# TOP axioms
# TOP problems

**TOP001-1.p** Topology generated by a basis forms a topological space, part 1
include('Axioms/TOP001-0.ax')
basis(cx, $f$)     cnf(lemma_1a$_1$, negated_conjecture)
¬ subset_sets(union_of_members(top_of_basis($f$)), cx)     cnf(lemma_1a$_2$, negated_conjecture)

**TOP001-2.p** Topology generated by a basis forms a topological space, part 1
element_of_set($u$, union_of_members(vf)) ⇒ element_of_set($u$, $f_1$(vf, $u$))     cnf(union_of_members$_1$, axiom)
element_of_set($u$, union_of_members(vf)) ⇒ element_of_collection($f_1$(vf, $u$), vf)     cnf(union_of_members$_2$, axiom)
(element_of_set($u$, uu$_1$) and element_of_collection(uu$_1$, vf)) ⇒ element_of_set($u$, union_of_members(vf))     cnf(union_of_mem
basis($x$, vf) ⇒ equal_sets(union_of_members(vf), $x$)     cnf(basis_for_topology$_{28}$, axiom)
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_set($x$, $f_{10}$(vf, $u$, $x$))     cnf(topology_genera
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_collection($f_{10}$(vf, $u$, $x$), vf)     cnf(topology
subset_sets($x$, $x$)     cnf(set_theory$_1$, axiom)
(subset_sets($x$, $y$) and element_of_set($u$, $x$)) ⇒ element_of_set($u$, $y$)     cnf(set_theory$_2$, axiom)
equal_sets($x$, $y$) ⇒ subset_sets($x$, $y$)     cnf(set_theory$_3$, axiom)
subset_sets($x$, $y$) or element_of_set(in_1st_set($x$, $y$), $x$)     cnf(set_theory$_4$, axiom)
element_of_set(in_1st_set($x$, $y$), $y$) ⇒ subset_sets($x$, $y$)     cnf(set_theory$_5$, axiom)
basis(cx, $f$)     cnf(lemma_1a$_1$, negated_conjecture)
¬ subset_sets(union_of_members(top_of_basis($f$)), cx)     cnf(lemma_1a$_2$, negated_conjecture)

**TOP002-1.p** Topology generated by a basis forms a topological space, part 2
include('Axioms/TOP001-0.ax')
basis(cx, $f$)     cnf(lemma_1b$_1$, negated_conjecture)
¬ element_of_collection(empty_set, top_of_basis($f$))     cnf(lemma_1b$_2$, negated_conjecture)

**TOP002-2.p** Topology generated by a basis forms a topological space, part 2
element_of_collection($u$, top_of_basis(vf)) or element_of_set($f_{11}$(vf, $u$), $u$)     cnf(topology_generated$_{40}$, axiom)
¬ element_of_set($x$, empty_set)     cnf(set_theory$_6$, axiom)
¬ element_of_collection(empty_set, top_of_basis($f$))     cnf(lemma_1b$_2$, negated_conjecture)

**TOP003-1.p** Topology generated by a basis forms a topological space, part 3
include('Axioms/TOP001-0.ax')
basis(cx, $f$)     cnf(lemma_1c$_1$, negated_conjecture)
¬ element_of_collection(cx, top_of_basis($f$))     cnf(lemma_1c$_2$, negated_conjecture)

**TOP003-2.p** Topology generated by a basis forms a topological space, part 3
element_of_set($u$, union_of_members(vf)) ⇒ element_of_set($u$, $f_1$(vf, $u$))     cnf(union_of_members$_1$, axiom)
element_of_set($u$, union_of_members(vf)) ⇒ element_of_collection($f_1$(vf, $u$), vf)     cnf(union_of_members$_2$, axiom)
basis($x$, vf) ⇒ equal_sets(union_of_members(vf), $x$)     cnf(basis_for_topology$_{28}$, axiom)
element_of_collection($u$, top_of_basis(vf)) or element_of_set($f_{11}$(vf, $u$), $u$)     cnf(topology_generated$_{40}$, axiom)
element_of_collection($x$, $y$) ⇒ subset_sets($x$, union_of_members($y$))     cnf(set_theory$_7$, axiom)
(subset_sets($x$, $y$) and element_of_set($u$, $x$)) ⇒ element_of_set($u$, $y$)     cnf(set_theory$_8$, axiom)
subset_sets($x$, $x$)     cnf(set_theory$_9$, axiom)
(equal_sets($x$, $y$) and subset_sets($z$, $x$)) ⇒ subset_sets($z$, $y$)     cnf(set_theory$_{10}$, axiom)
(equal_sets($x$, $y$) and subset_sets($x$, $z$)) ⇒ subset_sets($y$, $z$)     cnf(set_theory$_{11}$, axiom)
basis(cx, $f$)     cnf(lemma_1c$_1$, negated_conjecture)
¬ element_of_collection(cx, top_of_basis($f$))     cnf(lemma_1c$_2$, negated_conjecture)

**TOP004-1.p** Topology generated by a basis forms a topological space, part 4
include('Axioms/TOP001-0.ax')
basis(cx, $f$)     cnf(lemma_1d$_1$, negated_conjecture)
element_of_collection($u$, top_of_basis($f$))     cnf(lemma_1d$_2$, negated_conjecture)
element_of_collection($v$, top_of_basis($f$))     cnf(lemma_1d$_3$, negated_conjecture)
¬ element_of_collection(intersection_of_sets($u$, $v$), top_of_basis($f$))     cnf(lemma_1d$_4$, negated_conjecture)

**TOP004-2.p** Topology generated by a basis forms a topological space, part 4
(element_of_set($u$, uu$_1$) and element_of_collection(uu$_1$, vf)) ⇒ element_of_set($u$, union_of_members(vf))     cnf(union_of_mem
basis($x$, vf) ⇒ equal_sets(union_of_members(vf), $x$)     cnf(basis_for_topology$_{28}$, axiom)

(basis($x$, vf) and element_of_set($y$, $x$) and element_of_collection(vb$_1$, vf) and element_of_collection(vb$_2$, vf) and element_of_set(
element_of_set($y$, $f_6$($x$, vf, $y$, vb$_1$, vb$_2$))    cnf(basis_for_topology$_{29}$, axiom)
(basis($x$, vf) and element_of_set($y$, $x$) and element_of_collection(vb$_1$, vf) and element_of_collection(vb$_2$, vf) and element_of_set(
element_of_collection($f_6$($x$, vf, $y$, vb$_1$, vb$_2$), vf)    cnf(basis_for_topology$_{30}$, axiom)
(basis($x$, vf) and element_of_set($y$, $x$) and element_of_collection(vb$_1$, vf) and element_of_collection(vb$_2$, vf) and element_of_set(
subset_sets($f_6$($x$, vf, $y$, vb$_1$, vb$_2$), intersection_of_sets(vb$_1$, vb$_2$))    cnf(basis_for_topology$_{31}$, axiom)
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_set($x$, $f_{10}$(vf, $u$, $x$))    cnf(topology_genera
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_collection($f_{10}$(vf, $u$, $x$), vf)    cnf(topology
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ subset_sets($f_{10}$(vf, $u$, $x$), $u$)    cnf(topology_generated
element_of_collection($u$, top_of_basis(vf)) or element_of_set($f_{11}$(vf, $u$), $u$)    cnf(topology_generated$_{40}$, axiom)
(element_of_set($f_{11}$(vf, $u$), uu$_{11}$) and element_of_collection(uu$_{11}$, vf) and subset_sets(uu$_{11}$, $u$)) ⇒ element_of_collection($u$, top_
(subset_sets($x$, $y$) and subset_sets($y$, $z$)) ⇒ subset_sets($x$, $z$)    cnf(set_theory$_{12}$, axiom)
element_of_set($z$, intersection_of_sets($x$, $y$)) ⇒ element_of_set($z$, $x$)    cnf(set_theory$_{13}$, axiom)
element_of_set($z$, intersection_of_sets($x$, $y$)) ⇒ element_of_set($z$, $y$)    cnf(set_theory$_{14}$, axiom)
(element_of_set($z$, $x$) and element_of_set($z$, $y$)) ⇒ element_of_set($z$, intersection_of_sets($x$, $y$))    cnf(set_theory$_{15}$, axiom)
(subset_sets($x$, $y$) and subset_sets($u$, $v$)) ⇒ subset_sets(intersection_of_sets($x$, $u$), intersection_of_sets($y$, $v$))    cnf(set_theory
(equal_sets($x$, $y$) and element_of_set($z$, $x$)) ⇒ element_of_set($z$, $y$)    cnf(set_theory$_{17}$, axiom)
equal_sets(intersection_of_sets($x$, $y$), intersection_of_sets($y$, $x$))    cnf(set_theory$_{18}$, axiom)
basis(cx, $f$)    cnf(lemma_1d$_1$, negated_conjecture)
element_of_collection($u$, top_of_basis($f$))    cnf(lemma_1d$_2$, negated_conjecture)
element_of_collection($v$, top_of_basis($f$))    cnf(lemma_1d$_3$, negated_conjecture)
¬ element_of_collection(intersection_of_sets($u$, $v$), top_of_basis($f$))    cnf(lemma_1d$_4$, negated_conjecture)

**TOP005-1.p** Topology generated by a basis forms a topological space, part 5
include('Axioms/TOP001-0.ax')
basis(cx, $f$)    cnf(lemma_1e$_1$, negated_conjecture)
subset_collections($g$, top_of_basis($f$))    cnf(lemma_1e$_2$, negated_conjecture)
¬ element_of_collection(union_of_members($g$), top_of_basis($f$))    cnf(lemma_1e$_3$, negated_conjecture)

**TOP005-2.p** Topology generated by a basis forms a topological space, part 5
element_of_set($u$, union_of_members(vf)) ⇒ element_of_set($u$, $f_1$(vf, $u$))    cnf(union_of_members$_1$, axiom)
element_of_set($u$, union_of_members(vf)) ⇒ element_of_collection($f_1$(vf, $u$), vf)    cnf(union_of_members$_2$, axiom)
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_set($x$, $f_{10}$(vf, $u$, $x$))    cnf(topology_genera
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ element_of_collection($f_{10}$(vf, $u$, $x$), vf)    cnf(topology
(element_of_collection($u$, top_of_basis(vf)) and element_of_set($x$, $u$)) ⇒ subset_sets($f_{10}$(vf, $u$, $x$), $u$)    cnf(topology_generated
element_of_collection($u$, top_of_basis(vf)) or element_of_set($f_{11}$(vf, $u$), $u$)    cnf(topology_generated$_{40}$, axiom)
(element_of_set($f_{11}$(vf, $u$), uu$_{11}$) and element_of_collection(uu$_{11}$, vf) and subset_sets(uu$_{11}$, $u$)) ⇒ element_of_collection($u$, top_
element_of_set($u$, $x$) ⇒ (subset_sets($x$, $y$) or element_of_set($u$, $y$))    cnf(set_theory$_{19}$, axiom)
(subset_sets($x$, $y$) and element_of_collection($y$, $z$)) ⇒ subset_sets($x$, union_of_members($z$))    cnf(set_theory$_{20}$, axiom)
(subset_collections($x$, $y$) and element_of_collection($u$, $x$)) ⇒ element_of_collection($u$, $y$)    cnf(set_theory$_{21}$, axiom)
subset_collections($g$, top_of_basis($f$))    cnf(lemma_1e$_2$, negated_conjecture)
¬ element_of_collection(union_of_members($g$), top_of_basis($f$))    cnf(lemma_1e$_3$, negated_conjecture)

**TOP006-1.p** Topology generated by a basis forms a topological space
include('Axioms/TOP001-0.ax')
basis(cx, ct)    cnf(problem_1$_{110}$, negated_conjecture)
¬ topological_space(cx, top_of_basis(ct))    cnf(problem_1$_{111}$, negated_conjecture)

**TOP007-1.p** Property 1 of topological spaces
If (cx,ct) is a topological space, A is a subset of X, and every point in A has a neighborhood U that is a subset of A, then A is open in (cx,ct).
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)    cnf(problem_2$_{112}$, negated_conjecture)
subset_sets($a$, cx)    cnf(problem_2$_{113}$, negated_conjecture)
element_of_set($y$, $a$) ⇒ neighborhood($f_{30}$($y$), $y$, cx, ct)    cnf(problem_2$_{114}$, negated_conjecture)
element_of_set($y$, $a$) ⇒ subset_sets($f_{30}$($y$), $a$)    cnf(problem_2$_{115}$, negated_conjecture)
¬ open($a$, cx, ct)    cnf(problem_2$_{116}$, negated_conjecture)

**TOP008-1.p** The subspace topology gives rise to a topological space
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)    cnf(problem_3$_{117}$, negated_conjecture)
subset_sets(cy, cx)    cnf(problem_3$_{118}$, negated_conjecture)

¬ topological_space(cy, subspace_topology(cx, ct, cy))  cnf(problem_3$_{119}$, negated_conjecture)

**TOP009-1.p** If Y is open in X, and A is open in Y, then A is open in X
include('Axioms/TOP001-0.ax')
open(cy, cx, ct)  cnf(problem_4$_{120}$, negated_conjecture)
open($a$, cy, subspace_topology(cx, ct, cy))  cnf(problem_4$_{121}$, negated_conjecture)
¬ open($a$, cx, ct)  cnf(problem_4$_{122}$, negated_conjecture)

**TOP010-1.p** A finer topology induces a finer subspace topology
include('Axioms/TOP001-0.ax')
finer(ct$_1$, ct$_2$, cx)  cnf(problem_5$_{123}$, negated_conjecture)
subset_sets($a$, cx)  cnf(problem_5$_{124}$, negated_conjecture)
¬ finer(subspace_topology(cx, ct$_1$, $a$), subspace_topology(cx, ct$_2$, $a$), cx)  cnf(problem_5$_{125}$, negated_conjecture)

**TOP011-1.p** An alternative definition of top_of_basis
include('Axioms/TOP001-0.ax')
element_of_set(cu, top_of_basis($f$)) or subset_collections($g$, $f$)  cnf(problem_6$_{126}$, negated_conjecture)
element_of_set(cu, top_of_basis($f$)) or equal_sets(cu, union_of_members($g$))  cnf(problem_6$_{127}$, negated_conjecture)
(element_of_set(cu, top_of_basis($f$)) and subset_collections($x$, $f$)) ⇒ ¬ equal_sets(cu, union_of_members($x$))  cnf(problem_6

**TOP012-1.p** Intersections and finite unions of closed sets are closed
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)  cnf(problem_7$_{129}$, negated_conjecture)
(closed(empty_set, cx, ct) and closed(cx, cx, ct)) ⇒ (closed(cy$_1$, cx, ct) or subset_sets(union_of_members($f$), cx))  cnf(prob
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and element_of_collection($v$, $f$)) ⇒ (closed(cy$_1$, cx, ct) or closed($v$, cx, ct))
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and closed(intersection_of_members($f$), cx, ct)) ⇒ closed(cy$_1$, cx, ct)  cnf(p
(closed(empty_set, cx, ct) and closed(cx, cx, ct)) ⇒ (closed(cy$_2$, cx, ct) or subset_sets(union_of_members($f$), cx))  cnf(prob
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and element_of_collection($v$, $f$)) ⇒ (closed(cy$_2$, cx, ct) or closed($v$, cx, ct))
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and closed(intersection_of_members($f$), cx, ct)) ⇒ closed(cy$_2$, cx, ct)  cnf(p
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and closed(union_of_sets(cy$_1$, cy$_2$), cx, ct)) ⇒ subset_sets(union_of_members($f$
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and closed(union_of_sets(cy$_1$, cy$_2$), cx, ct) and element_of_collection($v$, $f$)) ⇒
closed($v$, cx, ct)  cnf(problem_7$_{137}$, negated_conjecture)
(closed(empty_set, cx, ct) and closed(cx, cx, ct) and closed(union_of_sets(cy$_1$, cy$_2$), cx, ct)) ⇒ ¬ closed(intersection_of_membe

**TOP013-1.p** Properties of interior and closure
The interior of A is a subset of A, which is a subset of the closure of A.
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)  cnf(problem_8$_{139}$, negated_conjecture)
subset_sets($a$, cx)  cnf(problem_8$_{140}$, negated_conjecture)
subset_sets(interior($a$, cx, ct), $a$) ⇒ ¬ subset_sets($a$, closure($a$, cx, ct))  cnf(problem_8$_{141}$, negated_conjecture)

**TOP014-1.p** Properties of open & interior and closed & closure
If A is open, the interior of A is A, and if A is closed, the closure of A is A.
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)  cnf(problem_9$_{142}$, negated_conjecture)
subset_sets($a$, cx)  cnf(problem_9$_{143}$, negated_conjecture)
open($a$, cx, ct) or equal_sets($a$, interior($a$, cx, ct)) or closed($a$, cx, ct) or equal_sets($a$, closure($a$, cx, ct))  cnf(problem_9$_{144}$, ne
(closed($a$, cx, ct) and equal_sets($a$, closure($a$, cx, ct))) ⇒ (open($a$, cx, ct) or equal_sets($a$, interior($a$, cx, ct)))  cnf(problem_9
(open($a$, cx, ct) and equal_sets($a$, interior($a$, cx, ct))) ⇒ (closed($a$, cx, ct) or equal_sets($a$, closure($a$, cx, ct)))  cnf(problem_9
(open($a$, cx, ct) and equal_sets($a$, interior($a$, cx, ct)) and closed($a$, cx, ct)) ⇒ ¬ equal_sets($a$, closure($a$, cx, ct))  cnf(problem

**TOP015-1.p** The interior and the boundary of a set are disjoint
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)  cnf(problem_10$_{148}$, negated_conjecture)
subset_sets($a$, cx)  cnf(problem_10$_{149}$, negated_conjecture)
¬ equal_sets(intersection_of_sets(interior($a$, cx, ct), boundary($a$, cx, ct)), empty_set)  cnf(problem_10$_{150}$, negated_conjecture)

**TOP016-1.p** The union of the interior and the boundary is the closure
include('Axioms/TOP001-0.ax')
topological_space(cx, ct)  cnf(problem_11$_{151}$, negated_conjecture)
subset_sets($a$, cx)  cnf(problem_11$_{152}$, negated_conjecture)
¬ equal_sets(union_of_sets(interior($a$, cx, ct), boundary($a$, cx, ct)), closure($a$, cx, ct))  cnf(problem_11$_{153}$, negated_conjecture)

**TOP017-1.p** If the boundary of A is empty, A is both open and closed
include('Axioms/TOP001-0.ax')

topological_space(cx, ct) cnf(problem_12$_{154}$, negated_conjecture)
subset_sets($a$, cx) cnf(problem_12$_{155}$, negated_conjecture)
equal_sets(boundary($a$, cx, ct), empty_set) or open($a$, cx, ct) cnf(problem_12$_{156}$, negated_conjecture)
equal_sets(boundary($a$, cx, ct), empty_set) or closed($a$, cx, ct) cnf(problem_12$_{157}$, negated_conjecture)
(equal_sets(boundary($a$, cx, ct), empty_set) and open($a$, cx, ct)) $\Rightarrow$ $\neg$ closed($a$, cx, ct) cnf(problem_12$_{158}$, negated_conjectu

**TOP018-1.p** Propoerty of limits points and connected sets
If limit points are added to a connected set, the result is connected.
include('Axioms/TOP001-0.ax')
connected_set($a$, cx, ct) cnf(problem_13$_{159}$, negated_conjecture)
element_of_set($y$, $b$) $\Rightarrow$ limit_point($y$, $a$, cx, ct) cnf(problem_13$_{160}$, negated_conjecture)
$\neg$ connected_set(union_of_sets($a$, $b$), cx, ct) cnf(problem_13$_{161}$, negated_conjecture)

**TOP019-1.p** The closure of a connected set is connected
include('Axioms/TOP001-0.ax')
connected_set($a$, cx, ct) cnf(problem_14$_{162}$, negated_conjecture)
$\neg$ connected_set(closure($a$, cx, ct), cx, ct) cnf(problem_14$_{163}$, negated_conjecture)

**TOP020+1.p** Property of a Hausdorff topological space
In a Hausdorff topological space, the diagonal of the space is closed in the product of the space with itself.
$\forall x, a$: ($\forall y$: ((a_member_of($y$, coerce_to_class($x$)) and $\neg$ a_member_of($y$, $a$)) $\Rightarrow$ $\exists g$: (a_member_of($y$, $g$) and open_in($g$, $x$) and d
closed_in($a$, $x$)) fof(closed_subset_thm, axiom)
$\forall x$: (a_hausdorff_top_space($x$) $\Rightarrow$ $\forall a, b$: ((a_member_of($a$, coerce_to_class($x$)) and a_member_of($b$, coerce_to_class($x$)) and $a \neq$
$b$) $\Rightarrow$ $\exists g_1, g_2$: (open_in($g_1$, $x$) and open_in($g_2$, $x$) and a_member_of($a$, $g_1$) and a_member_of($b$, $g_2$) and disjoint($g_1$, $g_2$)))) fo
$\forall a, x, b, y$: ((open_in($a$, $x$) and open_in($b$, $y$)) $\Rightarrow$ open_in(the_product_of($a$, $b$), the_product_top_space_of($x$, $y$))) fof(product
$\forall s, t, x$: (a_member_of($x$, coerce_to_class(the_product_top_space_of($s$, $t$))) $\Rightarrow$ $\exists a, b$: (a_member_of($a$, coerce_to_class($s$)) and a_
the_ordered_pair($a$, $b$))) fof(product_top, axiom)
$\forall x, s, t$: (a_member_of($x$, the_product_of($s$, $t$)) $\iff$ $\exists a, b$: (a_member_of($a$, $s$) and a_member_of($b$, $t$) and $x$ = the_ordered_pair
$\forall a, b$: (disjoint($a$, $b$) $\iff$ $\neg \exists y$: (a_member_of($y$, $a$) and a_member_of($y$, $b$))) fof(disjoint_defn, axiom)
$\forall a, b, c, d$: (the_ordered_pair($a$, $b$) = the_ordered_pair($c$, $d$) $\Rightarrow$ ($a = c$ and $b = d$)) fof(ordered_pair, axiom)
$\forall x, s$: (a_member_of($x$, coerce_to_class(the_diagonal_top($s$))) $\iff$ $\exists a$: (a_member_of($a$, coerce_to_class($s$)) and $x$ =
the_ordered_pair($a$, $a$))) fof(diagonal_top, axiom)
$\forall s$: (a_hausdorff_top_space($s$) $\Rightarrow$ closed_in(coerce_to_class(the_diagonal_top($s$)), the_product_top_space_of($s$, $s$))) fof(challe

**TOP021+1.p** Locally compact tological space
$\forall a, x, a_1$: a_continuous_function_from_onto(the_projection_function($a$, $x$, $a_1$), the_product_top_space_over($x$, $a_1$), apply($x$, $a$))
$\forall a, x, a_1, x_1$: an_open_function_from_onto(the_projection_function($a$, $x$, $a_1$), the_product_top_space_over($x_1$, $a_1$), apply($x_1$, $a$))
$\forall f, a, b$: ((an_open_function_from_onto($f$, $a$, $b$) and a_continuous_function_from_onto($f$, $a$, $b$) and a_locally_compact_top_space(
a_locally_compact_top_space($b$)) fof(kelley_p_147e, axiom)
$\forall x_1, a_1$: (a_locally_compact_top_space(the_product_top_space_over($x_1$, $a_1$)) $\Rightarrow$ $\forall a$: a_locally_compact_top_space(apply($x_1$, $a$)))

**TOP022+1.p** Homotopy groups
$\forall a, b$: (isomorphic_groups($a$, $b$) $\iff$ $\exists f$: a_group_isomorphism_from_to($f$, $a$, $b$)) fof(isomorphic_groups_defn, axiom)
$\forall x, x_0, x_1$: (path_connected($x$) $\iff$ ((a_member_of($x_0$, $x$) and a_member_of($x_1$, $x$)) $\Rightarrow$ $\exists p$: a_path_from_to_in($p$, $x_0$, $x_1$, $x$)))
$\forall a, x_0, x_1, x$: (a_path_from_to_in($a$, $x_0$, $x_1$, $x$) $\Rightarrow$ a_group_isomorphism_from_to(alpha_hat($a$), first_homotop_grp($x$, $x_0$), first_ho
$\forall x, x_0, x_1$: ((path_connected($x$) and a_member_of($x_0$, $x$) and a_member_of($x_1$, $x$)) $\Rightarrow$ isomorphic_groups(first_homotop_grp($x$,