KLE axioms

KLE001+0.ax Idempotent semirings
$\forall a, b$: $a + b = b + a$ fof(additive_commutativity, axiom)
$\forall c, b, a$: $a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)
$\forall a$: $a + 0 = a$ fof(additive_identity, axiom)
$\forall a$: $a + a = a$ fof(additive_idempotence, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
$\forall a$: $a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
$\forall a$: $1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
$\forall a, b, c$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(right_distributivity, axiom)
$\forall a, b, c$: $(a + b) \cdot c = a \cdot c + b \cdot c$ fof(left_distributivity, axiom)
$\forall a$: $a \cdot 0 = 0$ fof(right_annihilation, axiom)
$\forall a$: $0 \cdot a = 0$ fof(left_annihilation, axiom)
$\forall a, b$: $(a \leq b \iff a + b = b)$ fof(order, axiom)

KLE001+1.ax Characterisation of tests by complement predicate
$\forall x_0$: $(\text{test}(x_0) \iff \exists x_1 \colon x_1' = x_0)$ fof(test$_1$, axiom)
$\forall x_0, x_1$: $(x_1' = x_0 \iff (x_0 \cdot x_1 = 0 \text{ and } x_1 \cdot x_0 = 0 \text{ and } x_0 + x_1 = 1))$ fof(test$_2$, axiom)
$\forall x_0, x_1$: $(\text{test}(x_0) \Rightarrow (c(x_0) = x_1 \iff x_0' = x_1))$ fof(test$_3$, axiom)
$\forall x_0$: $(\neg \text{test}(x_0) \Rightarrow c(x_0) = 0)$ fof(test$_4$, axiom)

KLE001+2.ax de Morgan's laws for tests
$\forall x_0, x_1$: $((\text{test}(x_0) \text{ and } \text{test}(x_1)) \Rightarrow c(x_0 + x_1) = c(x_0) \cdot c(x_1))$ fof(test_deMorgan$_1$, axiom)
$\forall x_0, x_1$: $((\text{test}(x_0) \text{ and } \text{test}(x_1)) \Rightarrow c(x_0 \cdot x_1) = c(x_0) + c(x_1))$ fof(test_deMorgan$_2$, axiom)

KLE001+3.ax Universal characterisation of meet
$\forall x_0, x_1, x_2$: $(\text{ismeet}(x_2, x_0, x_1) \iff (x_2 \leq x_0 \text{ and } x_2 \leq x_1 \text{ and } \forall x_3 \colon ((x_3 \leq x_0 \text{ and } x_3 \leq x_1) \Rightarrow x_3 \leq x_2)))$ fof(ismeet, axiom)
$\forall x_0, x_1, x_2$: $(\text{ismeetu}(x_2, x_0, x_1) \iff \forall x_3 \colon ((x_3 \leq x_0 \text{ and } x_3 \leq x_1) \iff x_3 \leq x_2))$ fof(ismeetu, axiom)

KLE001+4.ax Boolean domain, antidomain, codomain, coantidomain
$\forall x_0$: $\text{ad}(x_0) \cdot x_0 = 0$ fof(domain$_1$, axiom)
$\forall x_0, x_1$: $\text{ad}(x_0 \cdot x_1) + \text{ad}(x_0 \cdot \text{ad}(\text{ad}(x_1))) = \text{ad}(x_0 \cdot \text{ad}(\text{ad}(x_1)))$ fof(domain$_2$, axiom)
$\forall x_0$: $\text{ad}(\text{ad}(x_0)) + \text{ad}(x_0) = 1$ fof(domain$_3$, axiom)
$\forall x_0$: $\text{dom}(x_0) = \text{ad}(\text{ad}(x_0))$ fof(domain$_4$, axiom)
$\forall x_0$: $x_0 \cdot \text{coad}(x_0) = 0$ fof(codomain$_1$, axiom)
$\forall x_0, x_1$: $\text{coad}(x_0 \cdot x_1) + \text{coad}(\text{coad}(\text{coad}(x_0)) \cdot x_1) = \text{coad}(\text{coad}(\text{coad}(x_0)) \cdot x_1)$ fof(codomain$_2$, axiom)
$\forall x_0$: $\text{coad}(\text{coad}(x_0)) + \text{coad}(x_0) = 1$ fof(codomain$_3$, axiom)
$\forall x_0$: $\text{cod}(x_0) = \text{coad}(\text{coad}(x_0))$ fof(codomain$_4$, axiom)

KLE001+5.ax Domain (not Boolean domain!)
$\forall x_0$: $x_0 + \text{dom}(x_0) \cdot x_0 = \text{dom}(x_0) \cdot x_0$ fof(domain$_1$, axiom)
$\forall x_0, x_1$: $\text{dom}(x_0 \cdot x_1) = \text{dom}(x_0 \cdot \text{dom}(x_1))$ fof(domain$_2$, axiom)
$\forall x_0$: $\text{dom}(x_0) + 1 = 1$ fof(domain$_3$, axiom)
$\text{dom}(0) = 0$ fof(domain$_4$, axiom)
$\forall x_0, x_1$: $\text{dom}(x_0 + x_1) = \text{dom}(x_0) + \text{dom}(x_1)$ fof(domain$_5$, axiom)

KLE001+6.ax Modal operators
$\forall x_0$: $c(x_0) = \text{ad}(\text{dom}(x_0))$ fof(complement, axiom)
$\forall x_0, x_1$: $\text{domain\_difference}(x_0, x_1) = \text{dom}(x_0) \cdot \text{ad}(x_1)$ fof(domain_difference, axiom)
$\forall x_0, x_1$: $\text{forward\_diamond}(x_0, x_1) = \text{dom}(x_0 \cdot \text{dom}(x_1))$ fof(forward_diamond, axiom)
$\forall x_0, x_1$: $\text{backward\_diamond}(x_0, x_1) = \text{cod}(\text{cod}(x_1) \cdot x_0)$ fof(backward_diamond, axiom)
$\forall x_0, x_1$: $\text{forward\_box}(x_0, x_1) = c(\text{forward\_diamond}(x_0, c(x_1)))$ fof(forward_box, axiom)
$\forall x_0, x_1$: $\text{backward\_box}(x_0, x_1) = c(\text{backward\_diamond}(x_0, c(x_1)))$ fof(backward_box, axiom)

KLE001+7.ax Divergence Kleene algebras
$\forall x_0$: $\text{forward\_diamond}(x_0, \text{divergence}(x_0)) = \text{divergence}(x_0)$ fof(divergence$_1$, axiom)
$\forall x_0, x_1, x_2$: $(\text{dom}(x_0) + (\text{forward\_diamond}(x_1, \text{dom}(x_0)) + \text{dom}(x_2)) = \text{forward\_diamond}(x_1, \text{dom}(x_0)) + \text{dom}(x_2) \Rightarrow \text{dom}(x_0) + (\text{divergence}(x_1) + \text{forward\_diamond}(x_1^*, \text{dom}(x_2))) = \text{divergence}(x_1) + \text{forward\_diamond}(x_1^*, \text{dom}(x_2)))$ fof(diverg

KLE002+0.ax Kleene algebra
$\forall a, b$: $a + b = b + a$ fof(additive_commutativity, axiom)
$\forall c, b, a$: $a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)

$\forall a$: $a + 0 = a$ fof(additive_identity, axiom)
$\forall a$: $a + a = a$ fof(additive_idempotence, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
$\forall a$: $a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
$\forall a$: $1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
$\forall a, b, c$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(right_distributivity, axiom)
$\forall a, b, c$: $(a + b) \cdot c = a \cdot c + b \cdot c$ fof(left_distributivity, axiom)
$\forall a$: $a \cdot 0 = 0$ fof(right_annihilation, axiom)
$\forall a$: $0 \cdot a = 0$ fof(left_annihilation, axiom)
$\forall a, b$: $(a \leq b \iff a + b = b)$ fof(order, axiom)
$\forall a$: $1 + a \cdot a^* \leq a^*$ fof(star_unfold_right, axiom)
$\forall a$: $1 + a^* \cdot a \leq a^*$ fof(star_unfold_left, axiom)
$\forall a, b, c$: $(a \cdot b + c \leq b \Rightarrow a^* \cdot c \leq b)$ fof(star_induction_left, axiom)
$\forall a, b, c$: $(a \cdot b + c \leq a \Rightarrow c \cdot b^* \leq a)$ fof(star_induction_right, axiom)

KLE003+0.ax Omega algebra
$\forall a, b$: $a + b = b + a$ fof(additive_commutativity, axiom)
$\forall c, b, a$: $a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)
$\forall a$: $a + 0 = a$ fof(additive_identity, axiom)
$\forall a$: $a + a = a$ fof(additive_idempotence, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
$\forall a$: $a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
$\forall a$: $1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
$\forall a, b, c$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(right_distributivity, axiom)
$\forall a, b, c$: $(a + b) \cdot c = a \cdot c + b \cdot c$ fof(left_distributivity, axiom)
$\forall a$: $a \cdot 0 = 0$ fof(right_annihilation, axiom)
$\forall a$: $0 \cdot a = 0$ fof(left_annihilation, axiom)
$\forall a, b$: $(a \leq b \iff a + b = b)$ fof(order, axiom)
$\forall a$: $1 + a \cdot a^* \leq a^*$ fof(star_unfold_right, axiom)
$\forall a$: $1 + a^* \cdot a \leq a^*$ fof(star_unfold_left, axiom)
$\forall a, b, c$: $(a \cdot b + c \leq b \Rightarrow a^* \cdot c \leq b)$ fof(star_induction_left, axiom)
$\forall a, b, c$: $(a \cdot b + c \leq a \Rightarrow c \cdot b^* \leq a)$ fof(star_induction_right, axiom)
$\forall a$: $a \cdot a^\omega = a^\omega$ fof(omega_unfold, axiom)
$\forall a, b, c$: $(a \leq b \cdot a + c \Rightarrow a \leq b^\omega + b^* \cdot c)$ fof(omega_co_induction, axiom)

KLE004+0.ax Demonic Refinement Algebra
$\forall a, b$: $a + b = b + a$ fof(additive_commutativity, axiom)
$\forall c, b, a$: $a + (b + c) = (a + b) + c$ fof(additive_associativity, axiom)
$\forall a$: $a + 0 = a$ fof(additive_identity, axiom)
$\forall a$: $a + a = a$ fof(idempotence, axiom)
$\forall a, b, c$: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ fof(multiplicative_associativity, axiom)
$\forall a$: $a \cdot 1 = a$ fof(multiplicative_right_identity, axiom)
$\forall a$: $1 \cdot a = a$ fof(multiplicative_left_identity, axiom)
$\forall a, b, c$: $a \cdot (b + c) = a \cdot b + a \cdot c$ fof(distributivity$_1$, axiom)
$\forall a, b, c$: $(a + b) \cdot c = a \cdot c + b \cdot c$ fof(distributivity$_2$, axiom)
$\forall a$: $0 \cdot a = 0$ fof(left_annihilation, axiom)
$\forall a$: $1 + a \cdot a^* = a^*$ fof(star_unfold$_1$, axiom)
$\forall a$: $1 + a^* \cdot a = a^*$ fof(star_unfold$_2$, axiom)
$\forall a, b, c$: $(a \cdot c + b \leq c \Rightarrow a^* \cdot b \leq c)$ fof(star_induction$_1$, axiom)
$\forall a, b, c$: $(c \cdot a + b \leq c \Rightarrow b \cdot a^* \leq c)$ fof(star_induction$_2$, axiom)
$\forall a$: $\text{strong\_iteration}(a) = a \cdot \text{strong\_iteration}(a) + 1$ fof(infty_unfold$_1$, axiom)
$\forall a, b, c$: $(c \leq a \cdot c + b \Rightarrow c \leq \text{strong\_iteration}(a) \cdot b)$ fof(infty_coinduction, axiom)
$\forall a$: $\text{strong\_iteration}(a) = a^* + \text{strong\_iteration}(a) \cdot 0$ fof(isolation, axiom)
$\forall a, b$: $(a \leq b \iff a + b = b)$ fof(order, axiom)

KLE problems

KLE001+1.p Addition is isotone
include('Axioms/KLE001+0.ax')
$\forall x_0, x_1, x_2$: $(x_0 \leq x_1 \Rightarrow x_0 + x_2 \leq x_0 + x_2)$ fof(goals, conjecture)

KLE002+1.p Multiplication is isotone
include('Axioms/KLE001+0.ax')
$\forall x_0, x_1, x_2\colon (x_0 \leq x_1 \Rightarrow x_0 \cdot x_2 \leq x_1 \cdot x_2)$ fof(goals, conjecture)

KLE003+1.p A semiring is idempotent iff 1 is idempotent
include('Axioms/KLE001+0.ax')
$\forall x_0\colon x_0 + x_0 = x_0 \iff 1 + 1 = 1$ fof(goals, conjecture)

KLE004+1.p Complement - 1 is the complement of 0
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$c(0) = 1$ fof(goals, conjecture)

KLE005+1.p Complement - 0 is the complement of 1
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$c(1) = 0$ fof(goals, conjecture)

KLE006+1.p Split 1 with p
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0\colon (\text{test}(x_0) \Rightarrow 1 = x_0 + c(x_0))$ fof(goals, conjecture)

KLE007+1.p Split 1 with q and split the parts with p
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1))$ fof(goals, conjecture)

KLE007+2.p Split 1 with q and split the parts with p
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (x_0+c(x_0))\cdot x_1+(x_0+c(x_0))\cdot c(x_1) \text{ and } (x_0+c(x_0))\cdot x_1+(x_0+c(x_0))\cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE007+3.p Split 1 with q and split the parts with p
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = (x_0 + c(x_0)) \cdot x_1 + (x_0 + c(x_0)) \cdot c(x_1))$ fof(goals, conjecture)

KLE007+4.p Split 1 with q and split the parts with p
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow (1 \leq (x_0+c(x_0))\cdot x_1+(x_0+c(x_0))\cdot c(x_1) \text{ and } (x_0+c(x_0))\cdot x_1+(x_0+c(x_0))\cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE008+1.p A simple law to eliminate the complement
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2\colon (\text{test}(x_2) \Rightarrow (x_0 \leq x_2 \cdot x_1 \iff (x_0 \leq x_1 \text{ and } c(x_2) \cdot x_0 \leq 0)))$ fof(goals, conjecture)

KLE009+1.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE009+2.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon ((\text{test}(x_1) \text{ and } \text{test}(x_0)) \Rightarrow 1 = ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE009+3.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')

$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE009+4.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq ((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $((x_0 \cdot x_1 + x_0 \cdot c(x_1)) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE010+1.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $1 = (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE010+2.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $(((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE010+3.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $1 = (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE010+4.p Split 1 into all combinations of p,q and their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq (((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $(((x_1 \cdot x_0 + c(x_1) \cdot x_0) + x_0 \cdot x_1) + c(x_0) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE011+1.p Split 1 into p,q and the product of their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $1 = ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE011+2.p Split 1 into p,q and the product of their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE011+3.p Split 1 into p,q and the product of their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $1 = ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1))$ fof(goals, conjecture)

KLE011+4.p Split 1 into p,q and the product of their complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $(1 \leq ((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1)$ and $((x_1 + c(x_1)) \cdot x_0 + (x_0 + c(x_0)) \cdot x_1) + c(x_0) \cdot c(x_1) \leq 1))$ fof(goals, conjecture)

KLE012+1.p The multiplication of tests is commutative
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $x_0 \cdot x_1 = x_1 \cdot x_0)$ fof(goals, conjecture)

KLE013+1.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')

$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $1 = (x_0 + x_1) + c(x_0) \cdot c(x_1)$) fof(goals, conjecture)

KLE014+2.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $c(x_0 + x_1) = c(x_0) \cdot c(x_1)$) fof(goals, conjecture)

KLE014+3.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ ($c(x_0{+}x_1) \leq c(x_0){\cdot}c(x_1)$ and $c(x_0){\cdot}c(x_1) \leq c(x_0{+}x_1)$)) fof(goals, conjecture)

KLE015+1.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $((x_0 + x_1) \cdot c(x_0)) \cdot c(x_1) = 0$) fof(goals, conjecture)

KLE016+2.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ $c(x_0 \cdot x_1) = c(x_0) + c(x_1)$) fof(goals, conjecture)

KLE016+3.p The complement of a product is the sum of the complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: ((test(x_1) and test(x_0)) $\Rightarrow$ ($c(x_0{\cdot}x_1) \leq c(x_0){+}c(x_1)$ and $c(x_0){+}c(x_1) \leq c(x_0{\cdot}x_1)$)) fof(goals, conjecture)

KLE017+1.p Product of tests is their meet
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_0) and test(x_1) and test(x_2)) $\Rightarrow$ ($x_2 \leq x_0{\cdot}x_1 \iff$ ($x_2 \leq x_0$ and $x_2 \leq x_1$))) fof(goals, conjecture)

KLE018+1.p Move a term from the left of an implication to the right and back
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_0) and test(x_1) and test(x_2)) $\Rightarrow$ ($x_0 \cdot c(x_1) \leq x_2 \Rightarrow x_0 \leq x_1 + x_2$)) fof(goals, conjecture)

KLE019+1.p Move a term from the left of an implication to the right and back
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_0) and test(x_1) and test(x_2)) $\Rightarrow$ ($x_0 \cdot c(x_1) \leq x_2 \Leftarrow x_0 \leq x_1 + x_2$)) fof(goals, conjecture)

KLE020+1.p On tests addition distributes over multiplication
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_0) and test(x_1) and test(x_2)) $\Rightarrow$ $x_0 + x_1 \cdot x_2 = (x_0 + x_1) \cdot (x_0 + x_2)$) fof(goals, conjecture)

KLE020+2.p On tests addition distributes over multiplication
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_0) and test(x_1) and test(x_2)) $\Rightarrow$ ($x_0 + x_1 \cdot x_2 \leq (x_0 + x_1) \cdot (x_0 + x_2)$ and $(x_0 + x_1) \cdot (x_0 + x_2) \leq x_0 + x_1 \cdot x_2$)) fof(goals, conjecture)

KLE021+1.p Decompose splitting
Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: (test(x_1) $\Rightarrow$ $x_0 = x_1 \cdot x_0 + c(x_1) \cdot x_0$) fof(goals, conjecture)

KLE021+2.p Decompoase splitting
Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1$: (test(x_1) $\Rightarrow$ ($x_0 \leq x_1 \cdot x_0 + c(x_1) \cdot x_0$ and $x_1 \cdot x_0 + c(x_1) \cdot x_0 \leq x_0$)) fof(goals, conjecture)

KLE021+3.p Decompose splitting
Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow x_0 = x_1 \cdot x_0 + c(x_1) \cdot x_0)$ fof(goals, conjecture)

KLE021+4.p Decompose splitting
Decompose x into the part starting in p and the one starting in p's complement; also the rule if p then x else x = x where if p then y else z = p;y + c(p);z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow (x_0 \leq x_1 \cdot x_0 + c(x_1) \cdot x_0 \text{ and } x_1 \cdot x_0 + c(x_1) \cdot x_0 \leq x_0))$ fof(goals, conjecture)

KLE022+1.p Decompose x into parts ending in p and p's complement
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow x_0 = x_0 \cdot x_1 + x_0 \cdot c(x_1))$ fof(goals, conjecture)

KLE022+2.p Decompose x into parts ending in p and p's complement
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow (x_0 \leq x_0 \cdot x_1 + x_0 \cdot c(x_1) \text{ and } x_0 \cdot x_1 + x_0 \cdot c(x_1) \leq x_0))$ fof(goals, conjecture)

KLE022+3.p Decompose x into parts ending in p and p's complement
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow x_0 = x_0 \cdot x_1 + x_0 \cdot c(x_1))$ fof(goals, conjecture)

KLE022+4.p Decompose x into parts ending in p and p's complement
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1\colon (\mathrm{test}(x_1) \Rightarrow (x_0 \leq x_0 \cdot x_1 + x_0 \cdot c(x_1) \text{ and } x_0 \cdot x_1 + x_0 \cdot c(x_1) \leq x_0))$ fof(goals, conjecture)

KLE023+1.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{test}(x_1) \text{ and } \mathrm{test}(x_2)) \Rightarrow (x_1 \cdot x_0 + x_0 \cdot x_2 = x_0 \cdot x_2 \Rightarrow x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0))$ fof(goals, conjecture)

KLE023+2.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{test}(x_1) \text{ and } \mathrm{test}(x_2)) \Rightarrow (x_1 \cdot x_0 + x_0 \cdot x_2 = x_0 \cdot x_2 \Rightarrow x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0))$ fof(goals, conjecture)

KLE024+1.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{test}(x_1) \text{ and } \mathrm{test}(x_2)) \Rightarrow (x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0 \Rightarrow (x_1 \cdot x_0) \cdot c(x_2) = 0))$ fof(goals, conjecture)

KLE024+2.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{test}(x_1) \text{ and } \mathrm{test}(x_2)) \Rightarrow (x_0 \cdot c(x_2) + c(x_1) \cdot x_0 = c(x_1) \cdot x_0 \Rightarrow (x_1 \cdot x_0) \cdot c(x_2) = 0))$ fof(goals, conjecture)

KLE025+1.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{test}(x_1) \text{ and } \mathrm{test}(x_2)) \Rightarrow ((x_1 \cdot x_0) \cdot c(x_2) = 0 \Rightarrow x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2))$ fof(goals, conjecture)

KLE025+2.p Two ways of expressing the Hoare triple pxq

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2$: ((test(x_1) and test(x_2)) $\Rightarrow$ $((x_1 \cdot x_0) \cdot c(x_2) = 0 \Rightarrow x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2))$ fof(goals, conjecture)

KLE026+1.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2$: ((test(x_1) and test(x_2)) $\Rightarrow$ $(x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2 \Rightarrow x_1 \cdot x_0 \leq x_0 \cdot x_2))$ fof(goals, conjecture)

KLE026+2.p Two ways of expressing the Hoare triple pxq
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2$: ((test(x_1) and test(x_2)) $\Rightarrow$ $(x_1 \cdot x_0 = (x_1 \cdot x_0) \cdot x_2 \Rightarrow x_1 \cdot x_0 \leq x_0 \cdot x_2))$ fof(goals, conjecture)

KLE027+1.p Simplify conditional
Simplify conditional: if p then (if p then x else y) else z = if p then x else z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_4$)) $\Rightarrow$ $x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 = x_3 \cdot x_0 + c(x_3) \cdot x_2)$ fof(goals, conjecture)

KLE027+2.p Simplify conditional
Simplify conditional: if p then (if p then x else y) else z = if p then x else z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_4$)) $\Rightarrow$ $(x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 \leq x_3 \cdot x_0 + c(x_3) \cdot x_2$ and $x_3 \cdot x_0 + c(x_3) \cdot x_2 \leq x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2))$ fof(goals, conjecture)

KLE027+3.p Simplify conditional
Simplify conditional: if p then (if p then x else y) else z = if p then x else z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_4$)) $\Rightarrow$ $x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 = x_3 \cdot x_0 + c(x_3) \cdot x_2)$ fof(goals, conjecture)

KLE027+4.p Simplify conditional
Simplify conditional: if p then (if p then x else y) else z = if p then x else z.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_4$)) $\Rightarrow$ $(x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2 \leq x_3 \cdot x_0 + c(x_3) \cdot x_2$ and $x_3 \cdot x_0 + c(x_3) \cdot x_2 \leq x_3 \cdot (x_3 \cdot x_0 + c(x_3) \cdot x_1) + c(x_3) \cdot x_2))$ fof(goals, conjecture)

KLE028+1.p Switch nested conditions and rearrange branches of conditional
If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: ((test($x_4$) and test($x_5$)) $\Rightarrow$ $x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) = x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3))$ fof(goals, conjecture)

KLE028+2.p Switch nested conditions and rearrange branches of conditional
If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: ((test($x_4$) and test($x_5$)) $\Rightarrow$ $(x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) \leq x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3)$ and $x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \leq x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3)))$ fof(goals, conjecture)

KLE028+3.p Switch nested conditions and rearrange branches of conditional
If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')

$\forall x_0, x_1, x_2, x_3, x_4, x_5$: ((test($x_4$) and test($x_5$)) $\Rightarrow$ $x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) = x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3)$) fof(goals, conjecture)

KLE028+4.p Switch nested conditions and rearrange branches of conditional
If p then (if q then u else x) else (if q then y else z) = if q then (if p then u else y) else (if p then x else z).
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: ((test($x_4$) and test($x_5$)) $\Rightarrow$ ($x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3) \leq x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3)$ and $x_5 \cdot (x_4 \cdot x_0 + c(x_4) \cdot x_2) + c(x_5) \cdot (x_4 \cdot x_1 + c(x_4) \cdot x_3) \leq x_4 \cdot (x_5 \cdot x_0 + c(x_5) \cdot x_1) + c(x_4) \cdot (x_5 \cdot x_2 + c(x_5) \cdot x_3)$)) fof(goals, conjecture)

KLE029+1.p Characterisations of meet
Equivalence of characterisation as greatest lower bound and universal characterisation.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+3.ax')
$\forall x_0, x_1, x_2$: (ismeet(x_0, x_1, x_2) $\iff$ ismeetu(x_0, x_1, x_2)) fof(goals, conjecture)

KLE030+1.p Restriction can be pulled out of meet
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+3.ax')
$\forall x_0, x_1, x_2, x_3$: ((test($x_3$) and ismeet($x_0, x_1, x_2$)) $\Rightarrow$ ismeet($x_3 \cdot x_0, x_3 \cdot x_1, x_2$)) fof(goals, conjecture)

KLE031+1.p Restriction distributes through meet
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+3.ax')
$\forall x_0, x_1, x_2, x_3$: ((test($x_3$) and ismeet($x_0, x_1, x_2$)) $\Rightarrow$ ismeet($x_3 \cdot x_0, x_3 \cdot x_1, x_3 \cdot x_2$)) fof(goals, conjecture)

KLE032+1.p Meet of restrictions is restriction by meet
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+3.ax')
$\forall x_0, x_1, x_2$: ((test(x_2) and test(x_1)) $\Rightarrow$ ismeet($(x_1 \cdot x_2) \cdot x_0, x_1 \cdot x_0, x_2 \cdot x_0$)) fof(goals, conjecture)

KLE033+1.p Disjoint tests induce disjoint restrictions
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+3.ax')
$\forall x_0, x_1, x_2$: ((test(x_2) and test(x_1) and ismeet($0, x_1, x_2$)) $\Rightarrow$ ismeet($0, x_1 \cdot x_0, x_2 \cdot x_0$)) fof(goals, conjecture)

KLE034+1.p Hoare rule product
Encoding of Hoare rule pxq <description> qyr → px;yr.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_2$) and test($x_4$) and $(x_2 \cdot x_0) \cdot c(x_3) \leq 0$ and $(x_3 \cdot x_1) \cdot c(x_4) \leq 0$) $\Rightarrow$ $((x_2 \cdot x_0) \cdot x_1) \cdot c(x_4) \leq 0$) fof(goals, conjecture)

KLE034+2.p Hoare rule product
Encoding of Hoare rule pxq <description> qyr → px;yr.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((test($x_3$) and test($x_2$) and test($x_4$) and $(x_2 \cdot x_0) \cdot c(x_3) \leq 0$ and $(x_3 \cdot x_1) \cdot c(x_4) \leq 0$) $\Rightarrow$ $((x_2 \cdot x_0) \cdot x_1) \cdot c(x_4) \leq 0$) fof(goals, conjecture)

KLE035+1.p Hoare rule sum
Encoding of Hoare rule pxq <description> qyr → px;yr.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3$: ((test($x_3$) and test($x_2$) and $(x_2 \cdot x_0) \cdot c(x_3) \leq 0$ and $(x_2 \cdot x_1) \cdot c(x_3) \leq 0$) $\Rightarrow$ $(x_2 \cdot (x_0 + x_1)) \cdot c(x_3) \leq 0$) fof(goals, conjecture)

KLE035+2.p Hoare rule sum

Encoding of Hoare rule pxq <description> qyr → px;yr.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3$: $((\text{test}(x_3) \text{ and } \text{test}(x_2) \text{ and } (x_2 \cdot x_0) \cdot c(x_3) \leq 0 \text{ and } (x_2 \cdot x_1) \cdot c(x_3) \leq 0) \Rightarrow (x_2 \cdot (x_0 + x_1)) \cdot c(x_3) \leq 0)$ fof(goals, conjecture)

KLE036+1.p Star recursion
Together with star unfold this yields the classical recursion equation for star.
include('Axioms/KLE002+0.ax')
$\forall x_0$: $x_0^* \leq 1 + x_0 \cdot x_0^*$ fof(goals, conjecture)

KLE037+1.p Star reflexivity
Star is reflexive, i.e., contains the identity.
include('Axioms/KLE002+0.ax')
$\forall x_0$: $1 \leq x_0^*$ fof(goals, conjecture)

KLE038+1.p Star extensivity
Star is extensive.
include('Axioms/KLE002+0.ax')
$\forall x_0$: $x_0 \leq x_0^*$ fof(goals, conjecture)

KLE039+1.p Star idempotence
The star operation is idempotent.
include('Axioms/KLE002+0.ax')
$\forall x_0$: $(x_0^*)^* = x_0^*$ fof(goals, conjecture)

KLE039+2.p Star idempotence
The star operation is idempotent.
include('Axioms/KLE002+0.ax')
$\forall x_0$: $((x_0^*)^* \leq x_0^* \text{ and } x_0^* \leq (x_0^*)^*)$ fof(goals, conjecture)

KLE040+1.p The star of an element is multiplicatively idempotent
include('Axioms/KLE002+0.ax')
$\forall x_0$: $x_0^* \cdot x_0^* = x_0^*$ fof(goals, conjecture)

KLE040+2.p The star of an element is multiplicatively idempotent
include('Axioms/KLE002+0.ax')
$\forall x_0$: $(x_0^* \cdot x_0^* \leq x_0^* \text{ and } x_0^* \leq x_0^* \cdot x_0^*)$ fof(goals, conjecture)

KLE041+1.p Star isotony
The star operation is isotone.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1$: $(x_0 \leq x_1 \Rightarrow x_0^* \leq x_1^*)$ fof(goals, conjecture)

KLE042+1.p Star sliding
Two ways of grouping an alternation of x's and y's that starts and ends with x.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1)^* \cdot x_0 = x_0 \cdot (x_1 \cdot x_0)^*$ fof(goals, conjecture)

KLE042+2.p Star sliding
Two ways of grouping an alternation of x's and y's that starts and ends with x.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1$: $((x_0 \cdot x_1)^* \cdot x_0 \leq x_0 \cdot (x_1 \cdot x_0)^* \text{ and } x_0 \cdot (x_1 \cdot x_0)^* \leq (x_0 \cdot x_1)^* \cdot x_0)$ fof(goals, conjecture)

KLE043+1.p Star recursion
Classical recursion equation for star of x followed by y.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1$: $x_0^* \cdot x_1 = x_1 + (x_0 \cdot x_0^*) \cdot x_1$ fof(goals, conjecture)

KLE043+2.p Star recursion
Classical recursion equation for star of x followed by y.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1$: $(x_0^* \cdot x_1 \leq x_1 + (x_0 \cdot x_0^*) \cdot x_1 \text{ and } x_1 + (x_0 \cdot x_0^*) \cdot x_1 \leq x_0^* \cdot x_1)$ fof(goals, conjecture)

KLE044+1.p Star simplification
Identity can be eliminated from the star of a sum.

include('Axioms/KLE002+0.ax')
$\forall x_0\colon (1+x_0)^* = x_0^*$ fof(goals, conjecture)

KLE044+2.p Star simplification
Identity can be eliminated from the star of a sum.
include('Axioms/KLE002+0.ax')
$\forall x_0\colon ((1+x_0)^* \leq x_0^* \text{ and } x_0^* \leq (1+x_0)^*)$ fof(goals, conjecture)

KLE045+1.p Semi-commutation
If x semi-commutes over y transforming into z then the same applies to x and y*.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1, x_2\colon (x_0 \cdot x_2 \leq x_2 \cdot x_1 \;\Rightarrow\; x_0^* \cdot x_2 \leq x_2 \cdot x_1^*)$ fof(goals, conjecture)

KLE046+1.p Church-Rosser theorem
Normalise arbitrary sequences of x's and y's into x's followed by y's.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1\colon (x_1^* \cdot x_0^* \leq x_0^* \cdot x_1^* \;\Rightarrow\; (x_0+x_1)^* \leq x_0^* \cdot x_1^*)$ fof(goals, conjecture)

KLE047+1.p Star simplification
Semi-commutation of x and y allows normalizing arbitrary sequences of x's and y's.
include('Axioms/KLE002+0.ax')
$\forall x_0, x_1\colon (x_1 \cdot x_0 \leq x_0 \cdot x_1 \;\Rightarrow\; (x_0+x_1)^* \leq x_0^* \cdot x_1^*)$ fof(goals, conjecture)

KLE048+1.p The star of a test is always 1
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0\colon (\text{test}(x_0) \;\Rightarrow\; x_0^* = 1)$ fof(goals, conjecture)

KLE049+1.p Hoare rule while
Encoding of Hoare rule p;qxq → q while p do x c(p);q, where while p do x = (p;x)*;c(p).
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2\colon ((\text{test}(x_1) \text{ and } \text{test}(x_2) \text{ and } ((x_2 \cdot x_1) \cdot x_0) \cdot c(x_2) \leq 0) \;\Rightarrow\; ((x_2 \cdot (x_1 \cdot x_0)^*) \cdot c(x_1)) \cdot c(x_2) \leq 0)$ fof(goals, conjecture)

KLE049+2.p Hoare rule while
Encoding of Hoare rule p;qxq → q while p do x c(p);q, where while p do x = (p;x)*;c(p).
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2\colon ((\text{test}(x_1) \text{ and } \text{test}(x_2) \text{ and } ((x_2 \cdot x_1) \cdot x_0) \cdot c(x_2) \leq 0) \;\Rightarrow\; ((x_2 \cdot (x_1 \cdot x_0)^*) \cdot c(x_1)) \cdot c(x_2) \leq 0)$ fof(goals, conjecture)

KLE050+1.p Loop denesting
Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3\colon ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \;\Rightarrow\; (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) = (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2))$ fof(goals, conjecture)

KLE050+2.p Loop denesting
Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
$\forall x_0, x_1, x_2, x_3\colon ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \;\Rightarrow\; ((((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \text{ and } (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2)))$ fof(goals, conjecture)

KLE050+3.p Loop denesting
Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3\colon ((\text{test}(x_2) \text{ and } \text{test}(x_3)) \;\Rightarrow\; (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) = (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2))$ fof(goals, conjecture)

KLE050+4.p Loop denesting
Encoding of the law while p do (x ; while q do y) = if p then (x ; while p+q do if q then y else x) else skip.

include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
$\forall x_0, x_1, x_2, x_3$: $((\text{test}(x_2) \text{ and } \text{test}(x_3)) \Rightarrow ((((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \text{ and } (((x_2 \cdot x_0) \cdot (x_2 + x_3)) \cdot (x_3 \cdot x_1 + c(x_3) \cdot x_0)^*) \cdot c(x_2 + x_3) + c(x_2) \leq (((x_2 \cdot x_0) \cdot (x_3 \cdot x_1)^*) \cdot c(x_3))^* \cdot c(x_2)))$ fof(goals, conjecture)

KLE051+1.p Every domain semiring is automatically isotone
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $x_0 + x_0 = x_0$ fof(goals, conjecture)

KLE052+1.p Domain is a left invariant
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $\text{dom}(x_0) \cdot x_0 = x_0$ fof(goals, conjecture)

KLE053+1.p Domain is a projection
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $\text{dom}(\text{dom}(x_0)) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE054+1.p Domain is prefix increasing
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1$: $\text{dom}(x_0 \cdot x_1) + \text{dom}(x_0) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE055+1.p Domain expands subidentities
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $(x_0 + 1 = 1 \Rightarrow x_0 + \text{dom}(x_0) = \text{dom}(x_0))$ fof(goals, conjecture)

KLE056+1.p Domain is very strict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $(\text{dom}(x_0) = 0 \Rightarrow x_0 = 0)$ fof(goals, conjecture)

KLE057+1.p Domain is very strict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $(\text{dom}(x_0) = 0 \Leftarrow x_0 = 0)$ fof(goals, conjecture)

KLE058+1.p Domain is costrict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\text{dom}(1) = 1$ fof(goals, conjecture)

KLE059+1.p Domain is isotone
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1$: $(x_0 + x_1 = x_1 \Rightarrow \text{dom}(x_0) + \text{dom}(x_1) = \text{dom}(x_1))$ fof(goals, conjecture)

KLE060+1.p Domain elements can be exported
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1$: $\text{dom}(\text{dom}(x_0) \cdot x_1) = \text{dom}(x_0) \cdot \text{dom}(x_1)$ fof(goals, conjecture)

KLE061+1.p Domain elements are multiplicatively idempotent
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0$: $\text{dom}(x_0) \cdot \text{dom}(x_0) = \text{dom}(x_0)$ fof(goals, conjecture)

KLE062+1.p Domain elements are multiplicatively commutative
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1$: $\text{dom}(x_0) \cdot \text{dom}(x_1) = \text{dom}(x_1) \cdot \text{dom}(x_0)$ fof(goals, conjecture)

KLE063+1.p Domain elements are least left preservers
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon (x_0 + \mathrm{dom}(x_1) \cdot x_0 = \mathrm{dom}(x_1) \cdot x_0 \;\Rightarrow\; \mathrm{dom}(x_0) + \mathrm{dom}(x_1) = \mathrm{dom}(x_1))$ fof(goals, conjecture)

KLE064+1.p Domain elements are least left preservers
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon (x_0 + \mathrm{dom}(x_1) \cdot x_0 = \mathrm{dom}(x_1) \cdot x_0 \;\Leftarrow\; \mathrm{dom}(x_0) + \mathrm{dom}(x_1) = \mathrm{dom}(x_1))$ fof(goals, conjecture)

KLE065+1.p Domain is weakly local
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon (x_0 \cdot x_1 = 0 \;\Rightarrow\; x_0 \cdot \mathrm{dom}(x_1) = 0)$ fof(goals, conjecture)

KLE066+1.p Domain is weakly local
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon (x_0 \cdot x_1 = 0 \;\Leftarrow\; x_0 \cdot \mathrm{dom}(x_1) = 0)$ fof(goals, conjecture)

KLE067+1.p Domain elements are closed under addition
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon \mathrm{dom}(\mathrm{dom}(x_0) + \mathrm{dom}(x_1)) = \mathrm{dom}(x_0) + \mathrm{dom}(x_1)$ fof(goals, conjecture)

KLE068+1.p Domain elements are closed under multiplication
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon \mathrm{dom}(\mathrm{dom}(x_0) \cdot \mathrm{dom}(x_1)) = \mathrm{dom}(x_0) \cdot \mathrm{dom}(x_1)$ fof(goals, conjecture)

KLE069+1.p Domain elements satisfy the first lattice absorption law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon \mathrm{dom}(x_0) \cdot (\mathrm{dom}(x_0) + \mathrm{dom}(x_1)) = \mathrm{dom}(x_0)$ fof(goals, conjecture)

KLE070+1.p Domain elements satisfy the second lattice absorption law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon \mathrm{dom}(x_0) + \mathrm{dom}(x_0) \cdot \mathrm{dom}(x_1) = \mathrm{dom}(x_0)$ fof(goals, conjecture)

KLE071+1.p Domain elements satisfy one of the lattice distributivity laws
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1, x_2\colon \mathrm{dom}(x_0) + \mathrm{dom}(x_1) \cdot \mathrm{dom}(x_2) = (\mathrm{dom}(x_0) + \mathrm{dom}(x_1)) \cdot (\mathrm{dom}(x_0) + \mathrm{dom}(x_2))$ fof(goals, conjecture)

KLE072+1.p Domain elements satisfy the first axiom of Kleene modules
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1, x_2\colon \mathrm{dom}((x_0 + x_1) \cdot \mathrm{dom}(x_2)) = \mathrm{dom}(x_0 \cdot \mathrm{dom}(x_2)) + \mathrm{dom}(x_1 \cdot \mathrm{dom}(x_2))$ fof(goals, conjecture)

KLE073+1.p Domain elements satisfy the second Kleene module axiom
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0, x_1, x_2\colon \mathrm{dom}(x_0 \cdot (\mathrm{dom}(x_1) + \mathrm{dom}(x_2))) = \mathrm{dom}(x_0 \cdot \mathrm{dom}(x_1)) + \mathrm{dom}(x_0 \cdot \mathrm{dom}(x_2))$ fof(goals, conjecture)

KLE074+1.p Domain elements satisfy the third Kleene module axiom.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1, x_2\colon \mathrm{dom}((x_0 \cdot x_1) \cdot \mathrm{dom}(x_2)) = \mathrm{dom}(x_0 \cdot \mathrm{dom}(x_1 \cdot \mathrm{dom}(x_2)))$ fof(goals, conjecture)

KLE075+1.p Domain elements satisfy the fourth Kleene module axiom
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon \mathrm{dom}(1 \cdot \mathrm{dom}(x_0)) = \mathrm{dom}(x_0)$ fof(goals, conjecture)

KLE076+1.p Domain elements satisfy the fifth Kleene module axiom
include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+5.ax')
$\forall x_0\colon \mathrm{dom}(x_0 \cdot \mathrm{dom}(0)) = 0 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE077+1.p Domain elements satisfy the sixth Kleene module axiom
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1, x_2\colon ((\mathrm{dom}(x_0)+\mathrm{dom}(x_1 \cdot \mathrm{dom}(x_2)))+\mathrm{dom}(x_2) = \mathrm{dom}(x_2) \Rightarrow \mathrm{dom}(x_1^* \cdot \mathrm{dom}(x_0))+\mathrm{dom}(x_2) = \mathrm{dom}(x_2)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{co}$

KLE078+1.p Antidomain elements are domain elements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon (\forall x_1\colon (\mathrm{dom}(x_1) + \mathrm{ad}(x_1) = 1 \text{ and } \mathrm{dom}(x_1) \cdot \mathrm{ad}(x_1) = 0) \Rightarrow \mathrm{dom}(\mathrm{ad}(x_0)) = \mathrm{ad}(x_0)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE079+1.p Domain and antidomain elements are complements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon (\forall x_1\colon (\mathrm{dom}(x_1) + \mathrm{ad}(x_1) = 1 \text{ and } \mathrm{dom}(x_1) \cdot \mathrm{ad}(x_1) = 0) \Rightarrow \mathrm{ad}(x_0) \cdot \mathrm{dom}(x_0) = 0) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE080+1.p Another complementation property of domain and antidomain elements
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon (\forall x_1\colon (\mathrm{dom}(x_1) + \mathrm{ad}(x_1) = 1 \text{ and } \mathrm{dom}(x_1) \cdot \mathrm{ad}(x_1) = 0) \Rightarrow \mathrm{ad}(\mathrm{ad}(x_0)) = \mathrm{dom}(x_0)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE081+1.p Antidomain elements are left annihilators
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon (\forall x_1\colon (\mathrm{dom}(x_1) + \mathrm{ad}(x_1) = 1 \text{ and } \mathrm{dom}(x_1) \cdot \mathrm{ad}(x_1) = 0) \Rightarrow \mathrm{ad}(x_0) \cdot x_0 = 0) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE082+1.p Antidomain is local with respect to multiplication
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0, x_1\colon (\forall x_2\colon (\mathrm{dom}(x_2) + \mathrm{ad}(x_2) = 1 \text{ and } \mathrm{dom}(x_2) \cdot \mathrm{ad}(x_2) = 0) \Rightarrow \mathrm{ad}(x_0 \cdot x_1) + \mathrm{ad}(x_0 \cdot \mathrm{dom}(x_1)) = \mathrm{ad}(x_0 \cdot \mathrm{dom}(x_1))) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE083+1.p Domain is a left invariant
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0\colon x_0 = \mathrm{dom}(x_0) \cdot x_0 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE084+1.p Domain is local with respect to multiplication
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0, x_1\colon \mathrm{dom}(x_0 \cdot x_1) = \mathrm{dom}(x_0 \cdot \mathrm{dom}(x_1)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE085+1.p Domain elements are subidenties
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0\colon \mathrm{dom}(x_0) + 1 = 1 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE086+1.p Domain is strict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\mathrm{dom}(0) = 0 \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE087+1.p Domain is additive
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0, x_1\colon \mathrm{dom}(x_0 + x_1) = \mathrm{dom}(x_0) + \mathrm{dom}(x_1) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE088+1.p Antidomain elements are greatest left annihilators
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0, x_1\colon (\mathrm{dom}(x_0) \cdot x_1 = 0 \Rightarrow \mathrm{dom}(x_0) + \mathrm{ad}(x_1) = \mathrm{ad}(x_1)) \qquad \mathrm{fof}(\mathrm{goals}, \mathrm{conjecture})$

KLE089+1.p Antidomain elements are greatest left annihilators
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')

$\forall x_0, x_1\colon (\text{dom}(x_0) \cdot x_1 = 0 \;\Leftarrow\; \text{dom}(x_0) + \text{ad}(x_1) = \text{ad}(x_1))$ fof(goals, conjecture)

KLE090+1.p Antidomain elements are antitone
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0, x_1\colon (x_0 + x_1 = x_1 \;\Rightarrow\; \text{ad}(x_1) + \text{ad}(x_0) = \text{ad}(x_0))$ fof(goals, conjecture)

KLE091+1.p Codomain closure
In a Boolean domain semiring, codomain elements are domain elements.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0\colon \text{dom}(\text{cod}(x_0)) = \text{cod}(x_0)$ fof(goals, conjecture)

KLE092+1.p Coantidomain closure
In a Boolean domain semiring, coantidomain elements are domain elements.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
$\forall x_0\colon \text{dom}(\text{coad}(x_0)) = \text{coad}(x_0)$ fof(goals, conjecture)

KLE093+1.p Domain of star
In a Kleene algebra with domain, the domain of a star is always one.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+5.ax')
$\forall x_0\colon \text{dom}(x_0^*) = 1$ fof(goals, conjecture)

KLE094+1.p Segerberg
Segerberg's formula holds in every Kleene algebra with Boolean domain.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1\colon \text{domain\_difference}(\text{forward\_diamond}(x_0^*, \text{dom}(x_1)), \text{dom}(x_1)) + \text{forward\_diamond}(x_0^*, \text{domain\_difference}(\text{forward\_diamo}$
$\text{forward\_diamond}(x_0^*, \text{domain\_difference}(\text{forward\_diamond}(x_0, \text{dom}(x_1)), \text{dom}(x_1)))$ fof(goals, conjecture)

KLE095+1.p Modal operators satisfy a star unfold law
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1\colon \text{dom}(x_0) + \text{forward\_diamond}(x_1, \text{forward\_diamond}(x_1^*, \text{dom}(x_0))) = \text{forward\_diamond}(x_1^*, \text{dom}(x_0))$ fof(goals, conje

KLE096+1.p Modal operators satisfy a star unfold law
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1\colon \text{dom}(x_0) + \text{forward\_diamond}(x_1^*, \text{forward\_diamond}(x_1, \text{dom}(x_0))) = \text{forward\_diamond}(x_1^*, \text{dom}(x_0))$ fof(goals, conje

KLE097+1.p Modal operators at left hand-sides can be eliminated
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2\colon (\text{forward\_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \;\Leftarrow\; x_0 \cdot \text{dom}(x_1) + \text{dom}(x_2) \cdot x_0 = \text{dom}(x_2) \cdot x_0)$ fof(goals, conjecture)

KLE098+1.p Modal operators at left hand-sides can be eliminated
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2\colon (\text{forward\_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \;\Rightarrow\; x_0 \cdot \text{dom}(x_1) + \text{dom}(x_2) \cdot x_0 = \text{dom}(x_2) \cdot x_0)$ fof(goals, conjecture)

KLE099+1.p Modal operators at left hand-sides can be eliminated
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2\colon (\text{forward\_diamond}(x_0, \text{dom}(x_1)) + \text{dom}(x_2) = \text{dom}(x_2) \;\Rightarrow\; \text{ad}(x_2) \cdot (x_0 \cdot \text{dom}(x_1)) = 0)$ fof(goals, conjecture)

KLE100+1.p Modal operators at left hand-sides can be eliminated

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_diamond(x_0, dom(x_1))+dom(x_2) = dom(x_2) $\Leftarrow$ ad(x_2)·(x_0·dom(x_1)) = 0) fof(goals, conjecture)

KLE101+1.p Forward and backward diamonds are conjugates
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_diamond(x_0, dom(x_1))·dom(x_2) = 0 $\Rightarrow$ dom(x_1)·backward_diamond(x_0, dom(x_2)) = 0) fof(goals, co

KLE102+1.p Forward and backward diamonds are conjugates
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_diamond(x_0, dom(x_1))·dom(x_2) = 0 $\Leftarrow$ dom(x_1)·backward_diamond(x_0, dom(x_2)) = 0) fof(goals, co

KLE103+1.p Forward and backward boxes are conjugates
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_box(x_0, dom(x_1))+dom(x_2) = 1 $\Rightarrow$ dom(x_1)+backward_box(x_0, dom(x_2)) = 1) fof(goals, conjecture

KLE104+1.p Forward and backward boxes are conjugates
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_box(x_0, dom(x_1))+dom(x_2) = 1 $\Leftarrow$ dom(x_1)+backward_box(x_0, dom(x_2)) = 1) fof(goals, conjecture

KLE105+1.p Galois
Forward diamonds and backward boxes are adjoints in a Galois connection.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_diamond(x_0, dom(x_1)) + dom(x_2) = dom(x_2) $\Rightarrow$ dom(x_1) + backward_box(x_0, dom(x_2)) = backward_box(x_0, dom(x_2))) fof(goals, conjecture)

KLE106+1.p Galois
Forward diamonds and backward boxes are adjoints in a Galois connection.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (forward_diamond(x_0, dom(x_1)) + dom(x_2) = dom(x_2) $\Leftarrow$ dom(x_1) + backward_box(x_0, dom(x_2)) = backward_box(x_0, dom(x_2))) fof(goals, conjecture)

KLE107+1.p Galois
Backward diamonds and forward boxes are adjoints of a Galois connection.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (backward_diamond(x_0, dom(x_1)) + dom(x_2) = dom(x_2) $\Rightarrow$ dom(x_1) + forward_box(x_0, dom(x_2)) = forward_box(x_0, dom(x_2))) fof(goals, conjecture)

KLE108+1.p Galois
Backward diamonds and forward boxes are adjoints of a Galois connection.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: (backward_diamond(x_0, dom(x_1)) + dom(x_2) = dom(x_2) $\Leftarrow$ dom(x_1) + forward_box(x_0, dom(x_2)) = forward_box(x_0, dom(x_2))) fof(goals, conjecture)

KLE109+1.p Forward diamonds and backward boxes satisfy a cancellation law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')

$\forall x_0, x_1$: forward_diamond(x_0, backward_box(x_0, dom(x_1))) + dom(x_1) = dom(x_1)   fof(goals, conjecture)

KLE110+1.p Forward diamonds and backward boxes satisfy a cancellation law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: dom(x_0)+backward_box(x_1, forward_diamond(x_1, dom(x_0))) = backward_box(x_1, forward_diamond(x_1, dom(x_0)))

KLE111+1.p Backward diamonds and forward boxes satisfy a cancellation law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: backward_diamond(x_0, forward_box(x_0, dom(x_1))) + dom(x_1) = dom(x_1)   fof(goals, conjecture)

KLE112+1.p Backward diamonds and forward boxes satisfy a cancellation law
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: dom(x_0)+forward_box(x_1, backward_diamond(x_1, dom(x_0))) = forward_box(x_1, backward_diamond(x_1, dom(x_0)))

KLE113+1.p Diamonds are strict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0$: forward_diamond(x_0, 0) = 0   fof(goals, conjecture)

KLE114+1.p Boxes are costrict
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0$: forward_box(x_0, 1) = 1   fof(goals, conjecture)

KLE115+1.p Diamonds are additive
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: forward_diamond(x_0, dom(x_1)+dom(x_2)) = forward_diamond(x_0, dom(x_1))+forward_diamond(x_0, dom(x_2))   fo

KLE116+1.p Boxes are multiplicative
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: forward_box(x_0, dom(x_1)·dom(x_2)) = forward_box(x_0, dom(x_1))·forward_box(x_0, dom(x_2))   fof(goals, conjectu

KLE117+1.p Diamonds are isotone
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: (x_0+x_1 = x_1 $\Rightarrow$ $\forall x_2$: forward_diamond(x_0, dom(x_2))+forward_diamond(x_1, dom(x_2)) = forward_diamond(x_1, dom(x

KLE118+1.p Boxes are antitone
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: (x_0+x_1 = x_1 $\Rightarrow$ $\forall x_2$: forward_box(x_0, dom(x_2))+forward_box(x_1, dom(x_2)) = forward_box(x_0, dom(x_2)))   fof(goa

KLE119+1.p Validity of abort rule
The abort rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: backward_diamond(0, dom(x_0)) + dom(x_1) = dom(x_1)   fof(goals, conjecture)

KLE120+1.p Validity of skip rule
The skip rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0$: backward_diamond$(1, \mathrm{dom}(x_0)) + \mathrm{dom}(x_0) = \mathrm{dom}(x_0)$ fof(goals, conjecture)

KLE121+1.p Validity of composition rule
The composition rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: ((backward_diamond$(x_3, \mathrm{dom}(x_0)) + \mathrm{dom}(x_1) = \mathrm{dom}(x_1)$ and backward_diamond$(x_4, \mathrm{dom}(x_1)) + \mathrm{dom}(x_2) = \mathrm{dom}(x_2)) \Rightarrow$ backward_diamond$(x_3 \cdot x_4, \mathrm{dom}(x_0)) + \mathrm{dom}(x_2) = \mathrm{dom}(x_2))$ fof(goals, conjecture)

KLE122+1.p Validity of conditional rule
The conditional rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: if_then_else$(x_0, x_1, x_2) = \mathrm{dom}(x_0) \cdot x_1 + \mathrm{ad}(x_0) \cdot x_2$ fof(if_then_else_definition, axiom)
$\forall x_0, x_1, x_2, x_3, x_4$: ((backward_diamond$(x_0, \mathrm{dom}(x_2) \cdot \mathrm{dom}(x_3)) + \mathrm{dom}(x_4) = \mathrm{dom}(x_4)$ and backward_diamond$(x_1, \mathrm{ad}(x_2) \cdot \mathrm{dom}(x_3)) + \mathrm{dom}(x_4) = \mathrm{dom}(x_4)) \Rightarrow$ backward_diamond(if_then_else$(x_2, x_0, x_1), \mathrm{dom}(x_3)) + \mathrm{dom}(x_4) = \mathrm{dom}(x_4))$ fof(goals

KLE123+1.p Validity of while rule
The while rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1$: while_do$(x_1, x_0) = (\mathrm{dom}(x_1) \cdot x_0)^* \cdot \mathrm{ad}(x_1)$ fof(while_do_definition, axiom)
$\forall x_0, x_1, x_2$: (backward_diamond$(x_0, \mathrm{dom}(x_1) \cdot \mathrm{dom}(x_2)) + \mathrm{dom}(x_2) = \mathrm{dom}(x_2) \Rightarrow$ backward_diamond(while_do(x_1, x_0), dom(x_2 $\mathrm{dom}(x_2) = \mathrm{dom}(x_2))$ fof(goals, conjecture)

KLE124+1.p Validity of weakening rule
The weakening rule of Hoare logic is valid with respect to the Kleene algebra semantics.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2, x_3, x_4$: $((\mathrm{dom}(x_2) + \mathrm{dom}(x_1) = \mathrm{dom}(x_1)$ and backward_diamond$(x_0, \mathrm{dom}(x_1)) + \mathrm{dom}(x_3) = \mathrm{dom}(x_3)$ and $\mathrm{dom}(x_3)+$ $\mathrm{dom}(x_4) = \mathrm{dom}(x_4)) \Rightarrow$ backward_diamond$(x_0, \mathrm{dom}(x_2)) + \mathrm{dom}(x_4) = \mathrm{dom}(x_4))$ fof(goals, conjecture)

KLE125+1.p Quasicommutation theorem
If x quasicommutes over y, then x+y terminates if x and y individually do.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 + x_1 \cdot (x_1 + x_0)^* = x_1 \cdot (x_1 + x_0)^* \Rightarrow$ (divergence$(x_1 + x_0) = 0 \iff$ divergence(x_1) + divergence$(x_0) = 0))$ fof(goals, conjecture)

KLE126+1.p Lazy commutation theorem
If x lazily commutes over y, then x+y terminates if x and y individually do.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 + x_1 \cdot (x_1 + x_0)^* = x_1 \cdot (x_1 + x_0)^* + x_0 \Rightarrow$ (divergence$(x_1 + x_0) = 0 \iff$ divergence(x_1) + divergence$(x_0) = 0))$ fof(goals, conjecture)

KLE127+1.p Loop refinement theorem
If x quasicommutes over y, then all infinite behaviours of x+y can be separated into infinite behaviours of x and a infinite behaviour of y after finitely many x steps.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')

$\forall x_0, x_1$: $(x_0{\cdot}x_1{+}x_1{\cdot}(x_1{+}x_0)^* = x_1{\cdot}(x_1{+}x_0)^* \Rightarrow$ divergence$(x_1{+}x_0) =$ divergence$(x_1){+}$forward_diamond$(x_1^*,$ divergence$(x_0)))$

KLE128+1.p Comparison of two different notions of termination
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (divergence$(x_0) = 0 \Rightarrow \forall x_1$: (dom$(x_1)$ + forward_diamond$(x_0,$ dom$(x_1))$ = forward_diamond$(x_0,$ dom$(x_1)) \Rightarrow$ dom$(x_1) = 0))$ fof(goals, conjecture)

KLE129+1.p Comparison of two different notions of termination
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (divergence$(x_0) = 0 \Leftarrow \forall x_1$: (dom$(x_1)$ + forward_diamond$(x_0,$ dom$(x_1))$ = forward_diamond$(x_0,$ dom$(x_1)) \Rightarrow$ dom$(x_1) = 0))$ fof(goals, conjecture)

KLE130+1.p Two notions of termination
The lemma compares absence of divergence with a third notion of termination.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (divergence$(x_0) = 0 \Rightarrow \forall x_1$: dom$(x_1)$+forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1))$
forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1)))))$ fof(goals, conjecture)

KLE131+1.p Two notions of termination
The lemma compares absence of divergence with a third notion of termination.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (divergence$(x_0) = 0 \Leftarrow \forall x_1$: dom$(x_1)$+forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1))$
forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1)))))$ fof(goals, conjecture)

KLE132+1.p Every element that satisfies Loeb's formula is wellfounded
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: ($\forall x_1$: forward_diamond$(x_0,$ dom$(x_1))$+forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1)))$
forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1)))) \Rightarrow \forall x_2$: (dom$(x_2)$+forward_diamond$(x_0,$ d
forward_diamond$(x_0,$ dom$(x_2)) \Rightarrow$ dom$(x_2) = 0))$ fof(goals, conjecture)

KLE133+1.p Loeb's formula and wellfoundedness
Every diamond transitive wellfounded element satisfies Loeb's formula.
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (($\forall x_1$: dom(x_1)+forward_diamond$(x_0^*,$ domain_difference(dom$(x_1),$ forward_diamond$(x_0,$ dom$(x_1))))$ = forward_diamond(
forward_diamond$(x_0,$ dom$(x_2))) \Rightarrow \forall x_3$: forward_diamond$(x_0,$ dom$(x_3))$+forward_diamond$(x_0^*,$ domain_difference(dom$(x_3),$ fo
forward_diamond$(x_0^*,$ domain_difference(dom$(x_3),$ forward_diamond$(x_0,$ dom$(x_3)))))$ fof(goals, conjecture)

KLE134+1.p A no name lemma
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
$\forall x_0, x_1, x_2$: domain_difference(forward_diamond$(x_0,$ dom$(x_1)),$ forward_diamond$(x_0,$ dom$(x_2)))$+forward_diamond$(x_0,$ domain
forward_diamond$(x_0,$ domain_difference(dom$(x_1),$ dom$(x_2)))$ fof(goals, conjecture)

KLE135+1.p Two notions of termination
If an element does not diverge, then it always becomes disabled after finitely many steps.

include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0$: (divergence(x_0) = 0 $\Rightarrow$ forward_diamond(x_0^*, ad(x_0)) = 1) fof(goals, conjecture)

KLE136+1.p Newman's lemma holds in divergence Kleene algebras
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')
$\forall x_0, x_1$: ((divergence(x_0+x_1) = 0 and $x_1 \cdot x_0 + x_0^* \cdot x_1^* = x_0^* \cdot x_1^*$) $\Rightarrow$ $(x_0+x_1)^* + x_0^* \cdot x_1^* = x_0^* \cdot x_1^*$) fof(goals, conjecture)

KLE137+1.p There is a greatest element, namely 1∧infty
include('Axioms/KLE004+0.ax')
$\forall x_0$: $x_0 \leq$ strong_iteration(1) fof(goals, conjecture)

KLE138+1.p Strong iteration of a abort is miracle
include('Axioms/KLE004+0.ax')
strong_iteration(0) = 1 fof(goals, conjecture)

KLE139+1.p Dual unfold
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration(x_0) = strong_iteration(x_0) $\cdot\, x_0 + 1$ fof(goals, conjecture)

KLE139+2.p Dual unfold
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration(x_0) $\leq$ strong_iteration(x_0)$\cdot x_0$+1 and strong_iteration(x_0)$\cdot x_0$+1 $\leq$ strong_iteration(x_0)) fof(goals, co

KLE140+1.p Isotonicity of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: ($x_0 \leq x_1$ $\Rightarrow$ strong_iteration(x_0) $\leq$ strong_iteration(x_1)) fof(goals, conjecture)

KLE141+1.p The greatest is left annihilator
The greatest is left annihilator; a miracle followed by anything is again a miracle.
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration(1) $\cdot\, x_0$ = strong_iteration(1) fof(goals, conjecture)

KLE141+2.p The greatest is left annihilator
The greatest is left annihilator; a miracle followed by anything is again a miracle.
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration(1)$\cdot x_0$ $\leq$ strong_iteration(1) and strong_iteration(1) $\leq$ strong_iteration(1)$\cdot x_0$) fof(goals, conjecture)

KLE142+1.p If strong iteration is applied twice, a miracle occurs.
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration(strong_iteration(x_0)) = strong_iteration(1) fof(goals, conjecture)

KLE142+2.p If strong iteration is applied twice, a miracle occurs.
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration(strong_iteration(x_0)) $\leq$ strong_iteration(1) and strong_iteration(1) $\leq$ strong_iteration(strong_iteration(

KLE143+1.p Strong iteration is idempotent w.r.t. multiplication
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration(x_0) $\cdot$ strong_iteration(x_0) = strong_iteration(x_0) fof(goals, conjecture)

KLE143+2.p Strong iteration is idempotent w.r.t. multiplication
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration(x_0)$\cdot$strong_iteration(x_0) $\leq$ strong_iteration(x_0) and strong_iteration(x_0) $\leq$ strong_iteration(x_0)$\cdot$ strong_iteration(x_0)) fof(goals, conjecture)

KLE144+1.p Strong iteration applied after finite iteration is magic.
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration(x_0^*) = strong_iteration(1) fof(goals, conjecture)

KLE144+2.p Strong iteration applied after finite iteration is magic.
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration(x_0^*) $\leq$ strong_iteration(1) and strong_iteration(1) $\leq$ strong_iteration(x_0^*)) fof(goals, conjecture)

KLE145+1.p Finite iteration after strong iteration is strong iteration

include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration$(x_0)^* =$ strong_iteration(x_0) fof(goals, conjecture)

KLE145+2.p Finite iteration after strong iteration is strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration$(x_0)^* \leq$ strong_iteration(x_0) and strong_iteration$(x_0) \leq$ strong_iteration$(x_0)^*$) fof(goals, conjecture)

KLE146+1.p Skip is part of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0$: $1 \leq$ strong_iteration(x_0) fof(goals, conjecture)

KLE147+1.p Sliding of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: strong_iteration$(x_0^* \cdot x_1) = x_1^* \cdot$ strong_iteration$(x_0^* \cdot x_1)$ fof(goals, conjecture)

KLE147+2.p Sliding of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: (strong_iteration$(x_0^* \cdot x_1) \leq x_1^* \cdot$strong_iteration$(x_0^* \cdot x_1)$ and $x_1^* \cdot$strong_iteration$(x_0^* \cdot x_1) \leq$ strong_iteration$(x_0^* \cdot x_1)$) fof(goals, conjecture)

KLE148+1.p Blocking law
If y is blocked by x then x before strong iteration of y reduces to x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot$ strong_iteration$(x_1) = x_0)$ fof(goals, conjecture)

KLE148+2.p Blocking law
If y is blocked by x then x before strong iteration of y reduces to x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $((x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot$ strong_iteration$(x_1) \leq x_0)$ and $x_0 \leq x_0 \cdot$ strong_iteration$(x_1))$ fof(goals, conjecture)

KLE149+1.p Strong version of unfold
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: strong_iteration$(x_0 + x_1) = (x_1^* \cdot x_0) \cdot$ strong_iteration$(x_0 + x_1) +$ strong_iteration(x_1) fof(goals, conjecture)

KLE149+2.p Strong version of unfold
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: (strong_iteration$(x_0 + x_1) \leq (x_1^* \cdot x_0) \cdot$strong_iteration$(x_0 + x_1) +$strong_iteration$(x_1)$ and $(x_1^* \cdot x_0) \cdot$strong_iteration$(x_0 + x_1) +$ strong_iteration$(x_1) \leq$ strong_iteration$(x_0 + x_1)$) fof(goals, conjecture)

KLE150+1.p Iterating non-terminating elements reduces to the element itself
include('Axioms/KLE004+0.ax')
$\forall x_0$: strong_iteration$(x_0 \cdot 0) = 1 + x_0 \cdot 0$ fof(goals, conjecture)

KLE150+2.p Iterating non-terminating elements reduces to the element itself
include('Axioms/KLE004+0.ax')
$\forall x_0$: (strong_iteration$(x_0 \cdot 0) \leq 1 + x_0 \cdot 0$ and $1 + x_0 \cdot 0 \leq$ strong_iteration$(x_0 \cdot 0)$) fof(goals, conjecture)

KLE151+1.p Sliding of strong iteration
Two ways of grouping an alternation of x's and y's that starts and ends with x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $x_0 \cdot$ strong_iteration$(x_1 \cdot x_0) \leq$ strong_iteration$(x_0 \cdot x_1) \cdot x_0$ fof(goals, conjecture)

KLE152+1.p Sliding of strong iteration
Two ways of grouping an alternation of x's and y's that starts and ends with x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: strong_iteration$(x_0 \cdot x_1) \cdot x_0 \leq x_0 \cdot$ strong_iteration$(x_1 \cdot x_0)$ fof(goals, conjecture)

KLE153+1.p Sliding of strong iteration
Two ways of grouping an alternation of x's and y's that starts and ends with x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $x_0 \cdot$ strong_iteration$(x_1 \cdot x_0) =$ strong_iteration$(x_0 \cdot x_1) \cdot x_0$ fof(goals, conjecture)

KLE154+1.p Denesting of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: strong_iteration$(x_0 + x_1) =$ strong_iteration$(x_0) \cdot$strong_iteration$(x_1 \cdot$strong_iteration$(x_0))$ fof(goals, conjecture)

KLE154+2.p Denesting of strong iteration
include('Axioms/KLE004+0.ax')

$\forall x_0, x_1$: (strong_iteration(x_0+x_1) $\leq$ strong_iteration(x_0)$\cdot$strong_iteration($x_1\cdot$strong_iteration(x_0)) and strong_iteration(x_0)$\cdot$ strong_iteration($x_1 \cdot$ strong_iteration(x_0)) $\leq$ strong_iteration($x_0 + x_1$)) fof(goals, conjecture)

KLE155+1.p Denesting of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: strong_iteration($x_0^* \cdot x_1$) $\cdot$ strong_iteration(x_0) = strong_iteration($x_0 + x_1$) fof(goals, conjecture)

KLE155+2.p Denesting of strong iteration
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: (strong_iteration($x_0 + x_1$) $\leq$ strong_iteration($x_0^* \cdot x_1$) $\cdot$ strong_iteration(x_0) and strong_iteration($x_0^* \cdot x_1$) $\cdot$ strong_iteration(x_0) $\leq$ strong_iteration($x_0 + x_1$)) fof(goals, conjecture)

KLE156+1.p Semicommuation law of finite iteration
If x semicommutes over y every finite sequence of x's and y's can be rearranged to a finite sequence of x's followed by finite sequence of y's.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow (x_1 + x_0)^* = x_1^* \cdot x_0^*)$ fof(goals, conjecture)

KLE156+2.p Semicommuation law of finite iteration
If x semicommutes over y every finite sequence of x's and y's can be rearranged to a finite sequence of x's followed by finite sequence of y's.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $((x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow (x_1 + x_0)^* \leq x_1^* \cdot x_0^*)$ and $x_1^* \cdot x_0^* \leq (x_1 + x_0)^*)$ fof(goals, conjecture)

KLE157+1.p Semicommuation law of finite iteration
If x semicommutes over y every sequence of x's and y's can be rearranged to a sequence of x's followed by sequence of y's.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: ($x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow$ strong_iteration(x_1+x_0) = strong_iteration(x_1)$\cdot$strong_iteration(x_0)) fof(goals, conjecture)

KLE157+2.p Semicommuation law of finite iteration
If x semicommutes over y every sequence of x's and y's can be rearranged to a sequence of x's followed by sequence of y's.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: (($x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow$ strong_iteration(x_1+x_0) $\leq$ strong_iteration(x_1)$\cdot$strong_iteration(x_0)) and strong_iteration(x_1)$\cdot$ strong_iteration(x_0) $\leq$ strong_iteration($x_1 + x_0$)) fof(goals, conjecture)

KLE158+1.p Simulation law for data refinement
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2$: ($x_0 \cdot x_1 \leq x_2 \cdot x_0 \Rightarrow x_0 \cdot$ strong_iteration(x_1) $\leq$ strong_iteration(x_2) $\cdot x_0$) fof(goals, conjecture)

KLE159+1.p Simulation law for data refinement
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2$: $(x_0 \cdot x_1 \leq x_2 \cdot x_0 \Rightarrow x_0 \cdot x_1^* \leq x_2^* \cdot x_0)$ fof(goals, conjecture)

KLE160+1.p Simulation law for data refinement
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2$: $(x_0 \cdot x_1 \leq x_1 \cdot x_2 \Rightarrow x_0^* \cdot x_1 \leq x_1 \cdot x_2^*)$ fof(goals, conjecture)

KLE161+1.p Data refinement
The first hypothesis says that b cannot loop infinitely. The second hypothesis says that a is data refined by aa w.r.t. upward simulation z. By the third hypothesis, 1 is data refined by b. The fourth and fifth condition expresses the standard data refinement of initialisations and finalisations.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7$: ((strong_iteration($x_7$) = x_7^* and $x_0 \cdot x_6 \leq x_5 \cdot x_0$ and $x_0 \cdot x_7 \leq x_0$ and $x_2 \leq x_1 \cdot x_0$ and $x_0 \cdot x_4 \leq x_3$) $\Rightarrow x_2 \cdot$ (strong_iteration($x_6 + x_7$) $\cdot x_4$) $\leq x_1 \cdot$ (strong_iteration(x_5) $\cdot x_3$)) fof(goals, conjecture)

KLE162+1.p Part 1 of Back's atomicity refinement theorem
Back's atomicity refinement theore is proved up to the reconstruction of concurrecy.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: (($x_0 \leq x_3 \cdot x_0$ and $x_4 \cdot x_1 \leq x_1 \cdot x_4$ and $(x_0+(x_4+x_1))\cdot x_2 \leq x_2 \cdot (x_0+(x_4+x_1))$ and $x_4 \cdot x_3 \leq x_3 \cdot x_4$ and x_4^* = strong_iteration(x_4) and $x_3 \cdot x_2 \leq x_2 \cdot x_3$ and $x_3 \cdot x_1 = 0$ and $x_5 \leq x_5 \cdot x_3$) $\Rightarrow x_5 \cdot$ (strong_iteration(($x_0 + (x_4+x_1))+x_2)\cdot x_3$) $\leq$ ($x_5\cdot$(strong_iteration(x_2)$\cdot x_3$))$\cdot$((strong_iteration(x_4)$\cdot x_3$)$\cdot$strong_iteration(($x_0\cdot$strong_iteration(x_1))$\cdot$ ($x_3 \cdot$ strong_iteration(x_4)))))) fof(goals, conjecture)

KLE163+1.p Part 2 of Back's atomicity refinement theorem
The concurrency bit of Back's atomicity refinement theorem.

include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: $((x_0 \leq x_3 \cdot x_0$ and $x_4 \cdot x_1 \leq x_1 \cdot x_4$ and $(x_0 + (x_4 + x_1)) \cdot x_2 \leq x_2 \cdot (x_0 + (x_4 + x_1))$ and $x_4 \cdot x_3 \leq x_3 \cdot x_4$ and $x_4^* = \text{strong\_iteration}(x_4)$ and $x_3 \cdot x_2 \leq x_2 \cdot x_3$ and $x_3 \cdot x_1 = 0$ and $x_5 \leq x_5 \cdot x_3$ and $x_3 \leq 1) \Rightarrow (x_5 \cdot (\text{strong\_iteration}(x_2) \cdot x_3)) \cdot ((\text{strong\_iteration}(x_4) \cdot x_3) \cdot \text{strong\_iteration}((x_0 \cdot \text{strong\_iteration}(x_1)) \cdot (x_3 \cdot \text{strong\_iteration}(x_4)))) \leq x_5 \cdot \text{strong\_iteration}(((x_0 \cdot \text{strong\_iteration}(x_1)) \cdot x_3 + x_4) + x_2))$ fof(goals, conjecture)

KLE164+1.p Back's atomicity refinement theorem
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1, x_2, x_3, x_4, x_5$: $((x_0 \leq x_3 \cdot x_0$ and $x_4 \cdot x_1 \leq x_1 \cdot x_4$ and $(x_0 + (x_4 + x_1)) \cdot x_2 \leq x_2 \cdot (x_0 + (x_4 + x_1))$ and $x_4 \cdot x_3 \leq x_3 \cdot x_4$ and $x_4^* = \text{strong\_iteration}(x_4)$ and $x_3 \cdot x_2 \leq x_2 \cdot x_3$ and $x_3 \cdot x_1 = 0$ and $x_5 \leq x_5 \cdot x_3$ and $x_3 \leq 1) \Rightarrow x_5 \cdot (\text{strong\_iteration}((x_0 + (x_4 + x_1)) + x_2) \cdot x_3) \leq x_5 \cdot \text{strong\_iteration}(((x_0 \cdot \text{strong\_iteration}(x_1)) \cdot x_3 + x_4) + x_2))$ fof(goals, conjecture)

KLE165+1.p Denest for weakly quasi commutation
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 \leq x_1 \cdot \text{strong\_iteration}(x_1 + x_0) \Rightarrow \text{strong\_iteration}(x_1 + x_0) = \text{strong\_iteration}(x_1) \cdot \text{strong\_iteration}(x_0))$ fof(

KLE165+2.p Denest for weakly quasi commutation
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $((x_0 \cdot x_1 \leq x_1 \cdot \text{strong\_iteration}(x_1 + x_0) \Rightarrow \text{strong\_iteration}(x_1 + x_0) \leq \text{strong\_iteration}(x_1) \cdot \text{strong\_iteration}(x_0))$ and st
$\text{strong\_iteration}(x_0) \leq \text{strong\_iteration}(x_1 + x_0))$ fof(goals, conjecture)

KLE166+1.p Strong iteration
If x weakly quasicommutes over y, x and y is strongly terminating iff x + y strongly terminates.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 \leq x_1 \cdot \text{strong\_iteration}(x_1 + x_0) \Rightarrow (\text{strong\_iteration}(x_1 + x_0) \cdot 0 = 0 \iff \text{strong\_iteration}(x_1) \cdot 0 + \text{strong\_iteration}(x_0) \cdot 0 = 0))$ fof(goals, conjecture)

KLE167+1.p Blocking law
If y is blocked by x then x before finite iteration of y reduces to x.
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $((x_0 \cdot x_1 = 0 \Rightarrow x_0 \cdot x_1^* \leq x_0)$ and $x_0 \leq x_0 \cdot x_1^*)$ fof(goals, conjecture)

KLE168+1.p If x quasicommutes over y then x∧infty quasicommutes over y∧infty
include('Axioms/KLE004+0.ax')
$\forall x_0, x_1$: $(x_0 \cdot x_1 \leq x_1 \cdot x_0 \Rightarrow \text{strong\_iteration}(x_0) \cdot \text{strong\_iteration}(x_1) \leq \text{strong\_iteration}(x_1) \cdot \text{strong\_iteration}(x_0))$ fof(goals

KLE169+1.p Exponential automata
Automata grows exponentially with N in sigma*.a.sigma∧N.a
include('Axioms/KLE002+0.ax')
$\text{sigma} = a + b$ fof(an, axiom)
$a \cdot (b \cdot a) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a))$ fof(a, conjecture)

KLE170+1.002.p a∧N ≤ a*, N=2
include('Axioms/KLE002+0.ax')
$a \cdot a \leq a^*$ fof(a, conjecture)

KLE170+1.004.p a∧N ≤ a*, N=4
include('Axioms/KLE002+0.ax')
$a \cdot (a \cdot (a \cdot a)) \leq a^*$ fof(a, conjecture)

KLE170+1.006.p a∧N ≤ a*, N=6
include('Axioms/KLE002+0.ax')
$a \cdot (a \cdot (a \cdot (a \cdot (a \cdot a)))) \leq a^*$ fof(a, conjecture)

KLE171+1.p Ben's problem 1
include('Axioms/KLE002+0.ax')
$\text{sigma} = a + b$ fof(an, axiom)
$a \cdot (b \cdot (b \cdot a)) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a))$ fof(a, conjecture)

KLE172+1.p Ben's problem 2
include('Axioms/KLE002+0.ax')
$\text{sigma} = a + b$ fof(an, axiom)
$a \cdot (b \cdot (b \cdot (a \cdot b))) \leq \text{sigma}^* \cdot (a \cdot (\text{sigma} \cdot a))$ fof(a, conjecture)

KLE173+1.p Idempotent semirings with tests
include('Axioms/KLE001+0.ax')

include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')

KLE174+1.p Idempotent semirings with domain/codomain, modal
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')

KLE175+1.p Idempotent semirings with tests
include('Axioms/KLE001+0.ax')
include('Axioms/KLE001+5.ax')

KLE176+1.p Kleene algebra
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')

KLE177+1.p Kleene algebra
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')

KLE178+1.p Kleene Algebra
include('Axioms/KLE002+0.ax')
include('Axioms/KLE001+5.ax')

KLE179+1.p Omega algebra
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+1.ax')
include('Axioms/KLE001+2.ax')
include('Axioms/KLE001+3.ax')

KLE180+1.p Omega algebra
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+4.ax')
include('Axioms/KLE001+6.ax')
include('Axioms/KLE001+7.ax')

KLE181+1.p Omega algebra
include('Axioms/KLE003+0.ax')
include('Axioms/KLE001+5.ax')

KLE182+1.p Demonic Refinement Algebra
include('Axioms/KLE004+0.ax')