d)$ fof(prove_th$_{117}$, conjecture)

**SET635∧5.p** TPS problem BOOL-PROP-117
Trybulec's 117th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}, y\colon a \to \$\text{o}, z\colon a \to \$\text{o}\colon (\lambda \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx and } \neg\, z@\text{xx})) = (\lambda \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx and } \neg\, x@\text{xx and } z@\text{xx}$

**SET636+3.p** X is disjoint from Y iff X ∧ Y = the empty set
X is disjoint from Y iff the intersection of X and Y is the empty set.
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \neg\, \text{intersect}(b, c))$ fof(disjoint_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (\text{disjoint}(b, c) \iff \text{intersection}(b, c) = \text{empty\_set})$ fof(prove_th$_{118}$, conjecture)

**SET636∧5.p** TPS problem BOOL-PROP-118
Trybulec's 118th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}, y\colon a \to \$\text{o}\colon (\neg \exists \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx}) \iff (\lambda \text{xx}\colon a\colon (x@\text{xx and } y@\text{xx})) = (\lambda \text{xx}\colon a\colon \$\text{false}))$ thf(cBOOL_PRO

**SET637+3.p** Trybulec's 119th Boolean property of sets
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \exists d\colon (d \in b \text{ and } d \in c))$ fof(intersect_defn, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon (\text{not\_equal}(b, c) \iff b \neq c)$ fof(not_equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \Rightarrow \text{intersect}(c, b))$ fof(symmetry_of_intersect, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (\text{intersect}(b, c) \iff \text{not\_equal}(\text{intersection}(b, c), \text{empty\_set}))$ fof(prove_th$_{119}$, conjecture)

**SET638+3.p** If X (= Y U Z and X ∧ Z = the empty set , then X (= Y
If X is a subset of the union of Y and Z and the intersection of X and Z is the empty set, then X is a subset of Y.
$\forall b, c\colon \text{intersection}(b, c) \subseteq b$ fof(intersection_is_subset, axiom)
$\forall b, c\colon (b \subseteq c \Rightarrow \text{intersection}(b, c) = b)$ fof(subset_intersection, axiom)
$\forall b\colon \text{union}(b, \text{empty\_set}) = b$ fof(union_empty_set, axiom)
$\forall b, c, d\colon \text{intersection}(b, \text{union}(c, d)) = \text{union}(\text{intersection}(b, c), \text{intersection}(b, d))$ fof(intersection_distributes_over_union, axi
$\forall b, c, d\colon (d \in \text{union}(b, c) \iff (d \in b \text{ or } d \in c))$ fof(union_defn, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{union}(b, c) = \text{union}(c, b)$ fof(commutativity_of_union, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c, d\colon ((b \subseteq \text{union}(c, d) \text{ and } \text{intersection}(b, d) = \text{empty\_set}) \Rightarrow b \subseteq c)$ fof(prove_th$_{120}$, conjecture)

**SET638^5.p** TPS problem BOOL-PROP-120
Trybulec's 120th Boolean property of sets
$a\colon \$\text{tType}$ thf(a_type, type)
$\forall x\colon a \to \$\text{o}, y\colon a \to \$\text{o}, z\colon a \to \$\text{o}\colon ((\forall \text{xx}\colon a\colon ((x@\text{xx}) \Rightarrow (y@\text{xx} \text{ or } z@\text{xx})) \text{ and } (\lambda \text{xx}\colon a\colon (x@\text{xx} \text{ and } z@\text{xx})) = (\lambda \text{xx}\colon a\colon \$\text{false})) \Rightarrow \forall \text{xx}\colon a\colon ((x@\text{xx}) \Rightarrow (y@\text{xx})))$ thf(cBOOL_PROP_120_pme, conjecture)

**SET639+3.p** Trybulec's 121th Boolean property of sets
$\forall b, c\colon (b \subseteq c \Rightarrow \text{intersection}(b, c) = b)$ fof(subset_intersection, axiom)
$\forall b\colon \neg\, b \in \text{empty\_set}$ fof(empty_set_defn, axiom)
$\forall b, c, d\colon (d \in \text{intersection}(b, c) \iff (d \in b \text{ and } d \in c))$ fof(intersection_defn, axiom)
$\forall b, c\colon (b \subseteq c \iff \forall d\colon (d \in b \Rightarrow d \in c))$ fof(subset_defn, axiom)
$\forall b, c\colon (b = c \iff (b \subseteq c \text{ and } c \subseteq b))$ fof(equal_defn, axiom)
$\forall b, c\colon \text{intersection}(b, c) = \text{intersection}(c, b)$ fof(commutativity_of_intersection, axiom)
$\forall b\colon b \subseteq b$ fof(reflexivity_of_subset, axiom)
$\forall b\colon (\text{empty}(b) \iff \forall c\colon \neg\, c \in b)$ fof(empty_defn, axiom)
$\forall b, c\colon (b = c \iff \forall d\colon (d \in b \iff d \in c))$ fof(equal_member_defn, axiom)
$\forall b, c\colon ((b \subseteq c \text{ and } \text{intersection}(c, b) = \text{empty\_set}) \Rightarrow b = \text{empty\_set})$ fof(prove_th$_{121}$, conjecture)

**SET640^3.p** A a subset of R (X to Y) => A a subset of X x Y
If A is a subset of a relation R from X to Y then A is a subset of X x Y.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r\colon \$\text{i} \to \$\text{i} \to \$\text{o}, q\colon \$\text{i} \to \$\text{i} \to \$\text{o}\colon ((\text{sub\_rel}@r@q) \Rightarrow (\text{sub\_rel}@r@(\text{cartesian\_product}@\lambda x\colon \$\text{i}\colon \$\text{true}@\lambda x\colon \$\text{i}\colon \$\text{true})))$ thf(

**SET646^3.p** If x is in X and y is in Y then <x,y> is from X to Y.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall x\colon \$\text{i}, y\colon \$\text{i}\colon (\text{sub\_rel}@(\text{pair\_rel}@x@y)@(\text{cartesian\_product}@\lambda x\colon \$\text{i}\colon \$\text{true}@\lambda x\colon \$\text{i}\colon \$\text{true}))$ thf(thm, conjecture)

**SET647^3.p** Domain of R (X to Y) a subset of X => R is (X to range of R)
If the domain of a relation R from X to Y is a subset of X, R is a relation from X to the range of R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r\colon \$\text{i} \to \$\text{i} \to \$\text{o}, x\colon \$\text{i} \to \$\text{o}\colon ((\subseteq @(\text{rel\_domain}@r)@x) \Rightarrow (\text{sub\_rel}@r@(\text{cartesian\_product}@x@(\text{rel\_codomain}@r))))$ thf(

**SET648^3.p** Range of R (X to Y) a subset of Y => R is (domain of R to Y)
If the range of a relation R from X to Y is a subset of Y, R is a relation from the domain of a relation R from X to Y and Y.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r\colon \$\text{i} \to \$\text{i} \to \$\text{o}, y\colon \$\text{i} \to \$\text{o}\colon ((\subseteq @(\text{rel\_codomain}@r)@y) \Rightarrow (\text{sub\_rel}@r@(\text{cartesian\_product}@(\text{rel\_domain}@r)@y)))$ thf(

**SET649^3.p** Domain R a subset of X & range R a subset of Y => R is (X to Y)
If the domain of a relation R from X to Y is a subset of X and the range of R is a subset of Y, R is a relation from X to Y.

include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((⊆ @(rel_domain@$r$)@$x$ and ⊆ @(rel_codomain@$r$)@$y$) ⇒ (sub_rel@$r$@(cartesian_p

**SET651∧3.p** Domain of R (X to Y) a subset of X1 => R is (X1 to Y)
If the domain of a relation R from X to Y is a subset of X1 then R is a relation from X1 to Y.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$a$: \$i → \$o  thf($a$, type)
$\forall r$: \$i → \$i → \$o: ((⊆ @(rel_domain@$r$)@$a$) ⇒ (sub_rel@$r$@(cartesian_product@$a$@$\lambda x$: \$i: \$true)))  thf(thm, conjecture)

**SET657∧3.p** The field of a relation R from X to Y is a subset of X union Y
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r$: \$i → \$i → \$o: (⊆ @(rel_field@$r$)@(union@$\lambda x$: \$i: \$true@$\lambda x$: \$i: \$true))  thf(thm, conjecture)

**SET669∧3.p** Id on Y subset of R => Y subset of domain R & Y is range R
If the identity relation on Y is a subset of a relation R from X to Y then Y is a subset of the domain of R and Y is the range of R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r$: \$i → \$i → \$o: ((sub_rel@(id_rel@$\lambda x$: \$i: \$true)@$r$) ⇒ (⊆ @$\lambda x$: \$i: \$true@(rel_domain@$r$) and ($\lambda x$: \$i: \$true) = (rel_codomain@$r$)))  thf(thm, conjecture)

**SET670∧3.p** R (X to Y) restricted to X1 is (X1 to Y)
A relation R from X to Y restricted to X1 is a relation from X1 to Y.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall z$: \$i → \$o, $r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$) ⇒ (is_rel_on@(restrict_rel_domain@$r$@$z$)@$z$@$y$))

**SET671∧3.p** X a subset of X1 => R (X to Y) restricted to X1 is R
If X is a subset of X1 then a relation R from X to Y restricted to X1 is R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall z$: \$i → \$o, $r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$ and ⊆ @$x$@$z$) ⇒ (restrict_rel_domain@$r$@$z$) = $r$)  thf(thm, conjecture)

**SET672∧3.p** Y1 restricted to R (X to Y) is (X to Y1)
Y1 restricted to a relation R from X to Y is a relation from X to Y1.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall z$: \$i → \$o, $r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$) ⇒ (is_rel_on@(restrict_rel_codomain@$r$@$z$)@$x$@$z$))

**SET673∧3.p** Y a subset of Y1 => Y1 restricted to R (X to Y) is R
If Y is a subset of Y1 then Y1 restricted to a relation R from X to Y is R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall z$: \$i → \$o, $r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$ and ⊆ @$y$@$z$) ⇒ (restrict_rel_codomain@$r$@$z$) = $r$)  thf(thm, conjecture)

**SET680∧3.p** !x in D, x the domain of R (X to Y) iff ?y in E : <x,y> in R
For every element x in D, x is in the domain of a relation R from X to Y iff there exists an element y in E such that <x,y> is in R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$) ⇒ $\forall u$: \$i: (($x$@$u$) ⇒ ((rel_domain@$r$@$u$) ⟺ $\exists v$: \$i: ($y$@$v$ and $r$@$u$@$v$))))  thf(thm, conjecture)

**SET683∧3.p** !y in E : y in range of R (X to Y) ?x in D : x in domain of R
For every element y in E such that y is in the range in a relation R from X to Y there exists an element x in D such that x is in the domain of R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall r$: \$i → \$i → \$o, $x$: \$i → \$o, $y$: \$i → \$o: ((is_rel_on@$r$@$x$@$y$) ⇒ $\forall v$: \$i: (($y$@$v$) ⇒ ((rel_codomain@$r$@$v$) ⇒ $\exists u$: \$i: ($x$@$u$ and rel_domain@$r$@$u$))))  thf(thm, conjecture)

**SET684^3.p** <x,z> in P(DtoE) o R(EtoF) iff ?y in E:<x,y> in P & <y,z> in R
Let P be a relation from D to E, R a relation from E to F, x an element of D, and z in F. Then <x,z> is in P composed with R if and only if there exists an element y in E such that <x,y> is in P and <y,z> is in R.
include('Axioms/SET008^0.ax')
include('Axioms/SET008^2.ax')
$\forall p$: \$i $\rightarrow$ \$i $\rightarrow$ \$o, $r$: \$i $\rightarrow$ \$i $\rightarrow$ \$o, $x$: \$i, $z$: \$i: ((rel_composition@$p$@$r$@$x$@$z$) $\iff$ $\exists y$: \$i: ($p$@$x$@$y$ and $r$@$y$@$z$)) thf(thm, c

**SET687+4.p** A set is a subset of itself
include('Axioms/SET006+0.ax')
$\forall a\colon a \subseteq a$ fof($\mathrm{thI}_{01}$, conjecture)

**SET688+4.p** Property of proper subset
If A is a proper subset of B and B a proper subset of C, then A is not equal to C.
include('Axioms/SET006+0.ax')
$\forall a, b, c\colon ((a \subseteq b \text{ and } \neg\, \mathrm{equal\_set}(a, b) \text{ and } b \subseteq c \text{ and } \neg\, \mathrm{equal\_set}(b, c)) \Rightarrow \neg\, \mathrm{equal\_set}(a, c))$ fof($\mathrm{thI}_{04}$, conjecture)

**SET689+4.p** Property of subset
If A is a subset of B,B a subset of C and C a subset of A, then A is equal to C.
include('Axioms/SET006+0.ax')
$\forall a, b, c\colon ((a \subseteq b \text{ and } b \subseteq c \text{ and } c \subseteq a) \Rightarrow \mathrm{equal\_set}(a, c))$ fof($\mathrm{thI}_{05}$, conjecture)

**SET690+4.p** Property of union and intersection
include('Axioms/SET006+0.ax')
$\forall a, b, c\colon (\mathrm{equal\_set}(\mathrm{union}(\mathrm{intersection}(a, b), c), \mathrm{intersection}(a, \mathrm{union}(b, c))) \iff c \subseteq a)$ fof($\mathrm{thI}_{12}$, conjecture)

**SET691+4.p** A set is a subset of empty set if and only if it is equal to it
include('Axioms/SET006+0.ax')
$\forall a\colon (a \subseteq \mathrm{empty\_set} \iff \mathrm{equal\_set}(a, \mathrm{empty\_set}))$ fof($\mathrm{thI}_{16}$, conjecture)

**SET692+4.p** A = A ∧ B iff A (= B
A is a subset of B if and only if it is equal to the intersection of A and B.
include('Axioms/SET006+0.ax')
$\forall a, b\colon (\mathrm{equal\_set}(a, \mathrm{intersection}(a, b)) \iff a \subseteq b)$ fof($\mathrm{thI}_{19}$, conjecture)

**SET693+4.p** Property of union
B is a subset of A if and only if A is equal to the union of A and B.
include('Axioms/SET006+0.ax')
$\forall a, b\colon (\mathrm{equal\_set}(a, \mathrm{union}(a, b)) \iff b \subseteq a)$ fof($\mathrm{thI}_{20}$, conjecture)

**SET694+4.p** Union of power sets is a subset of the power set of the union
The union of the power_set of A and the power_set of B is a subset of the power_set of the union of A and B.
include('Axioms/SET006+0.ax')
$\forall a, b\colon \mathrm{union}(\mathrm{power\_set}(a), \mathrm{power\_set}(b)) \subseteq \mathrm{power\_set}(\mathrm{union}(a, b))$ fof($\mathrm{thI}_{22}$, conjecture)

**SET695+4.p** Difference of subsets
A is a subset of B if and only if the difference of B is a subset of the difference of A.
include('Axioms/SET006+0.ax')
$\forall a, b, e\colon ((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff (e \setminus b) \subseteq (e \setminus a)))$ fof($\mathrm{thI}_{24}$, conjecture)

**SET696+4.p** If A (= E, then (E / A) ∧ A = empty set
include('Axioms/SET006+0.ax')
$\forall a, e\colon (a \subseteq e \Rightarrow \mathrm{equal\_set}(\mathrm{intersection}(e \setminus a, a), \mathrm{empty\_set}))$ fof($\mathrm{thI}_{28}$, conjecture)

**SET697+4.p** Property of intersection and difference
A is a subset of B if and only if the intersection of A and of the difference of B is empty.
include('Axioms/SET006+0.ax')
$\forall a, b, e\colon ((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff \mathrm{equal\_set}(\mathrm{intersection}(a, e \setminus b), \mathrm{empty\_set})))$ fof($\mathrm{thI}_{31}$, conjecture)

**SET698+4.p** Property of union and difference
A is a subset of B if and only if the union of A and of the difference of B in E is equal to E.
include('Axioms/SET006+0.ax')
$\forall a, b, e\colon ((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff \mathrm{equal\_set}(\mathrm{union}(e \setminus a, b), e)))$ fof($\mathrm{thI}_{32}$, conjecture)

**SET699+4.p** Property of intersection and difference 1
A is a subset of B if and only if the intersection of A and of the difference of B is a subset of the difference of A.
include('Axioms/SET006+0.ax')
$\forall a, b, e\colon ((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff \mathrm{intersection}(a, e \setminus b) \subseteq (e \setminus a)))$ fof($\mathrm{thI}_{33}$, conjecture)

**SET700+4.p** Property of intersection and difference 2
A is a subset of B if and only if the intersection of A and of the difference of B is a subset of B.
include('Axioms/SET006+0.ax')
$\forall a, b, e$: $((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff \text{intersection}(a, e \setminus b) \subseteq b))$ fof($\text{thI}_{34}$, conjecture)

**SET701+4.p** Property of intersection and difference 3
A is a subset of B if and only if the intersection of A and of the difference of B is a subset of the intersection of C and of the difference of C.
include('Axioms/SET006+0.ax')
$\forall a, b, c, e$: $((a \subseteq e \text{ and } b \subseteq e) \Rightarrow (a \subseteq b \iff \text{intersection}(a, e \setminus b) \subseteq \text{intersection}(c, e \setminus c)))$ fof($\text{thI}_{35}$, conjecture)

**SET702+4.p** Property of product and intersection
The intersection of product(A) and product(B) is a subset of the product of the intersection of A and B.
include('Axioms/SET006+0.ax')
$\forall a, b$: $\text{intersection}(\text{product}(a), \text{product}(b)) \subseteq \text{product}(\text{intersection}(a, b))$ fof($\text{thI}_{36}$, conjecture)

**SET703+4.p** Union of singletons
The union of singleton(A) and singleton(B) is equal to the unordered_pair(A,B)
include('Axioms/SET006+0.ax')
$\forall a, b$: $\text{equal\_set}(\text{union}(\text{singleton}(a), \text{singleton}(b)), \text{unordered\_pair}(a, b))$ fof($\text{thI}_{41}$, conjecture)

**SET704+4.p** If X is a member of A, then product(A) is a subset of X
include('Axioms/SET006+0.ax')
$\forall a, x$: $(x \in a \Rightarrow \text{product}(a) \subseteq x)$ fof($\text{thI}_{42}$, conjecture)

**SET705+4.p** A is a member of power_set(A)
include('Axioms/SET006+0.ax')
$\forall a$: $a \in \text{power\_set}(a)$ fof($\text{thI}_{48}$, conjecture)

**SET706+4.p** Property of difference
The difference of C in A is equal to the union of the difference of C in B and the difference of B in A.
include('Axioms/SET006+0.ax')
$\forall a, b, c$: $((c \subseteq b \text{ and } b \subseteq a) \Rightarrow \text{equal\_set}(a \setminus c, \text{union}(b \setminus c, a \setminus b)))$ fof($\text{thI}_{49}$, conjecture)

**SET707+4.p** Components of equal ordered pairs are equal
If A,A,B = U,U,V then A = U and B = V.
include('Axioms/SET006+0.ax')
$\forall a, b, u, v$: $(\text{equal\_set}(\text{unordered\_pair}(\text{singleton}(a), \text{unordered\_pair}(a, b)), \text{unordered\_pair}(\text{singleton}(u), \text{unordered\_pair}(u, v))) \Rightarrow$
$(a = u \text{ and } b = v))$ fof($\text{thI}_{50}$, conjecture)

**SET708+4.p** The composition of mappings is unique
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h_1, h_2, a, b, c$: $((\text{maps}(f, a, b) \text{ and } \text{maps}(g, b, c) \text{ and } \text{compose\_predicate}(h_1, g, f, a, b, c) \text{ and } \text{compose\_predicate}(h_2, g, f, a,$
$\text{equal\_maps}(h_1, h_2, a, c))$ fof(thII01a, conjecture)

**SET709+4.p** The composition of mappings is a mapping
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: $((\text{maps}(f, a, b) \text{ and } \text{maps}(g, b, c)) \Rightarrow \text{maps}(\text{compose\_function}(g, f, a, b, c), a, c))$ fof($\text{thII}_{01}$, conjecture)

**SET710+4.p** Associativity of composition
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c, d$: $((\text{maps}(f, a, b) \text{ and } \text{maps}(g, b, c) \text{ and } \text{maps}(h, c, d)) \Rightarrow \text{equal\_maps}(\text{compose\_function}(h, \text{compose\_function}(g$

**SET711+4.p** The inverse of a mapping is unique
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b$: $((\text{maps}(f, a, b) \text{ and } \text{one\_to\_one}(f, a, b) \text{ and } \text{inverse\_predicate}(g, f, a, b) \text{ and } \text{inverse\_predicate}(h, f, a, b)) \Rightarrow$
$\text{equal\_maps}(g, h, b, a))$ fof(thII03a, conjecture)

**SET712+4.p** The inverse of a one-to-one mapping is a mapping
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: $((\text{maps}(f, a, b) \text{ and } \text{one\_to\_one}(f, a, b)) \Rightarrow \text{maps}(\text{inverse\_function}(f, a, b), b, a))$ fof($\text{thII}_{03}$, conjecture)

**SET713+4.p** The inverse of a one-to-one mapping is one-to-one
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: ((maps($f, a, b$) and one_to_one($f, a, b$)) $\Rightarrow$ one_to_one(inverse_function($f, a, b$), $b, a$)) fof(thII$_{04}$, conjecture)

**SET714+4.p** The composition of inverse(F) and F is the identity
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: ((maps($f, a, b$) and one_to_one($f, a, b$)) $\Rightarrow$ identity(compose_function(inverse_function($f, a, b$), $f, a, b, a$), $a$)) fof(th

**SET715+4.p** The composition of F and its inverse is the identity
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: ((maps($f, a, b$) and one_to_one($f, a, b$)) $\Rightarrow$ identity(compose_function($f$, inverse_function($f, a, b$), $b, a, b$), $b$)) fof(th

**SET716+4.p** The composition of injective mappings is injective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and injective($f, a, b$) and injective($g, b, c$)) $\Rightarrow$ injective(compose_function($g, f, a$

**SET716^4.p** The composition of injective mappings is injective
include('Axioms/SET008^1.ax')
$\forall f$: \$i $\rightarrow$ \$i, $g$: \$i $\rightarrow$ \$i: ((fun_injective@$f$ and fun_injective@$g$) $\Rightarrow$ (fun_injective@(fun_composition@$f$@$g$))) thf(thm, con

**SET717+4.p** The composition of surjective mappings is surjective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and surjective($f, a, b$) and surjective($g, b, c$)) $\Rightarrow$ surjective(compose_function($g$,

**SET718+4.p** The composition of one-to-one mappings is one-to-one
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and one_to_one($f, a, b$) and one_to_one($g, b, c$)) $\Rightarrow$ one_to_one(compose_function

**SET719+4.p** Inverse of composition
The inverse of the composition of mappings is equal to the composition of the inverse mappings.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and one_to_one($f, a, b$) and one_to_one($g, b, c$)) $\Rightarrow$ equal_maps(inverse_function(

**SET720+4.p** The inverse of the inverse of a mapping is equal to the mapping
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: ((maps($f, a, b$) and one_to_one($f, a, b$)) $\Rightarrow$ equal_maps(inverse_function(inverse_function($f, a, b$), $b, a$), $f, a, b$)) fof(

**SET721+4.p** If the composition of F and G is injective, then F is injective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and injective(compose_function($g, f, a, b, c$), $a, c$)) $\Rightarrow$ injective($f, a, b$)) fof(th

**SET722+4.p** If the composition of F and G is surjective, then F is surjective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and surjective(compose_function($g, f, a, b, c$), $a, c$)) $\Rightarrow$ surjective($g, b, c$)) fof(

**SET723+4.p** If FoG = FoH and F is injective, then G = H
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, b, c$) and maps($g, a, b$) and maps($h, a, b$) and injective($f, b, c$) and equal_maps(compose_function($f, g$,
equal_maps($g, h, a, b$)) fof(thII$_{14}$, conjecture)

**SET724+4.p** If GoF = HoF and F is surjective, then G = H
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, b, c$) and surjective($f, a, b$) and equal_maps(compose_function($g$,
equal_maps($g, h, b, c$)) fof(thII$_{15}$, conjecture)

**SET724^4.p** If GoF = HoF and F is surjective, then G = H
include('Axioms/SET008ˆ1.ax')
$\forall f$: \$i → \$i, $g$: \$i → \$i, $h$: \$i → \$i: (((fun_composition@$f$@$g$) = (fun_composition@$f$@$h$) and fun_surjective@$f$) ⇒ $g = h$)    thf(thm, conjecture)

**SET725+4.p** If GoF and FoH are identities, then F is one-to-one
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b$: ((maps($f, a, b$) and maps($g, b, a$) and maps($h, b, a$) and identity(compose_function($g, f, a, b, a$), $a$) and identity(co
one_to_one($f, a, b$))    fof(thII$_{16}$, conjecture)

**SET726+4.p** If GoF and FoH are identities, then inverse(F) = G
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b$: ((maps($f, a, b$) and maps($g, b, a$) and maps($h, b, a$) and identity(compose_function($g, f, a, b, a$), $a$) and identity(co
equal_maps(inverse_function($f, a, b$), $g, b, a$))    fof(thII$_{17}$, conjecture)

**SET727+4.p** If GoF and FoH are identities, then inverse(F) = H
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b$: ((maps($f, a, b$) and maps($g, b, a$) and maps($h, b, a$) and identity(compose_function($g, f, a, b, a$), $a$) and identity(co
equal_maps(inverse_function($f, a, b$), $h, b, a$))    fof(thII$_{18}$, conjecture)

**SET728+4.p** If GoF and FoH are identities, then G = H
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b$: ((maps($f, a, b$) and maps($g, b, a$) and maps($h, b, a$) and identity(compose_function($g, f, a, b, a$), $a$) and identity(co
equal_maps($g, h, b, a$))    fof(thII$_{19}$, conjecture)

**SET729+4.p** F is one-to-one and inverse(F)=F iff FoF is the identity
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a$: (maps($f, a, a$) ⇒ ((one_to_one($f, a, a$) and inverse_predicate($f, f, a, a$)) ⟺ identity(compose_function($f, f, a, a, a$), $a$)

**SET730+4.p** Property of restriction 1
If F is a mapping from A to B,and G equal to F on A and C =image2(F,A), then G is a mapping from A to C.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and $c \subseteq b$ and image$_2$($f, a$) = $c$ and $\forall x, y$: (($x \in a$ and $y \in c$) ⇒ (apply($g, x, y$) ⟺ apply($f, x, y$)))) ⇒ maps($g, a, c$))    fof(thII$_{21}$, conjecture)

**SET731+4.p** Property of restriction 2
If F is a mapping from A to B,and G equal to F on A and C =image2(F,A), then G is surjective.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and $c \subseteq b$ and image$_2$($f, a$) = $c$ and $\forall x, y$: (($x \in a$ and $y \in c$) ⇒ (apply($g, x, y$) ⟺ apply($f, x, y$)))) ⇒ surjective($g, a, c$))    fof(thII$_{22}$, conjecture)

**SET732+4.p** Property of restriction 3
If F is a mapping from A to B,and G equal to F on A and C =image2(F,A) and F is injective, then G is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c$: ((maps($f, a, b$) and $c \subseteq b$ and image$_2$($f, a$) = $c$ and $\forall x, y$: (($x \in a$ and $y \in c$) ⇒ (apply($g, x, y$) ⟺ apply($f, x, y$))) and injective($f, a, b$)) ⇒ one_to_one($g, a, c$))    fof(thII$_{23}$, conjecture)

**SET733+4.p** If GoF is the identity, then F is injective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b$: ((maps($f, a, b$) and maps($g, b, a$) and identity(compose_function($g, f, a, b, a$), $a$)) ⇒ injective($f, a, b$))    fof(thII$_{24}$

**SET734+4.p** If GoF is the identity, then G is surjective
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b$: ((maps($g, a, b$) and maps($f, b, a$) and identity(compose_function($g, f, b, a, b$), $b$)) ⇒ surjective($g, a, b$))    fof(thII$_{25}$

**SET735+4.p** Property of mappings

If GoF1 and GoF2 are identities,and the images of A by F1 and F2 are equal, then F1 and F2 are equal.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f_1, f_2, g, a, b$: ((maps($f_1, a, b$) and maps($f_2, a, b$) and maps($g, b, a$) and identity(compose_function($g, f_1, a, b, a$)$, a$) and identit
equal_maps($f_1, f_2, a, b$)) fof(thII$_{26}$, conjecture)

**SET736+4.p** Problem on composition of mappings 1
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF and FoHoG are injective and GoFoH surjective, then F is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($f, a, b$)) fof(thII$_{27}$, conjecture)

**SET737+4.p** Problem on composition of mappings 2
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF and FoHoG are injective and GoFoH surjective, then G is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($g, b, c$)) fof(thII$_{28}$, conjecture)

**SET738+4.p** Problem on composition of mappings 3
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF and FoHoG are injective and GoFoH surjective, then H is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($h, c, a$)) fof(thII$_{29}$, conjecture)

**SET739+4.p** Problem on composition of mappings 4
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF is injective and FoHoG and GoFoH surjective, then F is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($f, a, b$)) fof(thII$_{30}$, conjecture)

**SET740+4.p** Problem on composition of mappings 5
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF is injective and FoHoG and GoFoH surjective, then G is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($g, b, c$)) fof(thII$_{31}$, conjecture)

**SET741+4.p** Problem on composition of mappings 6
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF is injective and FoHoG and GoFoH surjective, then H is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and injective(compose_function($h$, compose_function($g, f, a,$
one_to_one($h, c, a$)) fof(thII$_{32}$, conjecture)

**SET741^4.p** Problem on composition of mappings 6
Consider three mappings F from A to B,G from B to C,H from C to A. If HoGoF is injective and FoHoG and GoFoH surjective, then H is one-to-one.
include('Axioms/SET008^1.ax')
$\forall f$: \$i $\rightarrow$ \$i, $g$: \$i $\rightarrow$ \$i, $h$: \$i $\rightarrow$ \$i: ((fun_injective@(fun_composition@(fun_composition@$f$@$g$)@$h$) and fun_surjective@(fun_co
(fun_bijective@$h$)) thf(thm, conjecture)

**SET742+4.p** Problem on composition of mappings 7
Consider three mappings F from A to B,G from B to C,H from C to A. If GoF and HoG are one-to-one, then F is one-to-one.

include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and one_to_one(compose_function($g, f, a, b, c$), $a, c$) and one_
one_to_one($f, a, b$)) fof(thII$_{33}$, conjecture)

**SET743+4.p** Problem on composition of mappings 8
Consider three mappings F from A to B,G from B to C,H from C to A. If GoF and HoG are one-to-one, then G is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and one_to_one(compose_function($g, f, a, b, c$), $a, c$) and one_
one_to_one($g, b, c$)) fof(thII$_{34}$, conjecture)

**SET744+4.p** Problem on composition of mappings 9
Consider three mappings F from A to B,G from B to C,H from C to A. If GoF and HoG are one-to-one, then H is one-to-one.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, h, a, b, c$: ((maps($f, a, b$) and maps($g, b, c$) and maps($h, c, a$) and one_to_one(compose_function($g, f, a, b, c$), $a, c$) and one_
one_to_one($h, c, a$)) fof(thII$_{35}$, conjecture)

**SET745+4.p** Problem on composition of mappings 10
Consider three mappings F1 from A1 to B,F2 from A2 to B, F which is equal to F1 on A1 and to F2 on A2, then F is a mapping from union(A1,A2) to B if and only if F1 and F2 are equal on the intersection of A1 and A2.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f_1, f_2, f, a_1, a_2, b$: ((maps($f_1, a_1, b$) and maps($f_2, a_2, b$) and $\forall x, y$: (($x \in$ union($a_1, a_2$) and $y \in b$) $\Rightarrow$ (apply($f, x, y$) $\iff$ (($x \in a_1$ and apply($f_1, x, y$)) or ($x \in a_2$ and apply($f_2, x, y$)))))) $\Rightarrow$ (maps($f$, union($a_1, a_2$), $b$) $\iff$ $\forall x, y_1, y_2$: (($x \in a_1$ and $x \in a_2$ and $y_1 \in b$ and $y_2 \in b$ and apply($f_1, x, y_1$) and apply($f_2, x, y_2$)) $\Rightarrow$ $y_1 = y_2$))) fof(thII$_{36}$, conjecture)

**SET746+4.p** If F and G and increasing, then GoF is increasing
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c, r, s, t$: ((maps($f, a, b$) and maps($g, b, c$) and increasing($f, a, r, b, s$) and increasing($g, b, s, c, t$)) $\Rightarrow$ increasing(compo

**SET747+4.p** If F is increasing and G decreasing, then GoF is decreasing
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c, r, s, t$: ((maps($f, a, b$) and maps($g, b, c$) and increasing($f, a, r, b, s$) and decreasing($g, b, s, c, t$)) $\Rightarrow$ decreasing(comp

**SET747^4.p** If F is increasing and G decreasing, then GoF is decreasing
include('Axioms/SET008^1.ax')
$\forall f$: \$i $\rightarrow$ \$i, $g$: \$i $\rightarrow$ \$i, lESS: \$i $\rightarrow$ \$i $\rightarrow$ \$o: ((fun_increasing@$f$@lESS and fun_decreasing@$g$@lESS) $\Rightarrow$ (fun_decreasing@(fun

**SET748+4.p** If F is decreasing and G increasing, then GoF is decreasing
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c, r, s, t$: ((maps($f, a, b$) and maps($g, b, c$) and decreasing($f, a, r, b, s$) and increasing($g, b, s, c, t$)) $\Rightarrow$ decreasing(comp

**SET749+4.p** If F and G and decreasing, then GoF is increasing
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, g, a, b, c, r, s, t$: ((maps($f, a, b$) and maps($g, b, c$) and decreasing($f, a, r, b, s$) and decreasing($g, b, s, c, t$)) $\Rightarrow$ increasing(comp

**SET750+4.p** Property of isomorphism
If F is one-to-one, then F is an isomorphism if and only if F and its inverse are increasing.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, r, s$: ((maps($f, a, b$) and one_to_one($f, a, b$)) $\Rightarrow$ (isomorphism($f, a, r, b, s$) $\iff$ (increasing($f, a, r, b, s$) and increasing(

**SET751+4.p** If X is a subset of Y, then the image f(X) is a subset of f(Y)
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq a$ and $y \subseteq a$ and $x \subseteq y$) $\Rightarrow$ image$_2$($f, x$) $\subseteq$ image$_2$($f, y$)) fof(thIIa$_{01}$, conjecture)

**SET752+4.p** The image of union is equal to the union of images

include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq a$ and $y \subseteq a$) $\Rightarrow$ equal_set(image$_2$($f$, union($x, y$)), union(image$_2$($f, x$), image$_2$($f, y$))))

**SET752^4.p** The image of union is equal to the union of images
include('Axioms/SET008^0.ax')
include('Axioms/SET008^1.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $f$: \$i → \$i: (fun_image@$f$@(union@$x$@$y$)) = (union@(fun_image@$f$@$x$)@(fun_image@$f$@$y$)) thf(t

**SET753+4.p** Image of intersection is a subset of intersection of images
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq a$ and $y \subseteq a$) $\Rightarrow$ image$_2$($f$, intersection($x, y$)) $\subseteq$ intersection(image$_2$($f, x$), image$_2$($f, y$)))

**SET753^4.p** Image of intersection is a subset of intersection of images
include('Axioms/SET008^0.ax')
include('Axioms/SET008^1.ax')
$\forall x$: \$i → \$o, $y$: \$i → \$o, $f$: \$i → \$i: ($\subseteq$ @(fun_image@$f$@(intersection@$x$@$y$))@(intersection@(fun_image@$f$@$x$)@(fun_image@

**SET754+4.p** C is a subset of the inverse image of the image of C
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, c$: ((maps($f, a, b$) and $c \subseteq a$) $\Rightarrow$ $c \subseteq$ inverse_image$_2$($f$, image$_2$($f, c$))) fof(thIIa$_{04}$, conjecture)

**SET755+4.p** If X is a subset of Y, then f-1(X) is a subset of f-1(Y)
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq b$ and $y \subseteq b$ and $x \subseteq y$) $\Rightarrow$ inverse_image$_2$($f, x$) $\subseteq$ inverse_image$_2$($f, y$)) fof(thIIa$_{05}$, c

**SET756+4.p** Inverse image of union equals the union of inverse images
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq b$ and $y \subseteq b$) $\Rightarrow$ equal_set(inverse_image$_2$($f$, union($x, y$)), union(inverse_image$_2$($f, x$), inv

**SET757+4.p** Inverse image intersection equals intersection inverse images
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and $x \subseteq b$ and $y \subseteq b$) $\Rightarrow$ equal_set(inverse_image$_3$($f$, intersection($x, y$), $a$), intersection(inverse_im

**SET758+4.p** The image of the inverse image of Y is a subset of Y
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, y$: ((maps($f, a, b$) and $y \subseteq b$) $\Rightarrow$ image$_3$($f$, inverse_image$_3$($f, y, a$), $b$) $\subseteq y$) fof(thIIa$_{08}$, conjecture)

**SET759+4.p** Composition of images 1
If F is injective, then the inverse image of the image of X is equal to X.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x$: ((maps($f, a, b$) and injective($f, a, b$) and $x \subseteq a$) $\Rightarrow$ equal_set(inverse_image$_3$($f$, image$_3$($f, x, b$), $a$), $x$)) fof(thIIa

**SET760+4.p** Composition of images 2
If F is surjective, then the image of the inverse image of Y is equal to Y.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, y$: ((maps($f, a, b$) and surjective($f, a, b$) and $y \subseteq b$) $\Rightarrow$ equal_set(image$_3$($f$, inverse_image$_3$($f, y, a$), $b$), $y$)) fof(thIIa

**SET761+4.p** Intersection of images
If F is injective, then the image of intersection is equal to the intersection of images.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x, y$: ((maps($f, a, b$) and injective($f, a, b$) and $x \subseteq a$ and $y \subseteq a$) $\Rightarrow$ equal_set(image$_3$($f$, intersection($x, y$), $b$), intersecti

**SET762+4.p** The image of empty set is empty
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: (maps($f, a, b$) $\Rightarrow$ equal_set(image$_2$($f$, empty_set), empty_set)) fof(thIIa$_{12}$, conjecture)

**SET763+4.p** If the image of X is empty then X is empty
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b, x$: ((maps($f, a, b$) and $x \subseteq a$ and equal_set(image$_2$($f, x$), empty_set)) $\Rightarrow$ equal_set($x$, empty_set)) fof(thIIa$_{13}$, conje

**SET764+4.p** The inverse image of empty set is empty
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
$\forall f, a, b$: (maps($f, a, b$) $\Rightarrow$ equal_set(inverse_image$_2$($f$, empty_set), empty_set)) fof(thIIa$_{14}$, conjecture)

**SET764^4.p** The inverse image of empty set is empty
include('Axioms/SET008^0.ax')
include('Axioms/SET008^1.ax')
$\forall f$: \$i $\rightarrow$ \$i: (fun_inv_image@$f$@emptyset) = emptyset thf(thm, conjecture)

**SET765+4.p** The restriction of an equivalence relation is an equivalence
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, x$: ((equivalence($r, e$) and $x \subseteq e$) $\Rightarrow$ equivalence($r, x$)) fof(thIII$_{01}$, conjecture)

**SET766+4.p** A member belongs to its equivalence class
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, a$: ((equivalence($r, e$) and $a \in e$) $\Rightarrow$ $a \in$ equivalence_class($a, e, r$)) fof(thIII$_{02}$, conjecture)

**SET767+4.p** Equivalence classes on E are power_set E
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, a$: (equivalence($r, e$) $\Rightarrow$ equivalence_class($a, e, r$) $\subseteq e$) fof(thIII$_{03}$, conjecture)

**SET768+4.p** Equality of equivalence classes 1
Two equivalence classes are equal if and only if the members are equivalent.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, a, b$: ((equivalence($r, e$) and $a \in e$ and $b \in e$) $\Rightarrow$ (equal_set(equivalence_class($a, e, r$), equivalence_class($b, e, r$)) $\iff$ apply($r, a, b$))) fof(thIII$_{04}$, conjecture)

**SET769+4.p** Equality of equivalence classes 2
Two equivalence classes are equal if and only if they are not : disjoint.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, a, b$: ((equivalence($r, e$) and $a \in e$ and $b \in e$) $\Rightarrow$ (equal_set(equivalence_class($a, e, r$), equivalence_class($b, e, r$)) $\iff$ $\neg$ disjoint(equivalence_class($a, e, r$), equivalence_class($b, e, r$)))) fof(thIII$_{05}$, conjecture)

**SET770+4.p** Two equivalence classes are equal or disjoint
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, r, a, b$: ((equivalence($r, e$) and $a \in e$ and $b \in e$) $\Rightarrow$ (equal_set(equivalence_class($a, e, r$), equivalence_class($b, e, r$)) or disjoint

**SET771+4.p** Equality of images defines a equivalence relation
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')
include('Axioms/SET006+2.ax')
$\forall f, a, b, r$: ((maps($f, a, b$) and $\forall x_1, x_2$: (($x_1 \in a$ and $x_2 \in a$) $\Rightarrow$ (apply($r, x_1, x_2$) $\iff$ $\exists y$: ($y \in b$ and apply($f, x_1, y$) and appl equivalence($r, a$)) fof(thIII$_{07}$, conjecture)

**SET772+4.p** Belonging of the same member of a partition is an equivalence
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall a, e, r$: (partition($a, e$) $\Rightarrow$ ($\forall x, y$: (($x \in e$ and $y \in e$) $\Rightarrow$ (apply($r, x, y$) $\iff$ $\exists z$: ($z \in a$ and $x \in z$ and $y \in z$))) $\Rightarrow$ equivalence($r, e$))) fof(thIII$_{08}$, conjecture)

**SET773+4.p** Intersection of equivalence relations is an equivalence relation
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')

$\forall e, r_1, r_2, r$: ((equivalence($r_1, e$) and equivalence($r_2, e$) and $\forall a, b$: (($a \in e$ and $b \in e$) $\Rightarrow$ (apply($r, a, b$) $\iff$ (apply($r_1, a, b$) an
equivalence($r, e$)) fof(thIII$_{09}$, conjecture)

**SET774+4.p** The restriction of a pre-order relation is a pre-order relation
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, x, r$: ((pre_order($r, e$) and $x \subseteq e$) $\Rightarrow$ pre_order($r, x$)) fof(thIII$_{10}$, conjecture)

**SET775+4.p** Pre-order and equivalence
If P is a pre-order relation,and R defined by R(A,B) if and only if P(A,B) and P(B,A), then R is an equivalence relation.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, p, r$: ((pre_order($p, e$) and $\forall a, b$: (($a \in e$ and $b \in e$) $\Rightarrow$ (apply($r, a, b$) $\iff$ (apply($p, a, b$) and apply($p, b, a$))))) $\Rightarrow$ equivalence($r, e$)) fof(thIII$_{11}$, conjecture)

**SET776+4.p** Property of pre-order
If P is a pre-order relation,and R defined by R(A,B) iff P(A,B) and P(B,A), then R(X1,Y1) and R(X2,Y2) and P(X1,X2) implies P(Y1,Y2).
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall e, p, r$: ((pre_order($p, e$) and $\forall a, b$: (($a \in e$ and $b \in e$) $\Rightarrow$ (apply($r, a, b$) $\iff$ (apply($p, a, b$) and apply($p, b, a$))))) $\Rightarrow$ $\forall x_1, x_2, y_1, y_2$: (($x_1 \in e$ and $x_2 \in e$ and $y_1 \in e$ and $y_2 \in e$) $\Rightarrow$ ((apply($r, x_1, y_1$) and apply($r, x_2, y_2$) and apply($p, x_1, x_2$)) $\Rightarrow$ apply($p, y_1, y_2$)))) fof(thIII$_{12}$, conjecture)

**SET777-1.p** Set theory membership and subsets axioms
include('Axioms/SET001-0.ax')

**SET778-1.p** Set theory membership and union axioms
include('Axioms/SET001-0.ax')
include('Axioms/SET001-1.ax')

**SET779-1.p** Set theory membership and intersection axioms
include('Axioms/SET001-0.ax')
include('Axioms/SET001-2.ax')

**SET780-1.p** Set theory membership and difference axioms
include('Axioms/SET001-0.ax')
include('Axioms/SET001-3.ax')

**SET781+3.p** Set theory axioms based on NBG set theory
include('Axioms/SET005+0.ax')

**SET781-1.p** Set theory axioms
include('Axioms/SET002-0.ax')

**SET781-2.p** Set theory axioms based on Godel set theory
include('Axioms/SET003-0.ax')

**SET781-3.p** Set theory axioms based on NBG set theory
include('Axioms/SET004-0.ax')

**SET782-1.p** Set theory (Boolean algebra) axioms based on NBG set theory
include('Axioms/SET004-0.ax')
include('Axioms/SET004-1.ax')

**SET783+1.p** Naive set theory axioms based on Goedel's set theory
include('Axioms/SET006+0.ax')

**SET784+1.p** Mapping axioms for the SET006+0 set theory axioms
include('Axioms/SET006+0.ax')
include('Axioms/SET006+1.ax')

**SET785+1.p** Equivalence relation axioms for the SET006+0 set theory axioms
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')

**SET786+1.p** Peter Andrews Problem THM25
$\neg \exists y$: $\forall x$: (element($x, y$) $\iff$ $\neg \exists z$: (element($x, z$) and element($z, x$))) fof(thm$_{25}$, conjecture)

**SET786-1.p** Peter Andrews Problem THM25
$(\mathrm{element}(b, a) \text{ and } \mathrm{element}(a, b)) \Rightarrow \neg\,\mathrm{element}(a, \mathrm{sk}_1)$ $\quad$ cnf($\mathrm{thm25}_1$, negated_conjecture)
$\mathrm{element}(a, \mathrm{sk}_1) \text{ or } \mathrm{element}(a, \mathrm{sk}_2(a))$ $\quad$ cnf($\mathrm{thm25}_2$, negated_conjecture)
$\mathrm{element}(a, \mathrm{sk}_1) \text{ or } \mathrm{element}(\mathrm{sk}_2(a), a)$ $\quad$ cnf($\mathrm{thm25}_3$, negated_conjecture)

**SET787-1.p** un_eq_Union_2_c2
$u \in \mathrm{union}(v) \Rightarrow u \in \mathrm{unionE\_sk}_1(u, v)$ $\quad$ cnf($\mathrm{clause}_{119}$, axiom)
$u \in \mathrm{union}(v) \Rightarrow \mathrm{unionE\_sk}_1(u, v) \in v$ $\quad$ cnf($\mathrm{clause}_{120}$, axiom)
$\mathrm{subsetI\_sk}_1(a, b) \in a \text{ or } a \subseteq b$ $\quad$ cnf($\mathrm{clause}_{131}$, axiom)
$\mathrm{subsetI\_sk}_1(a, b) \in b \Rightarrow a \subseteq b$ $\quad$ cnf($\mathrm{clause}_{132}$, axiom)
$u \in \mathrm{cons}(v, w) \Rightarrow (u = v \text{ or } u \in w)$ $\quad$ cnf($\mathrm{clause}_{112}$, axiom)
$c \in a \Rightarrow c \in \mathrm{un}(a, b)$ $\quad$ cnf($\mathrm{unI}_1$, axiom)
$c \in b \Rightarrow c \in \mathrm{un}(a, b)$ $\quad$ cnf($\mathrm{unI}_2$, axiom)
$\neg\, x \in \mathrm{eptset}$ $\quad$ cnf($\mathrm{clause}_{158}$, axiom)
$\mathrm{pair}(u, v) \in w \Rightarrow \mathrm{pair}(v, u) \in w^{\smile}$ $\quad$ cnf($\mathrm{clause}_{10}$, axiom)
$\mathrm{pair}(a, b) \in r^{\smile} \Rightarrow \mathrm{pair}(b, a) \in r$ $\quad$ cnf(converseD, axiom)
$\mathrm{yX} \in r^{\smile} \Rightarrow \mathrm{yX} = \mathrm{pair}(\mathrm{converseE\_sk}_2(\mathrm{yX}), \mathrm{converseE\_sk}_1(\mathrm{yX}))$ $\quad$ cnf($\mathrm{converseE}_1$, axiom)
$\mathrm{yX} \in r^{\smile} \Rightarrow \mathrm{pair}(\mathrm{converse\_sk}_1(\mathrm{yX}), \mathrm{converse\_sk}_2(\mathrm{yX})) \in r$ $\quad$ cnf($\mathrm{converseE}_2$, axiom)
$\mathrm{sk}_2 \in \mathrm{union}(\mathrm{cons}(a, \mathrm{cons}(b, \mathrm{eptset})))$ $\quad$ cnf($\mathrm{un\_eq\_Union\_2\_c}_1$, negated_conjecture)
$\neg\,\mathrm{sk}_2 \in \mathrm{un}(a, b)$ $\quad$ cnf($\mathrm{un\_eq\_Union\_2\_c}_2$, negated_conjecture)

**SET788+1.p** Symmetry of equality from set membership
$\forall x, y\colon (x{=}y \iff \forall z\colon (\mathrm{a\_member\_of}(z, x) \iff \mathrm{a\_member\_of}(z, y))) \Rightarrow \forall x, y\colon (x{=}y \iff y{=}x)$ $\quad$ fof(prove_this, conjecture)

**SET789+4.p** The greatest element, if it existes, is unique
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{order}(r, e) \text{ and } \mathrm{greatest}(m, r, e)) \Rightarrow \forall x\colon (\mathrm{greatest}(x, r, e) \Rightarrow m = x))$ $\quad$ fof($\mathrm{thIV}_1$, conjecture)

**SET790+4.p** The least element, if it existes, is unique
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{order}(r, e) \text{ and } \mathrm{least}(m, r, e)) \Rightarrow \forall x\colon (\mathrm{least}(x, r, e) \Rightarrow m = x))$ $\quad$ fof($\mathrm{thIV}_2$, conjecture)

**SET791+4.p** The greatest element, if it exists, is maximal
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{order}(r, e) \text{ and } \mathrm{greatest}(m, r, e)) \Rightarrow \max(m, r, e))$ $\quad$ fof($\mathrm{thIV}_3$, conjecture)

**SET792+4.p** The least element, if it existes, is minimal
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{order}(r, e) \text{ and } \mathrm{least}(m, r, e)) \Rightarrow \min(m, r, e))$ $\quad$ fof($\mathrm{thIV}_4$, conjecture)

**SET793+4.p** If the order is total, a maximal element is the greatest element
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{total\_order}(r, e) \text{ and } \max(m, r, e)) \Rightarrow \mathrm{greatest}(m, r, e))$ $\quad$ fof($\mathrm{thIV}_5$, conjecture)

**SET794+4.p** If the order is total, a minimal element is the least element
include('Axioms/SET006+3.ax')
$\forall r, e, m\colon ((\mathrm{total\_order}(r, e) \text{ and } \min(m, r, e)) \Rightarrow \mathrm{least}(m, r, e))$ $\quad$ fof($\mathrm{thIV}_6$, conjecture)

**SET795+4.p** If R(a,b) then b is the least upper bound of unordered_pair(a,b)
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e, a, b\colon ((\mathrm{order}(r, e) \text{ and } a \in e \text{ and } b \in e \text{ and } \mathrm{apply}(r, a, b)) \Rightarrow \mathrm{least\_upper\_bound}(b, \mathrm{unordered\_pair}(a, b), r, e))$ $\quad$ fof(thIV

**SET796+4.p** If R(a,b) then a is the greatest lower bound of unordered_pair(a,b)
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e, a, b\colon ((\mathrm{order}(r, e) \text{ and } a \in e \text{ and } b \in e \text{ and } \mathrm{apply}(r, a, b)) \Rightarrow \mathrm{greatest\_lower\_bound}(a, \mathrm{unordered\_pair}(a, b), r, e))$ $\quad$ fof(t

**SET797+4.p** If X subset Y, then an upper bound of Y is an upper bound of X
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e\colon (\mathrm{order}(r, e) \Rightarrow \forall x, y\colon ((x \subseteq e \text{ and } y \subseteq e \text{ and } x \subseteq y) \Rightarrow \forall m\colon (b \Rightarrow b)))$ $\quad$ fof($\mathrm{thIV}_9$, conjecture)

**SET798+4.p** If X subset Y, then a lower bound of Y is a lower bound of X
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e\colon (\mathrm{order}(r, e) \Rightarrow \forall x, y\colon ((x \subseteq e \text{ and } y \subseteq e \text{ and } x \subseteq y) \Rightarrow \forall m\colon (a \Rightarrow a)))$ $\quad$ fof($\mathrm{thIV}_{10}$, conjecture)

**SET799+4.p** Least upper bounds of set in total order
If an order is total, the least upper bound of a set is less than the least upper bound of a subset of it.
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall x_1, x_2$: $((x_1 \subseteq e$ and $x_2 \subseteq e$ and $x_1 \subseteq x_2) \Rightarrow \forall m_1, m_2$: ((least_upper_bound$(m_1, x_1, r, e)$ and least_upp
apply$(r, m_1, m_2))))$ fof(thIV$_{11}$, conjecture)

**SET800+4.p** Greatest lower bound of sets in total order
If an order is total, the greatest lower bound of a set is greater than the greatest lower bound of a subset of it
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall x_1, x_2$: $((x_1 \subseteq e$ and $x_2 \subseteq e$ and $x_1 \subseteq x_2) \Rightarrow \forall m_1, m_2$: ((greatest_lower_bound$(m_1, x_1, r, e)$ and greates
apply$(r, m_2, m_1))))$ fof(thIV$_{12}$, conjecture)

**SET801+4.p** M is the greatest element iff it is a member and a LUB
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall x$: $(x \subseteq e \Rightarrow \forall m$: (greatest$(m, r, x) \iff (m \in x$ and least_upper_bound$(m, x, r, e)))))$ fof(thIV$_{13}$,

**SET802+4.p** M is the least of X iff it is a member and a GLB
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall x$: $(x \subseteq e \Rightarrow \forall m$: (least$(m, r, x) \iff (m \in x$ and greatest_lower_bound$(m, x, r, e)))))$ fof(thIV$_{14}$, c

**SET803+4.p** Two different maximal elements implies no greatest element
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall m_1, m_2$: ((max$(m_1, r, e)$ and max$(m_2, r, e)$ and $m_1 \neq m_2) \Rightarrow \neg \exists m$: greatest$(m, r, e)))$ fof(thIV$_{15}$, c

**SET804+4.p** Two different minimal elements implies no least element
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall m_1, m_2$: ((min$(m_1, r, e)$ and min$(m_2, r, e)$ and $m_1 \neq m_2) \Rightarrow \neg \exists m$: least$(m, r, e)))$ fof(thIV$_{16}$, conje

**SET805+4.p** Order relation on E is an order relation on a subset of E
include('Axioms/SET006+0.ax')
include('Axioms/SET006+3.ax')
$\forall r, e$: (order$(r, e) \Rightarrow \forall x$: $(x \subseteq e \Rightarrow$ order$(r, x)))$ fof(thIV$_{17}$, conjecture)

**SET806+4.p** Equality of sets defines a equivalence relation
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall x, y$: (apply(equal_set_predicate$, x, y) \iff$ equal_set$(x, y))$ fof(rel_equal_set, hypothesis)
$\forall e$: equivalence(equal_set_predicate, power_set$(e))$ fof(thIII$_{13}$, conjecture)

**SET807+4.p** Inclusion of sets defines a pre-order relation
include('Axioms/SET006+0.ax')
include('Axioms/SET006+2.ax')
$\forall x, y$: (apply(subset_predicate$, x, y) \iff x \subseteq y)$ fof(rel_subset, hypothesis)
$\forall e$: pre_order(subset_predicate, power_set$(e))$ fof(thIV18a, conjecture)

**SET808+4.p** The members of an ordinal number are ordinal numbers
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, b, c$: $((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(thI$_3$, axiom)
$\forall a$: $(a \in \text{on} \Rightarrow a \subseteq \text{on})$ fof(thV$_1$, conjecture)

**SET809+4.p** An ordinal number is not a member of itself
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a$: $(a \in \text{on} \Rightarrow \neg a \in a)$ fof(thV$_2$, conjecture)

**SET810+4.p** Ordinal numbers do not contain each other
If a and b are ordinal numbers, it is not possible that a belongs to b and b belongs to a
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, b$: $((a \in \text{on}$ and $b \in \text{on}) \Rightarrow \neg a \in b$ and $b \in a)$ fof(thV$_3$, conjecture)

**SET811+4.p** A member of an ordinal number is an initial segment

include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a$: ($a \in$ on $\Rightarrow$ $\forall x$: ($x \in a$ $\Rightarrow$ equal_set($x$, initial_segment($x$, member_predicate, $a$)))) fof(thV$_5$, conjecture)

**SET812+4.p** An ordinal A is equal to its intersection with its power-set
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a$: ($a \in$ on $\Rightarrow$ equal_set($a$, intersection($a$, power_set($a$)))) fof(thV$_{10}$, conjecture)

**SET813+4.p** An ordinal number is a member of its successor
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a$: ($a \in$ on $\Rightarrow$ $a \in$ suc($a$)) fof(thV$_{12}$, conjecture)

**SET814+4.p** The sum of an ordinal number is a subset of itself
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(thI$_3$, axiom)
$\forall a$: ($a \in$ on $\Rightarrow$ sum($a$) $\subseteq a$) fof(thV$_{14}$, conjecture)

**SET815+4.p** An ordinal number is equal to the sum of its successor
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, x$: ($x \in a$ $\Rightarrow$ singleton($x$) $\subseteq a$) fof(thI$_{44}$, axiom)
$\forall a$: ($a \in$ on $\Rightarrow$ equal_set(sum(suc($a$)), $a$)) fof(thV$_{15}$, conjecture)

**SET816+4.p** The sum of a collection of ordinal numbers is a collection
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(thI$_3$, axiom)
$\forall a$: ($a \subseteq$ on $\Rightarrow$ sum($a$) $\subseteq$ on) fof(thV$_{16}$, conjecture)

**SET817+4.p** The product of a nonempty set of ordinals is an ordinal
include('Axioms/SET006+0.ax')
include('Axioms/SET006+4.ax')
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(thI$_3$, axiom)
$\forall x$: (set($x$) $\Rightarrow$ set(product($x$))) fof(set_product, axiom)
$\forall a, x$: ($x \in a$ $\Rightarrow$ product($a$) $\subseteq x$) fof(thI$_{42}$, axiom)
$\forall a$: (($a \subseteq$ on and set($a$) and $\exists x$: $x \in a$) $\Rightarrow$ product($a$) $\in$ on) fof(thV$_{21}$, conjecture)

**SET818-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x(v_U), v_U, tc_IntDef_Oint) cnf(cls_conjecture$_0$, negated_conjecture)
$\neg$ c_lessequals(v_x(v_U), $c_0$, tc_IntDef_Oint) cnf(cls_conjecture$_1$, negated_conjecture)

**SET818-2.p** Problem about set theory
$\neg$ c_in(v_a, c_emptyset, t_a) cnf(cls_Set_OemptyE$_0$, axiom)
c_in(v_x(v_U), v_U, tc_IntDef_Oint) cnf(cls_conjecture$_0$, negated_conjecture)

**SET819-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_D, v_F, tc_set(t_a)) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x(v_U), v_U, tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_x(v_U), v_V, tc_set(t_a)) $\Rightarrow$ $\neg$ c_in(v_V, v_F, tc_set(t_a)) cnf(cls_conjecture$_2$, negated_conjecture)

**SET819-2.p** Problem about set theory
$\neg$ c_in(v_a, c_emptyset, t_a) cnf(cls_Set_OemptyE$_0$, axiom)
c_in(v_x(v_U), v_U, tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)

**SET820-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_P(v_a) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_V, v_U, tc_IntDef_Oint) $\Rightarrow$ c_in(v_x(v_U), v_U, tc_IntDef_Oint) cnf(cls_conjecture$_1$, negated_conjecture)

c_in(v_W, v_U, tc_IntDef_Oint) ⇒ ¬ v_P(v_x(v_U))  cnf(cls_conjecture$_{2}$, negated_conjecture)

**SET820-2.p** Problem about set theory
¬ c_in(v_a, c_emptyset, t_a)  cnf(cls_Set_OemptyE$_{0}$, axiom)
c_in(v_x, c_insert(v_x, v_B, t_a), t_a)  cnf(cls_Set_OinsertCI$_{1}$, axiom)
c_in(v_a, c_insert(v_b, v_A, t_a), t_a) ⇒ (c_in(v_a, v_A, t_a) or v_a = v_b)  cnf(cls_Set_OinsertE$_{0}$, axiom)
v_P(v_a)  cnf(cls_conjecture$_{0}$, negated_conjecture)
c_in(v_V, v_U, tc_IntDef_Oint) ⇒ c_in(v_x(v_U), v_U, tc_IntDef_Oint)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(v_W, v_U, tc_IntDef_Oint) ⇒ ¬ v_P(v_x(v_U))  cnf(cls_conjecture$_{2}$, negated_conjecture)

**SET821-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_less(v_a, v_b, tc_IntDef_Oint)  cnf(cls_conjecture$_{0}$, negated_conjecture)
c_less(v_b, v_c, tc_IntDef_Oint)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(v_b, v_U, tc_IntDef_Oint) ⇒ (c_in(v_c, v_U, tc_IntDef_Oint) or c_in(v_a, v_U, tc_IntDef_Oint))  cnf(cls_conjecture$_{2}$, neg

**SET821-2.p** Problem about set theory
class_Orderings_Oorder(t_a) ⇒ ¬ c_less(v_x, v_x, t_a)  cnf(cls_Orderings_Oorder__less__irrefl__iff1$_{0}$, axiom)
c_in(v_c, v_A, t_a) ⇒ ¬ c_in(v_c, c_uminus(v_A, tc_set(t_a)), t_a)  cnf(cls_Set_OComplD__dest$_{0}$, axiom)
c_in(v_c, v_A, t_a) or c_in(v_c, c_uminus(v_A, tc_set(t_a)), t_a)  cnf(cls_Set_OComplI$_{0}$, axiom)
c_in(v_a, v_B, t_a) ⇒ c_in(v_a, c_insert(v_b, v_B, t_a), t_a)  cnf(cls_Set_OinsertCI$_{0}$, axiom)
c_in(v_x, c_insert(v_x, v_B, t_a), t_a)  cnf(cls_Set_OinsertCI$_{1}$, axiom)
c_in(v_a, c_insert(v_b, v_A, t_a), t_a) ⇒ (c_in(v_a, v_A, t_a) or v_a = v_b)  cnf(cls_Set_OinsertE$_{0}$, axiom)
c_less(v_a, v_b, tc_IntDef_Oint)  cnf(cls_conjecture$_{0}$, negated_conjecture)
c_less(v_b, v_c, tc_IntDef_Oint)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(v_b, v_U, tc_IntDef_Oint) ⇒ (c_in(v_c, v_U, tc_IntDef_Oint) or c_in(v_a, v_U, tc_IntDef_Oint))  cnf(cls_conjecture$_{2}$, neg
class_Orderings_Oorder(tc_IntDef_Oint)  cnf(clsarity_IntDef__Oint$_{31}$, axiom)

**SET822-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_P(v_f(v_b))  cnf(cls_conjecture$_{0}$, negated_conjecture)
c_in(v_f(v_U), v_V, tc_IntDef_Oint) ⇒ c_in(v_x(v_U, v_V), v_V, tc_IntDef_Oint)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(v_f(v_U), v_V, tc_IntDef_Oint) ⇒ ¬ v_P(v_x(v_U, v_V))  cnf(cls_conjecture$_{2}$, negated_conjecture)

**SET822-2.p** Problem about set theory
¬ c_in(v_a, c_emptyset, t_a)  cnf(cls_Set_OemptyE$_{0}$, axiom)
c_in(v_x, c_insert(v_x, v_B, t_a), t_a)  cnf(cls_Set_OinsertCI$_{1}$, axiom)
c_in(v_a, c_insert(v_b, v_A, t_a), t_a) ⇒ (c_in(v_a, v_A, t_a) or v_a = v_b)  cnf(cls_Set_OinsertE$_{0}$, axiom)
v_P(v_f(v_b))  cnf(cls_conjecture$_{0}$, negated_conjecture)
c_in(v_f(v_U), v_V, tc_IntDef_Oint) ⇒ c_in(v_x(v_U, v_V), v_V, tc_IntDef_Oint)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(v_f(v_U), v_V, tc_IntDef_Oint) ⇒ ¬ v_P(v_x(v_U, v_V))  cnf(cls_conjecture$_{2}$, negated_conjecture)

**SET824-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_a, v_U, tc_IntDef_Oint)  cnf(cls_conjecture$_{0}$, negated_conjecture)

**SET824-2.p** Problem about set theory
c_in(v_c, v_A, t_a) ⇒ ¬ c_in(v_c, c_uminus(v_A, tc_set(t_a)), t_a)  cnf(cls_Set_OComplD__dest$_{0}$, axiom)
c_in(v_a, v_U, tc_IntDef_Oint)  cnf(cls_conjecture$_{0}$, negated_conjecture)

**SET825-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_Q(v_n)  cnf(cls_conjecture$_{0}$, negated_conjecture)
¬ v_Q(v_m)  cnf(cls_conjecture$_{1}$, negated_conjecture)
c_in(c_Pair($c_0$, $c_0$, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat)) ⇒ (c_in(c_Pair(v_n, v_m, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc.
(c_in(c_Pair(c_Suc(v_x(v_U)), c_Suc(v_xa(v_U)), tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat)) and c_in(c_Pair($c_0$, $c_0$, tc_nat, tc_n
c_in(c_Pair(v_n, v_m, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat))  cnf(cls_conjecture$_{3}$, negated_conjecture)

**SET825-2.p** Problem about set theory
c_in(c_Pair(v_a, v_a, t_a, t_a), c_Relation_OId, tc_prod(t_a, t_a))  cnf(cls_Relation_OIdI$_{0}$, axiom)
c_in(c_Pair(v_a, v_b, t_a, t_a), c_Relation_OId, tc_prod(t_a, t_a)) ⇒ v_a = v_b  cnf(cls_Relation_Opair__in__Id__conv__iff1$_{0}$, ax

v_Q(v_n)    cnf(cls_conjecture$_0$, negated_conjecture)
¬ v_Q(v_m)    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(c_Pair($c_0$, $c_0$, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat)) ⇒ (c_in(c_Pair(v_n, v_m, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc.
(c_in(c_Pair(c_Suc(v_x(v_U)), c_Suc(v_xa(v_U)), tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat)) and c_in(c_Pair($c_0$, $c_0$, tc_nat, tc_n
c_in(c_Pair(v_n, v_m, tc_nat, tc_nat), v_U, tc_prod(tc_nat, tc_nat))    cnf(cls_conjecture$_3$, negated_conjecture)

**SET826-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_V, v_Y, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_V, v_Z, tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET826-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_V, v_Y, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET827-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_V, v_Y, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_V, v_Z, tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET827-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_V, v_Z, tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET828-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) ⇒ c_lessequals(v_U, v_X, tc_set(t_a))    cnf(cls_co

**SET828-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET829-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) ⇒ c_lessequals(v_U, v_X, tc_set(t_a))    cnf(cls_co

**SET829-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET830-1.p** Problem about set theory

include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)
$\neg$ c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) $\Rightarrow$ c_lessequals(v_U, v_X, tc_set(t_a))    cnf(cls_co

**SET830-2.p** Problem about set theory
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) $\Rightarrow$ c_in(v_c, v_B, t_a)    cnf(cls_Set_OIntE$_0$, axiom)
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) $\Rightarrow$ c_in(v_c, v_A, t_a)    cnf(cls_Set_OIntE$_1$, axiom)
(c_in(v_c, v_B, t_a) and c_in(v_c, v_A, t_a)) $\Rightarrow$ c_in(v_c, c_inter(v_A, v_B, t_a), t_a)    cnf(cls_Set_OIntI$_0$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a))    cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) $\Rightarrow$ c_lessequals(v_A, v_B, tc_set(t_a))    cnf(cls_Set_OsubsetI$_1$, axiom)
c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_x, v_Z, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)
$\neg$ c_in(v_x, v_X, t_a)    cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) $\Rightarrow$ c_lessequals(v_U, v_X, tc_set(t_a))    cnf(cls_co

**SET831-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
(c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a)) and v_X = c_inter(v_Y, v_Z, t_a)) $\Rightarrow$ c_lessequals(v_
(c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a)) and v_X = c_inter(v_Y, v_Z, t_a)) $\Rightarrow$ c_lessequals(v_
(c_lessequals(v_x, v_X, tc_set(t_a)) and c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a))) $\Rightarrow$ v_X $\neq$
c_inter(v_Y, v_Z, t_a)    cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) $\Rightarrow$ (v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_U

**SET831-2.p** Problem about set theory
v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_X, v_Y, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_X, v_Z, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
(c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a)) and v_X = c_inter(v_Y, v_Z, t_a)) $\Rightarrow$ c_lessequals(v_
(c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a)) and v_X = c_inter(v_Y, v_Z, t_a)) $\Rightarrow$ c_lessequals(v_
(c_lessequals(v_x, v_X, tc_set(t_a)) and c_lessequals(v_X, v_Z, tc_set(t_a)) and c_lessequals(v_X, v_Y, tc_set(t_a))) $\Rightarrow$ v_X $\neq$
c_inter(v_Y, v_Z, t_a)    cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_U, v_Z, tc_set(t_a)) and c_lessequals(v_U, v_Y, tc_set(t_a))) $\Rightarrow$ (v_X = c_inter(v_Y, v_Z, t_a) or c_lessequals(v_U
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) $\Rightarrow$ c_in(v_c, v_B, t_a)    cnf(cls_Set_OIntE$_0$, axiom)
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) $\Rightarrow$ c_in(v_c, v_A, t_a)    cnf(cls_Set_OIntE$_1$, axiom)
(c_in(v_c, v_B, t_a) and c_in(v_c, v_A, t_a)) $\Rightarrow$ c_in(v_c, c_inter(v_A, v_B, t_a), t_a)    cnf(cls_Set_OIntI$_0$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a))    cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) $\Rightarrow$ c_lessequals(v_A, v_B, tc_set(t_a))    cnf(cls_Set_OsubsetI$_1$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ v_A = v_B    cnf(cls_Set_Osubset__antisym$_0$, a

**SET832-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_Y, v_V, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Z, v_V, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
$\neg$ c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET832-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_Y, v_V, tc_set(t_a))    cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x, v_Y, t_a)    cnf(cls_conjecture$_2$, negated_conjecture)
$\neg$ c_in(v_x, v_V, t_a)    cnf(cls_conjecture$_3$, negated_conjecture)

**SET833-1.p** Problem about set theory

include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_Y, v_V, tc_set(t_a))  cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Z, v_V, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_V, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)

**SET833-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)  cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_Z, v_V, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_V, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)

**SET834-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_Y, v_X, tc_set(t_a))  cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Z, v_X, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_X, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)  cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ c_lessequals(v_X, v_U, tc_set(t_a))  cnf(cls_co

**SET834-2.p** Problem about set theory
c_in(v_c, v_B, t_a) ⇒ c_in(v_c, c_union(v_A, v_B, t_a), t_a)  cnf(cls_Set_OUnCI$_0$, axiom)
c_in(v_c, v_A, t_a) ⇒ c_in(v_c, c_union(v_A, v_B, t_a), t_a)  cnf(cls_Set_OUnCI$_1$, axiom)
c_in(v_c, c_union(v_A, v_B, t_a), t_a) ⇒ (c_in(v_c, v_B, t_a) or c_in(v_c, v_A, t_a))  cnf(cls_Set_OUnE$_0$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)  cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a))  cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a))  cnf(cls_Set_OsubsetI$_1$, axiom)
c_in(v_x, v_X, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_in(v_x, v_Y, t_a)  cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ c_lessequals(v_X, v_U, tc_set(t_a))  cnf(cls_co

**SET835-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_Y, v_X, tc_set(t_a))  cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Z, v_X, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Y, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_X, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ c_lessequals(v_X, v_U, tc_set(t_a))  cnf(cls_co

**SET835-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)  cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_Y, v_X, tc_set(t_a))  cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x, v_Y, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_X, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)

**SET836-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_Y, v_X, tc_set(t_a))  cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Z, v_X, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_in(v_x, v_X, t_a)  cnf(cls_conjecture$_3$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ c_lessequals(v_X, v_U, tc_set(t_a))  cnf(cls_co

**SET836-2.p** Problem about set theory
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)  cnf(cls_Set_OsubsetD$_0$, axiom)
c_lessequals(v_Z, v_X, tc_set(t_a))  cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Z, t_a)  cnf(cls_conjecture$_2$, negated_conjecture)

¬ c_in(v_x, v_X, t_a) cnf(cls_conjecture$_3$, negated_conjecture)

**SET837-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_Y, v_X, tc_set(t_a)) cnf(cls_conjecture$_0$, negated_conjecture)
v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_Z, v_X, tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
(c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a)) and v_X = c_union(v_Y, v_Z, t_a)) ⇒ c_lessequals(v
(c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a)) and v_X = c_union(v_Y, v_Z, t_a)) ⇒ c_lessequals(v
(c_lessequals(v_X, v_x, tc_set(t_a)) and c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a))) ⇒ v_X ≠ c_union(v_Y, v_Z, t_a) cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ (v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_

**SET837-2.p** Problem about set theory
v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_Y, v_X, tc_set(t_a)) cnf(cls_conjecture$_0$, negated_conjecture)
v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_Z, v_X, tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
(c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a)) and v_X = c_union(v_Y, v_Z, t_a)) ⇒ c_lessequals(v
(c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a)) and v_X = c_union(v_Y, v_Z, t_a)) ⇒ c_lessequals(v
(c_lessequals(v_X, v_x, tc_set(t_a)) and c_lessequals(v_Z, v_X, tc_set(t_a)) and c_lessequals(v_Y, v_X, tc_set(t_a))) ⇒ v_X ≠ c_union(v_Y, v_Z, t_a) cnf(cls_conjecture$_4$, negated_conjecture)
(c_lessequals(v_Z, v_U, tc_set(t_a)) and c_lessequals(v_Y, v_U, tc_set(t_a))) ⇒ (v_X = c_union(v_Y, v_Z, t_a) or c_lessequals(v_
c_in(v_c, v_B, t_a) ⇒ c_in(v_c, c_union(v_A, v_B, t_a), t_a) cnf(cls_Set_OUnCI$_0$, axiom)
c_in(v_c, v_A, t_a) ⇒ c_in(v_c, c_union(v_A, v_B, t_a), t_a) cnf(cls_Set_OUnCI$_1$, axiom)
c_in(v_c, c_union(v_A, v_B, t_a), t_a) ⇒ (c_in(v_c, v_B, t_a) or c_in(v_c, v_A, t_a)) cnf(cls_Set_OUnE$_0$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a

**SET838-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
v_f(v_g(v_x)) = v_x cnf(cls_conjecture$_0$, negated_conjecture)
v_f(v_g(v_U)) = v_U ⇒ v_U = v_x cnf(cls_conjecture$_1$, negated_conjecture)
v_g(v_f(v_U)) = v_U ⇒ v_g(v_f(v_xa(v_U))) = v_xa(v_U) cnf(cls_conjecture$_2$, negated_conjecture)
v_xa(v_U) = v_U ⇒ v_g(v_f(v_U)) ≠ v_U cnf(cls_conjecture$_3$, negated_conjecture)

**SET838-2.p** Problem about set theory
v_f(v_g(v_x)) = v_x cnf(cls_conjecture$_0$, negated_conjecture)
v_f(v_g(v_U)) = v_U ⇒ v_U = v_x cnf(cls_conjecture$_1$, negated_conjecture)
v_g(v_f(v_U)) = v_U ⇒ v_g(v_f(v_xa(v_U))) = v_xa(v_U) cnf(cls_conjecture$_2$, negated_conjecture)
v_xa(v_U) = v_U ⇒ v_g(v_f(v_U)) ≠ v_U cnf(cls_conjecture$_3$, negated_conjecture)

**SET839-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
¬ c_lessequals(v_S, c_insert(v_U, c_emptyset, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
(c_in(v_V, v_S, tc_set(t_a)) and c_in(v_U, v_S, tc_set(t_a))) ⇒ c_lessequals(v_U, v_V, tc_set(t_a)) cnf(cls_conjecture$_1$, negate

**SET839-2.p** Problem about set theory
¬ c_lessequals(v_S, c_insert(v_U, c_emptyset, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
(c_in(v_V, v_S, tc_set(t_a)) and c_in(v_U, v_S, tc_set(t_a))) ⇒ c_lessequals(v_U, v_V, tc_set(t_a)) cnf(cls_conjecture$_1$, negate
c_in(v_x, c_insert(v_x, v_B, t_a), t_a) cnf(cls_Set_OinsertCI$_1$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a

**SET840-1.p** Problem about set theory
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
¬ c_lessequals(v_S, c_insert(v_U, c_emptyset, tc_set(t_b)), tc_set(tc_set(t_b))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_b)) ⇒ c_lessequals(c_Union(v_S, t_b), v_U, tc_set(t_b)) cnf(cls_conjecture$_1$, negated_conjecture)

**SET840-2.p** Problem about set theory

¬ c_lessequals(v_S, c_insert(v_U, c_emptyset, tc_set(t_b)), tc_set(tc_set(t_b))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_b)) ⇒ c_lessequals(c_Union(v_S, t_b), v_U, tc_set(t_b)) cnf(cls_conjecture$_1$, negated_conjecture)
(c_in(v_A, v_X, t_a) and c_in(v_X, v_C, tc_set(t_a))) ⇒ c_in(v_A, c_Union(v_C, t_a), t_a) cnf(cls_Set_OUnionI$_0$, axiom)
c_in(v_a, v_B, t_a) ⇒ c_in(v_a, c_insert(v_b, v_B, t_a), t_a) cnf(cls_Set_OinsertCI$_0$, axiom)
c_in(v_x, c_insert(v_x, v_B, t_a), t_a) cnf(cls_Set_OinsertCI$_1$, axiom)
c_in(v_a, c_insert(v_b, v_A, t_a), t_a) ⇒ (c_in(v_a, v_A, t_a) or v_a = v_b) cnf(cls_Set_OinsertE$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a

**SET841-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
(c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_m, tc_set(tc_set(t_a))) or v_n = v_m) cnf(cls_Zorn_OTFin__subsetD$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
c_lessequals(v_x, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_x, t_a), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)

**SET841-2.p** Problem about Zorn's lemma
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
(c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_m, tc_set(tc_set(t_a))) or v_n = v_m) cnf(cls_Zorn_OTFin__subsetD$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
c_lessequals(v_x, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_x, t_a), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)

**SET842-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
(c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_m, tc_set(tc_set(t_a))) or v_n = v_m) cnf(cls_Zorn_OTFin__subsetD$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(c_Union(v_Y, tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_Y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_U, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)

**SET842-2.p** Problem about Zorn's lemma
¬ c_lessequals(c_Union(v_Y, tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_Y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_U, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)
c_in(v_A, c_Union(v_C, t_a), t_a) ⇒ c_in(c_Main_OUnionE$_{-1}$(v_A, v_C, t_a), v_C, tc_set(t_a)) cnf(cls_Set_OUnionE$_0$, axiom)
c_in(v_A, c_Union(v_C, t_a), t_a) ⇒ c_in(v_A, c_Main_OUnionE$_{-1}$(v_A, v_C, t_a), t_a) cnf(cls_Set_OUnionE$_1$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)

**SET843-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Main_OUnion__least$_{-1}$(v_A, v_C, t_a), v_A, tc_set(t_a)) or c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cnf(cls_Set
c_lessequals(c_Main_OUnion__least$_{-1}$(v_A, v_C, t_a), v_C, tc_set(t_a)) ⇒ c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cn
c_in(v_B, v_A, tc_set(t_a)) ⇒ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) ⇒ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) ⇒ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)

c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) $\Rightarrow$ v_m $\neq$ c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_co
v_m = c_Zorn_Osucc(v_S, v_m, t_a) or v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) cnf(cls_conjecture$_2$, negated_con

**SET843-2.p** Problem about Zorn's lemma
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) $\Rightarrow$ v_m $\neq$ c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_co
v_m = c_Zorn_Osucc(v_S, v_m, t_a) or v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) cnf(cls_conjecture$_2$, negated_con

**SET844-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Main_OUnion__least$_1$(v_A, v_C, t_a), v_A, tc_set(t_a)) or c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cnf(cls_Set
c_lessequals(c_Main_OUnion__least$_1$(v_A, v_C, t_a), v_C, tc_set(t_a)) $\Rightarrow$ c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cn
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_conjecture)
$\neg$ c_lessequals(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjectur

**SET844-2.p** Problem about Zorn's lemma
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_1$, negated_conjecture)
$\neg$ c_lessequals(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjectur

**SET845-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Main_OUnion__least$_1$(v_A, v_C, t_a), v_A, tc_set(t_a)) or c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cnf(cls_Set
c_lessequals(c_Main_OUnion__least$_1$(v_A, v_C, t_a), v_C, tc_set(t_a)) $\Rightarrow$ c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cn
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
v_m $\neq$ c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_2$, negated_conjecture)

**SET845-2.p** Problem about Zorn's lemma
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)

v_m = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
v_m $\neq$ c_Zorn_Osucc(v_S, v_m, t_a) cnf(cls_conjecture$_2$, negated_conjecture)

**SET846-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Main_OUnion__least_$_1$(v_A, v_C, t_a), v_A, tc_set(t_a)) or c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cnf(cls_Set
c_lessequals(c_Main_OUnion__least_$_1$(v_A, v_C, t_a), v_C, tc_set(t_a)) $\Rightarrow$ c_lessequals(c_Union(v_A, t_a), v_C, tc_set(t_a)) cn
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m =
c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_Zorn_Oeq__succ__upper$_0$, axiom)
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
$\neg$ c_lessequals(c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a), c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a

**SET846-2.p** Problem about Zorn's lemma
c_in(v_B, v_A, tc_set(t_a)) $\Rightarrow$ c_lessequals(v_B, c_Union(v_A, t_a), tc_set(t_a)) cnf(cls_Set_OUnion__upper$_0$, axiom)
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
$\neg$ c_lessequals(c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a), c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a

**SET847-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(v_c, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m = c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ v_m = c_Union(c_Zorn_OT
c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_Union(c_Zorn_OTFin(v_S
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) cnf(cls_Zorn_Oequal__succ__Union$_1$, axiom)
c_in(v_c, c_minus(c_Zorn_Ochain(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))), tc_set(tc_set(t_a))) $\Rightarrow$
c_Zorn_Osucc(v_S, v_c, t_a) $\neq$ v_c cnf(cls_Zorn_Osucc__not__equals$_0$, axiom)
$\neg$ c_in(v_U, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)

**SET847-2.p** Problem about Zorn's lemma
c_in(v_c, v_A, t_a) $\Rightarrow$ (c_in(v_c, v_B, t_a) or c_in(v_c, c_minus(v_A, v_B, tc_set(t_a)), t_a)) cnf(cls_Set_ODiffI$_0$, axiom)
$\neg$ c_in(v_a, c_emptyset, t_a) cnf(cls_Set_OemptyE$_0$, axiom)
c_lessequals(c_emptyset, v_A, tc_set(t_a)) cnf(cls_Set_Oempty__subsetI$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) $\Rightarrow$ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(v_c, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_Union(c_Zorn_OTFin(v_S
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) cnf(cls_Zorn_Oequal__succ__Union$_1$, axiom)
c_in(v_c, c_minus(c_Zorn_Ochain(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))), tc_set(tc_set(t_a))) $\Rightarrow$
c_Zorn_Osucc(v_S, v_c, t_a) $\neq$ v_c cnf(cls_Zorn_Osucc__not__equals$_0$, axiom)
$\neg$ c_in(v_U, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)

**SET848-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) $\Rightarrow$ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(v_c, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m = c_Zorn_Osucc(v_S, v_m, t_a)) $\Rightarrow$ v_m = c_Union(c_Zorn_OT
c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) $\Rightarrow$ c_Union(c_Zorn_OTFin(v_S
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) cnf(cls_Zorn_Oequal__succ__Union$_1$, axiom)

c_in(v_c, c_minus(c_Zorn_Ochain(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))), tc_set(tc_set(t_a))) ⇒
c_Zorn_Osucc(v_S, v_c, t_a) ≠ v_c    cnf(cls_Zorn_Osucc__not__equals$_0$, axiom)
¬ c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, n
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a))    cnf(c
¬ c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_2$, n

**SET848-2.p** Problem about Zorn's lemma
c_in(v_c, v_A, t_a) ⇒ (c_in(v_c, v_B, t_a) or c_in(v_c, c_minus(v_A, v_B, tc_set(t_a)), t_a))    cnf(cls_Set_ODiffI$_0$, axiom)
c_lessequals(v_A, v_A, tc_set(t_a))    cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) ⇒ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(v_c, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn
c_in(v_c, c_minus(c_Zorn_Ochain(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))), tc_set(tc_set(t_a))) ⇒
c_Zorn_Osucc(v_S, v_c, t_a) ≠ v_c    cnf(cls_Zorn_Osucc__not__equals$_0$, axiom)
¬ c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, n
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) = c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a))    cnf(c

**SET849-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(c_Zorn_Osucc(v_S, v_x, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) ⇒ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(v_c, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and v_m = c_Zorn_Osucc(v_S, v_m, t_a)) ⇒ v_m = c_Union(c_Zorn_OT
c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_Union(c_Zorn_OTFin(v_S
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a)    cnf(cls_Zorn_Oequal__succ__Union$_1$, axiom)
c_in(v_c, c_minus(c_Zorn_Ochain(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))), tc_set(tc_set(t_a))) ⇒
c_Zorn_Osucc(v_S, v_c, t_a) ≠ v_c    cnf(cls_Zorn_Osucc__not__equals$_0$, axiom)
¬ c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, n
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) ≠ c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a))    cnf(c

**SET849-2.p** Problem about Zorn's lemma
c_lessequals(v_A, v_A, tc_set(t_a))    cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) ⇒ c_in(c_Union(v_Y, tc_set(t_a)), c_Zorn_OTFin(v_S, t_a
c_in(c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_Union(c_Zorn_OTFin(v_S
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a)    cnf(cls_Zorn_Oequal__succ__Union$_1$, axiom)
c_Zorn_Osucc(v_S, c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a)), t_a) ≠ c_Union(c_Zorn_OTFin(v_S, t_a), tc_set(t_a))    cnf(c

**SET850-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a))    cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_lessequals(v_x, v_y, tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, negated_conjecture)
¬ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_1$, negated_conjecture)

**SET850-2.p** Problem about Zorn's lemma
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a))    cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_lessequals(v_x, v_y, tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, negated_conjecture)
¬ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_1$, negated_conjecture)

**SET851-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_C, tc_set(t_a)) or c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) or c_les
c_lessequals(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_A, tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_B, c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn_C
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_x, t_a), v_m, tc_set(tc_set(t_a)))    cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_3$, negated_con
c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_x, tc_set(tc_set(t_a))) or c_lessequals(v_x, v_m, tc_set(tc_set(t_a)))    cnf(cls_conje

(c_lessequals(v_U, v_m, tc_set(tc_set(t_a))) and c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_lessequals(c_Zorn_
v_m) cnf(cls_conjecture$_5$, negated_conjecture)

**SET851-2.p** Problem about Zorn's lemma
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_x, t_a), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_co
c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_x, tc_set(tc_set(t_a))) or c_lessequals(v_x, v_m, tc_set(tc_set(t_a))) cnf(cls_conje
(c_lessequals(v_U, v_m, tc_set(tc_set(t_a))) and c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_lessequals(c_Zorn_
v_m) cnf(cls_conjecture$_5$, negated_conjecture)

**SET852-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_C, tc_set(t_a)) or c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) or c_les
c_lessequals(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_A, tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_B, c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_lessequals(c_Union(v_Y, tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conje
c_in(v_U, v_Y, tc_set(tc_set(t_a))) ⇒ (c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_U, tc_set(tc_set(t_a))) or c_lessequals(v_U, v_
(c_lessequals(v_U, v_m, tc_set(tc_set(t_a))) and c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_lessequals(c_Zorn_
v_m) cnf(cls_conjecture$_5$, negated_conjecture)

**SET852-2.p** Problem about Zorn's lemma
¬ c_lessequals(c_Union(v_Y, tc_set(t_a)), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conje
c_in(v_U, v_Y, tc_set(tc_set(t_a))) ⇒ (c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_U, tc_set(tc_set(t_a))) or c_lessequals(v_U, v_
c_in(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_C, tc_set(t_a)) or c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) or c_les
c_lessequals(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_A, tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_B, c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a

**SET853-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zor
c_in(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_C, tc_set(t_a)) or c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) or c_les
c_lessequals(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_A, tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_B, c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_xa, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_xa, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
v_xa ≠ c_Zorn_Osucc(v_S, v_x, t_a) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_xa, t_a), c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_co
(c_lessequals(v_U, v_x, tc_set(tc_set(t_a))) and c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_lessequals(c_Zorn_
v_x) cnf(cls_conjecture$_5$, negated_conjecture)

**SET853-2.p** Problem about Zorn's lemma
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)

c_in(v_xa, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_xa, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
v_xa ≠ c_Zorn_Osucc(v_S, v_x, t_a) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_xa, t_a), c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_co
(c_lessequals(v_U, v_x, tc_set(tc_set(t_a))) and c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_lessequals(c_Zorn_
v_x) cnf(cls_conjecture$_5$, negated_conjecture)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a
c_lessequals(v_A, v_A, tc_set(t_a)) cnf(cls_Set_Osubset__refl$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zorn
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_

**SET854-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zorn
c_in(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_C, tc_set(t_a)) or c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) or c_les
c_lessequals(c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), v_A, tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_B, c_Zorn_OUnion__lemma0$_{-1}$(v_A, v_B, v_C, t_a), tc_set(t_a)) ⇒ (c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a
c_lessequals(v_x, v_y, tc_set(tc_set(t_a))) ⇒ c_lessequals(v_x, c_Zorn_Osucc(v_S, v_y, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_x, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(v_x, c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
v_x ≠ c_Union(v_Y, tc_set(t_a)) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_x, t_a), c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conje
(c_lessequals(v_V, v_U, tc_set(tc_set(t_a))) and c_in(v_V, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_U, v_Y, tc_set(
(c_lessequals(c_Zorn_Osucc(v_S, v_V, t_a), v_U, tc_set(tc_set(t_a))) or v_V = v_U) cnf(cls_conjecture$_5$, negated_conjecture)

**SET855-2.p** Problem about Zorn's lemma
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a
c_lessequals(v_n, c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
c_lessequals(c_Union(v_Y, tc_set(t_a)), v_n, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
v_n ≠ c_Union(v_Y, tc_set(t_a)) cnf(cls_conjecture$_4$, negated_conjecture)

**SET856-2.p** Problem about Zorn's lemma
c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjectu
¬ c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), c_Union(v_Y, tc_set(t_a)), tc_set(tc_set(t_a))) cnf(cls_conjecture$_5$, negated_conje

**SET857-2.p** Problem about Zorn's lemma
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_

(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zor
c_lessequals(v_Y, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_Y, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(v_x, v_n, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)
¬ c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_x, tc_set(tc_set(t_a))) cnf(cls_conjecture$_5$, negated_conjecture)
(c_lessequals(v_V, v_U, tc_set(tc_set(t_a))) and c_in(v_V, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_U, v_Y, tc_set(
(c_lessequals(c_Zorn_Osucc(v_S, v_V, t_a), v_U, tc_set(tc_set(t_a))) or v_V = v_U) cnf(cls_conjecture$_6$, negated_conjecture)

**SET858-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a)) cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zor
(c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequ
(c_lessequals(c_Zorn_Osucc(v_S, v_U, t_a), v_m, tc_set(tc_set(t_a))) or v_U = v_m) cnf(cls_Zorn_OTFin__linear__lemma2$_0$, ax
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(v_m, v_n, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)

**SET858-2.p** Problem about Zorn's lemma
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
¬ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(v_m, v_n, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a)) cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zor
(c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequ
(c_lessequals(c_Zorn_Osucc(v_S, v_U, t_a), v_m, tc_set(tc_set(t_a))) or v_U = v_m) cnf(cls_Zorn_OTFin__linear__lemma2$_0$, ax

**SET859-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a)) cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_in(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) or c_lessequals(v_n, v_m, tc_s
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a)))) ⇒
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_OTFin__linear__lemma1$_{-1}$(v_S, v_m, t_a), v_m, tc_set(tc_set(t_
(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_Zorn_O
v_m) ⇒ (c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cn

(c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequa
(c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) or c_lessequals(c_Zorn_Osucc(v_S, v_m, t_a), v_n, tc_set(tc_set(t_a)))) cnf(cls_Zor
(c_in(v_U, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequ
(c_lessequals(c_Zorn_Osucc(v_S, v_U, t_a), v_m, tc_set(tc_set(t_a))) or v_U = v_m) cnf(cls_Zorn_OTFin__linear__lemma2$_0$, a
c_in(v_m, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_n, c_Zorn_OTFin(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)
¬ c_lessequals(v_m, v_n, tc_set(tc_set(t_a))) cnf(cls_conjecture$_4$, negated_conjecture)

**SET859-2.p** Problem about Zorn's lemma
(c_lessequals(v_B, v_C, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_lessequals(v_A, v_C, tc_set(t_a)) cnf(cls_Se
c_lessequals(v_x, c_Zorn_Osucc(v_S, v_x, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OAbrial__axiom1$_0$, axiom)
c_lessequals(c_Zorn_Osucc(v_S, v_n, t_a), v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_2$, negated_conjecture)
¬ c_lessequals(v_n, v_m, tc_set(tc_set(t_a))) cnf(cls_conjecture$_3$, negated_conjecture)

**SET860-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
¬ c_lessequals(c_Union(v_C, t_a), v_A, tc_set(t_a)) cnf(cls_conjecture$_0$, negated_conjecture)
¬ c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_C, tc_set(t_a)) ⇒ (c_lessequals(v_B, v_U, tc_set(t_a)) or c_lessequals(v_U, v_A, tc_set(t_a))) cnf(cls_conjecture$_2$

**SET860-2.p** Problem about Zorn's lemma
¬ c_lessequals(c_Union(v_C, t_a), v_A, tc_set(t_a)) cnf(cls_conjecture$_0$, negated_conjecture)
¬ c_lessequals(v_B, c_Union(v_C, t_a), tc_set(t_a)) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_C, tc_set(t_a)) ⇒ (c_lessequals(v_B, v_U, tc_set(t_a)) or c_lessequals(v_U, v_A, tc_set(t_a))) cnf(cls_conjecture$_2$
c_in(v_c, v_A, t_a) ⇒ ¬ c_in(v_c, c_uminus(v_A, tc_set(t_a)), t_a) cnf(cls_Set_OComplD__dest$_0$, axiom)
c_in(v_c, v_A, t_a) or c_in(v_c, c_uminus(v_A, tc_set(t_a)), t_a) cnf(cls_Set_OComplI$_0$, axiom)
c_lessequals(c_uminus(v_A, tc_set(t_a)), c_uminus(v_B, tc_set(t_a)), tc_set(t_a)) ⇒ c_lessequals(v_B, v_A, tc_set(t_a)) cnf(cl
c_lessequals(v_B, v_A, tc_set(t_a)) ⇒ c_lessequals(c_uminus(v_A, tc_set(t_a)), c_uminus(v_B, tc_set(t_a)), tc_set(t_a)) cnf(cl
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OIntE$_0$, axiom)
c_in(v_c, c_inter(v_A, v_B, t_a), t_a) ⇒ c_in(v_c, v_A, t_a) cnf(cls_Set_OIntE$_1$, axiom)
(c_in(v_c, v_B, t_a) and c_in(v_c, v_A, t_a)) ⇒ c_in(v_c, c_inter(v_A, v_B, t_a), t_a) cnf(cls_Set_OIntI$_0$, axiom)
c_in(v_x, c_UNIV, t_a) cnf(cls_Set_OUNIV__I$_0$, axiom)
c_in(v_c, v_B, t_a) ⇒ c_in(v_c, c_union(v_A, v_B, t_a), t_a) cnf(cls_Set_OUnCI$_0$, axiom)
c_in(v_c, c_union(v_A, v_B, t_a), t_a) ⇒ (c_in(v_c, v_B, t_a) or c_in(v_c, v_A, t_a)) cnf(cls_Set_OUnE$_0$, axiom)
c_in(v_A, c_Union(v_C, t_a), t_a) ⇒ c_in(c_Main_OUnionE$_{-1}$(v_A, v_C, t_a), v_C, tc_set(t_a)) cnf(cls_Set_OUnionE$_0$, axiom)
c_in(v_A, c_Union(v_C, t_a), t_a) ⇒ c_in(v_A, c_Main_OUnionE$_{-1}$(v_A, v_C, t_a), t_a) cnf(cls_Set_OUnionE$_1$, axiom)
(c_in(v_A, v_X, t_a) and c_in(v_X, v_C, tc_set(t_a))) ⇒ c_in(v_A, c_Union(v_C, t_a), t_a) cnf(cls_Set_OUnionI$_0$, axiom)
¬ c_in(v_a, c_emptyset, t_a) cnf(cls_Set_OemptyE$_0$, axiom)
c_in(v_a, v_B, t_a) ⇒ c_in(v_a, c_insert(v_b, v_B, t_a), t_a) cnf(cls_Set_OinsertCI$_0$, axiom)
c_in(v_x, c_insert(v_x, v_B, t_a), t_a) cnf(cls_Set_OinsertCI$_1$, axiom)
c_in(v_a, c_insert(v_b, v_A, t_a), t_a) ⇒ (c_in(v_a, v_A, t_a) or v_a = v_b) cnf(cls_Set_OinsertE$_0$, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI$_{-1}$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B cnf(cls_Set_Osubset__antisym$_0$, a

**SET861-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OHausdorff$_{-1}$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend$_{-1}$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend$_{-1}$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_Zorn_Omaxchain__sub
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma$_{-1}$(v_c, v_y, t_a), v_c, tc_set(t_a))) cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a) cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)

c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))     cnf(cls_conjecture$_0$, negated_conje
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_xa(v_U), v_S, tc_set(t_a))     cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_xa(v_U), tc_set(t_a))     cnf(cls_conjecture$_2$, negated_conjecture)
v_U = v_xa(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))     cnf(cls_conjecture$_3$, negated_conjecture)
(c_in(v_V, v_U, tc_set(t_a)) and c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ c_lessequals(v_V, v_x(v_U), tc_set(t_

**SET861-2.p** Problem about Zorn's lemma
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B     cnf(cls_Set_Osubset__antisym$_0$, a
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)     cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a))     cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a))     cnf(cls_Set_OsubsetI$_1$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend_$_1$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend_$_1$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_Zorn
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_Zorn_OHausdorff$_0$, axiom)
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a))))     cnf(cls_Zorn_Omaxchain__sub
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma_$_1$(v_c, v_y, t_a), v_c, tc_set(t_a)))     cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a)     cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)
c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))     cnf(cls_conjecture$_0$, negated_conje
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_xa(v_U), v_S, tc_set(t_a))     cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_xa(v_U), tc_set(t_a))     cnf(cls_conjecture$_2$, negated_conjecture)
v_U = v_xa(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))     cnf(cls_conjecture$_3$, negated_conjecture)
(c_in(v_V, v_U, tc_set(t_a)) and c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ c_lessequals(v_V, v_x(v_U), tc_set(t_

**SET862-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend_$_1$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend_$_1$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_Zorn
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a))))     cnf(cls_Zorn_Omaxchain__sub
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma_$_1$(v_c, v_y, t_a), v_c, tc_set(t_a)))     cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a)     cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_y, v_S, tc_set(t_a))     cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_U, v_c, tc_set(t_a)) ⇒ c_lessequals(v_U, v_y, tc_set(t_a))     cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))     cnf(cls_conjecture$_4$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a))     cnf(cls_conjecture$_5$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))     cnf(cls_conjecture$_6$, negated_conjecture)

**SET862-2.p** Problem about Zorn's lemma
(c_lessequals(v_B, v_A, tc_set(t_a)) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ v_A = v_B     cnf(cls_Set_Osubset__antisym$_0$, a
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)     cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a))     cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a))     cnf(cls_Set_OsubsetI$_1$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend_$_1$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend_$_1$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_Zorn
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma_$_1$(v_c, v_y, t_a), v_c, tc_set(t_a)))     cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a)     cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))     cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_y, v_S, tc_set(t_a))     cnf(cls_conjecture$_2$, negated_conjecture)

c_in(v_U, v_c, tc_set(t_a)) ⇒ c_lessequals(v_U, v_y, tc_set(t_a)) cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a)) cnf(cls_conjecture$_4$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a)) cnf(cls_conjecture$_5$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a)) cnf(cls_conjecture$_6$, negated_conjecture)

**SET863-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend_$_1$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend_$_1$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_Zorn_Omaxchain__sub
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma_$_1$(v_c, v_y, t_a), v_c, tc_set(t_a))) cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a) cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_y, v_S, tc_set(t_a)) cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_x, v_S, tc_set(t_a)) cnf(cls_conjecture$_3$, negated_conjecture)
c_lessequals(v_y, v_x, tc_set(t_a)) cnf(cls_conjecture$_4$, negated_conjecture)
c_in(v_xa, v_x, t_a) cnf(cls_conjecture$_5$, negated_conjecture)
¬ c_in(v_xa, v_y, t_a) cnf(cls_conjecture$_6$, negated_conjecture)
c_in(v_U, v_c, tc_set(t_a)) ⇒ c_lessequals(v_U, v_y, tc_set(t_a)) cnf(cls_conjecture$_7$, negated_conjecture)

**SET863-2.p** Problem about Zorn's lemma
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_A, t_a) or c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_0$, axiom)
c_in(c_Main_OsubsetI_$_1$(v_A, v_B, t_a), v_B, t_a) ⇒ c_lessequals(v_A, v_B, tc_set(t_a)) cnf(cls_Set_OsubsetI$_1$, axiom)
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a)))) ⇒ (c_in(c_Zorn_Ochain__extend_$_1$(v_c, v_
(c_in(v_z, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Zorn_Ochain__extend_$_1$(
c_in(c_union(c_insert(v_z, c_emptyset, tc_set(t_a)), v_c, tc_set(t_a)), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
(c_in(v_z, v_y, t_a) or c_in(c_Zorn_Omaxchain__super__lemma_$_1$(v_c, v_y, t_a), v_c, tc_set(t_a))) cnf(cls_Zorn_Omaxchain__su
(c_in(v_z, v_x, t_a) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_in(c_union(c_insert(v_x, c_emptyset, tc_s
c_in(v_z, v_y, t_a) cnf(cls_Zorn_Omaxchain__super__lemma$_1$, axiom)
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_x, v_S, tc_set(t_a)) cnf(cls_conjecture$_3$, negated_conjecture)
c_lessequals(v_y, v_x, tc_set(t_a)) cnf(cls_conjecture$_4$, negated_conjecture)
c_in(v_xa, v_x, t_a) cnf(cls_conjecture$_5$, negated_conjecture)
¬ c_in(v_xa, v_y, t_a) cnf(cls_conjecture$_6$, negated_conjecture)
c_in(v_U, v_c, tc_set(t_a)) ⇒ c_lessequals(v_U, v_y, tc_set(t_a)) cnf(cls_conjecture$_7$, negated_conjecture)

**SET864-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_u, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Union(v_c, t_a), v_u
c_Union(v_c, t_a) = v_u cnf(cls_Zorn_Omaxchain__Zorn$_0$, axiom)
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a)))) cnf(cls_Zorn_Omaxchain__sub
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_0$, negated_conjecture)
c_in(v_c, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) cnf(cls_conjecture$_1$, negated_conjecture)
c_in(c_Union(v_c, t_a), v_S, tc_set(t_a)) cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a)) cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a)) cnf(cls_conjecture$_4$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a)) cnf(cls_conjecture$_5$, negated_conjecture)

**SET864-2.p** Problem about Zorn's lemma

(c_in(v_u, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Union(v_c, t_a), v_u
c_Union(v_c, t_a) = v_u    cnf(cls_Zorn_Omaxchain__Zorn$_0$, axiom)
c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_conjecture$_0$, negated_conjecture)
c_in(c_Union(v_c, t_a), v_S, tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))    cnf(cls_conjecture$_3$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a))    cnf(cls_conjecture$_4$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))    cnf(cls_conjecture$_5$, negated_conjecture)

**SET865-1.p** Problem about Zorn's lemma
include('Axioms/MSC001-2.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_u, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Union(v_c, t_a), v_u
c_Union(v_c, t_a) = v_u    cnf(cls_Zorn_Omaxchain__Zorn$_0$, axiom)
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a))))    cnf(cls_Zorn_Omaxchain__sub
c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(c_Union(v_U, t_a), v_S, tc_set(t_a))    cnf(cls_conjecture$_0$, negat
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))    cnf(cls_conjecture$_3$, negated_conjecture)

**SET865-2.p** Problem about Zorn's lemma
c_in(v_U, c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(t_a))) ⇒ c_in(c_Union(v_U, t_a), v_S, tc_set(t_a))    cnf(cls_conjecture$_0$, negat
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_in(v_x(v_U), v_S, tc_set(t_a))    cnf(cls_conjecture$_1$, negated_conjecture)
c_in(v_U, v_S, tc_set(t_a)) ⇒ c_lessequals(v_U, v_x(v_U), tc_set(t_a))    cnf(cls_conjecture$_2$, negated_conjecture)
v_U = v_x(v_U) ⇒ ¬ c_in(v_U, v_S, tc_set(t_a))    cnf(cls_conjecture$_3$, negated_conjecture)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a)    cnf(cls_Set_OsubsetD$_0$, axiom)
c_in(c_Zorn_OHausdorff_$_1$(v_S, t_a), c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a)))    cnf(cls_Zorn_OHausdorff$_0$, axiom)
(c_in(v_u, v_S, tc_set(t_a)) and c_in(v_c, c_Zorn_Omaxchain(v_S, t_a), tc_set(tc_set(t_a))) and c_lessequals(c_Union(v_c, t_a), v_u
c_Union(v_c, t_a) = v_u    cnf(cls_Zorn_Omaxchain__Zorn$_0$, axiom)
c_lessequals(c_Zorn_Omaxchain(v_S, t_a), c_Zorn_Ochain(v_S, t_a), tc_set(tc_set(tc_set(t_a))))    cnf(cls_Zorn_Omaxchain__sub

**SET867+1.p** union(empty_set) = empty_set
∀$a, b$: (in($a, b$) ⇒ ¬ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
∀$a$: ($a$ = empty_set ⟺ ∀$b$: ¬ in($b, a$))    fof(d1_xboole$_0$, axiom)
∀$a, b$: ($b$ = union($a$) ⟺ ∀$c$: (in($c, b$) ⟺ ∃$d$: (in($c, d$) and in($d, a$))))    fof(d4_tarski, axiom)
empty(empty_set)    fof(fc1_xboole$_0$, axiom)
∃$a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
∃$a$: ¬ empty($a$)    fof(rc2_xboole$_0$, axiom)
union(empty_set) = empty_set    fof(t2_zfmisc$_1$, conjecture)

**SET872+1.p** subset(singleton(A),unordered_pair(A,B))
∀$a, b$: (in($a, b$) ⇒ ¬ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
∀$a, b$: unordered_pair($a, b$) = unordered_pair($b, a$)    fof(commutativity_k2_tarski, axiom)
∀$a, b$: ($b$ = singleton($a$) ⟺ ∀$c$: (in($c, b$) ⟺ $c = a$))    fof(d1_tarski, axiom)
∀$a, b, c$: ($c$ = unordered_pair($a, b$) ⟺ ∀$d$: (in($d, c$) ⟺ ($d = a$ or $d = b$)))    fof(d2_tarski, axiom)
∀$a, b$: ($a \subseteq b$ ⟺ ∀$c$: (in($c, a$) ⇒ in($c, b$)))    fof(d3_tarski, axiom)
∃$a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
∃$a$: ¬ empty($a$)    fof(rc2_xboole$_0$, axiom)
∀$a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)
∀$a, b$: singleton($a$) ⊆ unordered_pair($a, b$)    fof(t12_zfmisc$_1$, conjecture)

**SET873+1.p** union(singleton(A),singleton(B)) = singleton(A) => A = B
∀$a, b$: (in($a, b$) ⇒ ¬ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
∀$a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$)    fof(commutativity_k2_xboole$_0$, axiom)
∀$a, b$: ($b$ = singleton($a$) ⟺ ∀$c$: (in($c, b$) ⟺ $c = a$))    fof(d1_tarski, axiom)
∀$a, b$: (¬ empty($a$) ⇒ ¬ empty(set_union$_2$($a, b$)))    fof(fc2_xboole$_0$, axiom)
∀$a, b$: (¬ empty($a$) ⇒ ¬ empty(set_union$_2$($b, a$)))    fof(fc3_xboole$_0$, axiom)
∀$a, b$: set_union$_2$($a, a$) = $a$    fof(idempotence_k2_xboole$_0$, axiom)
∀$a, b$: (set_union$_2$(singleton($a$), $b$) ⊆ $b$ ⇒ in($a, b$))    fof(l21_zfmisc$_1$, axiom)
∃$a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
∃$a$: ¬ empty($a$)    fof(rc2_xboole$_0$, axiom)
∀$a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)

$\forall a, b$: (set_union$_2$(singleton($a$), singleton($b$)) = singleton($a$) $\Rightarrow$ $a = b$) fof(t13_zfmisc$_1$, conjecture)

**SET874+1.p** union(singleton(A),unordered_pair(A,B)) = unordered_pair(A,B)
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = unordered_pair($a, b$) $\iff$ $\forall d$: (in($d, c$) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) = $a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: (in($a, b$) $\Rightarrow$ set_union$_2$(singleton($a$), $b$) = $b$) fof(l23_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(singleton($a$), unordered_pair($a, b$)) = unordered_pair($a, b$) fof(t14_zfmisc$_1$, conjecture)

**SET875+1.p** ( disjoint(singleton(A),singleton(B)) & A = B )
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b$ = singleton($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: $\neg$ disjoint(singleton($a$), $b$) and in($a, b$) fof(l25_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint($a, b$) $\Rightarrow$ disjoint($b, a$)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ disjoint(singleton($a$), singleton($b$)) and $a = b$ fof(t16_zfmisc$_1$, conjecture)

**SET876+1.p** A != B => disjoint(singleton(A),singleton(B))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b$ = singleton($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: ($\neg$ in($a, b$) $\Rightarrow$ disjoint(singleton($a$), $b$)) fof(l28_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint($a, b$) $\Rightarrow$ disjoint($b, a$)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: ($a \neq b$ $\Rightarrow$ disjoint(singleton($a$), singleton($b$))) fof(t17_zfmisc$_1$, conjecture)

**SET877+1.p** intersection(singleton(A),singleton(B)) = singleton(A) => A = B
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($b$ = singleton($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) = $a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: (set_intersection$_2$($a$, singleton($b$)) = singleton($b$) $\Rightarrow$ in($b, a$)) fof(l30_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_intersection$_2$(singleton($a$), singleton($b$)) = singleton($a$) $\Rightarrow$ $a = b$) fof(t18_zfmisc$_1$, conjecture)

**SET878+1.p** intersection(singleton(A),unordered_pair(A,B)) = singleton(A)
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = unordered_pair($a, b$) $\iff$ $\forall d$: (in($d, c$) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) = $a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: (in($a, b$) $\Rightarrow$ set_intersection$_2$($b$, singleton($a$)) = singleton($a$)) fof(l32_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(singleton($a$), unordered_pair($a, b$)) = singleton($a$) fof(t19_zfmisc$_1$, conjecture)

**SET879+1.p** difference(singleton(A),singleton(B)) = singleton(A) <=> A != B
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b$ = singleton($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: (set_difference(singleton($a$), $b$) = singleton($a$) $\iff$ $\neg$ in($a, b$)) fof(l34_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_difference(singleton($a$), singleton($b$)) = singleton($a$) $\iff$ $a \neq b$) fof(t20_zfmisc$_1$, conjecture)

**SET880+1.p** difference(singleton(A),singleton(B)) = empty_set => A = B

$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{set\_difference}(\mathrm{singleton}(a), b) = \mathrm{empty\_set} \iff \mathrm{in}(a, b))$ fof(l36_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{set\_difference}(\mathrm{singleton}(a), \mathrm{singleton}(b)) = \mathrm{empty\_set} \Rightarrow a = b)$ fof(t21_zfmisc$_1$, conjecture)

**SET881+1.p** difference(singleton(A),unordered_pair(A,B)) = empty_set
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{set\_difference}(\mathrm{singleton}(a), b) = \mathrm{empty\_set} \iff \mathrm{in}(a, b))$ fof(l36_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_difference}(\mathrm{singleton}(a), \mathrm{unordered\_pair}(a, b)) = \mathrm{empty\_set}$ fof(t22_zfmisc$_1$, conjecture)

**SET882+1.p** A != B => diff(unordered_pair(A,B),singleton(B)) = singleton(A)
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(\mathrm{set\_difference}(\mathrm{unordered\_pair}(a, b), c) = \mathrm{singleton}(a) \iff (\neg\,\mathrm{in}(a, c) \text{ and } (\mathrm{in}(b, c) \text{ or } a = b)))$ fof(l39_zfmisc$_1$, axi
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(a \neq b \Rightarrow \mathrm{set\_difference}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(b)) = \mathrm{singleton}(a))$ fof(t23_zfmisc$_1$, conjecture)

**SET883+1.p** subset(singleton(A),singleton(B)) => A = B
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{singleton}(a) \subseteq \mathrm{singleton}(b) \Rightarrow a = b)$ fof(t24_zfmisc$_1$, conjecture)
$\forall a, b$: $(\mathrm{singleton}(a) \subseteq \mathrm{singleton}(b) \Rightarrow a = b)$ fof(t6_zfmisc$_1$, axiom)

**SET884+1.p** ( subset(singleton(A),unordered_pair(B,C)) & A != B & A != C )
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: $\neg\,\mathrm{singleton}(a) \subseteq \mathrm{unordered\_pair}(b, c) \text{ and } a \neq b \text{ and } a \neq c$ fof(t25_zfmisc$_1$, conjecture)

**SET885+1.p** subset(unordered_pair(A,B),singleton(C)) => A = C
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: $(\mathrm{unordered\_pair}(a, b) \subseteq \mathrm{singleton}(c) \Rightarrow a = c)$ fof(t26_zfmisc$_1$, conjecture)

**SET886+1.p** subset(uno_pair(A,B),singleton(C)) => uno_pair(A,B) = singleton(C)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: $(\mathrm{unordered\_pair}(a, b) \subseteq \mathrm{singleton}(c) \Rightarrow a = c)$ fof(t26_zfmisc$_1$, axiom)
$\forall a, b, c$: $(\mathrm{unordered\_pair}(a, b) \subseteq \mathrm{singleton}(c) \Rightarrow \mathrm{unordered\_pair}(a, b) = \mathrm{singleton}(c))$ fof(t27_zfmisc$_1$, conjecture)

$\forall a$: unordered_pair$(a, a)$ = singleton$(a)$  fof(t69_enumset$_1$, axiom)

**SET887+1.p** ( subset(uno_pair(A,B),uno_pair(C,D)) & A != C & A != D )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$)  fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$  fof(commutativity_k2_tarski, axiom)
$\forall a$: ($a$ = empty_set $\iff \forall b$: $\neg$ in$(b, a)$)  fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b$)))  fof(d2_tarski, axiom)
empty(empty_set)  fof(fc1_xboole$_0$, axiom)
$\forall a, b, c$: ($a \subseteq$ unordered_pair$(b, c) \iff \neg\, a \neq$ empty_set and $a \neq$ singleton$(b)$ and $a \neq$ singleton$(c)$ and $a \neq$ unordered_pair$(b, c)$)  fof(l46_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$  fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$  fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$  fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: $\neg$ unordered_pair$(a, b)$ = unordered_pair$(c, d)$ and $a \neq c$ and $a \neq d$  fof(t10_zfmisc$_1$, axiom)
$\forall a, b, c, d$: $\neg$ unordered_pair$(a, b) \subseteq$ unordered_pair$(c, d)$ and $a \neq c$ and $a \neq d$  fof(t28_zfmisc$_1$, conjecture)
$\forall a, b, c$: (singleton$(a)$ = unordered_pair$(b, c) \Rightarrow a = b$)  fof(t8_zfmisc$_1$, axiom)

**SET888+1.p** Basic properties of sets, theorem 29
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$)  fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$  fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b)$ = set_union$_2(b, a)$  fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: symmetric_difference$(a, b)$ = symmetric_difference$(b, a)$  fof(commutativity_k5_xboole$_0$, axiom)
$\forall a, b$: ($b$ = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$))  fof(d1_tarski, axiom)
$\forall a, b, c$: ($c$ = unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b$)))  fof(d2_tarski, axiom)
$\forall a, b$: symmetric_difference$(a, b)$ = set_union$_2$(set_difference$(a, b)$, set_difference$(b, a)$)  fof(d6_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(a, b)$))  fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(b, a)$))  fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$  fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty$(a)$  fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$  fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (in$(a$, symmetric_difference$(b, c)) \iff \neg$ in$(a, b) \iff$ in$(a, c)$)  fof(t1_xboole$_0$, axiom)
$\forall a, b$: ($a \neq b \Rightarrow$ symmetric_difference(singleton$(a)$, singleton$(b)$) = unordered_pair$(a, b)$)  fof(t29_zfmisc$_1$, conjecture)

**SET889+1.p** powerset(singleton(A)) = unordered_pair(empty_set,singleton(A))
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$)  fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$  fof(commutativity_k2_tarski, axiom)
$\forall a, b$: ($b$ = powerset$(a) \iff \forall c$: (in$(c, b) \iff c \subseteq a$))  fof(d1_zfmisc$_1$, axiom)
$\forall a, b, c$: ($c$ = unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b$)))  fof(d2_tarski, axiom)
empty(empty_set)  fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq$ singleton$(b) \iff (a$ = empty_set or $a$ = singleton$(b)$))  fof(l4_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$  fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$  fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$  fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($\forall c$: (in$(c, a) \iff$ in$(c, b)$) $\Rightarrow a = b$)  fof(t2_tarski, axiom)
$\forall a$: powerset(singleton$(a)$) = unordered_pair(empty_set, singleton$(a)$)  fof(t30_zfmisc$_1$, conjecture)

**SET890+1.p** union(singleton(A)) = A
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$)  fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a = b \iff (a \subseteq b$ and $b \subseteq a$))  fof(d10_xboole$_0$, axiom)
$\forall a, b$: ($b$ = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$))  fof(d1_tarski, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in$(c, b)$))  fof(d3_tarski, axiom)
$\forall a, b$: ($b$ = union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$)))  fof(d4_tarski, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow a \subseteq$ union$(b)$)  fof(l50_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$  fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$  fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$  fof(reflexivity_r1_tarski, axiom)
$\forall a$: union(singleton$(a)$) = $a$  fof(t31_zfmisc$_1$, conjecture)

**SET891+1.p** union(uno_pair(singleton(A),singleton(B))) = uno_pair(A,B)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$  fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b)$ = set_union$_2(b, a)$  fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(a, b)$))  fof(fc2_xboole$_0$, axiom)

$\forall a, b$: $(\neg\,\text{empty}(a) \;\Rightarrow\; \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: $\text{union}(\text{unordered\_pair}(a, b)) = \text{set\_union}_2(a, b)$ fof(l52_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $\text{union}(\text{unordered\_pair}(\text{singleton}(a), \text{singleton}(b))) = \text{unordered\_pair}(a, b)$ fof(t32_zfmisc$_1$, conjecture)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{set\_union}_2(\text{singleton}(a), \text{singleton}(b))$ fof(t41_enumset$_1$, axiom)

**SET893+1.p** in(o_pair(A,B),cart_prod(sgtn(C),sgtn(D))) <=> ( A = C & B = D )
$\forall a, b$: $(\text{in}(a, b) \;\Rightarrow\; \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c\colon (\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c) \text{ and } \text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(\text{singleton}(c), \text{singleton}(d))) \iff (a = c \text{ and } b = d))$ fof(t34_zfmisc$_1$, c

**SET894+1.p** cart_prod(singleton(A),singleton(B)) = singleton(o_pair(A,B))
$\forall a, b$: $(\text{in}(a, b) \;\Rightarrow\; \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c\colon (\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \text{cartesian\_product}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff \exists e, f\colon (\text{in}(e, a) \text{ and } \text{in}(f, b) \text{ and } d = \text{ordered\_pair}(e, f))))$ fof
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c) \text{ and } \text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\forall c\colon (\text{in}(c, a) \iff \text{in}(c, b)) \;\Rightarrow\; a = b)$ fof(t2_tarski, axiom)
$\forall a, b$: $\text{cartesian\_product}_2(\text{singleton}(a), \text{singleton}(b)) = \text{singleton}(\text{ordered\_pair}(a, b))$ fof(t35_zfmisc$_1$, conjecture)

**SET895+1.p** Basic properties of sets, theorem 36
cartesian_product2(singleton(A),unordered_pair(B,C)) = unordered_pair(ordered_pair(A,B),ordered_pair(A,C)) & cartesian_product2(unordered_pair(A,B),singleton(C)) = unordered_pair(ordered_pair(A,C),ordered_pair(B,C))
$\forall a, b$: $(\text{in}(a, b) \;\Rightarrow\; \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c\colon (\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \text{unordered\_pair}(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c$: $(c = \text{cartesian\_product}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff \exists e, f\colon (\text{in}(e, a) \text{ and } \text{in}(f, b) \text{ and } d = \text{ordered\_pair}(e, f))))$ fof
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c) \text{ and } \text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\forall c\colon (\text{in}(c, a) \iff \text{in}(c, b)) \;\Rightarrow\; a = b)$ fof(t2_tarski, axiom)
$\forall a, b, c$: $(\text{cartesian\_product}_2(\text{singleton}(a), \text{unordered\_pair}(b, c)) = \text{unordered\_pair}(\text{ordered\_pair}(a, b), \text{ordered\_pair}(a, c)) \text{ and ca}$
$\text{unordered\_pair}(\text{ordered\_pair}(a, c), \text{ordered\_pair}(b, c)))$ fof(t36_zfmisc$_1$, conjecture)

**SET899+1.p** subset(A,B) => ( in(C,A) — subset(A,difference(B,singleton(C))) )
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\text{in}(a, b) \;\Rightarrow\; \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $(a \subseteq b \;\Rightarrow\; (\text{in}(c, a) \text{ or } a \subseteq \text{set\_difference}(b, \text{singleton}(c))))$ fof(t40_zfmisc$_1$, conjecture)
$\forall a, b, c$: $(a \subseteq b \;\Rightarrow\; (\text{in}(c, a) \text{ or } a \subseteq \text{set\_difference}(b, \text{singleton}(c))))$ fof(l3_zfmisc$_1$, axiom)

**SET900+1.p** ( A != singleton(B) & A != empty_set & ( in(C,A) & C != B ) )
$\forall a, b$: $(\text{in}(a, b) \;\Rightarrow\; \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)

$\forall a, b$: $\neg\, a \neq$ singleton($b$) and $a \neq$ empty_set and $\forall c$: $\neg$ in($c, a$) and $c \neq b$ fof(t41_zfmisc$_1$, conjecture)
$\forall a, b$: $\neg\, a \neq$ singleton($b$) and $a \neq$ empty_set and $\forall c$: $\neg$ in($c, a$) and $c \neq b$ fof(l45_zfmisc$_1$, axiom)

**SET901+1.p** Basic properties of sets, theorem 42
subset(A,unordered_pair(B,C)) <=> ( A != empty_set & A != singleton(B) & A != singleton(C) & A != unordered_pair(B,C) )
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: ($a \subseteq$ unordered_pair($b, c$) $\iff$ $\neg\, a \neq$ empty_set and $a \neq$ singleton($b$) and $a \neq$ singleton($c$) and $a \neq$ unordered_pair($b, c$)) fof(t42_zfmisc$_1$, conjecture)
$\forall a, b, c$: ($a \subseteq$ unordered_pair($b, c$) $\iff$ $\neg\, a \neq$ empty_set and $a \neq$ singleton($b$) and $a \neq$ singleton($c$) and $a \neq$ unordered_pair($b, c$)) fof(l46_zfmisc$_1$, axiom)

**SET902+1.p** Basic properties of sets, theorem 43
( singleton(A) = set_union2(B,C) & ( B = singleton(A) & C = singleton(A) ) & ( B = empty_set & C = singleton(A) ) & ( B = singleton(A) & C = empty_set ) )
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) = $a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a$: singleton($a$) $\neq$ empty_set fof(l1_zfmisc$_1$, axiom)
$\forall a, b$: ($a \subseteq$ singleton($b$) $\iff$ ($a$ = empty_set or $a$ = singleton($b$))) fof(l4_zfmisc$_1$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: $\neg$ singleton($a$) = set_union$_2$($b, c$) and $\neg\, b$ = singleton($a$) and $c$ = singleton($a$) and $\neg\, b$ = empty_set and $c$ = singleton($a$) and $\neg\, b$ = singleton($a$) and $c$ = empty_set fof(t43_zfmisc$_1$, conjecture)
$\forall a, b$: $a \subseteq$ set_union$_2$($a, b$) fof(t7_xboole$_1$, axiom)

**SET903+1.p** ( sgtn(A) = union(B,C) & B != C & B != empty & C != empty )
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) = $a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $\neg$ singleton($a$) = set_union$_2$($b, c$) and $\neg\, b$ = singleton($a$) and $c$ = singleton($a$) and $\neg\, b$ = empty_set and $c$ = singleton($a$) and $\neg\, b$ = singleton($a$) and $c$ = empty_set fof(t43_zfmisc$_1$, axiom)
$\forall a, b, c$: $\neg$ singleton($a$) = set_union$_2$($b, c$) and $b \neq c$ and $b \neq$ empty_set and $c \neq$ empty_set fof(t44_zfmisc$_1$, conjecture)

**SET904+1.p** subset(set_union2(singleton(A),B),B) => in(A,B)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) = $a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_union$_2$(singleton($a$), $b$) $\subseteq b$ $\Rightarrow$ in($a, b$)) fof(t45_zfmisc$_1$, conjecture)
$\forall a, b$: (set_union$_2$(singleton($a$), $b$) $\subseteq b$ $\Rightarrow$ in($a, b$)) fof(l21_zfmisc$_1$, axiom)

**SET906+1.p** subset(set_union2(unordered_pair(A,B),C),C) => in(A,C)
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = unordered_pair($a, b$) $\iff$ $\forall d$: (in($d, c$) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)

$\forall a, b, c$: $(c = \text{set\_union}_2(a, b) \iff \forall d\text{: } (\text{in}(d, c) \iff (\text{in}(d, a) \text{ or } \text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c\text{: } (\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: $(\text{set\_union}_2(\text{unordered\_pair}(a, b), c) \subseteq c \Rightarrow \text{in}(a, c))$ fof(t47_zfmisc$_1$, conjecture)

**SET907+1.p** ( in(A,B) & in(C,B) ) => set_union2(unordered_pair(A,C),B) = B
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $\text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(a \subseteq b \Rightarrow \text{set\_union}_2(a, b) = b)$ fof(t12_xboole$_1$, axiom)
$\forall a, b, c$: $(\text{unordered\_pair}(a, b) \subseteq c \iff (\text{in}(a, c) \text{ and } \text{in}(b, c)))$ fof(t38_zfmisc$_1$, axiom)
$\forall a, b, c$: $((\text{in}(a, b) \text{ and } \text{in}(c, b)) \Rightarrow \text{set\_union}_2(\text{unordered\_pair}(a, c), b) = b)$ fof(t48_zfmisc$_1$, conjecture)

**SET908+1.p** union(singleton(A),B) != empty_set
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c\text{: } (\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a$: $(a = \text{empty\_set} \iff \forall b\text{: } \neg\,\text{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: $(c = \text{set\_union}_2(a, b) \iff \forall d\text{: } (\text{in}(d, c) \iff (\text{in}(d, a) \text{ or } \text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(\text{singleton}(a), b) \neq \text{empty\_set}$ fof(t49_zfmisc$_1$, conjecture)

**SET909+1.p** union(unordered_pair(A,B),C) != empty_set
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $\text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: $(a = \text{empty\_set} \iff \forall b\text{: } \neg\,\text{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: $(c = \text{unordered\_pair}(a, b) \iff \forall d\text{: } (\text{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c$: $(c = \text{set\_union}_2(a, b) \iff \forall d\text{: } (\text{in}(d, c) \iff (\text{in}(d, a) \text{ or } \text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $\text{set\_union}_2(\text{unordered\_pair}(a, b), c) \neq \text{empty\_set}$ fof(t50_zfmisc$_1$, conjecture)

**SET910+1.p** intersection(A,singleton(B)) = singleton(B) => in(B,A)
$\forall a, b$: $\text{set\_intersection}_2(a, b) = \text{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $\text{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\text{set\_intersection}_2(a, \text{singleton}(b)) = \text{singleton}(b) \Rightarrow \text{in}(b, a))$ fof(t51_zfmisc$_1$, conjecture)
$\forall a, b$: $(\text{set\_intersection}_2(a, \text{singleton}(b)) = \text{singleton}(b) \Rightarrow \text{in}(b, a))$ fof(l30_zfmisc$_1$, axiom)

**SET911+1.p** in(A,B) => set_intersection2(B,singleton(A)) = singleton(A)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ set_intersection$_2(b,$ singleton$(a)) =$ singleton$(a)$) fof(t52_zfmisc$_1$, conjecture)
$\forall a, b$: (in$(a, b) \Rightarrow$ set_intersection$_2(b,$ singleton$(a)) =$ singleton$(a)$) fof(l32_zfmisc$_1$, axiom)

**SET912+1.p** ( in(A,B) & in(C,B) ) => intsctn(uno_pair(A,C),B) = uno_pair(A,C)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($a \subseteq b \Rightarrow$ set_intersection$_2(a, b) = a$) fof(t28_xboole$_1$, axiom)
$\forall a, b, c$: (unordered_pair$(a, b) \subseteq c \iff$ (in$(a, c)$ and in$(b, c)$)) fof(t38_zfmisc$_1$, axiom)
$\forall a, b, c$: ((in$(a, b)$ and in$(c, b)$) $\Rightarrow$ set_intersection$_2($unordered_pair$(a, c), b) =$ unordered_pair$(a, c)$) fof(t53_zfmisc$_1$, conject

**SET913+1.p** ( disjoint(singleton(A),B) & in(A,B) )
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $\neg$ disjoint(singleton$(a), b)$ and in$(a, b)$ fof(t54_zfmisc$_1$, conjecture)
$\forall a, b$: $\neg$ disjoint(singleton$(a), b)$ and in$(a, b)$ fof(l25_zfmisc$_1$, axiom)

**SET914+1.p** ( disjoint(unordered_pair(A,B),C) & in(A,C) )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: ($a =$ empty_set $\iff \forall b$: $\neg$ in$(b, a)$) fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b)$)) fof(d2_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ and in$(d, b)$))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: (disjoint$(a, b) \iff$ set_intersection$_2(a, b) =$ empty_set) fof(d7_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: $\neg$ disjoint(unordered_pair$(a, b), c)$ and in$(a, c)$ fof(t55_zfmisc$_1$, conjecture)

**SET915+1.p** in(A,B) => disjoint(singleton(A),B)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ in$(a, b) \Rightarrow$ disjoint(singleton$(a), b)$) fof(t56_zfmisc$_1$, conjecture)
$\forall a, b$: ($\neg$ in$(a, b) \Rightarrow$ disjoint(singleton$(a), b)$) fof(l28_zfmisc$_1$, axiom)

**SET916+1.p** ( in(A,B) & in(C,B) & disjoint(unordered_pair(A,C),B) )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b)$)) fof(d2_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ and in$(d, b)$))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)

$\forall a, b$: $(\neg\neg \mathrm{disjoint}(a, b)$ and $\forall c$: $\neg \mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\neg \exists c$: $\mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\mathrm{disjoint}(a, b))$ fof(
$\forall a, b, c$: $\neg\neg \mathrm{in}(a, b)$ and $\neg \mathrm{in}(c, b)$ and $\neg \mathrm{disjoint}(\mathrm{unordered\_pair}(a, c), b)$ fof(t57_zfmisc$_1$, conjecture)

**SET917+1.p** disjoint(sgtn(A),B) — intersection(sgtn(A),B) = sgtn(A)
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: $(\neg \mathrm{in}(a, b) \Rightarrow \mathrm{disjoint}(\mathrm{singleton}(a), b))$ fof(l28_zfmisc$_1$, axiom)
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \mathrm{set\_intersection}_2(b, \mathrm{singleton}(a)) = \mathrm{singleton}(a))$ fof(l32_zfmisc$_1$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{disjoint}(\mathrm{singleton}(a), b)$ or $\mathrm{set\_intersection}_2(\mathrm{singleton}(a), b) = \mathrm{singleton}(a))$ fof(t58_zfmisc$_1$, conjecture)

**SET918+1.p** ( intersection(uno_pair(A,B),C) = sgtn(A) & in(B,C) & A != B )
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a$ or $d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_intersection}_2(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $\neg \mathrm{set\_intersection}_2(\mathrm{unordered\_pair}(a, b), c) = \mathrm{singleton}(a)$ and $\mathrm{in}(b, c)$ and $a \neq b$ fof(t59_zfmisc$_1$, conjecture)

**SET919+1.p** in(A,B) => ((in(C,B) & A!=C) — intsctn(uno_pair(A,C),B) = sgtn(A))
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a$ or $d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_intersection}_2(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $(\mathrm{in}(a, b) \Rightarrow ((\mathrm{in}(c, b)$ and $a \neq c)$ or $\mathrm{set\_intersection}_2(\mathrm{unordered\_pair}(a, c), b) = \mathrm{singleton}(a)))$ fof(t60_zfmisc$_1$, conj

**SET920+1.p** intersection(uno_pair(A,B),C) = uno_pair(A,B) => in(A,C)
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c$: $(c = \mathrm{unordered\_pair}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (d = a$ or $d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_intersection}_2(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $(\mathrm{set\_intersection}_2(\mathrm{unordered\_pair}(a, b), c) = \mathrm{unordered\_pair}(a, b) \Rightarrow \mathrm{in}(a, c))$ fof(t63_zfmisc$_1$, conjecture)

**SET921+1.p** in(A,difference(B,singleton(C))) <=> ( in(A,B) & A != C )
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $(b = \mathrm{singleton}(a) \iff \forall c$: $(\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_difference}(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\neg \mathrm{in}(d, b))))$ fof(d4_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $(\mathrm{in}(a, \mathrm{set\_difference}(b, \mathrm{singleton}(c))) \iff (\mathrm{in}(a, b)$ and $a \neq c))$ fof(t64_zfmisc$_1$, conjecture)

**SET923+1.p** ( difference(A,sgtn(B)) = empty & A != empty & A != sgtn(B) )
$\mathrm{empty}(\mathrm{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq \mathrm{singleton}(b) \iff (a = \mathrm{empty\_set}$ or $a = \mathrm{singleton}(b)))$ fof(l4_zfmisc$_1$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

$\forall a, b$: (set_difference$(a, b)$ = empty_set $\iff a \subseteq b$) fof(t37_xboole$_1$, axiom)
$\forall a, b$: $\neg$ set_difference$(a$, singleton$(b))$ = empty_set and $a \neq$ empty_set and $a \neq$ singleton$(b)$ fof(t66_zfmisc$_1$, conjecture)

**SET924+1.p** difference(singleton(A),B) = singleton(A) <=> in(A,B)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_difference(singleton$(a), b)$ = singleton$(a) \iff \neg$ in$(a, b)$) fof(t67_zfmisc$_1$, conjecture)
$\forall a, b$: (set_difference(singleton$(a), b)$ = singleton$(a) \iff \neg$ in$(a, b)$) fof(l34_zfmisc$_1$, axiom)

**SET925+1.p** difference(singleton(A),B) = empty_set <=> in(A,B)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_difference(singleton$(a), b)$ = empty_set $\iff$ in$(a, b)$) fof(t68_zfmisc$_1$, conjecture)
$\forall a, b$: (set_difference(singleton$(a), b)$ = empty_set $\iff$ in$(a, b)$) fof(l36_zfmisc$_1$, axiom)

**SET926+1.p** difference(sgtn(A),B) = empty — difference(sgtn(A),B) = sgtn(A)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (set_difference(singleton$(a), b)$ = singleton$(a) \iff \neg$ in$(a, b)$) fof(l34_zfmisc$_1$, axiom)
$\forall a, b$: (set_difference(singleton$(a), b)$ = empty_set $\iff$ in$(a, b)$) fof(l36_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (set_difference(singleton$(a), b)$ = empty_set or set_difference(singleton$(a), b)$ = singleton$(a)$) fof(t69_zfmisc$_1$, conjectu

**SET927+1.p** diff(uno_pair(A,B),C) = sgtn(A) <=> ( in(A,C) & (in(B,C) — A = B))
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (set_difference(unordered_pair$(a, b), c)$ = singleton$(a) \iff (\neg$ in$(a, c)$ and (in$(b, c)$ or $a = b$))) fof(t70_zfmisc$_1$, con
$\forall a, b, c$: (set_difference(unordered_pair$(a, b), c)$ = singleton$(a) \iff (\neg$ in$(a, c)$ and (in$(b, c)$ or $a = b$))) fof(l39_zfmisc$_1$, axi

**SET928+1.p** diff(uno_pair(A,B),C) = uno_pair(A,B) <=> ( in(A,C) & in(B,C))
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: $\neg$ disjoint(unordered_pair$(a, b), c)$ and in$(a, c)$ fof(t55_zfmisc$_1$, axiom)
$\forall a, b, c$: $\neg \neg$ in$(a, b)$ and $\neg$ in$(c, b)$ and $\neg$ disjoint(unordered_pair$(a, c), b)$ fof(t57_zfmisc$_1$, axiom)
$\forall a, b, c$: (set_difference(unordered_pair$(a, b), c)$ = unordered_pair$(a, b) \iff (\neg$ in$(a, c)$ and $\neg$ in$(b, c)$)) fof(t72_zfmisc$_1$, con
$\forall a, b$: (disjoint$(a, b) \iff$ set_difference$(a, b) = a$) fof(t83_xboole$_1$, axiom)

**SET929+1.p** diff(uno_pair(A,B),C) = empty <=> ( in(A,C) & in(B,C) )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (set_difference$(a, b)$ = empty_set $\iff a \subseteq b$) fof(t37_xboole$_1$, axiom)
$\forall a, b, c$: (unordered_pair$(a, b) \subseteq c \iff$ (in$(a, c)$ and in$(b, c)$)) fof(t38_zfmisc$_1$, axiom)
$\forall a, b, c$: (set_difference(unordered_pair$(a, b), c)$ = empty_set $\iff$ (in$(a, c)$ and in$(b, c)$)) fof(t73_zfmisc$_1$, conjecture)

**SET930+1.p** Basic properties of sets, theorem 74
( difference(unordered_pair(A,B),C) != empty_set & difference(unordered_pair(A,B),C) != singleton(A) & difference(unordered_pair(A,B),C) != singleton(B) & difference(unordered_pair(A,B),C) != unordered_pair(A,B) )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)

$\forall a, b, c$: (set_difference(unordered_pair($a, b$), $c$) = singleton($a$) $\iff$ ($\neg$ in($a, c$) and (in($b, c$) or $a = b$)))   fof(l39_zfmisc$_1$, axi
$\exists a$: empty($a$)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)   fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (set_difference(unordered_pair($a, b$), $c$) = unordered_pair($a, b$) $\iff$ ($\neg$ in($a, c$) and $\neg$ in($b, c$)))   fof(t72_zfmisc$_1$, axio
$\forall a, b, c$: (set_difference(unordered_pair($a, b$), $c$) = empty_set $\iff$ (in($a, c$) and in($b, c$)))   fof(t73_zfmisc$_1$, axiom)
$\forall a, b, c$: $\neg$ set_difference(unordered_pair($a, b$), $c$) $\neq$ empty_set and set_difference(unordered_pair($a, b$), $c$) $\neq$ singleton($a$) and set
singleton($b$) and set_difference(unordered_pair($a, b$), $c$) $\neq$ unordered_pair($a, b$)   fof(t74_zfmisc$_1$, conjecture)

**SET931+1.p** Basic properties of sets, theorem 75
difference(A,unordered_pair(B,C)) = empty_set <=> ( A != empty_set & A != singleton(B) & A != singleton(C) & A != unordered_pair(B,C) )
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$)   fof(commutativity_k2_tarski, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b, c$: ($a \subseteq$ unordered_pair($b, c$) $\iff$ $\neg\, a \neq$ empty_set and $a \neq$ singleton($b$) and $a \neq$ singleton($c$) and $a \neq$ unordered_pair($b, c$))   fof(l46_zfmisc$_1$, axiom)
$\exists a$: empty($a$)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$   fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (set_difference($a, b$) = empty_set $\iff$ $a \subseteq b$)   fof(t37_xboole$_1$, axiom)
$\forall a, b, c$: (set_difference($a$, unordered_pair($b, c$)) = empty_set $\iff$ $\neg\, a \neq$ empty_set and $a \neq$ singleton($b$) and $a \neq$ singleton($c$) and $a \neq$ unordered_pair($b, c$))   fof(t75_zfmisc$_1$, conjecture)

**SET932+1.p** subset(A,B) => subset(powerset(A),powerset(B))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c \subseteq a$))   fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in($c, a$) $\Rightarrow$ in($c, b$)))   fof(d3_tarski, axiom)
$\exists a$: empty($a$)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$   fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$)   fof(t1_xboole$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ powerset($a$) $\subseteq$ powerset($b$))   fof(t79_zfmisc$_1$, conjecture)

**SET933+1.p** subset(singleton(A),powerset(A))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c \subseteq a$))   fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: (singleton($a$) $\subseteq b$ $\iff$ in($a, b$))   fof(l2_zfmisc$_1$, axiom)
$\exists a$: empty($a$)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$   fof(reflexivity_r1_tarski, axiom)
$\forall a$: singleton($a$) $\subseteq$ powerset($a$)   fof(t80_zfmisc$_1$, conjecture)

**SET934+1.p** subset(union(powerset(A),powerset(B)),powerset(union(A,B)))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$)   fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c \subseteq a$))   fof(d1_zfmisc$_1$, axiom)
$\forall a, b, c$: ($c$ = set_union$_2$($a, b$) $\iff$ $\forall d$: (in($d, c$) $\iff$ (in($d, a$) or in($d, b$))))   fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in($c, a$) $\Rightarrow$ in($c, b$)))   fof(d3_tarski, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$)))   fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$)))   fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) = $a$   fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty($a$)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$   fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$)   fof(t1_xboole$_1$, axiom)
$\forall a, b$: $a \subseteq$ set_union$_2$($a, b$)   fof(t7_xboole$_1$, axiom)
$\forall a, b$: set_union$_2$(powerset($a$), powerset($b$)) $\subseteq$ powerset(set_union$_2$($a, b$))   fof(t81_zfmisc$_1$, conjecture)

**SET935+1.p** union(powset(A),powset(B)) = powset(union(A,B)) => inc_comp(A,B)
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$)   fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: ($a = b$ $\iff$ ($a \subseteq b$ and $b \subseteq a$))   fof(d10_xboole$_0$, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff$ $\forall c$: (in($c, b$) $\iff$ $c \subseteq a$))   fof(d1_zfmisc$_1$, axiom)

$\forall a, b, c$: ($c = \mathrm{set\_union}_2(a, b) \iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) or in($d, b$)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: (inclusion_comparable($a, b$) $\iff$ ($a \subseteq b$ or $b \subseteq a$)) fof(d9_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: inclusion_comparable($a, a$) fof(reflexivity_r3_xboole$_0$, axiom)
$\forall a, b$: (inclusion_comparable($a, b$) $\Rightarrow$ inclusion_comparable($b, a$)) fof(symmetry_r3_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq$ set_union$_2$($a, b$) fof(t7_xboole$_1$, axiom)
$\forall a, b$: (set_union$_2$(powerset($a$), powerset($b$)) = powerset(set_union$_2$($a, b$)) $\Rightarrow$ inclusion_comparable($a, b$)) fof(t82_zfmisc$_1$, c

**SET936+1.p** powset(intersection(A,B)) = intersection(powset(A),powset(B))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff \forall c$: (in($c, b$) $\iff c \subseteq a$)) fof(d1_zfmisc$_1$, axiom)
$\forall a, b, c$: ($c$ = set_intersection$_2$($a, b$) $\iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) and in($d, b$)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) $= a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) $\subseteq a$ fof(t17_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $a \subseteq c$) $\Rightarrow$ $a \subseteq$ set_intersection$_2$($b, c$)) fof(t19_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(t1_xboole$_1$, axiom)
$\forall a, b$: ($\forall c$: (in($c, a$) $\iff$ in($c, b$)) $\Rightarrow$ $a = b$) fof(t2_tarski, axiom)
$\forall a, b$: powerset(set_intersection$_2$($a, b$)) = set_intersection$_2$(powerset($a$), powerset($b$)) fof(t83_zfmisc$_1$, conjecture)

**SET937+1.p** subset(pset(diff(A,B)),union(sgtn(empty),diff(pset(A),pset(B))))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($b$ = singleton($a$) $\iff \forall c$: (in($c, b$) $\iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff \forall c$: (in($c, b$) $\iff c \subseteq a$)) fof(d1_zfmisc$_1$, axiom)
$\forall a, b, c$: ($c$ = set_union$_2$($a, b$) $\iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) or in($d, b$)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in($c, b$))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c$ = set_difference($a, b$) $\iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) and $\neg$ in($d, b$)))) fof(d4_xboole$_0$, axiom)
$\forall a, b$: (disjoint($a, b$) $\iff$ set_intersection$_2$($a, b$) = empty_set) fof(d7_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($a, b$))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty($a$) $\Rightarrow$ $\neg$ empty(set_union$_2$($b, a$))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$($a, a$) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) $= a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty($a$) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint($a, b$) $\Rightarrow$ disjoint($b, a$)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(t1_xboole$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ set_intersection$_2$($a, b$) $= a$) fof(t28_xboole$_1$, axiom)
$\forall a, b$: set_difference($a, b$) $\subseteq a$ fof(t36_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and disjoint($b, c$)) $\Rightarrow$ disjoint($a, c$)) fof(t63_xboole$_1$, axiom)
$\forall a, b$: disjoint(set_difference($a, b$), $b$) fof(t79_xboole$_1$, axiom)
$\forall a, b$: powerset(set_difference($a, b$)) $\subseteq$ set_union$_2$(singleton(empty_set), set_difference(powerset($a$), powerset($b$))) fof(t84_zf

**SET938+1.p** subset(union(pset(diff(A,B)),pset(diff(B,A))),pset(symdiff(A,B)))
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: symmetric_difference($a, b$) = symmetric_difference($b, a$) fof(commutativity_k5_xboole$_0$, axiom)
$\forall a, b$: ($b$ = powerset($a$) $\iff \forall c$: (in($c, b$) $\iff c \subseteq a$)) fof(d1_zfmisc$_1$, axiom)
$\forall a, b, c$: ($c$ = set_union$_2$($a, b$) $\iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) or in($d, b$)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in($c, b$))) fof(d3_tarski, axiom)

$\forall a, b$: symmetric_difference$(a, b)$ = set_union$_2$(set_difference$(a, b)$, set_difference$(b, a)$) fof(d6_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: set_union$_2$(powerset(set_difference$(a, b)$), powerset(set_difference$(b, a)$)) $\subseteq$ powerset(symmetric_difference$(a, b)$) fof(t

**SET940+1.p** union(unordered_pair(A,B)) = union(A,B)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: unordered_pair$(a, b)$ = unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b)$ = set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: union(unordered_pair$(a, b)$) = set_union$_2(a, b)$ fof(t93_zfmisc$_1$, conjecture)
$\forall a, b$: union(unordered_pair$(a, b)$) = set_union$_2(a, b)$ fof(l52_zfmisc$_1$, axiom)

**SET941+1.p** ( in(C,A) => subset(C,B) ) => subset(union(A),B)
$\forall a, b$: (in$(a, b)$ $\Rightarrow$ $\neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a)$ $\Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b$: ($b$ = union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($\forall c$: (in$(c, a)$ $\Rightarrow$ $c \subseteq b$) $\Rightarrow$ union$(a) \subseteq b$) fof(t94_zfmisc$_1$, conjecture)

**SET942+1.p** subset(A,B) => subset(union(A),union(B))
$\forall a, b$: (in$(a, b)$ $\Rightarrow$ $\neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a)$ $\Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b$: ($b$ = union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ union$(a) \subseteq$ union$(b)$) fof(t95_zfmisc$_1$, conjecture)

**SET943+1.p** union(union(A,B)) = union(union(A),union(B))
$\forall a, b$: (in$(a, b)$ $\Rightarrow$ $\neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2(a, b)$ = set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: ($a = b \iff (a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = set_union$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ or in$(d, b)$))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a)$ $\Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b$: ($b$ = union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a)$ $\Rightarrow$ $\neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $a \subseteq$ set_union$_2(a, b)$ fof(t7_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $c \subseteq b$) $\Rightarrow$ set_union$_2(a, c) \subseteq b$) fof(t8_xboole$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ union$(a) \subseteq$ union$(b)$) fof(t95_zfmisc$_1$, axiom)
$\forall a, b$: union(set_union$_2(a, b)$) = set_union$_2$(union$(a)$, union$(b)$) fof(t96_zfmisc$_1$, conjecture)

**SET944+1.p** subset(union(intersection(A,B)),intersection(union(A),union(B)))
$\forall a, b$: (in$(a, b)$ $\Rightarrow$ $\neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2(a, b)$ = set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a)$ $\Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c$ = set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ and in$(d, b)$))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: ($b$ = union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)

$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: union(set_intersection$_2(a, b)) \subseteq$ set_intersection$_2$(union$(a)$, union$(b)$) fof(t97_zfmisc$_1$, conjecture)

**SET945+1.p** ( in(C,A) => disjoint(C,B) ) => disjoint(union(A),B)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ and in$(d, b)$))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: ($b =$ union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: ($\neg\neg$ disjoint$(a, b)$ and $\forall c$: $\neg$ in($c$, set_intersection$_2(a, b)$) and $\neg \exists c$: in($c$, set_intersection$_2(a, b)$) and disjoint$(a, b)$) fof(
$\forall a, b$: ($\forall c$: (in$(c, a) \Rightarrow$ disjoint$(c, b)$) $\Rightarrow$ disjoint(union$(a)$, $b$)) fof(t98_zfmisc$_1$, conjecture)

**SET947+1.p** subset(A,powerset(union(A)))
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($b =$ powerset$(a) \iff \forall c$: (in$(c, b) \iff c \subseteq a$)) fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow a \subseteq$ union$(b)$) fof(l50_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: $a \subseteq$ powerset(union$(a)$) fof(t100_zfmisc$_1$, conjecture)

**SET948+1.p** Basic properties of sets, theorem 101
((in(C,union(A,B)) & in(D,union(A,B)) ) => (C=D — disjoint(C,D)) ) => union(intersection(A,B)) = intersection(union(A),union(B))
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b \iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ set_union$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ or in$(d, b)$))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in$(c, b)$)) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in$(d, a)$ and in$(d, b)$))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: ($b =$ union$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: (in$(c, d)$ and in$(d, a)$))) fof(d4_tarski, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint$(b, a)$) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: ($\forall c, d$: ((in($c$, set_union$_2(a, b)$) and in($d$, set_union$_2(a, b)$)) $\Rightarrow$ ($c = d$ or disjoint$(c, d)$)) $\Rightarrow$ union(set_intersection$_2(a, b)$)
set_intersection$_2$(union$(a)$, union$(b)$)) fof(t101_zfmisc$_1$, conjecture)
$\forall a, b$: ($\neg\neg$ disjoint$(a, b)$ and $\forall c$: $\neg$ in($c$, set_intersection$_2(a, b)$) and $\neg \exists c$: in($c$, set_intersection$_2(a, b)$) and disjoint$(a, b)$) fof(
$\forall a, b$: union(set_intersection$_2(a, b)) \subseteq$ set_intersection$_2$(union$(a)$, union$(b)$) fof(t97_zfmisc$_1$, axiom)

**SET949+1.p** ( in(A,cartesian_product(B,C)) & ordered_pair(D,E) != A )
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in$(b, a)$) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair$(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: ($c =$ cartesian_product$_2(a, b) \iff \forall d$: (in$(d, c) \iff \exists e, f$: (in$(e, a)$ and in$(f, b)$ and $d =$ ordered_pair$(e, f)$))) fof
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair$(a, b)$, singleton$(a)$) fof(d5_tarski, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair$(a, b)$) fof(fc1_zfmisc$_1$, axiom)
$\exists a$: empty$(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty$(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: $\neg$ in($a$, cartesian_product$_2(b, c)$) and $\forall d, e$: ordered_pair$(d, e) \neq a$ fof(t102_zfmisc$_1$, conjecture)

**SET950+1.p** Basic properties of sets, theorem 103
( subset(A,cart_prod(B,C)) & in(D,A) & ( in(E,B) & in(F,C) & D = ordered_pair(E,F) ) )
∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
∀a, b, c: (c = cartesian_product$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ ∃e, f: (in(e, a) and in(f, b) and d = ordered_pair(e, f)))) fof
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b))) fof(d3_tarski, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
∃a: empty(a) fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a fof(reflexivity_r1_tarski, axiom)
∀a, b, c, d: ¬ a ⊆ cartesian_product$_2$(b, c) and in(d, a) and ∀e, f: ¬ in(e, b) and in(f, c) and d = ordered_pair(e, f) fof(t103_

**SET951+1.p** Basic properties of sets, theorem 104
( in(A,intersection(cart_product(B,C),cart_product(D,E))) & ( A = ordered_pair(F,G) & in(F,intersection(B,D)) & in(G,set_intersection2(C,E)) ) )
∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
∀a, b, c: (c = cartesian_product$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ ∃e, f: (in(e, a) and in(f, b) and d = ordered_pair(e, f)))) fof
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
∀a, b: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a) fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
∀a, b, c, d, e: ¬ in(a, set_intersection$_2$(cartesian_product$_2$(b, c), cartesian_product$_2$(d, e))) and ∀f, g: ¬ a = ordered_pair(f, g) an
∀a, b, c, d: (ordered_pair(a, b) = ordered_pair(c, d) ⇒ (a = c and b = d)) fof(t33_zfmisc$_1$, axiom)

**SET952+1.p** subset(cartesian_product(A,B),powerset(powerset(union(A,B))))
∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
∀a, b: set_union$_2$(a, b) = set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
∀a, b: (b = powerset(a) ⟺ ∀c: (in(c, b) ⟺ c ⊆ a)) fof(d1_zfmisc$_1$, axiom)
∀a, b, c: (c = unordered_pair(a, b) ⟺ ∀d: (in(d, c) ⟺ (d = a or d = b))) fof(d2_tarski, axiom)
∀a, b, c: (c = set_union$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
∀a, b, c: (c = cartesian_product$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ ∃e, f: (in(e, a) and in(f, b) and d = ordered_pair(e, f)))) fof
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b))) fof(d3_tarski, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
∀a, b: set_union$_2$(a, a) = a fof(idempotence_k2_xboole$_0$, axiom)
∀a, b: (singleton(a) ⊆ b ⟺ in(a, b)) fof(l2_zfmisc$_1$, axiom)
∃a: empty(a) fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a fof(reflexivity_r1_tarski, axiom)
∀a, b: cartesian_product$_2$(a, b) ⊆ powerset(powerset(set_union$_2$(a, b))) fof(t105_zfmisc$_1$, conjecture)
∀a, b, c: ((a ⊆ b and b ⊆ c) ⇒ a ⊆ c) fof(t1_xboole$_1$, axiom)
∀a, b, c: (unordered_pair(a, b) ⊆ c ⟺ (in(a, c) and in(b, c))) fof(t38_zfmisc$_1$, axiom)
∀a, b: a ⊆ set_union$_2$(a, b) fof(t7_xboole$_1$, axiom)

**SET954+1.p** in(o_pair(A,B),cart_prod(C,D)) => in(o_pair(B,A),cart_prod(D,C))
∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
∀a, b, c, d: (in(ordered_pair(a, b), cartesian_product$_2$(c, d)) ⟺ (in(a, c) and in(b, d))) fof(l55_zfmisc$_1$, axiom)
∃a: empty(a) fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
∀a, b, c, d: (in(ordered_pair(a, b), cartesian_product$_2$(c, d)) ⇒ in(ordered_pair(b, a), cartesian_product$_2$(d, c))) fof(t107_zfm

**SET955+1.p** Basic properties of sets, theorem 108
( in(ordered_pair(E,F),cartesian_product2(A,B)) <=> in(ordered_pair(E,F),cartesian_product2(C,D)) ) => cartesian_product2(
= cartesian_product2(C,D)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b, c\colon (c = \mathrm{cartesian\_product}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff \exists e, f\colon (\mathrm{in}(e, a) \text{ and } \mathrm{in}(f, b) \text{ and } d = \mathrm{ordered\_pair}(e, f))))$ fof
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c, d\colon (\forall e, f\colon (\mathrm{in}(\mathrm{ordered\_pair}(e, f), \mathrm{cartesian\_product}_2(a, b)) \iff \mathrm{in}(\mathrm{ordered\_pair}(e, f), \mathrm{cartesian\_product}_2(c, d))) \Rightarrow$
$\mathrm{cartesian\_product}_2(a, b) = \mathrm{cartesian\_product}_2(c, d))$ fof(t108_zfmisc$_1$, conjecture)
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ fof(t2_tarski, axiom)

**SET956+1.p** Basic properties of sets, theorem 109
( subset(A,cartesian_product2(B,C)) & ( in(ordered_pair(E,F),A) => in(ordered_pair(E,F),D) ) ) => subset(A,D)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d\colon \neg\, a \subseteq \mathrm{cartesian\_product}_2(b, c) \text{ and } \mathrm{in}(d, a) \text{ and } \forall e, f\colon \neg\,\mathrm{in}(e, b) \text{ and } \mathrm{in}(f, c) \text{ and } d = \mathrm{ordered\_pair}(e, f)$ fof(t103_
$\forall a, b, c, d\colon ((a \subseteq \mathrm{cartesian\_product}_2(b, c) \text{ and } \forall e, f\colon (\mathrm{in}(\mathrm{ordered\_pair}(e, f), a) \Rightarrow \mathrm{in}(\mathrm{ordered\_pair}(e, f), d))) \Rightarrow a \subseteq$
$d)$ fof(t109_zfmisc$_1$, conjecture)

**SET957+1.p** Basic properties of sets, theorem 110
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d\colon \neg\, a \subseteq \mathrm{cartesian\_product}_2(b, c) \text{ and } \mathrm{in}(d, a) \text{ and } \forall e, f\colon \neg\,\mathrm{in}(e, b) \text{ and } \mathrm{in}(f, c) \text{ and } d = \mathrm{ordered\_pair}(e, f)$ fof(t103_
$\forall a, b, c, d, e, f\colon ((a \subseteq \mathrm{cartesian\_product}_2(b, c) \text{ and } d \subseteq \mathrm{cartesian\_product}_2(e, f) \text{ and } \forall g, h\colon (\mathrm{in}(\mathrm{ordered\_pair}(g, h), a) \iff$
$\mathrm{in}(\mathrm{ordered\_pair}(g, h), d))) \Rightarrow a = d)$ fof(t110_zfmisc$_1$, conjecture)
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ fof(t2_tarski, axiom)

**SET958+1.p** Basic properties of sets, theorem 111
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon ((\forall c\colon \neg\,\mathrm{in}(c, a) \text{ and } \forall d, e\colon c \neq \mathrm{ordered\_pair}(d, e) \text{ and } \forall c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(c, d), a) \Rightarrow \mathrm{in}(\mathrm{ordered\_pair}(c, d), b))) \Rightarrow$
$a \subseteq b)$ fof(t111_zfmisc$_1$, conjecture)

**SET959+1.p** Basic properties of sets, theorem 112
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)

$\forall a, b\colon ((\forall c\colon \neg\, \mathrm{in}(c, a)$ and $\forall d, e\colon c \neq \mathrm{ordered\_pair}(d, e)$ and $\forall c\colon \neg\, \mathrm{in}(c, b)$ and $\forall d, e\colon c \neq \mathrm{ordered\_pair}(d, e)$ and $\forall c, d\colon (\mathrm{in}(\mathrm{ordered}$
$\mathrm{in}(\mathrm{ordered\_pair}(c, d), b))) \Rightarrow a = b)$ $\quad$ $\mathrm{fof}(\mathrm{t112\_zfmisc}_1, \mathrm{conjecture})$
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ $\quad$ $\mathrm{fof}(\mathrm{t2\_tarski}, \mathrm{axiom})$

**SET960+1.p** cart_prod(A,B) = empty <=> ( A = empty — B = empty )
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ $\quad$ $\mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a\colon (a = \mathrm{empty\_set} \iff \forall b\colon \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{d1\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c\colon (c = \mathrm{cartesian\_product}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff \exists e, f\colon (\mathrm{in}(e, a)$ and $\mathrm{in}(f, b)$ and $d = \mathrm{ordered\_pair}(e, f))))$ $\quad$ fof
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ $\quad$ $\mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set})$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon \neg\, \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{cartesian\_product}_2(a, b) = \mathrm{empty\_set} \iff (a = \mathrm{empty\_set}$ or $b = \mathrm{empty\_set}))$ $\quad$ $\mathrm{fof}(\mathrm{t113\_zfmisc}_1, \mathrm{conjecture})$

**SET961+1.p** cart_prod(A,B) = cart_prod(B,A) => ( A=empty — B=empty — A = B )
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ $\quad$ $\mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a\colon (a = \mathrm{empty\_set} \iff \forall b\colon \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{d1\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ $\quad$ $\mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set})$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c)$ and $\mathrm{in}(b, d)))$ $\quad$ $\mathrm{fof}(\mathrm{l55\_zfmisc}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon \neg\, \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{cartesian\_product}_2(a, b) = \mathrm{cartesian\_product}_2(b, a) \Rightarrow (a = \mathrm{empty\_set}$ or $b = \mathrm{empty\_set}$ or $a = b))$ $\quad$ fof(t114_zfmisc
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ $\quad$ $\mathrm{fof}(\mathrm{t2\_tarski}, \mathrm{axiom})$

**SET962+1.p** cartesian_product(A,A) = cartesian_product(B,B) => A = B
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ $\quad$ $\mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ $\quad$ $\mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c)$ and $\mathrm{in}(b, d)))$ $\quad$ $\mathrm{fof}(\mathrm{l55\_zfmisc}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon \neg\, \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{cartesian\_product}_2(a, a) = \mathrm{cartesian\_product}_2(b, b) \Rightarrow a = b)$ $\quad$ $\mathrm{fof}(\mathrm{t115\_zfmisc}_1, \mathrm{conjecture})$
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ $\quad$ $\mathrm{fof}(\mathrm{t2\_tarski}, \mathrm{axiom})$

**SET963+1.p** Basic properties of sets, theorem 116
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ $\quad$ $\mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ $\quad$ $\mathrm{fof}(\mathrm{commutativity\_k2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (b = \mathrm{singleton}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff c = a))$ $\quad$ $\mathrm{fof}(\mathrm{d1\_tarski}, \mathrm{axiom})$
$\forall a\colon (a = \mathrm{empty\_set} \iff \forall b\colon \neg\, \mathrm{in}(b, a))$ $\quad$ $\mathrm{fof}(\mathrm{d1\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c\colon (c = \mathrm{unordered\_pair}(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (d = a$ or $d = b)))$ $\quad$ $\mathrm{fof}(\mathrm{d2\_tarski}, \mathrm{axiom})$
$\forall a, b, c\colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ $\quad$ $\mathrm{fof}(\mathrm{d2\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c\colon (c = \mathrm{cartesian\_product}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff \exists e, f\colon (\mathrm{in}(e, a)$ and $\mathrm{in}(f, b)$ and $d = \mathrm{ordered\_pair}(e, f))))$ $\quad$ fof
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ $\quad$ $\mathrm{fof}(\mathrm{d3\_tarski}, \mathrm{axiom})$
$\forall a, b\colon (b = \mathrm{union}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff \exists d\colon (\mathrm{in}(c, d)$ and $\mathrm{in}(d, a))))$ $\quad$ $\mathrm{fof}(\mathrm{d4\_tarski}, \mathrm{axiom})$
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ $\quad$ $\mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set})$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ $\quad$ $\mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ $\quad$ $\mathrm{fof}(\mathrm{fc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ $\quad$ $\mathrm{fof}(\mathrm{fc3\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ $\quad$ $\mathrm{fof}(\mathrm{idempotence\_k2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ $\quad$ $\mathrm{fof}(\mathrm{l50\_zfmisc}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon \neg\, \mathrm{empty}(a)$ $\quad$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon a \subseteq a$ $\quad$ $\mathrm{fof}(\mathrm{reflexivity\_r1\_tarski}, \mathrm{axiom})$

$\forall a, b, c\colon \neg\, \mathrm{in}(a, \mathrm{cartesian\_product}_2(b, c))$ and $\forall d, e\colon \mathrm{ordered\_pair}(d, e) \neq a$ fof(t102_zfmisc$_1$, axiom)
$\forall a\colon (a \subseteq \mathrm{cartesian\_product}_2(a, a) \Rightarrow a = \mathrm{empty\_set})$ fof(t116_zfmisc$_1$, conjecture)
$\forall a, b\colon \neg\, \mathrm{in}(a, b)$ and $\forall c\colon \neg\, \mathrm{in}(c, b)$ and $\forall d\colon \neg\, \mathrm{in}(d, b)$ and $\mathrm{in}(d, c)$ fof(t7_tarski, axiom)

**SET964+1.p** Basic properties of sets, theorem 117
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a\colon (a = \mathrm{empty\_set} \iff \forall b\colon \neg\, \mathrm{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{cartesian\_product}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff \exists e, f\colon (\mathrm{in}(e, a)$ and $\mathrm{in}(f, b)$ and $d = \mathrm{ordered\_pair}(e, f))))$ fof
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\mathrm{empty}(\mathrm{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c)$ and $\mathrm{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon \neg\, a \neq \mathrm{empty\_set}$ and $(\mathrm{cartesian\_product}_2(b, a) \subseteq \mathrm{cartesian\_product}_2(c, a)$ or $\mathrm{cartesian\_product}_2(a, b) \subseteq$ cartesian_prod
$c$ fof(t117_zfmisc$_1$, conjecture)

**SET967+1.p** Basic properties of sets, theorem 120
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c)$ and $\mathrm{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c\colon \neg\, \mathrm{in}(a, \mathrm{cartesian\_product}_2(b, c))$ and $\forall d, e\colon \mathrm{ordered\_pair}(d, e) \neq a$ fof(t102_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \Rightarrow \mathrm{in}(\mathrm{ordered\_pair}(b, a), \mathrm{cartesian\_product}_2(d, c)))$ fof(t107_zfm
$\forall a, b\colon ((\forall c\colon \neg\, \mathrm{in}(c, a)$ and $\forall d, e\colon c \neq \mathrm{ordered\_pair}(d, e)$ and $\forall c\colon \neg\, \mathrm{in}(c, b)$ and $\forall d, e\colon c \neq \mathrm{ordered\_pair}(d, e)$ and $\forall c, d\colon$ (in(ordered
$\mathrm{in}(\mathrm{ordered\_pair}(c, d), b))) \Rightarrow a = b)$ fof(t112_zfmisc$_1$, axiom)
$\forall a, b, c\colon (\mathrm{cartesian\_product}_2(\mathrm{set\_union}_2(a, b), c) = \mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(a, c), \mathrm{cartesian\_product}_2(b, c))$ and cartesian_
$\mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(c, a), \mathrm{cartesian\_product}_2(c, b)))$ fof(t120_zfmisc$_1$, conjecture)

**SET968+1.p** Basic properties of sets, theorem 121
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c\colon (\mathrm{cartesian\_product}_2(\mathrm{set\_union}_2(a, b), c) = \mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(a, c), \mathrm{cartesian\_product}_2(b, c))$ and cartesian_
$\mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(c, a), \mathrm{cartesian\_product}_2(c, b)))$ fof(t120_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon \mathrm{cartesian\_product}_2(\mathrm{set\_union}_2(a, b), \mathrm{set\_union}_2(c, d)) = \mathrm{set\_union}_2(\mathrm{set\_union}_2(\mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(a, c)$, c
$\forall a, b, c\colon \mathrm{set\_union}_2(\mathrm{set\_union}_2(a, b), c) = \mathrm{set\_union}_2(a, \mathrm{set\_union}_2(b, c))$ fof(t4_xboole$_1$, axiom)

**SET969+1.p** Basic properties of sets, theorem 122
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_intersection}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b, c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c)$ and $\mathrm{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)

$\exists a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)    fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: (in(ordered_pair($a, b$), cartesian_product$_2$($c, d$)) $\Rightarrow$ in(ordered_pair($b, a$), cartesian_product$_2$($d, c$)))    fof(t107_zfm
$\forall a, b, c, d, e, f$: (($a \subseteq$ cartesian_product$_2$($b, c$) and $d \subseteq$ cartesian_product$_2$($e, f$) and $\forall g, h$: (in(ordered_pair($g, h$), $a$) $\iff$ in(ordered_pair($g, h$), $d$))) $\Rightarrow$ $a = d$)    fof(t110_zfmisc$_1$, axiom)
$\forall a, b, c$: (cartesian_product$_2$(set_intersection$_2$($a, b$), $c$) = set_intersection$_2$(cartesian_product$_2$($a, c$), cartesian_product$_2$($b, c$)) an set_intersection$_2$(cartesian_product$_2$($c, a$), cartesian_product$_2$($c, b$)))    fof(t122_zfmisc$_1$, conjecture)
$\forall a, b$: set_intersection$_2$($a, b$) $\subseteq a$    fof(t17_xboole$_1$, axiom)

**SET970+1.p** Basic properties of sets, theorem 123
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$)    fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$)    fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b \iff (a \subseteq b$ and $b \subseteq a$))    fof(d10_xboole$_0$, axiom)
$\forall a, b, c$: ($c$ = cartesian_product$_2$($a, b$) $\iff \forall d$: (in($d, c$) $\iff \exists e, f$: (in($e, a$) and in($f, b$) and $d$ = ordered_pair($e, f$))))    fof
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in($c, b$)))    fof(d3_tarski, axiom)
$\forall a, b$: ordered_pair($a, b$) = unordered_pair(unordered_pair($a, b$), singleton($a$))    fof(d5_tarski, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair($a, b$))    fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) = $a$    fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)    fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d, e$: $\neg$ in($a$, set_intersection$_2$(cartesian_product$_2$($b, c$), cartesian_product$_2$($d, e$))) and $\forall f, g$: $\neg\, a$ = ordered_pair($f, g$) and
$\forall a, b, c, d$: (($a \subseteq b$ and $c \subseteq d$) $\Rightarrow$ cartesian_product$_2$($a, c$) $\subseteq$ cartesian_product$_2$($b, d$))    fof(t119_zfmisc$_1$, axiom)
$\forall a, b, c, d$: cartesian_product$_2$(set_intersection$_2$($a, b$), set_intersection$_2$($c, d$)) = set_intersection$_2$(cartesian_product$_2$($a, c$), cartes
$\forall a, b$: set_intersection$_2$($a, b$) $\subseteq a$    fof(t17_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $a \subseteq c$) $\Rightarrow$ $a \subseteq$ set_intersection$_2$($b, c$))    fof(t19_xboole$_1$, axiom)

**SET971+1.p** Basic properties of sets, theorem 124
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$)    fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$($a, a$) = $a$    fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)    fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: cartesian_product$_2$(set_intersection$_2$($a, b$), set_intersection$_2$($c, d$)) = set_intersection$_2$(cartesian_product$_2$($a, c$), cartes
$\forall a, b, c, d$: (($a \subseteq b$ and $c \subseteq d$) $\Rightarrow$ set_intersection$_2$(cartesian_product$_2$($a, d$), cartesian_product$_2$($b, c$)) = cartesian_product$_2$($a$, $c$
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ set_intersection$_2$($a, b$) = $a$)    fof(t28_xboole$_1$, axiom)

**SET972+1.p** Basic properties of sets, theorem 125
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$)    fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: ($c$ = set_difference($a, b$) $\iff \forall d$: (in($d, c$) $\iff$ (in($d, a$) and $\neg$ in($d, b$))))    fof(d4_xboole$_0$, axiom)
$\forall a, b$: ordered_pair($a, b$) = unordered_pair(unordered_pair($a, b$), singleton($a$))    fof(d5_tarski, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair($a, b$))    fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d$: (in(ordered_pair($a, b$), cartesian_product$_2$($c, d$)) $\iff$ (in($a, c$) and in($b, d$)))    fof(l55_zfmisc$_1$, axiom)
$\exists a$: empty($a$)    fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty($a$)    fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$    fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: (in(ordered_pair($a, b$), cartesian_product$_2$($c, d$)) $\Rightarrow$ in(ordered_pair($b, a$), cartesian_product$_2$($d, c$)))    fof(t107_zfm
$\forall a, b, c, d, e, f$: (($a \subseteq$ cartesian_product$_2$($b, c$) and $d \subseteq$ cartesian_product$_2$($e, f$) and $\forall g, h$: (in(ordered_pair($g, h$), $a$) $\iff$ in(ordered_pair($g, h$), $d$))) $\Rightarrow$ $a = d$)    fof(t110_zfmisc$_1$, axiom)
$\forall a, b, c$: (cartesian_product$_2$(set_difference($a, b$), $c$) = set_difference(cartesian_product$_2$($a, c$), cartesian_product$_2$($b, c$)) and cart set_difference(cartesian_product$_2$($c, a$), cartesian_product$_2$($c, b$)))    fof(t125_zfmisc$_1$, conjecture)
$\forall a, b$: set_difference($a, b$) $\subseteq a$    fof(t36_xboole$_1$, axiom)

**SET973+1.p** Basic properties of sets, theorem 126
$\forall a, b$: (in($a, b$) $\Rightarrow$ $\neg$ in($b, a$))    fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair($a, b$) = unordered_pair($b, a$)    fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2$($a, b$) = set_union$_2$($b, a$)    fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$($a, b$) = set_intersection$_2$($b, a$)    fof(commutativity_k3_xboole$_0$, axiom)

$\forall a, b \colon (a = b \iff (a \subseteq b \text{ and } b \subseteq a)) \quad \mathrm{fof}(\mathrm{d10\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c \colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d \colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a) \text{ or } \mathrm{in}(d, b)))) \quad \mathrm{fof}(\mathrm{d2\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c \colon (c = \mathrm{cartesian\_product}_2(a, b) \iff \forall d \colon (\mathrm{in}(d, c) \iff \exists e, f \colon (\mathrm{in}(e, a) \text{ and } \mathrm{in}(f, b) \text{ and } d = \mathrm{ordered\_pair}(e, f)))) \quad \mathrm{fof}$
$\forall a, b \colon (a \subseteq b \iff \forall c \colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b))) \quad \mathrm{fof}(\mathrm{d3\_tarski}, \mathrm{axiom})$
$\forall a, b, c \colon (c = \mathrm{set\_difference}(a, b) \iff \forall d \colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a) \text{ and } \neg\, \mathrm{in}(d, b)))) \quad \mathrm{fof}(\mathrm{d4\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a)) \quad \mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b)) \quad \mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b \colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(a, b))) \quad \mathrm{fof}(\mathrm{fc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon (\neg\, \mathrm{empty}(a) \Rightarrow \neg\, \mathrm{empty}(\mathrm{set\_union}_2(b, a))) \quad \mathrm{fof}(\mathrm{fc3\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon \mathrm{set\_union}_2(a, a) = a \quad \mathrm{fof}(\mathrm{idempotence\_k2\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon \mathrm{set\_intersection}_2(a, a) = a \quad \mathrm{fof}(\mathrm{idempotence\_k3\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c, d \colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c) \text{ and } \mathrm{in}(b, d))) \quad \mathrm{fof}(\mathrm{l55\_zfmisc}_1, \mathrm{axiom})$
$\exists a \colon \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a \colon \neg\, \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon a \subseteq a \quad \mathrm{fof}(\mathrm{reflexivity\_r1\_tarski}, \mathrm{axiom})$
$\forall a, b, c, d \colon ((a \subseteq b \text{ and } c \subseteq d) \Rightarrow \mathrm{cartesian\_product}_2(a, c) \subseteq \mathrm{cartesian\_product}_2(b, d)) \quad \mathrm{fof}(\mathrm{t119\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d \colon \mathrm{cartesian\_product}_2(\mathrm{set\_intersection}_2(a, b), \mathrm{set\_intersection}_2(c, d)) = \mathrm{set\_intersection}_2(\mathrm{cartesian\_product}_2(a, c), \mathrm{cartes}$
$\forall a, b, c \colon (\mathrm{cartesian\_product}_2(\mathrm{set\_difference}(a, b), c) = \mathrm{set\_difference}(\mathrm{cartesian\_product}_2(a, c), \mathrm{cartesian\_product}_2(b, c)) \text{ and } \mathrm{cart}$
$\mathrm{set\_difference}(\mathrm{cartesian\_product}_2(c, a), \mathrm{cartesian\_product}_2(c, b))) \quad \mathrm{fof}(\mathrm{t125\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d \colon \mathrm{set\_difference}(\mathrm{cartesian\_product}_2(a, b), \mathrm{cartesian\_product}_2(c, d)) = \mathrm{set\_union}_2(\mathrm{cartesian\_product}_2(\mathrm{set\_difference}(a, c$
$\forall a, b \colon \mathrm{set\_intersection}_2(a, b) \subseteq a \quad \mathrm{fof}(\mathrm{t17\_xboole}_1, \mathrm{axiom})$
$\forall a, b, c \colon (a \subseteq b \Rightarrow \mathrm{set\_difference}(c, b) \subseteq \mathrm{set\_difference}(c, a)) \quad \mathrm{fof}(\mathrm{t34\_xboole}_1, \mathrm{axiom})$
$\forall a, b, c \colon \mathrm{set\_difference}(a, \mathrm{set\_intersection}_2(b, c)) = \mathrm{set\_union}_2(\mathrm{set\_difference}(a, b), \mathrm{set\_difference}(a, c)) \quad \mathrm{fof}(\mathrm{t54\_xboole}_1, \mathrm{axiom}$

**SET974+1.p** Basic properties of sets, theorem 127
$\forall a, b \colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a)) \quad \mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a) \quad \mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a) \quad \mathrm{fof}(\mathrm{commutativity\_k3\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a)) \quad \mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b)) \quad \mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b \colon \mathrm{set\_intersection}_2(a, a) = a \quad \mathrm{fof}(\mathrm{idempotence\_k3\_xboole}_0, \mathrm{axiom})$
$\exists a \colon \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a \colon \neg\, \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b \colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a)) \quad \mathrm{fof}(\mathrm{symmetry\_r1\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c, d, e \colon \neg\, \mathrm{in}(a, \mathrm{set\_intersection}_2(\mathrm{cartesian\_product}_2(b, c), \mathrm{cartesian\_product}_2(d, e))) \text{ and } \forall f, g \colon \neg\, a = \mathrm{ordered\_pair}(f, g) \text{ an}$
$\forall a, b, c, d \colon ((\mathrm{disjoint}(a, b) \text{ or } \mathrm{disjoint}(c, d)) \Rightarrow \mathrm{disjoint}(\mathrm{cartesian\_product}_2(a, c), \mathrm{cartesian\_product}_2(b, d))) \quad \mathrm{fof}(\mathrm{t127\_zfmisc}_1$
$\forall a, b \colon (\neg\, \neg\, \mathrm{disjoint}(a, b) \text{ and } \forall c \colon \neg\, \mathrm{in}(c, \mathrm{set\_intersection}_2(a, b)) \text{ and } \neg\, \exists c \colon \mathrm{in}(c, \mathrm{set\_intersection}_2(a, b)) \text{ and } \mathrm{disjoint}(a, b)) \quad \mathrm{fof}($

**SET975+1.p** in(o_pair(A,B),cart_prod(sgtn(C),D)) <=> ( A = C & in(B,D) )
$\forall a, b \colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a)) \quad \mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a) \quad \mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a, b \colon (b = \mathrm{singleton}(a) \iff \forall c \colon (\mathrm{in}(c, b) \iff c = a)) \quad \mathrm{fof}(\mathrm{d1\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a)) \quad \mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b)) \quad \mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d \colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c) \text{ and } \mathrm{in}(b, d))) \quad \mathrm{fof}(\mathrm{l55\_zfmisc}_1, \mathrm{axiom})$
$\exists a \colon \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a \colon \neg\, \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c, d \colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(\mathrm{singleton}(c), d)) \iff (a = c \text{ and } \mathrm{in}(b, d))) \quad \mathrm{fof}(\mathrm{t128\_zfmisc}_1, \mathrm{conjectur}$

**SET976+1.p** in(o_pair(A,B),cart_prod(C,sgtn(D))) <=> ( in(A,C) & B = D )
$\forall a, b \colon (\mathrm{in}(a, b) \Rightarrow \neg\, \mathrm{in}(b, a)) \quad \mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a) \quad \mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a, b \colon (b = \mathrm{singleton}(a) \iff \forall c \colon (\mathrm{in}(c, b) \iff c = a)) \quad \mathrm{fof}(\mathrm{d1\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a)) \quad \mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$\forall a, b \colon \neg\, \mathrm{empty}(\mathrm{ordered\_pair}(a, b)) \quad \mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a, b, c, d \colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, d)) \iff (\mathrm{in}(a, c) \text{ and } \mathrm{in}(b, d))) \quad \mathrm{fof}(\mathrm{l55\_zfmisc}_1, \mathrm{axiom})$
$\exists a \colon \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a \colon \neg\, \mathrm{empty}(a) \quad \mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b, c, d \colon (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{cartesian\_product}_2(c, \mathrm{singleton}(d))) \iff (\mathrm{in}(a, c) \text{ and } b = d)) \quad \mathrm{fof}(\mathrm{t129\_zfmisc}_1, \mathrm{conjectur}$

**SET977+1.p** Basic properties of sets, theorem 130

$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a\colon (a = \text{empty\_set} \iff \forall b\colon \neg\,\text{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b\colon \text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c, d\colon (\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(\text{singleton}(c), d)) \iff (a = c \text{ and } \text{in}(b, d)))$ fof(t128_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, \text{singleton}(d))) \iff (\text{in}(a, c) \text{ and } b = d))$ fof(t129_zfmisc$_1$, axiom)
$\forall a, b\colon (a \neq \text{empty\_set} \Rightarrow (\text{cartesian\_product}_2(\text{singleton}(b), a) \neq \text{empty\_set and cartesian\_product}_2(a, \text{singleton}(b)) \neq \text{empty\_set}))$ fof(t130_zfmisc$_1$, conjecture)

**SET978+1.p** Basic properties of sets, theorem 131
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\text{disjoint}(a, b) \Rightarrow \text{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c, d\colon ((\text{disjoint}(a, b) \text{ or } \text{disjoint}(c, d)) \Rightarrow \text{disjoint}(\text{cartesian\_product}_2(a, c), \text{cartesian\_product}_2(b, d)))$ fof(t127_zfmisc$_1$
$\forall a, b, c, d\colon (a \neq b \Rightarrow (\text{disjoint}(\text{cartesian\_product}_2(\text{singleton}(a), c), \text{cartesian\_product}_2(\text{singleton}(b), d)) \text{ and disjoint(cartesian\_p}$
$\forall a, b\colon (a \neq b \Rightarrow \text{disjoint}(\text{singleton}(a), \text{singleton}(b)))$ fof(t17_zfmisc$_1$, axiom)

**SET979+1.p** Basic properties of sets, theorem 132
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b, c\colon (\text{cartesian\_product}_2(\text{set\_union}_2(a, b), c) = \text{set\_union}_2(\text{cartesian\_product}_2(a, c), \text{cartesian\_product}_2(b, c)) \text{ and cartesian\_}$
$\text{set\_union}_2(\text{cartesian\_product}_2(c, a), \text{cartesian\_product}_2(c, b)))$ fof(t120_zfmisc$_1$, axiom)
$\forall a, b, c\colon (\text{cartesian\_product}_2(\text{unordered\_pair}(a, b), c) = \text{set\_union}_2(\text{cartesian\_product}_2(\text{singleton}(a), c), \text{cartesian\_product}_2(\text{singl}$
$\text{set\_union}_2(\text{cartesian\_product}_2(c, \text{singleton}(a)), \text{cartesian\_product}_2(c, \text{singleton}(b))))$ fof(t132_zfmisc$_1$, conjecture)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{set\_union}_2(\text{singleton}(a), \text{singleton}(b))$ fof(t41_enumset$_1$, axiom)

**SET980+1.p** Basic properties of sets, theorem 134
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a\colon (a = \text{empty\_set} \iff \forall b\colon \neg\,\text{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b\colon \text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c) \text{ and } \text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\text{cartesian\_product}_2(a, b) = \text{empty\_set} \iff (a = \text{empty\_set} \text{ or } b = \text{empty\_set}))$ fof(t113_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\text{cartesian\_product}_2(a, b) = \text{cartesian\_product}_2(c, d) \Rightarrow (a = \text{empty\_set} \text{ or } b = \text{empty\_set} \text{ or } (a = c \text{ and } b = d)))$ fof(t134_zfmisc$_1$, conjecture)
$\forall a, b\colon (\forall c\colon (\text{in}(c, a) \iff \text{in}(c, b)) \Rightarrow a = b)$ fof(t2_tarski, axiom)

**SET981+1.p** Basic properties of sets, theorem 135
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon (b = \text{singleton}(a) \iff \forall c\colon (\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a\colon (a = \text{empty\_set} \iff \forall b\colon \neg\,\text{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \text{unordered\_pair}(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (d = a \text{ or } d = b)))$ fof(d2_tarski, axiom)
$\forall a, b, c\colon (c = \text{set\_union}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a) \text{ or } \text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \text{cartesian\_product}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff \exists e, f\colon (\text{in}(e, a) \text{ and } \text{in}(f, b) \text{ and } d = \text{ordered\_pair}(e, f))))$ fof
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon (b = \text{union}(a) \iff \forall c\colon (\text{in}(c, b) \iff \exists d\colon (\text{in}(c, d) \text{ and } \text{in}(d, a))))$ fof(d4_tarski, axiom)

∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))      fof(d5_tarski, axiom)
empty(empty_set)      fof(fc1_xboole$_0$, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))      fof(fc1_zfmisc$_1$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(a, b)))      fof(fc2_xboole$_0$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(b, a)))      fof(fc3_xboole$_0$, axiom)
∀a, b: set_union$_2$(a, a) = a      fof(idempotence_k2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ a ⊆ union(b))      fof(l50_zfmisc$_1$, axiom)
∃a: empty(a)      fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)      fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a      fof(reflexivity_r1_tarski, axiom)
∀a: ∃b: ∀c: (in(c, b) ⟺ (in(c, union(a)) and ∃d: (c = d and ∃e: (in(e, d) and in(e, a)))))      fof(s1_xboole_0__e2_121_2__zfmis
∀a, b, c: ¬ in(a, cartesian_product$_2$(b, c)) and ∀d, e: ordered_pair(d, e) ≠ a      fof(t102_zfmisc$_1$, axiom)
∀a, b: ((a ⊆ cartesian_product$_2$(a, b) or a ⊆ cartesian_product$_2$(b, a)) ⇒ a = empty_set)      fof(t135_zfmisc$_1$, conjecture)
∀a, b: (set_union$_2$(a, b) = empty_set ⇒ a = empty_set)      fof(t15_xboole$_1$, axiom)
∀a, b: ¬ in(a, b) and ∀c: ¬ in(c, b) and ∀d: ¬ in(d, b) and in(d, c)      fof(t7_tarski, axiom)

**SET983+1.p** Basic properties of sets, theorem 137
∀a, b: (in(a, b) ⇒ ¬ in(b, a))      fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)      fof(commutativity_k3_xboole$_0$, axiom)
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b))))      fof(d3_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a      fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)      fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)      fof(rc2_xboole$_0$, axiom)
∀a, b, c, d: cartesian_product$_2$(set_intersection$_2$(a, b), set_intersection$_2$(c, d)) = set_intersection$_2$(cartesian_product$_2$(a, c), cartes
∀a, b, c, d, e: ((in(a, cartesian_product$_2$(b, c)) and in(a, cartesian_product$_2$(d, e))) ⇒ in(a, cartesian_product$_2$(set_intersection$_2$

**SET984+1.p** Basic properties of sets, theorem 138
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)      fof(commutativity_k3_xboole$_0$, axiom)
empty(empty_set)      fof(fc1_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a      fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)      fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)      fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a      fof(reflexivity_r1_tarski, axiom)
∀a, b: (cartesian_product$_2$(a, b) = empty_set ⟺ (a = empty_set or b = empty_set))      fof(t113_zfmisc$_1$, axiom)
∀a, b, c, d: cartesian_product$_2$(set_intersection$_2$(a, b), set_intersection$_2$(c, d)) = set_intersection$_2$(cartesian_product$_2$(a, c), cartes
∀a, b, c, d: (cartesian_product$_2$(a, b) = cartesian_product$_2$(c, d) ⇒ (a = empty_set or b = empty_set or (a = c and b = d)))      fof(t134_zfmisc$_1$, axiom)
∀a, b, c, d: (cartesian_product$_2$(a, b) ⊆ cartesian_product$_2$(c, d) ⇒ (cartesian_product$_2$(a, b) = empty_set or (a ⊆ c and b ⊆ d)))      fof(t138_zfmisc$_1$, conjecture)
∀a, b: set_intersection$_2$(a, b) ⊆ a      fof(t17_xboole$_1$, axiom)
∀a, b: (a ⊆ b ⇒ set_intersection$_2$(a, b) = a)      fof(t28_xboole$_1$, axiom)

**SET985+1.p** Basic properties of sets, theorem 139
empty(empty_set)      fof(fc1_xboole$_0$, axiom)
∃a: empty(a)      fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)      fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a      fof(reflexivity_r1_tarski, axiom)
∀a, b: (cartesian_product$_2$(a, b) = empty_set ⟺ (a = empty_set or b = empty_set))      fof(t113_zfmisc$_1$, axiom)
∀a, b, c, d: (cartesian_product$_2$(a, b) ⊆ cartesian_product$_2$(c, d) ⇒ (cartesian_product$_2$(a, b) = empty_set or (a ⊆ c and b ⊆ d)))      fof(t138_zfmisc$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∀b, c, d: ((cartesian_product$_2$(a, b) ⊆ cartesian_product$_2$(c, d) or cartesian_product$_2$(b, a) ⊆ cartesian_product$_2$(d, c)) ⇒ b ⊆ d))      fof(t139_zfmisc$_1$, conjecture)
∀a: empty_set ⊆ a      fof(t2_xboole$_1$, axiom)

**SET986+1.p** in(A,B) => union(difference(B,singleton(A)),singleton(A)) = B
∀a, b: (in(a, b) ⇒ ¬ in(b, a))      fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_union$_2$(a, b) = set_union$_2$(b, a)      fof(commutativity_k2_xboole$_0$, axiom)
∀a, b: (a = b ⟺ (a ⊆ b and b ⊆ a))      fof(d10_xboole$_0$, axiom)
∀a, b: (b = singleton(a) ⟺ ∀c: (in(c, b) ⟺ c = a))      fof(d1_tarski, axiom)
∀a, b, c: (c = set_union$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) or in(d, b))))      fof(d2_xboole$_0$, axiom)
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b)))      fof(d3_tarski, axiom)

$\forall a, b, c\colon (c = \text{set\_difference}(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a) \text{ and } \neg\,\text{in}(d, b))))$ fof(d4_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \text{set\_union}_2(\text{set\_difference}(b, \text{singleton}(a)), \text{singleton}(a)) = b)$ fof(t140_zfmisc$_1$, conjecture)
$\forall a, b, c\colon (\text{in}(a, \text{set\_difference}(b, \text{singleton}(c))) \iff (\text{in}(a, b) \text{ and } a \neq c))$ fof(t64_zfmisc$_1$, axiom)

**SET987+1.p** in(A,B) => difference(union(B,singleton(A)),singleton(A)) = B
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\text{in}(a, b) \Rightarrow \text{set\_difference}(\text{set\_union}_2(b, \text{singleton}(a)), \text{singleton}(a)) = b)$ fof(t141_zfmisc$_1$, conjecture)
$\forall a, b\colon \text{set\_difference}(\text{set\_union}_2(a, b), b) = \text{set\_difference}(a, b)$ fof(t40_xboole$_1$, axiom)
$\forall a, b\colon (\text{set\_difference}(a, \text{singleton}(b)) = a \iff \neg\,\text{in}(b, a))$ fof(t65_zfmisc$_1$, axiom)

**SET988+1.p** Functions and their basic properties, theorem 2
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\text{empty}(\text{empty\_set}) \text{ and } \text{relation}(\text{empty\_set})$ fof(fc4_relat$_1$, axiom)
$\text{empty}(\text{empty\_set}) \text{ and } \text{relation}(\text{empty\_set}) \text{ and } \text{relation\_empty\_yielding}(\text{empty\_set})$ fof(fc12_relat$_1$, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a\colon \exists b\colon \text{element}(b, a)$ fof(existence_m1_subset$_1$, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow \text{function}(a))$ fof(cc1_funct$_1$, axiom)
$\forall a\colon \neg\,\text{empty}(\text{powerset}(a))$ fof(fc1_subset$_1$, axiom)
$\forall a\colon \neg\,\text{empty}(\text{singleton}(a))$ fof(fc2_subset$_1$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{unordered\_pair}(a, b))$ fof(fc3_subset$_1$, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow \text{relation}(a))$ fof(cc1_relat$_1$, axiom)
$\forall a, b\colon (\text{element}(a, b) \Rightarrow (\text{empty}(b) \text{ or } \text{in}(a, b)))$ fof(t2_subset, axiom)
$\forall a, b\colon (\text{element}(a, \text{powerset}(b)) \iff a \subseteq b)$ fof(t3_subset, axiom)
$\forall a, b, c\colon ((\text{in}(a, b) \text{ and } \text{element}(b, \text{powerset}(c))) \Rightarrow \text{element}(a, c))$ fof(t4_subset, axiom)
$\forall a, b, c\colon \neg\,\text{in}(a, b) \text{ and } \text{element}(b, \text{powerset}(c)) \text{ and } \text{empty}(c)$ fof(t5_subset, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\,\text{empty}(a) \text{ and } a \neq b \text{ and } \text{empty}(b)$ fof(t8_boole, axiom)
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a\colon (\text{relation}(a) \text{ and } \text{function}(a))$ fof(rc1_funct$_1$, axiom)
$\forall a\colon (\neg\,\text{empty}(a) \Rightarrow \exists b\colon (\text{element}(b, \text{powerset}(a)) \text{ and } \neg\,\text{empty}(b)))$ fof(rc1_subset$_1$, axiom)
$\forall a\colon \exists b\colon (\text{element}(b, \text{powerset}(a)) \text{ and } \text{empty}(b))$ fof(rc2_subset$_1$, axiom)
$\exists a\colon (\text{empty}(a) \text{ and } \text{relation}(a))$ fof(rc1_relat$_1$, axiom)
$\exists a\colon (\neg\,\text{empty}(a) \text{ and } \text{relation}(a))$ fof(rc2_relat$_1$, axiom)
$\exists a\colon (\text{relation}(a) \text{ and } \text{relation\_empty\_yielding}(a))$ fof(rc3_relat$_1$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \text{element}(a, b))$ fof(t1_subset, axiom)
$\forall a, b\colon \neg\,\text{in}(a, b) \text{ and } \text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon \text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a\colon ((\forall b\colon \neg\,\text{in}(b, a) \text{ and } \forall c, d\colon \text{ordered\_pair}(c, d) \neq b \text{ and } \forall b, c, d\colon ((\text{in}(\text{ordered\_pair}(b, c), a) \text{ and } \text{in}(\text{ordered\_pair}(b, d), a)) \Rightarrow c = d)) \Rightarrow (\text{relation}(a) \text{ and } \text{function}(a)))$ fof(t2_funct$_1$, conjecture)
$\forall a\colon (\text{function}(a) \iff \forall b, c, d\colon ((\text{in}(\text{ordered\_pair}(b, c), a) \text{ and } \text{in}(\text{ordered\_pair}(b, d), a)) \Rightarrow c = d))$ fof(d1_funct$_1$, axiom)
$\forall a\colon (\text{relation}(a) \iff \forall b\colon \neg\,\text{in}(b, a) \text{ and } \forall c, d\colon b \neq \text{ordered\_pair}(c, d))$ fof(d1_relat$_1$, axiom)

**SET991+1.p** Functions and their basic properties, theorem 12
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\text{empty}(\text{empty\_set}) \text{ and } \text{relation}(\text{empty\_set})$ fof(fc4_relat$_1$, axiom)

empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set)  fof(fc12_relat$_1$, axiom)
empty(empty_set)  fof(fc1_xboole$_0$, axiom)
∀a: ∃b: element(b, a)  fof(existence_m1_subset$_1$, axiom)
∀a: (empty(a) ⇒ function(a))  fof(cc1_funct$_1$, axiom)
∀a: ¬ empty(powerset(a))  fof(fc1_subset$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a)))  fof(fc5_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_rng(a)))  fof(fc6_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a))))  fof(fc7_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_rng(a)) and relation(relation_rng(a))))  fof(fc8_relat$_1$, axiom)
∀a: (empty(a) ⇒ relation(a))  fof(cc1_relat$_1$, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))  fof(t2_subset, axiom)
∀a, b: (element(a, powerset(b)) ⟺ a ⊆ b)  fof(t3_subset, axiom)
∀a, b, c: ((in(a, b) and element(b, powerset(c))) ⇒ element(a, c))  fof(t4_subset, axiom)
∀a, b, c: ¬ in(a, b) and element(b, powerset(c)) and empty(c)  fof(t5_subset, axiom)
∀a: (empty(a) ⇒ a = empty_set)  fof(t6_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)  fof(t8_boole, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))  fof(antisymmetry_r2_hidden, axiom)
∃a: (relation(a) and function(a))  fof(rc1_funct$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b)))  fof(rc1_subset$_1$, axiom)
∀a: ∃b: (element(b, powerset(a)) and empty(b))  fof(rc2_subset$_1$, axiom)
∃a: (empty(a) and relation(a))  fof(rc1_relat$_1$, axiom)
∃a: (¬ empty(a) and relation(a))  fof(rc2_relat$_1$, axiom)
∃a: (relation(a) and relation_empty_yielding(a))  fof(rc3_relat$_1$, axiom)
∃a: empty(a)  fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)  fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ element(a, b))  fof(t1_subset, axiom)
∀a, b: ¬ in(a, b) and empty(b)  fof(t7_boole, axiom)
∀a, b: ((relation(b) and function(b)) ⇒ (in(a, relation_dom(b)) ⇒ in(apply(b, a), relation_rng(b))))  fof(t12_funct$_1$, conject
∀a: ((relation(a) and function(a)) ⇒ ∀b: (b = relation_rng(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: (in(d, relation_dom(a)) and c = apply(a, d)))))  fof(d5_funct$_1$, axiom)

**SET994+1.p** Functions and their basic properties, theorem 16
∀a, b: (in(a, b) ⇒ ¬ in(b, a))  fof(antisymmetry_r2_hidden, axiom)
∀a: (empty(a) ⇒ function(a))  fof(cc1_funct$_1$, axiom)
∀a: (empty(a) ⇒ relation(a))  fof(cc1_relat$_1$, axiom)
∀a: ∃b: element(b, a)  fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set)  fof(fc12_relat$_1$, axiom)
∀a: ¬ empty(powerset(a))  fof(fc1_subset$_1$, axiom)
empty(empty_set)  fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set)  fof(fc4_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a)))  fof(fc5_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a))))  fof(fc7_relat$_1$, axiom)
∃a: (relation(a) and function(a))  fof(rc1_funct$_1$, axiom)
∃a: (empty(a) and relation(a))  fof(rc1_relat$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b)))  fof(rc1_subset$_1$, axiom)
∃a: empty(a)  fof(rc1_xboole$_0$, axiom)
∃a: (¬ empty(a) and relation(a))  fof(rc2_relat$_1$, axiom)
∀a: ∃b: (element(b, powerset(a)) and empty(b))  fof(rc2_subset$_1$, axiom)
∃a: ¬ empty(a)  fof(rc2_xboole$_0$, axiom)
∃a: (relation(a) and relation_empty_yielding(a))  fof(rc3_relat$_1$, axiom)
∀a, b: a ⊆ a  fof(reflexivity_r1_tarski, axiom)
∀a: ∃b: (relation(b) and function(b) and relation_dom(b) = a and ∀c: (in(c, a) ⇒ apply(b, c) = $n_0$))  fof(s3_funct_1__e4_14_
∀a: ∃b: (relation(b) and function(b) and relation_dom(b) = a and ∀c: (in(c, a) ⇒ apply(b, c) = $n_1$))  fof(s3_funct_1__e7_14_
empty($n_0$)  fof(spc0_boole, axiom)
¬ empty($n_1$)  fof(spc1_boole, axiom)
∀a: (∀b: ((relation(b) and function(b)) ⇒ ∀c: ((relation(c) and function(c)) ⇒ ((relation_dom(b) = a and relation_dom(c) = a) ⇒ b = c))) ⇒ a = empty_set)  fof(t16_funct$_1$, conjecture)
∀a, b: (in(a, b) ⇒ element(a, b))  fof(t1_subset, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))  fof(t2_subset, axiom)
∀a, b: (element(a, powerset(b)) ⟺ a ⊆ b)  fof(t3_subset, axiom)

$\forall a, b, c$: $((\text{in}(a,b)$ and $\text{element}(b, \text{powerset}(c))) \;\Rightarrow\; \text{element}(a,c))$ fof(t4_subset, axiom)
$\forall a, b, c$: $\neg\,\text{in}(a,b)$ and $\text{element}(b, \text{powerset}(c))$ and $\text{empty}(c)$ fof(t5_subset, axiom)
$\forall a$: $(\text{empty}(a) \;\Rightarrow\; a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\text{in}(a,b)$ and $\text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b$: $\neg\,\text{empty}(a)$ and $a \neq b$ and $\text{empty}(b)$ fof(t8_boole, axiom)